Fixed Order Calculations

part 3

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- Hadron collisions
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- Glimpse beyond NLO

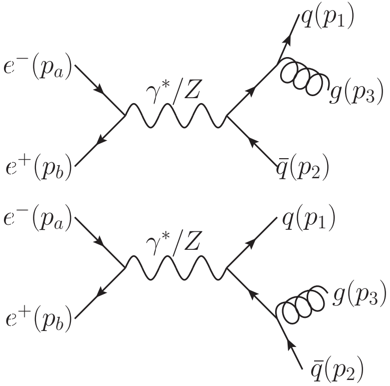
Subtractions – an example

Subtractions – an example

- Stick to our theme example: $\mathrm{e}^+\mathrm{e}^- \to \text{hadrons}$
- To set up subtractions need to know singular regions:

$$\frac{1}{2E_i E_j(1-\cos\theta_{ij})} \to \begin{cases} \xrightarrow{E_i\to 0} & \text{soft}\,, \\ \xrightarrow{E_j\to 0} & \text{soft}\,, \\ \xrightarrow{\theta_{ij}\to 0} & \text{collinear}\,. \end{cases}$$

$$\left|\mathcal{M}^{(0)}_{\mathrm{e}^+\mathrm{e}^-\to \mathrm{q}\,\bar{\mathrm{q}}\,\mathrm{g}}\right|^2 \to \begin{cases} \mathrm{g}\to 0 & \mathrm{g}\ \text{soft}\,, \\ \mathrm{q}||\mathrm{g} & \mathrm{q},\ \mathrm{g}\ \text{collinear}\,, \\ \bar{\mathrm{q}}||\mathrm{g} & \bar{\mathrm{q}},\ \mathrm{g}\ \text{collinear} \end{cases}$$



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Subtractions – an example

- These limits should be subtracted from real contribution:
  - In a unified way a'la Catani-Seymour
  - Or separately (soft and collinear limits), e.g. in ColorFul or LASS

$$\int \mathrm{d}\Phi_3^{(\mathsf{d})}\mathcal{R} \to \int \mathrm{d}\Phi_3^{(4)} \left(\mathcal{R} - \mathcal{S}_{\mathsf{g}} - \mathcal{C}_{\mathrm{q}\,\mathsf{g}} - \mathcal{C}_{\bar{\mathrm{q}}\,\mathsf{g}} + \mathcal{CS}_{\mathrm{q}\,\mathsf{g}} + \mathcal{CS}_{\bar{\mathrm{q}}\,\mathsf{g}}\right)$$

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Subtractions – an example

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  - In a unified way a'la Catani-Seymour
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$$\int \mathrm{d}\Phi_3^{(\mathrm{d})}\mathcal{R} \to \int \mathrm{d}\Phi_3^{(4)} \left(\mathcal{R} - \underbrace{\mathcal{S}_{\mathrm{g}}}_{\text{soft}} - \underbrace{\mathcal{C}_{\mathrm{q\,g}} - \mathcal{C}_{\bar{\mathrm{q}}\,\mathrm{g}}}_{\text{collinear}} + \underbrace{\mathcal{CS}_{\mathrm{q\,g}} + \mathcal{CS}_{\bar{\mathrm{q}}\,\mathrm{g}}}_{\text{soft}-\text{collinear}}\right)$$

- Last two terms: limits are overlapping $\Rightarrow$ have to avoid double counting

$$\int \mathrm{d}\Phi_3^{(\mathrm{d})}\mathcal{R} \to \int \mathrm{d}\Phi_3^{(4)} \left(\mathcal{R} - \mathcal{S}_{\mathrm{g}} - \mathcal{C}^{\mathrm{hc}}_{\mathrm{q\,g}} - \mathcal{C}^{\mathrm{hc}}_{\bar{\mathrm{q}}\,\mathrm{g}}\right)$$

