The muon anomalous magnetic moment

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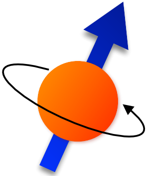
What is a muon?



- Elementary point-like particle
- Same electric charge as an electron
- Approximately **200** times heavier than an electron
- Like the electron, behaves as if it was intrinsically spinning about a vector $\vec{S}$

These properties combine to give it a magnetic moment

$$\vec{\mu} = g \left(\frac{e}{2m} \right) \vec{S}$$

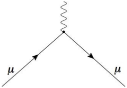
such that when put in a magnetic field, it exhibits precession similar to a spinning top.

We can measure this precession **very** precisely.

The magnetic moment and quantum corrections

The g-factor in $\vec{\mu} = g \left(\frac{e}{2m}\right) \vec{S}$ describes the strength of coupling to a magnetic field, which can be measured and computed from theory **very** precisely.

Dirac: $g = 2$

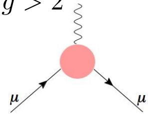


Anomalous magnetic moment

$$a = (g-2)/2$$

quantum effects →

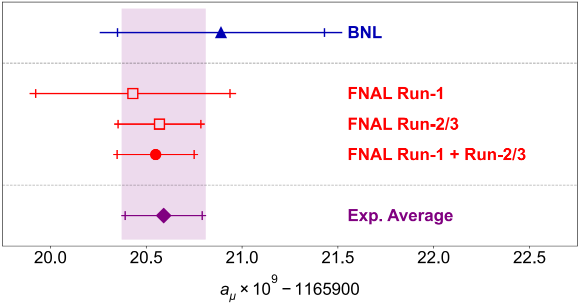
$g > 2$

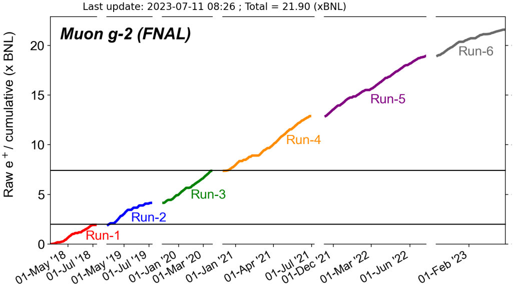


The quantum effects arise from virtual particle contributions from all known **and unknown** particles.

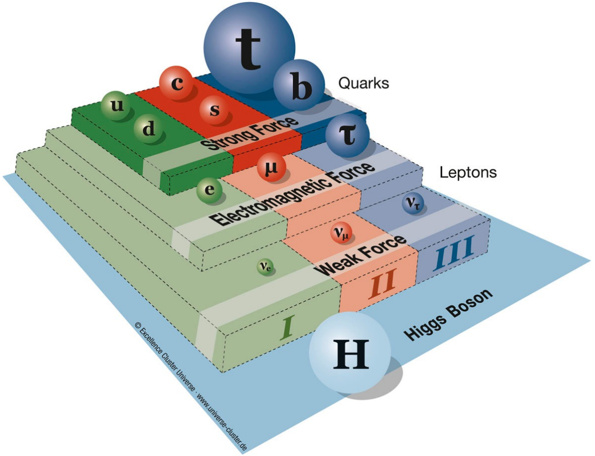
By comparing high-precision experiments and theory, we have the potential to learn about such contributions of new particles.

Experimental status (PRL 131 (2023) 16, 161802)





Contributions from known particles: The Standard Model

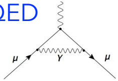


Open questions: dark matter, size of matter-antimatter asymmetry, origin of neutrino masses, . . . ⇒ Standard Model is incomplete

Contributions from known particles: The Standard Model

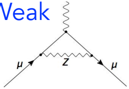
$$a_\mu(\text{SM}) = a_\mu(\text{QED}) + a_\mu(\text{Weak}) + a_\mu(\text{Hadronic})$$

QED



$+\ldots$ $116\,584\,718.9\,(1) \times 10^{-11}$ 0.001 ppm

Weak

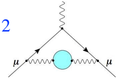


$+\ldots$ $153.6\,(1.0) \times 10^{-11}$ 0.01 ppm

Hadronic...

...Vacuum Polarization (HVP)

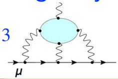
α^2



$+\ldots$ $6845\,(40) \times 10^{-11}$ [0.6%] 0.37 ppm

...Light-by-Light (HLbL)

