QCD at the Extremes

From DIS to $pp \to jets$

Jet production in pp

- x-section (i.e. for light and heavy quark ($t\bar{t}$) production)

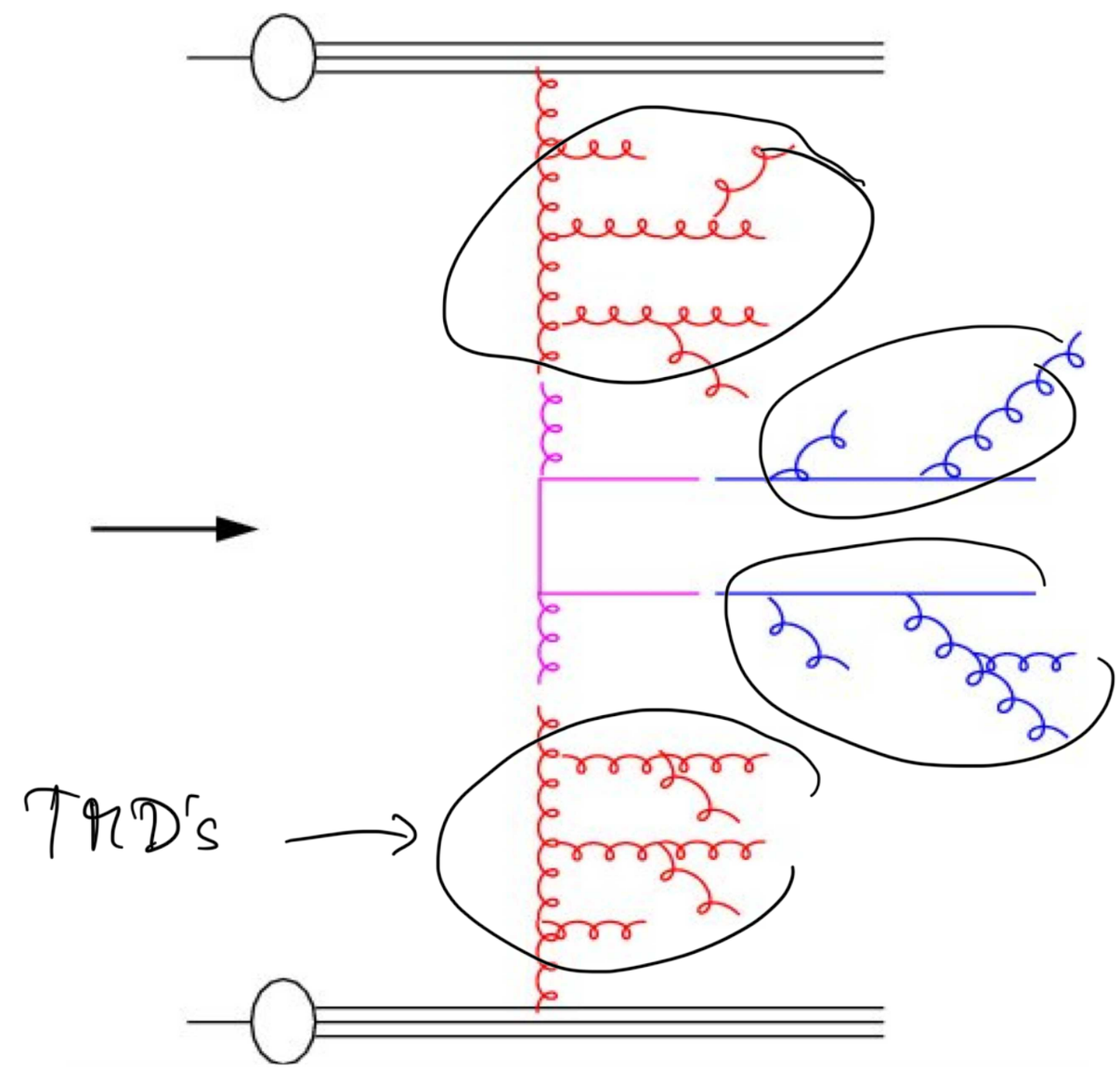
$$\sigma(\mathrm{pp} \to \mathrm{q\bar{q}X}) \quad = \quad \int \frac{dx_1}{x_1}\frac{dx_2}{x_2} x_1 G(x_1,\bar{q}) x_2 G(x_2,\bar{q}) \times \hat{\sigma}(\hat{s},\bar{q})$$

- with gluon densities

$$xG(x,\bar{q})$$

- hard x-section:

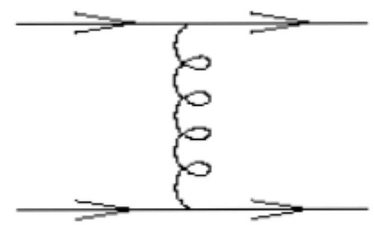
$$\frac{d\sigma}{dt} = \frac{1}{64\hat{s}^2}|M_{ij}|^2$$



Lowest Order Diagrams

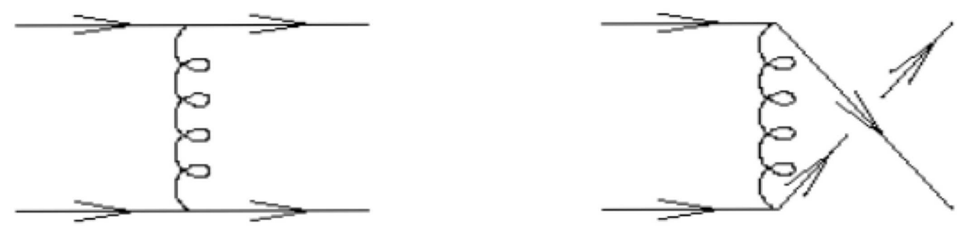
Ellis, Stirling, Webber
QCD & collider physics p248

$qq' \to qq'$



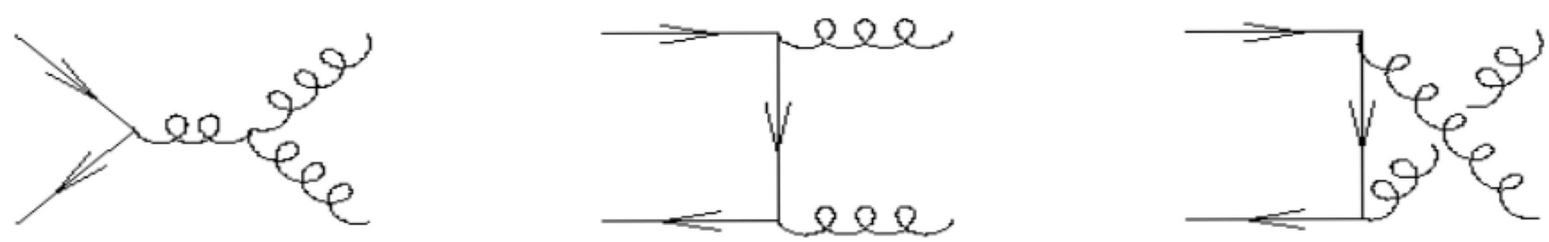
(a)

$qq \to qq$



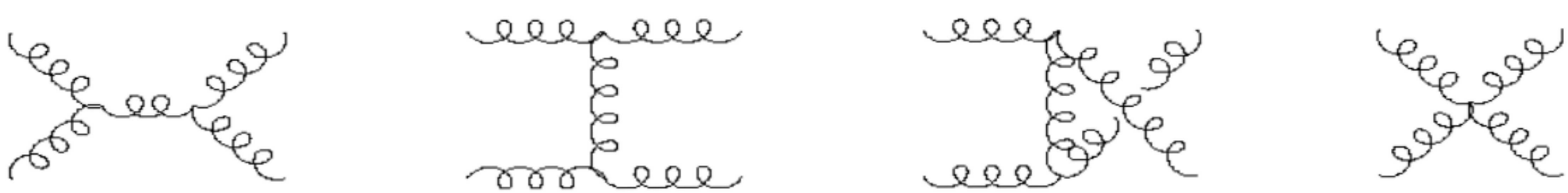
(b)