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Subtractions – an example

- The full NLO correction is combination of real ($\mathcal{R}$) and virtual ($\mathcal{V}$) emissions:

$$\sigma_{\mathrm{NLO}} = \int \mathrm{d}\Phi_3^{(\mathsf{d})}\mathcal{R} + \int \mathrm{d}\Phi_2^{(\mathsf{d})}\mathcal{V}$$

- Via subtractions we add zero:

$$\int \mathrm{d}\Phi_3^{(4)} \left(\mathcal{R} - \mathcal{S}_{\mathsf{g}} - \mathcal{C}_{\mathrm{q}\,\mathsf{g}}^{\mathrm{hc}} - \mathcal{C}_{\bar{\mathrm{q}}\,\mathsf{g}}^{\mathrm{hc}}\right) + \int \mathrm{d}\Phi_2^{(\mathsf{d})}\mathcal{V} + \int \mathrm{d}\Phi_3^{(\mathsf{d})} \left(\mathcal{S}_{\mathsf{g}} + \mathcal{C}_{\mathrm{q}\,\mathsf{g}}^{\mathrm{hc}} + \mathcal{C}_{\bar{\mathrm{q}}\,\mathsf{g}}^{\mathrm{hc}}\right)$$

- Subterms needs to be partially integrable analytically:

$$\int \mathrm{d}\Phi_3^{(4)} \left(\mathcal{R} - \mathcal{S}_{\mathsf{g}} - \mathcal{C}_{\mathrm{q}\,\mathsf{g}}^{\mathrm{hc}} - \mathcal{C}_{\bar{\mathrm{q}}\,\mathsf{g}}^{\mathrm{hc}}\right) + \int \mathrm{d}\Phi_2^{(\mathsf{d})}\mathcal{V} + \int_1 \int \mathrm{d}\Phi_2^{(\mathsf{d})} \left(\mathcal{S}_{\mathsf{g}} + \mathcal{C}_{\mathrm{q}\,\mathsf{g}}^{\mathrm{hc}} + \mathcal{C}_{\bar{\mathrm{q}}\,\mathsf{g}}^{\mathrm{hc}}\right)$$

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Subtractions – an example

- The second instances of subterms can be combined with the virtual:

$$\int \mathrm{d}\Phi_3^{(4)} \left(\mathcal{R} - \mathcal{S}_{\mathrm{g}} - \mathcal{C}^{\mathrm{hc}}_{\mathrm{q\,g}} - \mathcal{C}^{\mathrm{hc}}_{\bar{\mathrm{q}}\,\mathrm{g}}\right) + \int \mathrm{d}\Phi_2^{(\mathrm{d})} \left[\mathcal{V} + \underbrace{\int_1 \left(\mathcal{S}_{\mathrm{g}} + \mathcal{C}^{\mathrm{hc}}_{\mathrm{q\,g}} + \mathcal{C}^{\mathrm{hc}}_{\bar{\mathrm{q}}\,\mathrm{g}}\right)}_{\mathcal{I}^{(1)}}\right]$$

- Integrated subterms cancel poles of the virtual:

$$\int \mathrm{d}\Phi_3^{(4)} \left(\mathcal{R} - \mathcal{S}_{\mathrm{g}} - \mathcal{C}^{\mathrm{hc}}_{\mathrm{q\,g}} - \mathcal{C}^{\mathrm{hc}}_{\bar{\mathrm{q}}\,\mathrm{g}}\right) + \int \mathrm{d}\Phi_2^{(4)} \left[\mathcal{V} + \mathcal{I}^{(1)}\right]$$

$\Rightarrow$ Separately finite, can be delegated to MC!