α^3

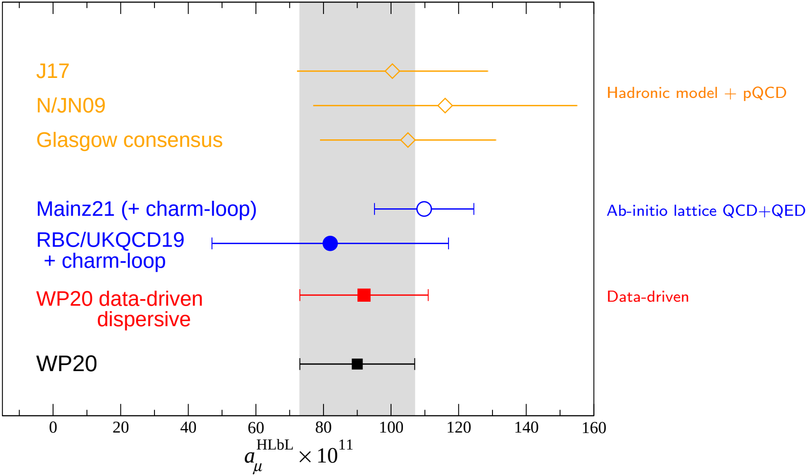


$+\ldots$ $92\,(18) \times 10^{-11}$ [20%] 0.15 ppm

Numbers from Theory Initiative Whitepaper

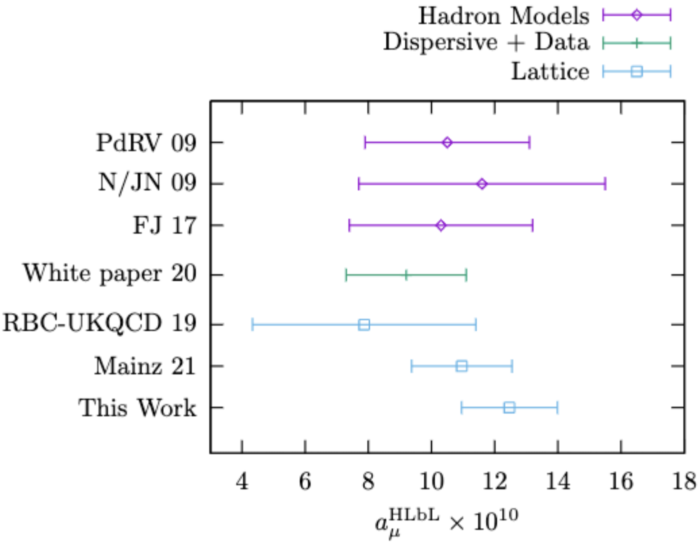
Uncertainty dominated by hadronic contributions

Status of hadronic light-by-light contribution

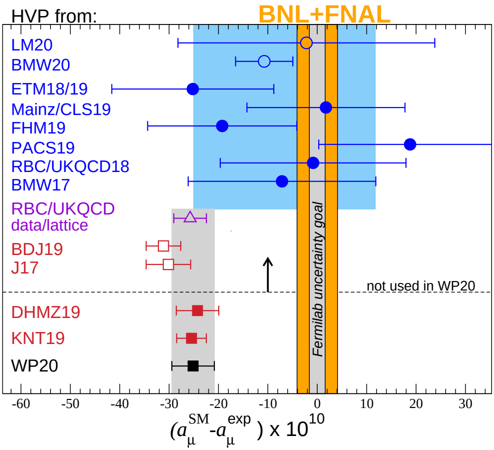


Systematically improvable methods are maturing; uncertainty to a_μ controlled at 0.15ppm; cross-checks detailed in Theory Initiative whitepaper

RBC/UKQCD 23 update (2304.04423)



Status and impact of hadronic vacuum polarization contribution



Ab-initio lattice QCD(+QED) calculations are maturing

Difficult problem: scales from $2m_\pi$ to several GeV enter; cross-checks needed at high precision

Hybrid window method restricts scales that enter from lattice/dispersive data

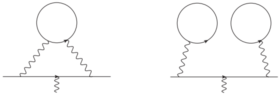
Dispersive, $e^+e^- \to$ hadrons (20+ years of experiments, however, unresolved tensions of experimental data sets)

Now first published lattice result with sub-percent precision available (BMW20), cross-checks are crucial to establish or refute high-precision lattice methodology