$q\bar{q} \to gg$



(c)

$gg \to gg$



(d)

$2 \to 2$ processes

invariant matrix elements for $2 \to 2$ processes with massless partons

Process	$\bar{\sum}\lvert\mathcal{M}\rvert^2/g^4$	$\theta^* = \pi/2$
$qq' \to qq'$	$\frac{4}{9}\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2}$	2.22
$q\bar{q}' \to q\bar{q}'$	$\frac{4}{9}\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2}$	2.22
$qq \to qq$	$\frac{4}{9}\left(\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2}+\frac{\hat{s}^2+\hat{t}^2}{\hat{u}^2}\right)-\frac{8}{27}\frac{\hat{s}^2}{\hat{u}\hat{t}}$	3.26
$q\bar{q} \to q'\bar{q}'$	$\frac{4}{9}\frac{\hat{t}^2+\hat{u}^2}{\hat{s}^2}$	0.22
$q\bar{q} \to q\bar{q}$	$\frac{4}{9}\left(\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2}+\frac{\hat{t}^2+\hat{u}^2}{\hat{s}^2}\right)-\frac{8}{27}\frac{\hat{u}^2}{\hat{s}\hat{t}}$	2.59
$q\bar{q} \to gg$	$\frac{32}{27}\frac{\hat{t}^2+\hat{u}^2}{\hat{t}\hat{u}}-\frac{8}{3}\frac{\hat{t}^2+\hat{u}^2}{\hat{s}^2}$	1.04
$gg \to q\bar{q}$	$\frac{1}{6}\frac{\hat{t}^2+\hat{u}^2}{\hat{t}\hat{u}}-\frac{3}{8}\frac{\hat{t}^2+\hat{u}^2}{\hat{s}^2}$	0.15
$gq \to gq$	$-\frac{4}{9}\frac{\hat{s}^2+\hat{u}^2}{\hat{s}\hat{u}}+\frac{\hat{u}^2+\hat{s}^2}{\hat{t}^2}$	6.11
$gg \to gg$	$\frac{9}{2}\left(3-\frac{\hat{t}\hat{u}}{\hat{s}^2}-\frac{\hat{s}\hat{u}}{\hat{t}^2}-\frac{\hat{s}\hat{t}}{\hat{u}^2}\right)$	30.4

Ellis, Stirling, Webber
QCD & collider physics p249

Remember: W+jet and dijets

- check on propagator:

CDF Collaboration (F. Abe et al.).Phys.Rev.Lett.73:2296-2300,1994.