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Subtractions – an example

No subterm:

```
430.282664 +/- 113.967916
863.479534 +/- 416.143910
2153.29605 +/- 1523.17197
1693.97151 +/- 1142.45845
1397.71728 +/- 913.977829
1214.47253 +/- 761.770102
1077.28686 +/- 652.998983
1001.59986 +/- 571.960136
914.445041 +/- 508.417978
843.204778 +/- 457.580479
```

With subterms:

```
-0.66552 +/- 0.52457E-3
-0.66759 +/- 0.63545E-3
-0.66691 +/- 0.51648E-3
-0.66680 +/- 0.52139E-3
-0.66598 +/- 0.53662E-3
-0.66752 +/- 0.63090E-3
-0.66672 +/- 0.56636E-3
-0.66724 +/- 0.52904E-3
-0.66671 +/- 0.63949E-3
-0.66849 +/- 0.66734E-3
```

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Subtractions – an example

The general NLO case:

- For an observable O the NLO calculation takes the form of:

$$\sigma_{\mathrm{NLO}}(\mathsf{O}) = \int \mathrm{d}\Phi^{(4)}_{\mathsf{n+1}} \left(\mathcal{R} \cdot \mathsf{O}(\Phi_{\mathsf{n+1}}) - \mathsf{K}^{(1)}(\mathsf{O})\right) + \int \mathrm{d}\Phi^{(4)}_{\mathsf{n}} \left[\mathcal{V} + \mathcal{I}^{(1)}\right] \mathsf{O}(\Phi_{\mathsf{n}})$$

- In the subterms multiple phase spaces can be present ⇒ many calls to evaluate O ⇒ complicated analysis is slow!

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Observables

Observables

- Not any observable will do for fixed order calculations (beyond LO)!
- It should be infra-red finite!

$$O_{n+1}(\underbrace{\{p_1, \dots, p_i, \dots, p_j, \dots, p_{n+1}\}}_{n+1}) \xrightarrow{i||j} O_n(\underbrace{\{p_1, \dots, p_{ij}, \dots, p_{n+1}\}}_{n}),$$

$$O_{n+1}(\underbrace{\{p_1, \dots, p_i, \dots, p_{n+1}\}}_{n+1}) \xrightarrow{i\to 0} O_n(\underbrace{\{p_1, \dots, \not{p}_i, \dots, p_{n+1}\}}_{n})$$

$\Rightarrow$ Otherwise the result can be anything!

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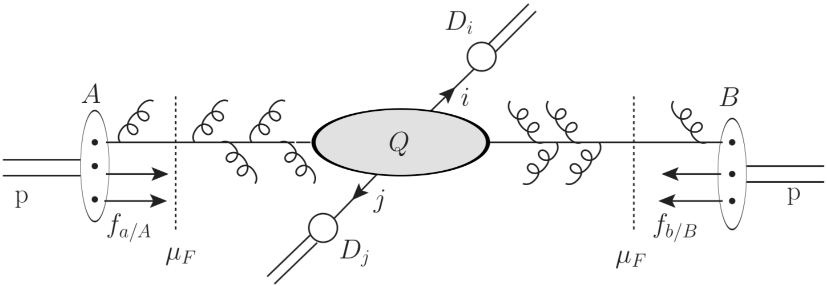
Hadron collisions

Hadron collisions

- Same particles can be created but initial state is more complicated:



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Hadron collisions

- Hadrons collide but actual collision happens between partons:

$$\sigma_{A\,B\to n} = \sum_{a,\,b} \int_0^1 dx_a dx_b f_{a/A}(x_a) f_{b/B}(x_b)\, \sigma_{a\,b\to n}(x_a x_b s)$$

- Always taking a look at hadrons at some energy scale μ_F, probabilities will depend:

$$\sigma_{A\,B\to n} = \sum_{a,\,b} \int_0^1 dx_a dx_b f_{a/A}(x_a, \mu_F) f_{b/B}(x_b, \mu_F)\, \sigma_{a\,b\to n}(x_a x_b s, \mu_F)$$

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Hadron collisions

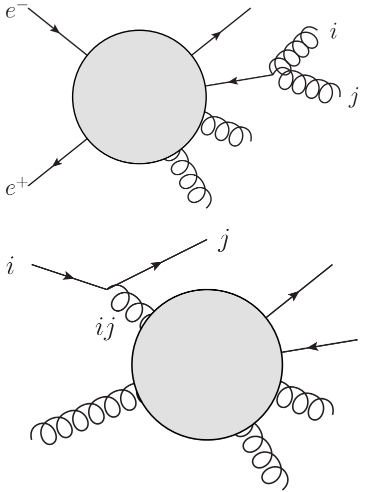
Implications for fixed order calculations?