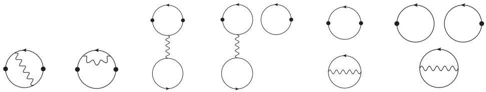
Consistency of lattice results

Diagrams

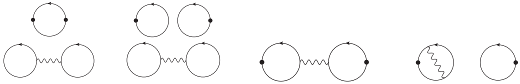
Isospin limit



QED corrections

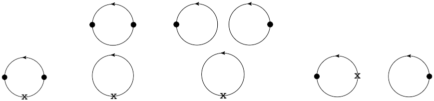


(a) V (b) S (c) T (d) T_d (e) D1 (f) $D1_d$



(g) D2 (h) $D2_d$ (i) F (j) D3

Strong isospin breaking



(a) M (b) R (c) R_d (d) O

Overview of individual contributions

Diagrams – Isospin limit

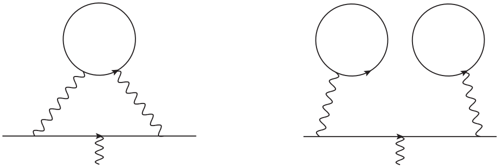
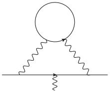
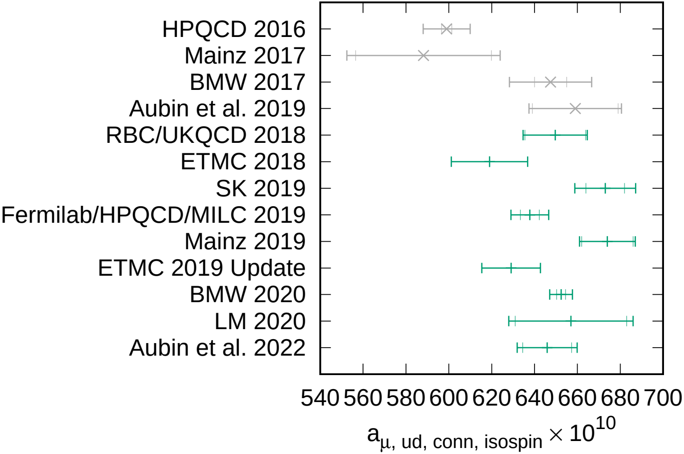


FIG. 1. Quark-connected (left) and quark-disconnected (right) diagram for the calculation of $a_\mu^{\mathrm{HVP\ LO}}$. We do not draw gluons but consider each diagram to represent all orders in QCD.

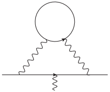
Up, down; isospin symmetric limit; $m_\pi = m_\pi^0$

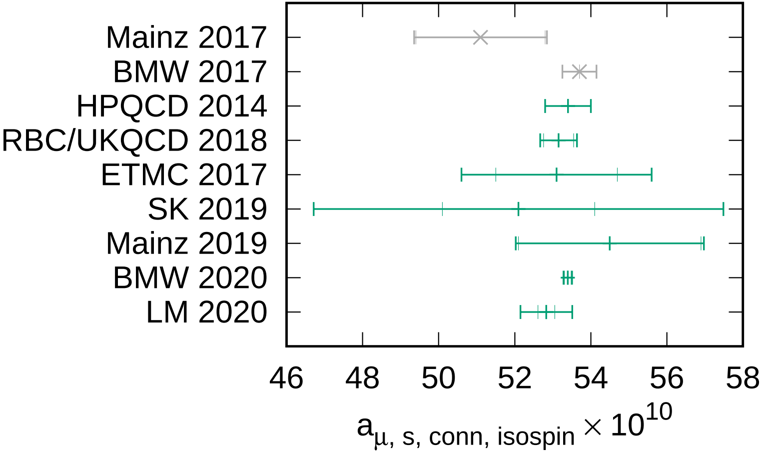




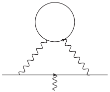
Some tensions to be understood

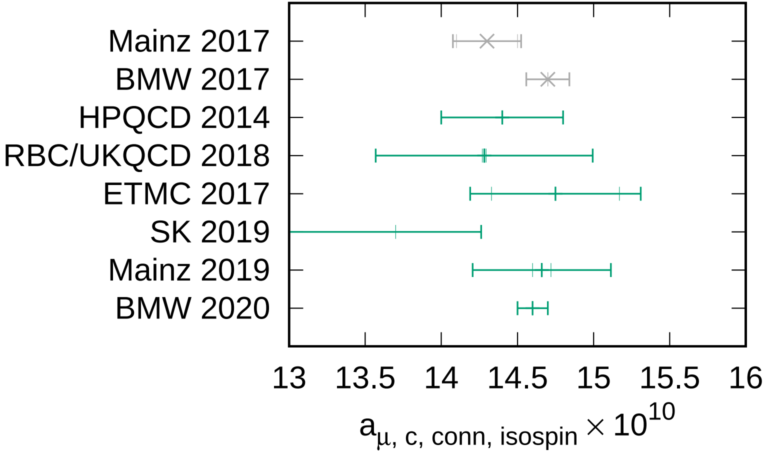
Strange

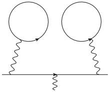


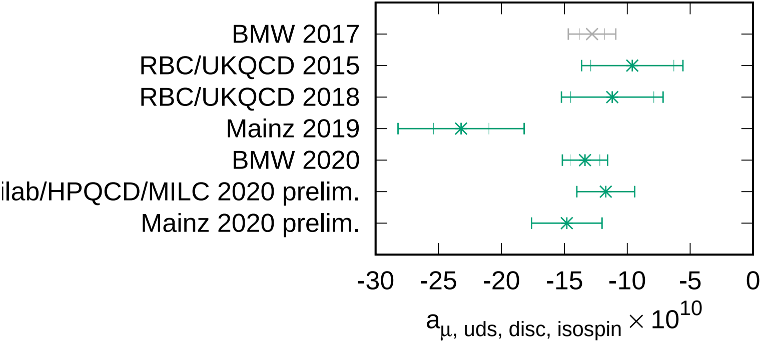

Mainz 2017
BMW 2017
HPQCD 2014
RBC/UKQCD 2018
ETMC 2017
SK 2019
Mainz 2019
BMW 2020
LM 2020
46
48
50
52
54
56
58
$a_{\mu,\,s,\,conn,\,isospin} \times 10^{10}$