Ellis, Stirling, Webber
QCD & collider physics p248

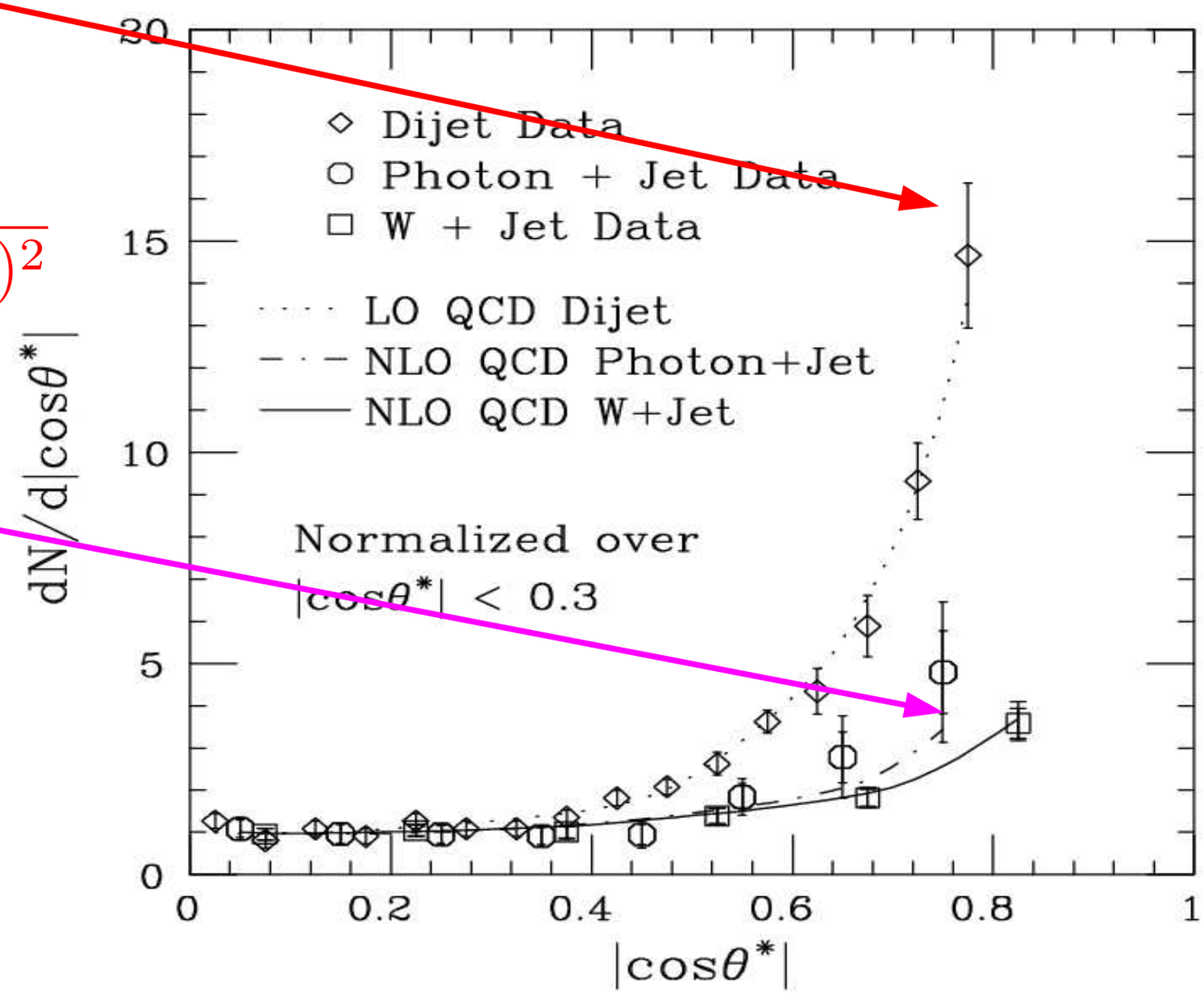
$$qq \to qq$$

$$|M|^2 \sim \frac{s^2+u^2}{t^2}$$

$$\sim \frac{1}{t^2} \sim \frac{1}{(1-cos\theta^*)^2}$$

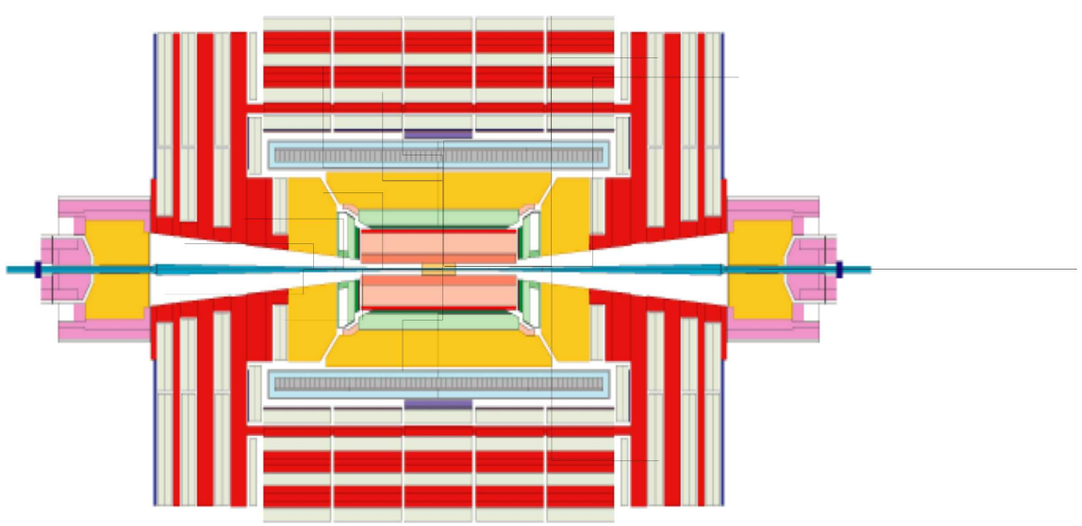
$$qq \to W+g:$$

$$|M|^2 \sim \frac{1}{t} \sim \frac{1}{(1-cos\theta^*)}$$

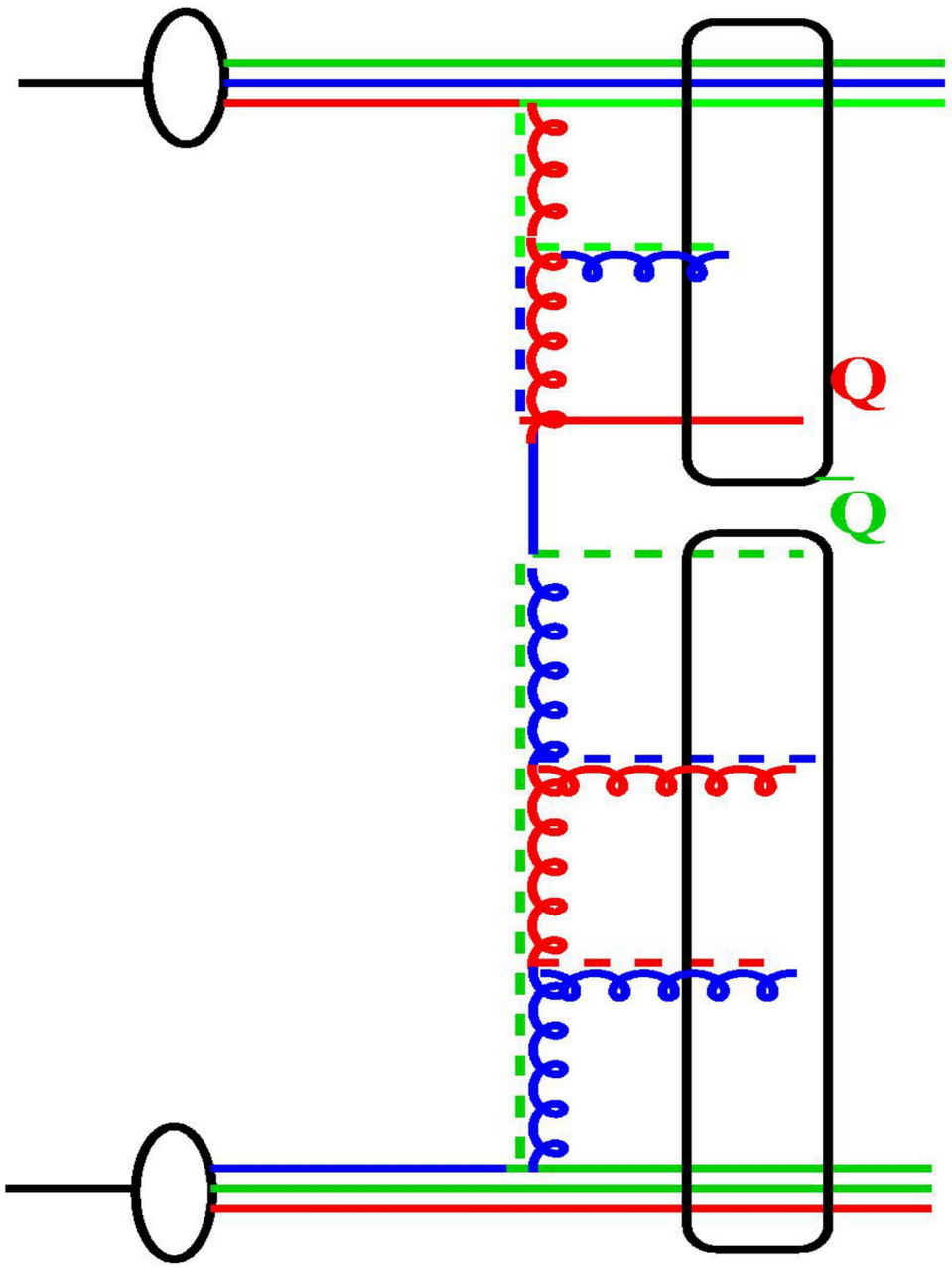


Color Flow in pp

- quarks carry color
- anti-quarks carry anti-color
- gluons carry color – anti-color
  - connect to color singlet systems
  - watch out pp or $p\bar{p}$



$$pp \to q\bar{q} + X$$

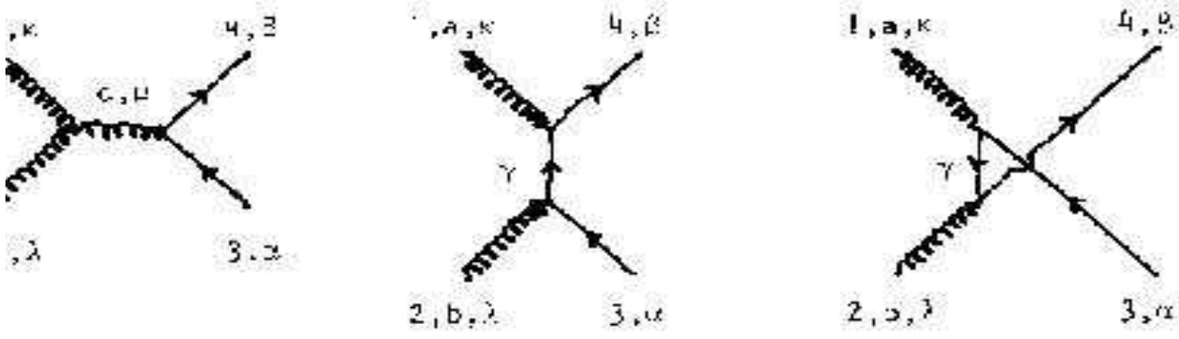


Color Flow in pp

The Lund Monte Carlo For High P(T) Physics H.U. Bengtsson
Comput.Phys.Commun.31:323,1984.

cess: $gg \to q_i \bar{q}_i$

grams:



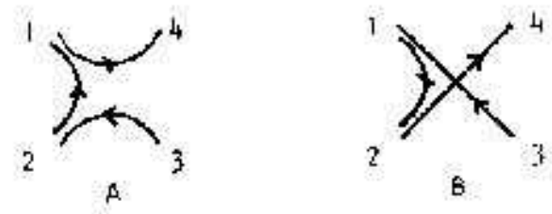
plitudes:

$$g^2 f^{abc} T^c_{\alpha\beta} \bar{u}_i^\beta(q_4) \frac{\epsilon_1^\kappa \epsilon_2^\lambda \gamma^\mu}{\hat{s}} C_{\kappa\lambda\mu}(q_1, q_2, -q_1 - q_2) v_i^\alpha(q_3)$$

$$-ig^2 T^b_{\alpha\gamma} T^a_{\gamma\beta} \bar{u}_i^\beta(q_4) \not\epsilon_1 \frac{\not q_1 - \not q_4}{\hat{t}} \not\epsilon_2 v_i^\alpha(q_3)$$

$$-ig^2 T^a_{\alpha\gamma} T^b_{\gamma\beta} \bar{u}_i^\beta(q_4) \not\epsilon_2 \frac{\not q_1 - \not q_3}{\hat{u}} \not\epsilon_1 v_i^\alpha(q_3)$$

Colour flows:



String configurations:

Colour factors: A: $T^b_{\alpha\gamma} T^a_{\gamma\beta}$; B: $T^a_{\alpha\gamma} T^b_{\gamma\beta}$
Amplitudes:

$$\text{A:} \quad -ig^2 \bar{u}_i^\beta(q_4) \left[\not\epsilon_1 \frac{\not q_1 - \not q_4}{\hat{t}} \not\epsilon_2 - \frac{\epsilon_1^\kappa \epsilon_2^\lambda \gamma^\mu}{\hat{s}} C_{\kappa\lambda\mu}(q_1, q_2, -q_1 - q_2) \right] v_i^\alpha(q_3)$$

$$\text{B:} \quad -ig^2 \bar{u}_i^\beta(q_4) \left[\not\epsilon_2 \frac{\not q_1 - \not q_3}{\hat{u}} \epsilon_1 + \frac{\epsilon_1^\kappa \epsilon_2^\lambda \gamma^\mu}{\hat{s}} C_{\kappa\lambda\mu}(q_1, q_2, -q_1 - q_2) \right] v_i^\alpha(q_3)$$

Cross-sections:

$$\text{A:} \frac{\pi\alpha_s^2}{\hat{s}^2} \frac{1}{6} \left(\frac{\hat{u}}{\hat{t}} - 2\frac{\hat{u}^2}{\hat{s}^2} \right); \quad \text{B:} \frac{\pi\alpha_s^2}{\hat{s}^2} \frac{1}{6} \left(\frac{\hat{t}}{\hat{u}} - 2\frac{\hat{t}^2}{\hat{s}^2} \right)$$

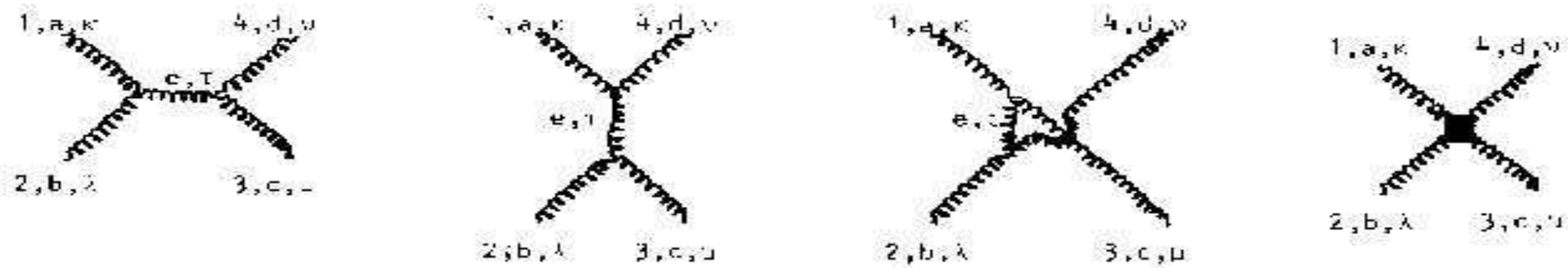
$$A + B = \ldots \left(\frac{\hat{u}}{\hat{t}} - 2\frac{\hat{u}^2 + \hat{t}^2}{\hat{s}^2} + \frac{\hat{t}}{\hat{u}} \right) = \frac{\hat{u}^2 + \hat{t}^2}{\hat{t}\hat{u}} - 2\frac{\hat{u}^2 + \hat{t}^2}{\hat{s}^2}$$

Color Flow in pp

The Lund Monte Carlo For High P(T) Physics H.U. Bengtsson
Comput.Phys.Commun.31:323,1984.

Process: gg → gg

Diagrams:



Amplitudes:

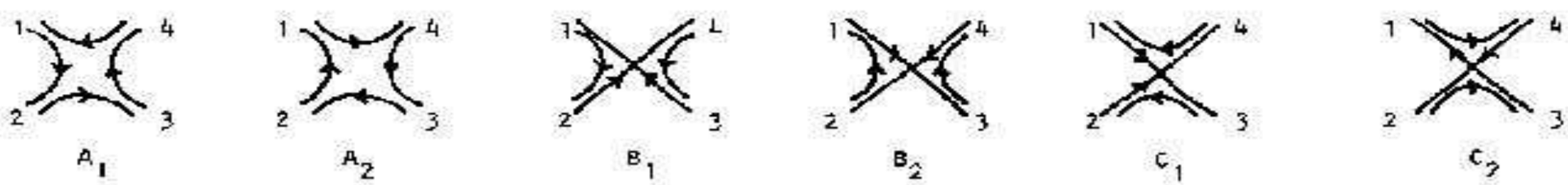
$$s: \quad -ig^2\frac{1}{\hat{s}} f^{abe} f^{cde} \epsilon_1^{\kappa}\epsilon_2^{\lambda}\epsilon_3^{\mu}\epsilon_4^{\nu} C_{\kappa\lambda\tau}(-q_1, -q_2, q_1+q_2) C_{\mu\nu}{}^{\tau}(q_3, q_4, -q_3-q_4)$$

$$t: \quad -ig^2\frac{1}{\hat{t}} f^{dae} f^{bce} \epsilon_1^{\kappa}\epsilon_2^{\lambda}\epsilon_3^{\mu}\epsilon_4^{\nu} C_{\nu\kappa\tau}(q_4, -q_1, q_1-q_4) C_{\lambda\mu}{}^{\tau}(-q_2, q_3, q_2-q_3)$$

$$u: \quad -ig^2\frac{1}{\hat{u}} f^{cae} f^{bde} \epsilon_1^{\kappa}\epsilon_2^{\lambda}\epsilon_3^{\mu}\epsilon_4^{\nu} C_{\mu\kappa\tau}(q_3, -q_1, q_1-q_3) C_{\lambda\nu}{}^{\tau}(-q_2, q_4, q_2-q_4)$$

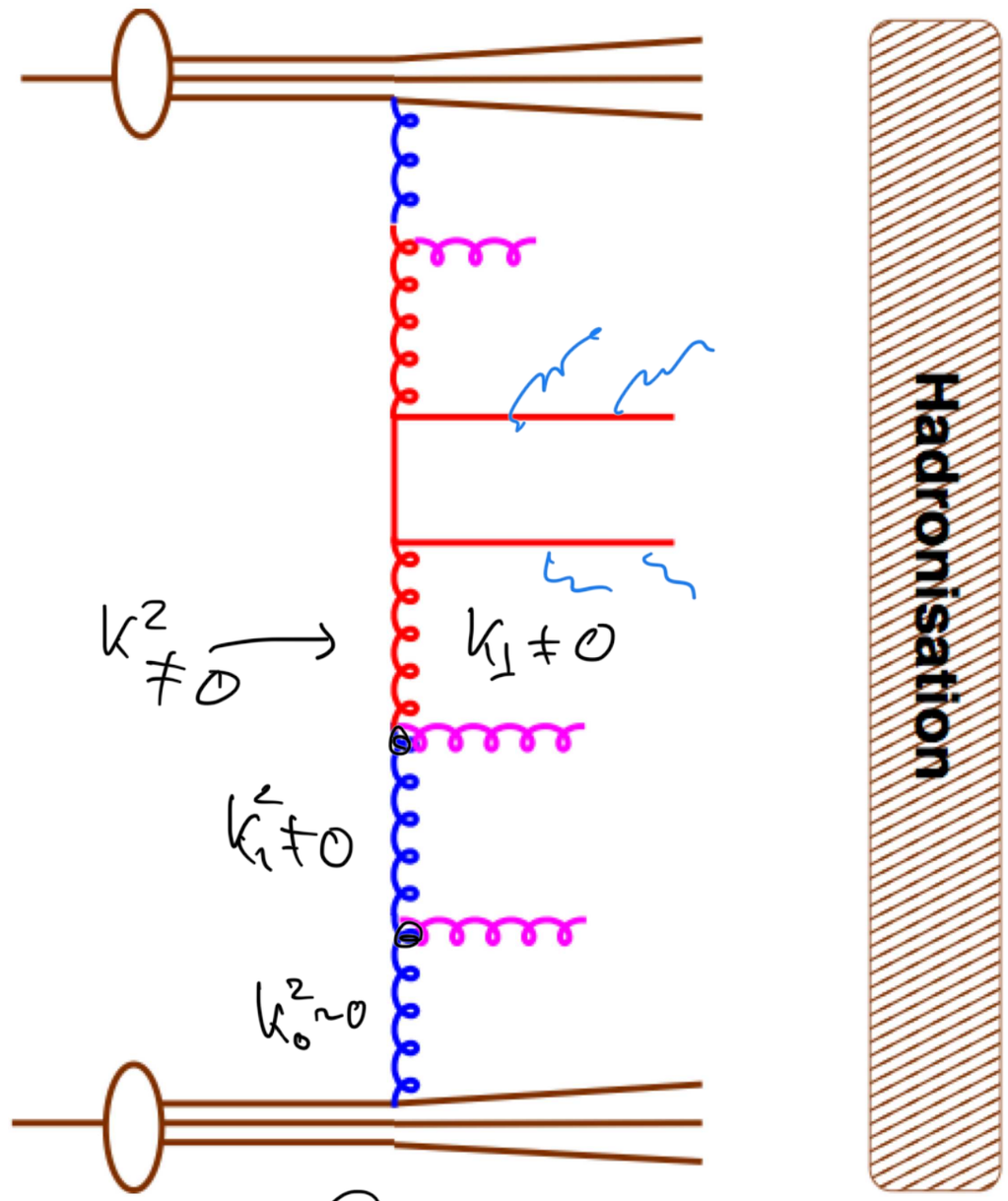
$$4: \quad -ig^2 f^{abe} f^{cde}(g_{\kappa\mu}g_{\lambda\nu} - g_{\kappa\nu}g_{\lambda\mu}) - ig^2 f^{dae} f^{bce}$$
$$\times(g_{\kappa\mu}g_{\lambda\nu} - g_{\kappa\lambda}g_{\nu\mu}) + ig^2 f^{cae} f^{bde}(g_{\kappa\lambda}g_{\mu\nu} - g_{\kappa\nu}g_{\lambda\mu})$$