- New singularities associated to initial state

⇒ Methods designed for $e^+ e^-$ collisions need extension

- Analysis and PDF evaluation should be run multiple times per event

⇒ Extra constraint on PDF provider (need to be fast)



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Hadron collisions

- PDFs are integral part of the calculations!
- Obtained from fitting to data using standard processes (Drell-Yen, etc.)

⇒ Get associated to orderedness in perturbation theory:

$$\begin{aligned}
\sigma^{\mathrm{LO}} &= \sigma_{\mathrm{LO}} \quad \Leftarrow \quad \mathrm{LO\,PDF}\,, \\
\sigma^{\mathrm{NLO}} &= \sigma_{\mathrm{LO}} + \sigma_{\mathrm{NLO}} \quad \Leftarrow \quad \mathrm{NLO\,PDF}\,, \\
\sigma^{\mathrm{NNLO}} &= \sigma_{\mathrm{LO}} + \sigma_{\mathrm{NLO}} + \sigma_{\mathrm{NNLO}} \quad \Leftarrow \quad \mathrm{NNLO\,PDF}\,, \\
&\dots
\end{aligned}$$

- Your NLO prediction will not become better if you use an NNLO PDF, it becomes worse!

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Scales

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Scales

- Performing perturbative expansion of cross section:

$$\sigma = \sigma_{\mathrm{LO}} + \sigma_{\mathrm{NLO}} + \sigma_{\mathrm{NNLO}} + \ldots$$

- Not all orders are considered!

⇒ Truncation introduce dependence on non-physical scales cancelling only when whole series is considered

$$\sigma(\not{\mu}_{\mathsf{R}}, \not{\mu}_{\mathsf{F}}) = \sigma_{\mathrm{LO}}(\mu_{\mathsf{R}}, \mu_{\mathsf{F}}) + \sigma_{\mathrm{NLO}}(\mu_{\mathsf{R}}, \mu_{\mathsf{F}}) + \sigma_{\mathrm{NNLO}}(\mu_{\mathsf{R}}, \mu_{\mathsf{F}}) + \ldots$$

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Scales

⇒ Truncation introduce dependence on non-physical scales cancelling only when whole series is considered

$$\sigma(\not{\mu}_{\mathsf{R}}, \not{\mu}_{\mathsf{F}}) = \sigma_{\mathrm{LO}}(\mu_{\mathsf{R}}, \mu_{\mathsf{F}}) + \sigma_{\mathrm{NLO}}(\mu_{\mathsf{R}}, \mu_{\mathsf{F}}) + \sigma_{\mathrm{NNLO}}(\mu_{\mathsf{R}}, \mu_{\mathsf{F}}) + \dots$$

$$\begin{aligned}
\sigma^{\mathrm{LO}}(\mu_{\mathsf{R}}, \mu_{\mathsf{F}}) &= \sigma_{\mathrm{LO}}(\mu_{\mathsf{R}}, \mu_{\mathsf{F}})\,, \\
\sigma^{\mathrm{NLO}}(\mu_{\mathsf{R}}, \mu_{\mathsf{F}}) &= \sigma_{\mathrm{LO}}(\mu_{\mathsf{R}}, \mu_{\mathsf{F}}) + \sigma_{\mathrm{NLO}}(\mu_{\mathsf{R}}, \mu_{\mathsf{F}})\,, \\
\sigma^{\mathrm{NNLO}}(\mu_{\mathsf{R}}, \mu_{\mathsf{F}}) &= \sigma_{\mathrm{LO}}(\mu_{\mathsf{R}}, \mu_{\mathsf{F}}) + \sigma_{\mathrm{NLO}}(\mu_{\mathsf{R}}, \mu_{\mathsf{F}}) + \sigma_{\mathrm{NNLO}}(\mu_{\mathsf{R}}, \mu_{\mathsf{F}})
\end{aligned}$$