Charm

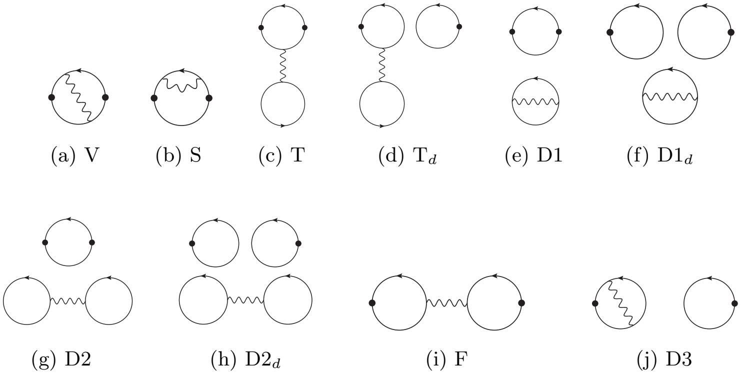



Mainz 2017
BMW 2017
HPQCD 2014
RBC/UKQCD 2018
ETMC 2017
SK 2019
Mainz 2019
BMW 2020
13
13.5
14
14.5
15
15.5
16
$a_{\mu,\text{ c, conn, isospin}} \times 10^{10}$

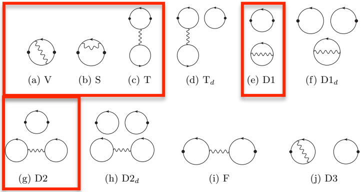



BMW 2017
RBC/UKQCD 2015
RBC/UKQCD 2018
Mainz 2019
BMW 2020
ilab/HPQCD/MILC 2020 prelim.
Mainz 2020 prelim.
-30
-25
-20
-15
-10
-5
0
$a_{\mu,\text{ uds, disc, isospin}} \times 10^{10}$

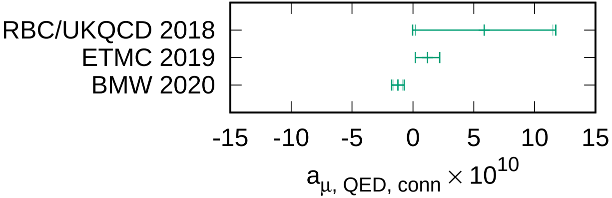
Diagrams – QED corrections



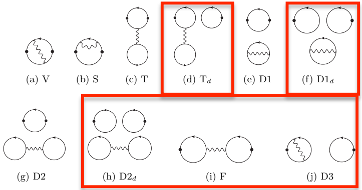
For diagram F we enforce exchange of gluons between the quark loops as otherwise a cut through a single photon line would be possible. This single-photon contribution is counted as part of the HVP NLO and not included for the HVP LO.

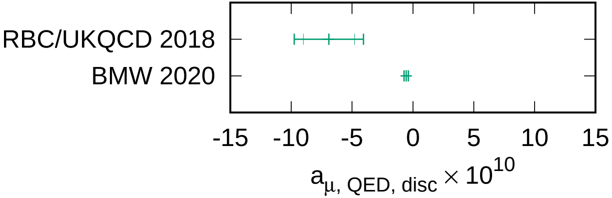


(a) V
(b) S
(c) T
(d) T_d
(e) D1
(f) $D1_d$
(g) D2
(h) $D2_d$
(i) F
(j) D3



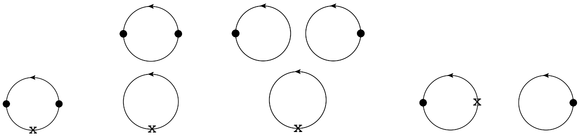
RBC/UKQCD 2018
ETMC 2019
BMW 2020
-15
-10
-5
0
5
10
15
$a_{\mu,\text{ QED, conn}} \times 10^{10}$





Attention needed

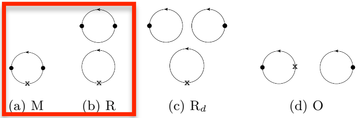
Diagrams – Strong isospin breaking

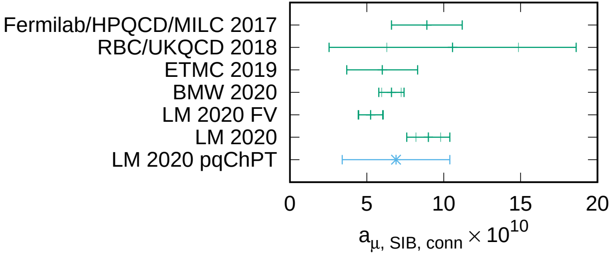


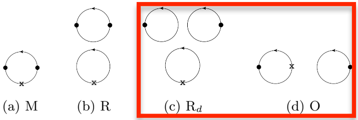
(a) M (b) R (c) R_d (d) O

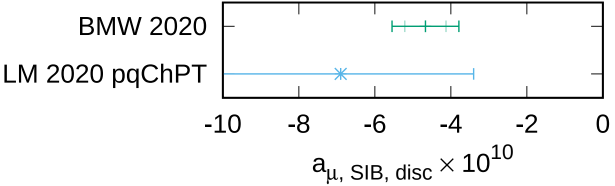
For the HVP R is negligible since $\Delta m_u \approx -\Delta m_d$ and O is SU(3) and $1/N_c$ suppressed.