Colour flows:



The general pp case

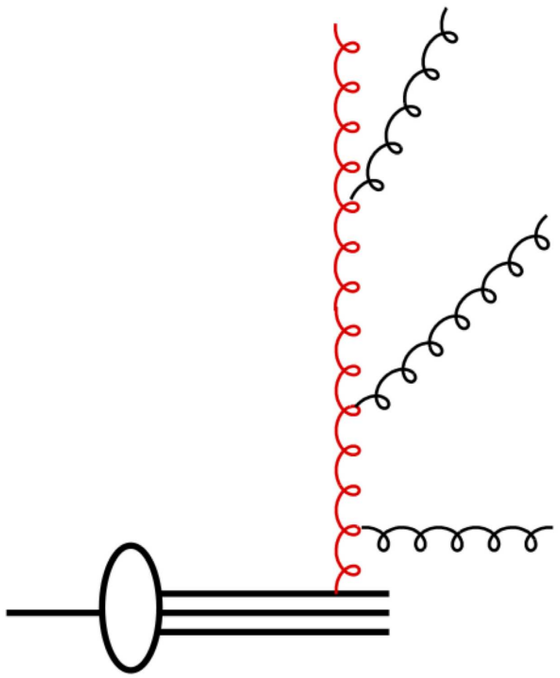
- basic elements are:
  - Matrix Elements:
    - ➔ on shell/off shell
  - PDFs
    - ➔ TMD PDFs
  - Parton Shower
    - ➔ angular ordering
- Proton remnant and hadronization



$$\sigma(pp \to q\bar{q} + X) = \int \frac{dx_{g1}}{x_{g1}} \frac{dx_{g2}}{x_{g2}} \int d^2k_{t1} d^2k_{t2} \hat{\sigma}(\hat{s}, k_t, \mu)$$
$$\times x_{g1}\mathcal{A}(x_{g1}, k_{t1}, \mu) x_{g2}\mathcal{A}(x_{g2}, k_{t2}, \mu)$$

Parton shower

- simulate explicitly parton radiation with evolution of parton densities
- advantage to include properly energy momentum conservation in each step
- perform resummation numerically



- Parton shower follows backward evolution:

$$\Pi = \exp\left[-\int_{\mu_l^2}^{\mu_h^2} \frac{d\mu'^2}{\mu'^2} \int^{z_{\rm dyn}} \frac{dz}{z} \hat{P}(z) \frac{f(x/z,\mu^2)}{f(x,\mu^2)}\right]$$

- Emitted partons should have *resolvable* energy (or p_T) with: $p_T > q_{t\,\rm cut} \sim 1{\rm GeV}$

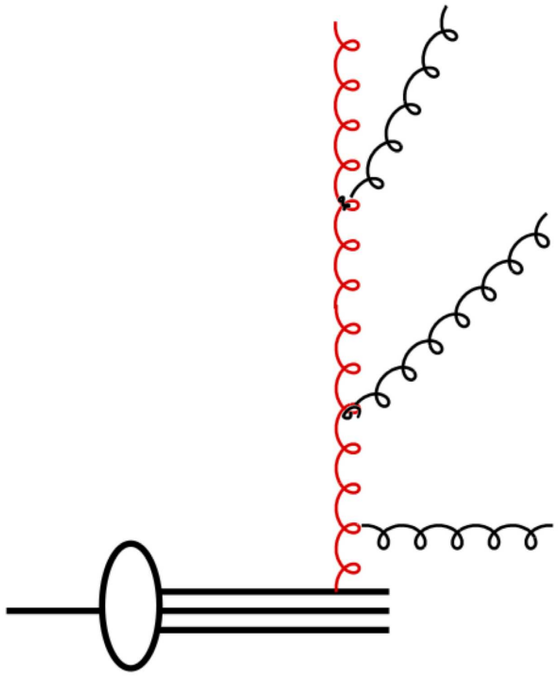
$$z_{\rm dyn} = 1 - \frac{q_{t\ \rm cut}}{\mu'}$$

- With $z_{\rm dyn} \ll 1$ soft gluons with $p_T < 1{\rm GeV}$ are neglected.

Parton shower

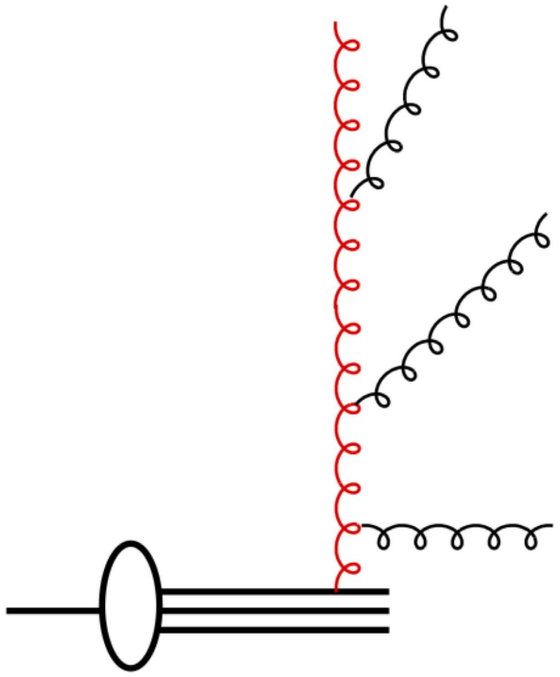
- Parton shower with collinear parton densities adds new information on transverse momentum of partons, by explicitly undoing the evolution.
  - not necessarily same parameters as in evolution:
    - $z_{\rm dyn} \ll 1$ versus $z \to 1$
    - order of splitting functions
    - α_s, heavy flavors
    - ordering conditions (angular ordering ?)
    - etc
  - inconsistencies arise from energy momentum conservation
  - special treatment needs to be done to conserve energy momentum at the end
    - boost, rotation of hard process