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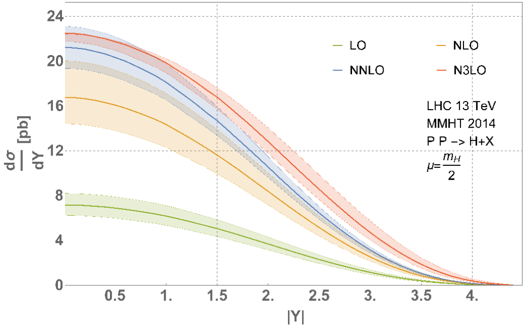
Scales

- Scale dependence hints on missing higher-order corrections
- Beware of new channels at higher orders!
- LO cross section gives just the order of magnitude
  - Still useful for BSM
  - Can we have enough events?
  - Is there any hope to see it?
- Precision starts at NLO



Dulat et al., arXiv:1710.03016

Glimpse beyond NLO

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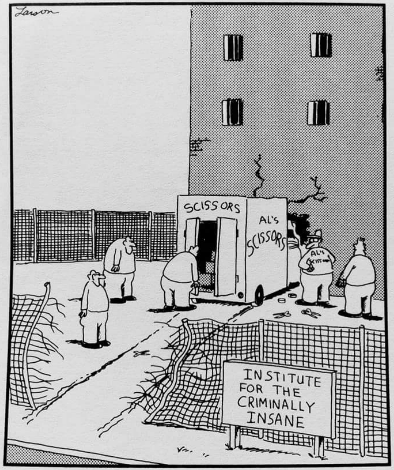
Glimpse beyond NLO

- At NNLO up to three partons can become unresolved
- More complicated singularity structure

⇒ More subtractions

⇒ More stress on numerical cancellations

⇒ CPU time drastically increases



And then Al realized his problems were much bigger than just a smashed truck.

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Glimpse beyond NLO

- Much richer singularity structure:
  - NLO:

$$\begin{aligned} \text{soft}: &\quad E_i \to 0\,, \\ \text{collinear}: &\quad i||j \end{aligned}$$

  - NNLO:

$$\begin{aligned} \text{double}-\text{soft}: &\quad E_i\,, E_j \to 0\,, \\ \text{triple}-\text{collinear}: &\quad i||j||k\,, \\ \text{soft}-\text{collinear}: &\quad E_i \to 0,\ j||k\,, \\ \text{double}-\text{collinear}: &\quad i||j,\ k||l \end{aligned}$$

  - NNNLO:

$$\ldots$$

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Glimpse beyond NLO

- Good news: NNLO calculations are so complicated experimentalists seldom do them themselves

$\Rightarrow$ Loan machines to theorists

- Next five years should witness a change in this trend...

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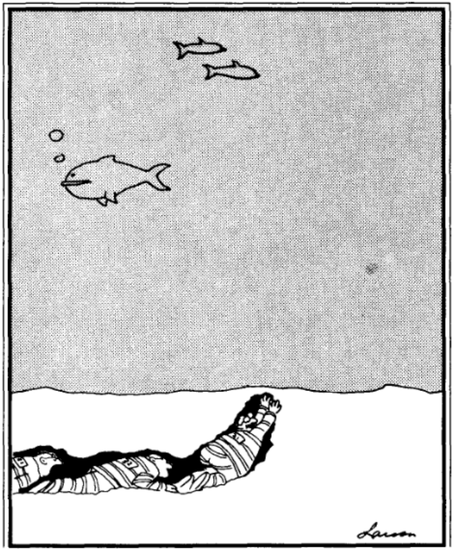
Glimpse beyond NLO



"We're almost free, everyone! . . . I just felt the first drop of rain."

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Thank you for your attention!