Lehner, Meyer 2020: NLO PQChPT: FV effects in connected and disconnected cancel but are each significant $O(4 \times 10^{-10})$; PQChPT expects cancellation between connected and disconnected contribution $a_\mu^{\mathrm{SIB,\ conn.}} = -a_\mu^{\mathrm{SIB,\ disc.}} = 6.9 \times 10^{-10}$


(a) M
(b) R
(c) R_d
(d) O


Fermilab/HPQCD/MILC 2017
RBC/UKQCD 2018
ETMC 2019
BMW 2020
LM 2020 FV
LM 2020
LM 2020 pqChPT
0
5
10
15
20
$a_{\mu,\,\mathrm{SIB,\,conn}} \times 10^{10}$


(a) M
(b) R
(c) R$_d$
(d) O


BMW 2020
LM 2020 pqChPT
-10
-8
-6
-4
-2
0
$a_{\mu,\text{ SIB, disc}} \times 10^{10}$

Attention on light-quark isospin-symmetric contribution and QED disconnected contribution

Lattice QCD – Time-Moment Representation

Starting from the vector current $J_\mu(x) = i\sum_f Q_f \overline{\Psi}_f(x)\gamma_\mu \Psi_f(x)$ we may write

$$a_\mu^{\mathrm{HVP\ LO}} = \sum_{t=0}^{\infty} w_t C(t)$$

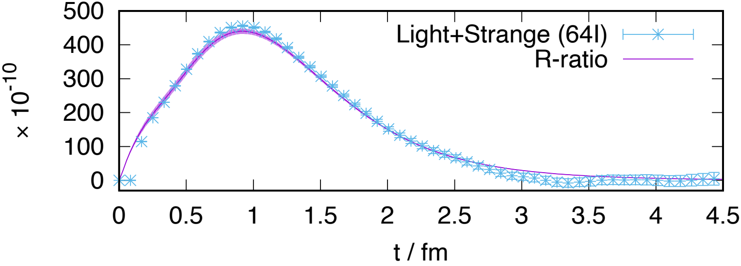
with

$$C(t) = \frac{1}{3}\sum_{\vec{x}} \sum_{j=0,1,2} \langle J_j(\vec{x},t) J_j(0) \rangle$$

and w_t capturing the photon and muon part of the HVP diagrams (Bernecker-Meyer 2011).

The correlator $C(t)$ is computed in lattice QCD+QED at physical pion mass with non-degenerate up and down quark masses including up, down, strange, and charm quark contributions. The missing bottom quark contributions are computed in pQCD.

Lattice QCD – Example of correlation function $C(t)$ (RBC/UKQCD18)



Large discretization errors at short distance, large finite-volume errors and statistical errors at large distance

Window method (introduced in RBC/UKQCD 2018)

We therefore also consider a window method. Following Meyer-Bernecker 2011 and smearing over t to define the continuum limit we write

$$a_\mu = a_\mu^{\mathrm{SD}} + a_\mu^{\mathrm{W}} + a_\mu^{\mathrm{LD}}$$

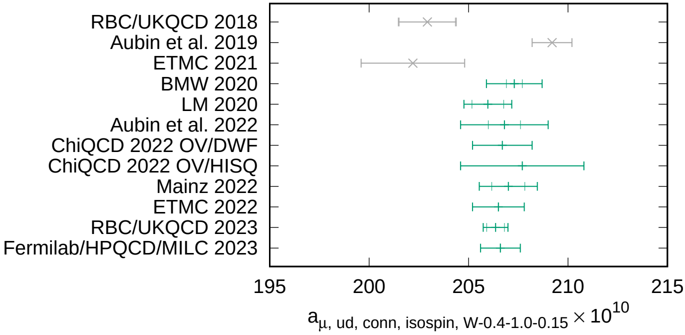
with

$$\begin{aligned} a_\mu^{\mathrm{SD}} &= \sum_t C(t) w_t [1 - \Theta(t, t_0, \Delta)]\,, \\ a_\mu^{\mathrm{W}} &= \sum_t C(t) w_t [\Theta(t, t_0, \Delta) - \Theta(t, t_1, \Delta)]\,, \\ a_\mu^{\mathrm{LD}} &= \sum_t C(t) w_t \Theta(t, t_1, \Delta)\,, \\ \Theta(t, t', \Delta) &= [1 + \tanh[(t - t')/\Delta]]/2\,. \end{aligned}$$

All contributions are well-defined individually and can be computed from lattice or R-ratio via $C(t) = \frac{1}{12\pi^2} \int_0^\infty d(\sqrt{s}) R(s) s e^{-\sqrt{s}t}$ with $R(s) = \frac{3s}{4\pi\alpha^2} \sigma(s, e^+e^- \to \mathrm{had})$.