$\alpha_s^{PS}(p_\perp)$



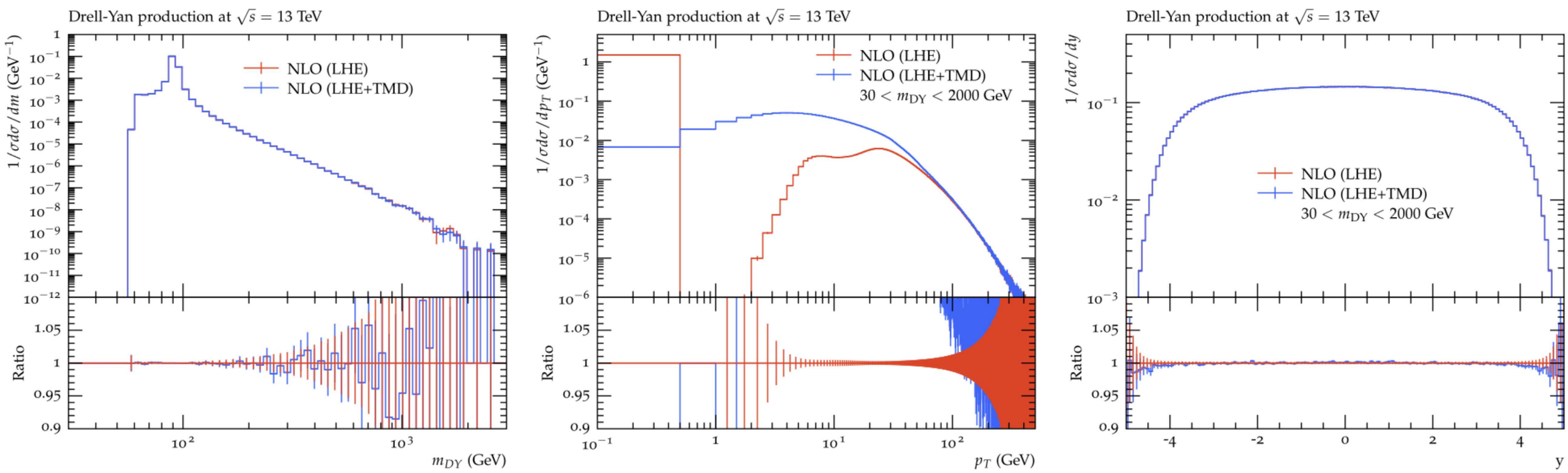
TMD parton shower

- TMD parton shower uses information from TMDs (obtained from fits to inclusive data)
  - uses exactly the same parameters as in evolution
    - cut on z values
    - order of splitting functions
    - α_s, heavy flavors
    - ordering conditions (angular ordering…)
  - inconsistencies arise from energy momentum conservation stay, when using collinear ME
  - special treatment needs to be done to conserve energy momentum at the end
    - boost, rotation of hard process



Parton shower and TMDs

- Check that including initial state PS and TMDs does not change the x-section of the hard process.



How to calculate x-sections then ?

MCatNLO method

- NLO calculation of dijets:

$$d\sigma = \left[B(\Phi_B) + \hat{V}(\Phi_B)\right] + R(\Phi_R)d\Phi_r$$

  - diverging spectrum of gluon radiation
  - cancelled by virtual corrections
  - spectrum at low p_T of gluon (or p_T of dijet system) unphysical
- introduce auxiliary counter term $C(\Phi_R)$

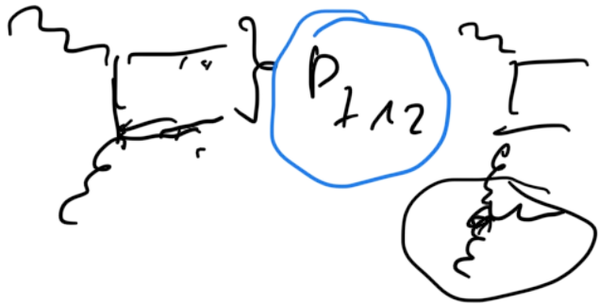
$$d\sigma = \left[B(\Phi_B) + V(\Phi_B) + \int d\Phi C\right] + R(\Phi_R)d\Phi_r - Cd\Phi$$

  - subtract and add collinear,soft divergent term
    - calculated matrix element with subtraction
    - physical quantity obtained only if $C(\Phi_R)$ is added back by TMD or parton shower
  - method used in MCatNLO with CASCASDE3 for TMD or with HERWIG/PYTHIA for parton shower

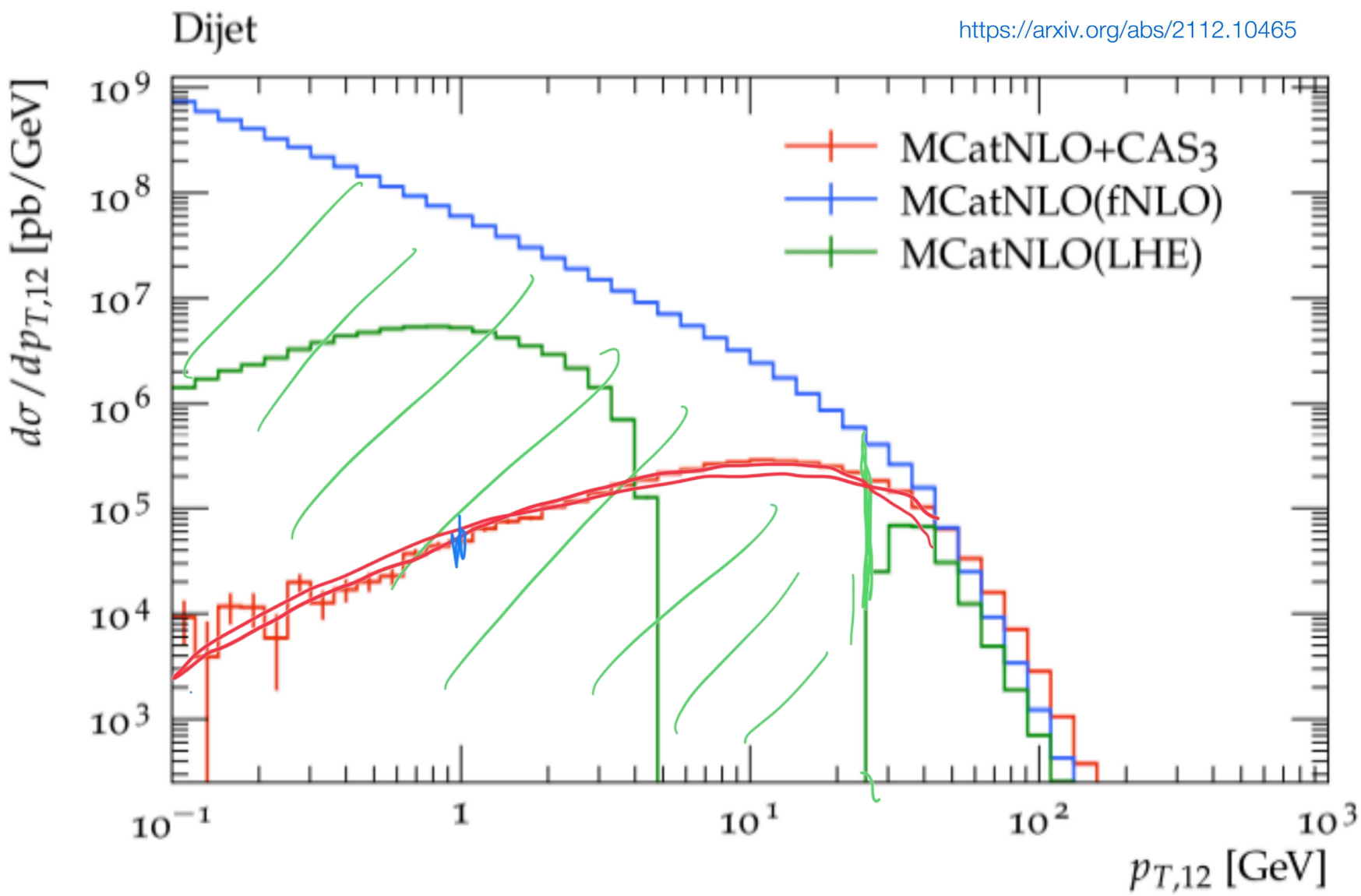
MCatNLO method

- **MCatNLO** method (*Frixione, S. and Webber, B. R. Matching NLO QCD computations and parton shower simulations, JHEP, 06(2002), 029*)):
  - subtract collinear and soft part
  - add it back by using TMD