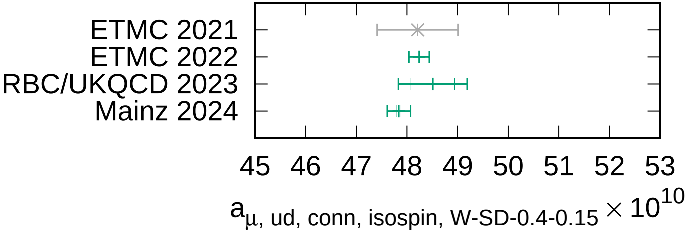
a_μ^{W} has small statistical and systematic errors on lattice!

Use these windows as a lattice internal cross-check



Isospin-symmetric light quark-connected contribution to a_μ^{W} for $t_0 = 0.4$ fm, $t_1 = 1.0$ fm;

Use these windows as a lattice internal cross-check

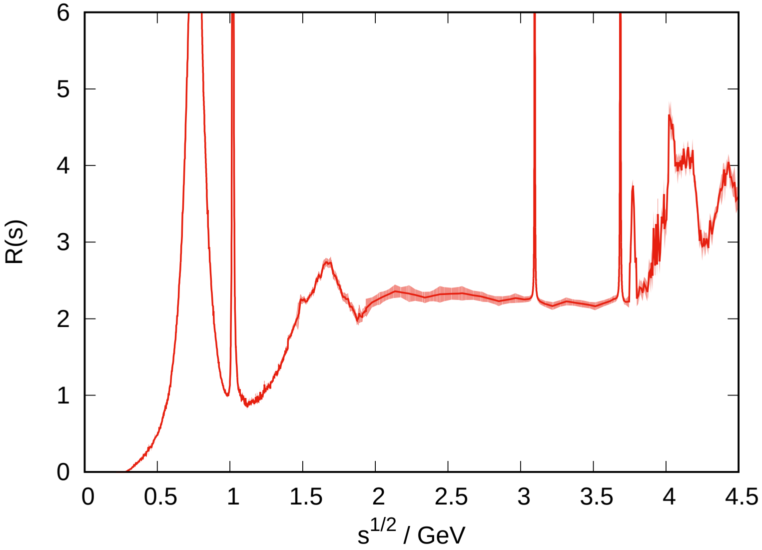


Isospin-symmetric light quark-connected contribution to a_μ^{SD} for $t_0 =$ fm; consistent with pQCD (RBC/UKQCD 2023)

Summary of current status

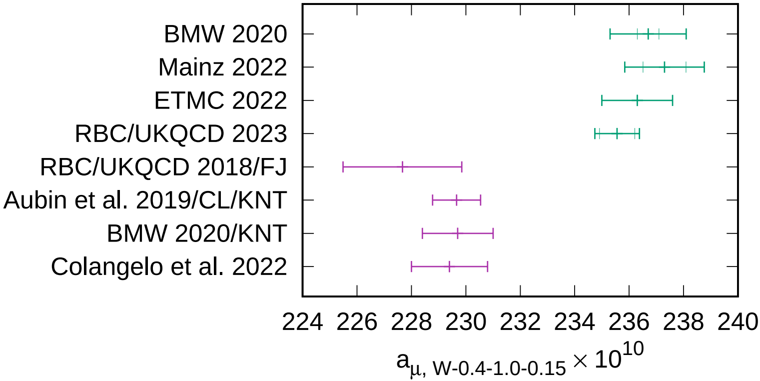
- Short distance window (up to $t_0 = 0.4$ fm) dominated by pQCD; consistency between LQCD and LQCD/pQCD
- Intermediate window ($t_0 = 0.4$ fm, $t_1 = 1.0$ fm); consistency between different LQCD results established
- The long-distance window is at this point not yet independently checked! Only BMW20 result at sub-percent precision. This is expected to change over the next year!

Consistency of lattice results with R-ratio

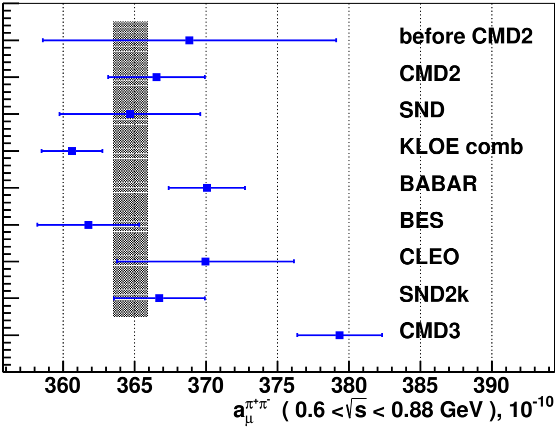


$$R(s) = \frac{3s}{4\pi\alpha^2}\sigma(s, e^+e^- \to \mathrm{had}), \quad C(t) = \frac{1}{12\pi^2}\int_0^\infty d(\sqrt{s})R(s)se^{-\sqrt{s}t}$$

Situation before CMD3:



Tensions in dispersive two-pion channel: From arXiv:2302.08834



Lattice has now converged for short-distance and intermediate windows. Difficult to come up with single dispersive number at this point. If fluctuation up to CMD3 is taken as systematic error for dispersive result, tension in intermediate window between lattice and dispersive disappears.

Once R-ratio is understood again compare LQCD/R-ratio

If there is a shift in R-ratio, it crucially depends on which energy to understand what the impact on $\Delta\alpha$ and EW precision physics is.

Express Euclidean Windows in time-like region:

$$a_\mu = \int_0^\infty ds\, R(s)K(s) \tag{1}$$

and window

$$a_\mu^{\mathrm{W}} = \int_0^\infty ds\, R(s)K(s)P(s)\,. \tag{2}$$

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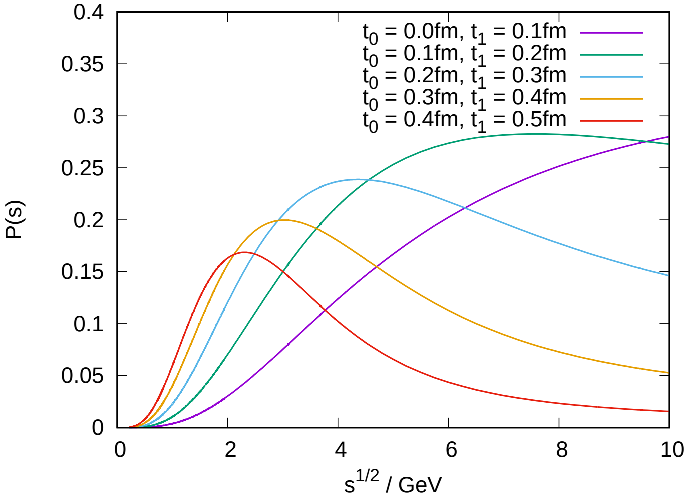
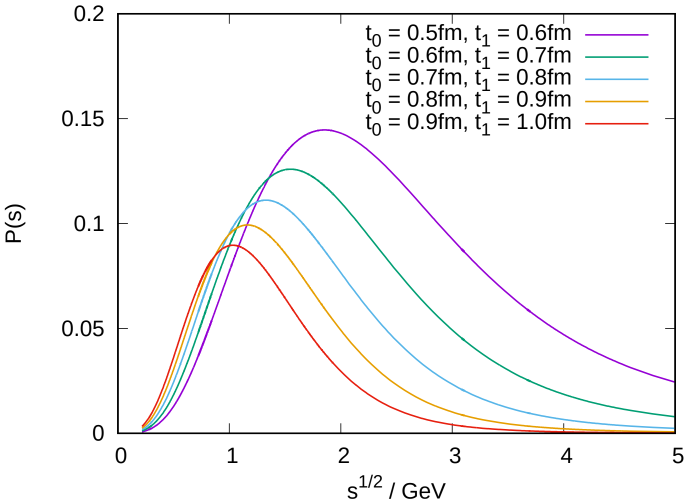
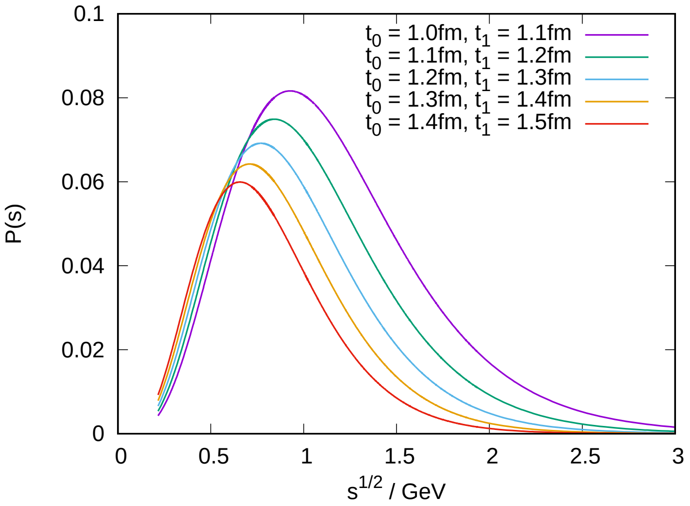


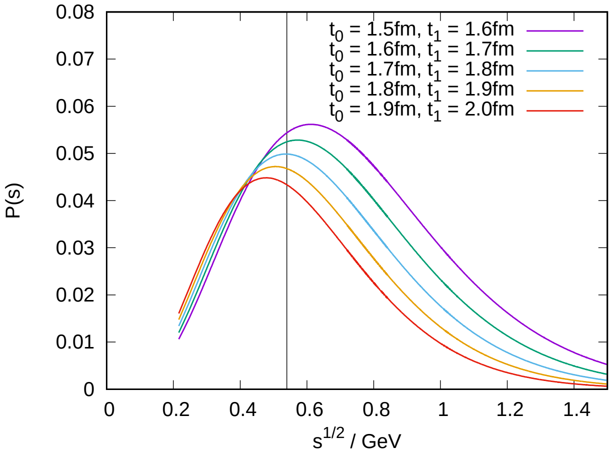
Study of windows for different t_0 and t_1 can give some energy resolution!



Study of windows for different t_0 and t_1 can give some energy resolution!