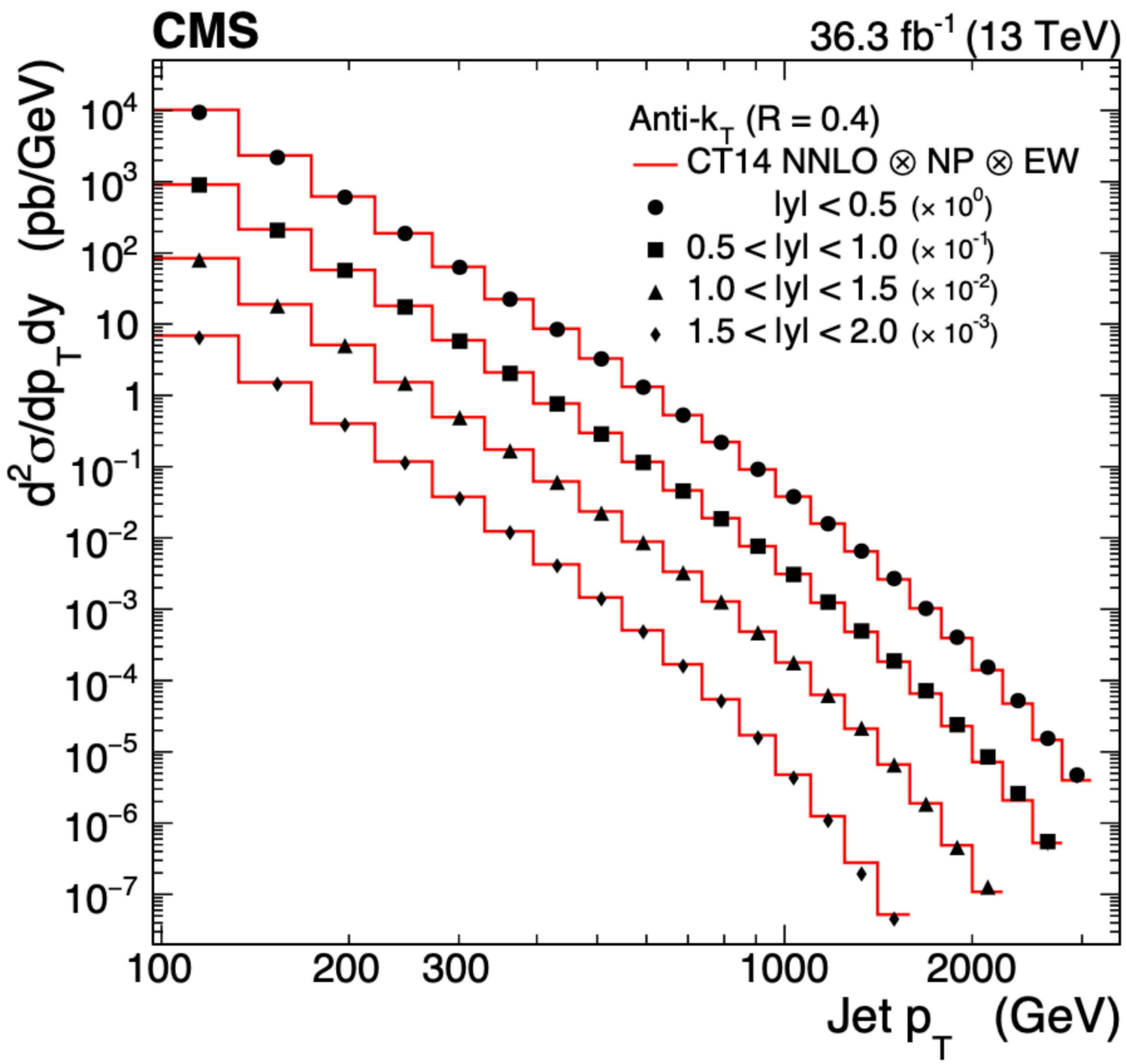
$p_{\perp 1/2} > 100$ GeV



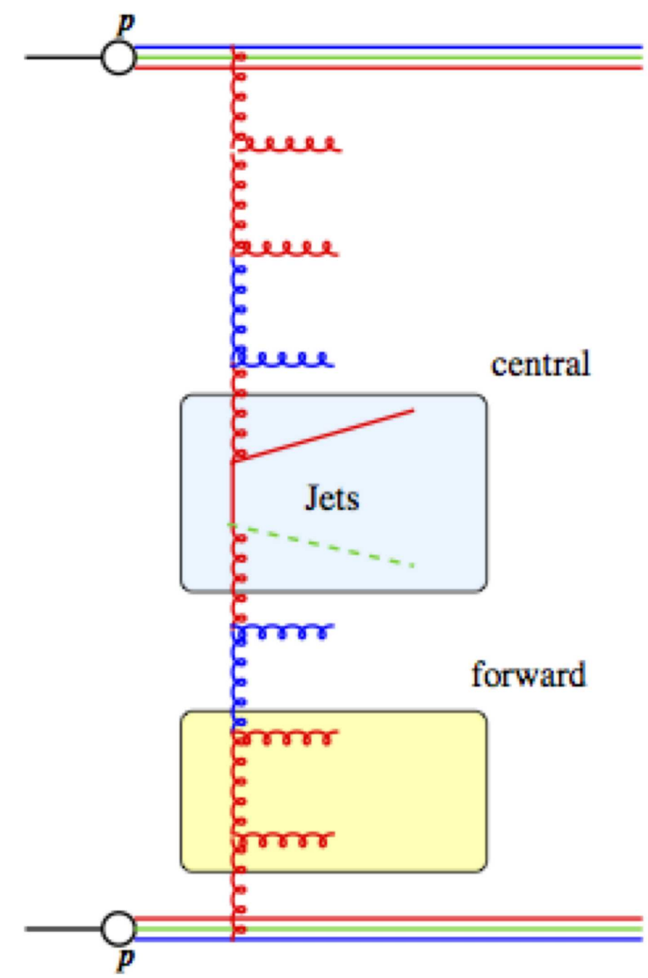
https://arxiv.org/abs/2112.10465



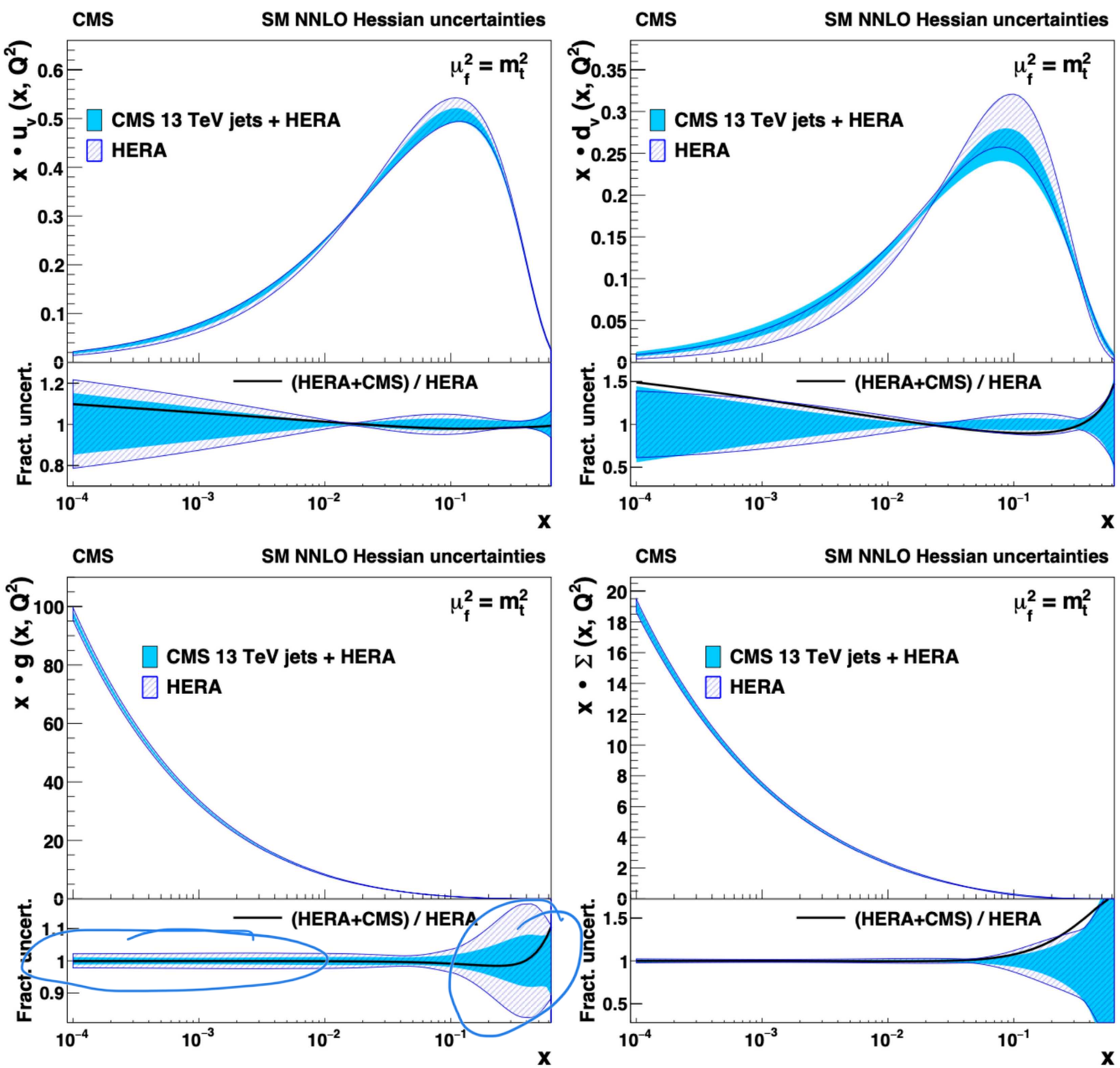
Jet production at the LHC



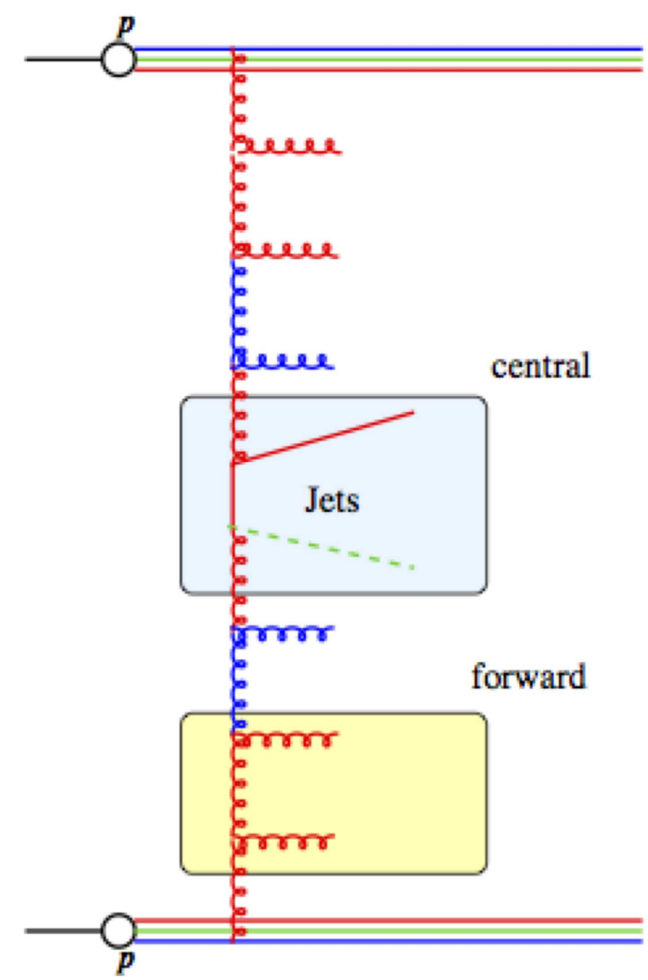
Tumasyan, A. and others Measurement and QCD analysis of double-differential inclusive jet cross