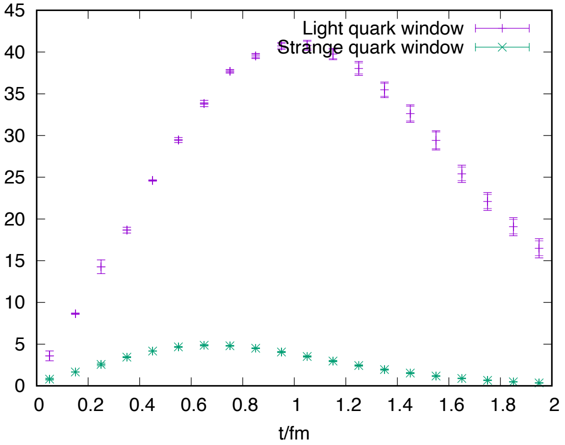


Study of windows for different t_0 and t_1 can give some energy resolution!



Below black line, we can use Lellouch-Lüscher-Meyer formalism to get $R(s)$ from lattice directly! Programs for this by FHM, Mainz, and RBC/UKQCD.

First results for more windows already available - Lehner & Meyer 2020



Here: $t_0 = t$, $t_1 = t + 0.1\text{fm}$

No results for QED, SIB, and charm contribution yet available.

First results for more windows already available - Lehner & Meyer 2020

t_0/fm	t_1/fm	Δ/fm	$a_\mu^{\rm ud,conn.,isospin}10^{10}$	$a_\mu^{\rm s,conn.,isospin}10^{10}$
Total			657(26)(12)	52.83(22)(65)
0.0	0.1	0.15	3.60(00)(59)	0.81(00)(12)
0.1	0.2	0.15	8.649(03)(73)	1.666(01)(12)
0.2	0.3	0.15	14.27(01)(82)	2.57(00)(16)
0.3	0.4	0.15	18.67(02)(35)	3.448(05)(65)
0.4	0.5	0.15	24.617(35)(63)	4.170(07)(20)
0.5	0.6	0.15	29.47(06)(29)	4.666(10)(59)
0.6	0.7	0.15	33.85(10)(37)	4.866(13)(74)
0.7	0.8	0.15	37.71(14)(15)	4.799(16)(39)
0.8	0.9	0.15	39.55(20)(21)	4.505(17)(44)
0.9	1.0	0.15	40.77(27)(31)	4.058(19)(65)
1.0	1.1	0.15	40.86(44)(41)	3.527(19)(76)
1.1	1.2	0.15	39.81(54)(42)	2.973(19)(75)
1.2	1.3	0.15	38.10(65)(51)	2.441(18)(77)
1.3	1.4	0.15	35.54(77)(53)	1.955(17)(67)
1.4	1.5	0.15	32.70(88)(56)	1.534(15)(60)
1.5	1.6	0.15	29.50(100)(58)	1.181(13)(52)
1.6	1.7	0.15	25.51(81)(66)	0.894(12)(44)
1.7	1.8	0.15	22.20(85)(66)	0.667(10)(37)
1.8	1.9	0.15	19.18(86)(67)	0.491(08)(30)
1.9	2.0	0.15	16.59(89)(75)	0.357(07)(24)

0.0	0.2	0.15	12.25(00)(52)	2.48(00)(11)
0.2	0.4	0.15	32.95(03)(48)	6.02(01)(10)
0.4	0.6	0.15	54.08(10)(29)	8.837(18)(74)
0.6	0.8	0.15	71.55(24)(38)	9.666(29)(91)
0.8	1.0	0.15	80.33(47)(44)	8.56(04)(10)
0.3	1.0	0.15	224.6(0.8)(1.1)	30.51(08)(25)
0.3	1.3	0.15	343.1(2.6)(2.0)	39.45(13)(35)
0.3	1.6	0.15	441.0(5.1)(3.4)	44.12(17)(49)
0.4	1.0	0.15	205.97(79)(90)	27.06(08)(21)
0.4	1.3	0.15	324.6(2.6)(1.9)	36.01(13)(36)
0.4	1.6	0.15	422.4(5.1)(3.5)	40.68(17)(51)
0.4	1.0	0.05	216.5(0.8)(6.2)	27.9(0.1)(1.1)
0.4	1.0	0.1	209.80(77)(79)	27.70(08)(21)
0.4	1.0	0.2	202.10(82)(91)	26.24(08)(21)

More results expected by other collaborations soon! See also study of $a_\mu^{\rm SD}(t_0)$ for different t_0 by Fermilab/HPQCD/MILC22.

Summary

- Lattice QCD making steady progress towards full first-principles determination of HVP and HLbL at FNAL E989 target precision
- HLbL converged, no surprises
- HVP a_μ^{SD} contribution converged between LQCD and pQCD, no surprises
- HVP a_μ^{W} contribution converged in LQCD, $O(6 \times 10^{-10})$ higher than previous dispersive consensus (before CMD3)
- HVP a_μ^{LD} next focus of LQCD community. We may expect high-precision results in 2024!
- Further reduction in experimental uncertainty for a_μ expected by upcoming FNAL E989 Run 4-6 data release