sections in proton-proton collisions at $ s $ = 13 TeV, JHEP, 022022(), 142



Jet production to constrain PDFs

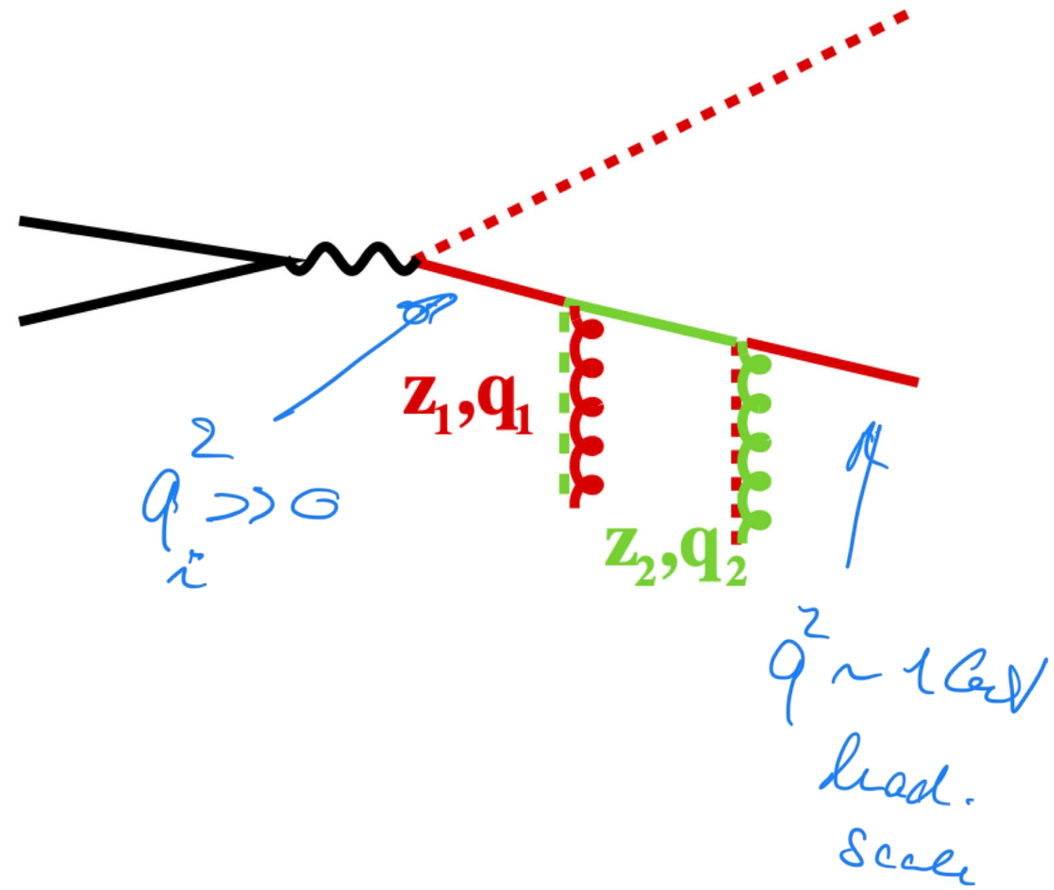


Tumasyan, A. and others Measurement and QCD analysis of double-differential inclusive jet cross sections in proton-proton collisions at $ s $ = 13 TeV, JHEP, 022022(), 142



Jet Mass – Sudakov peak

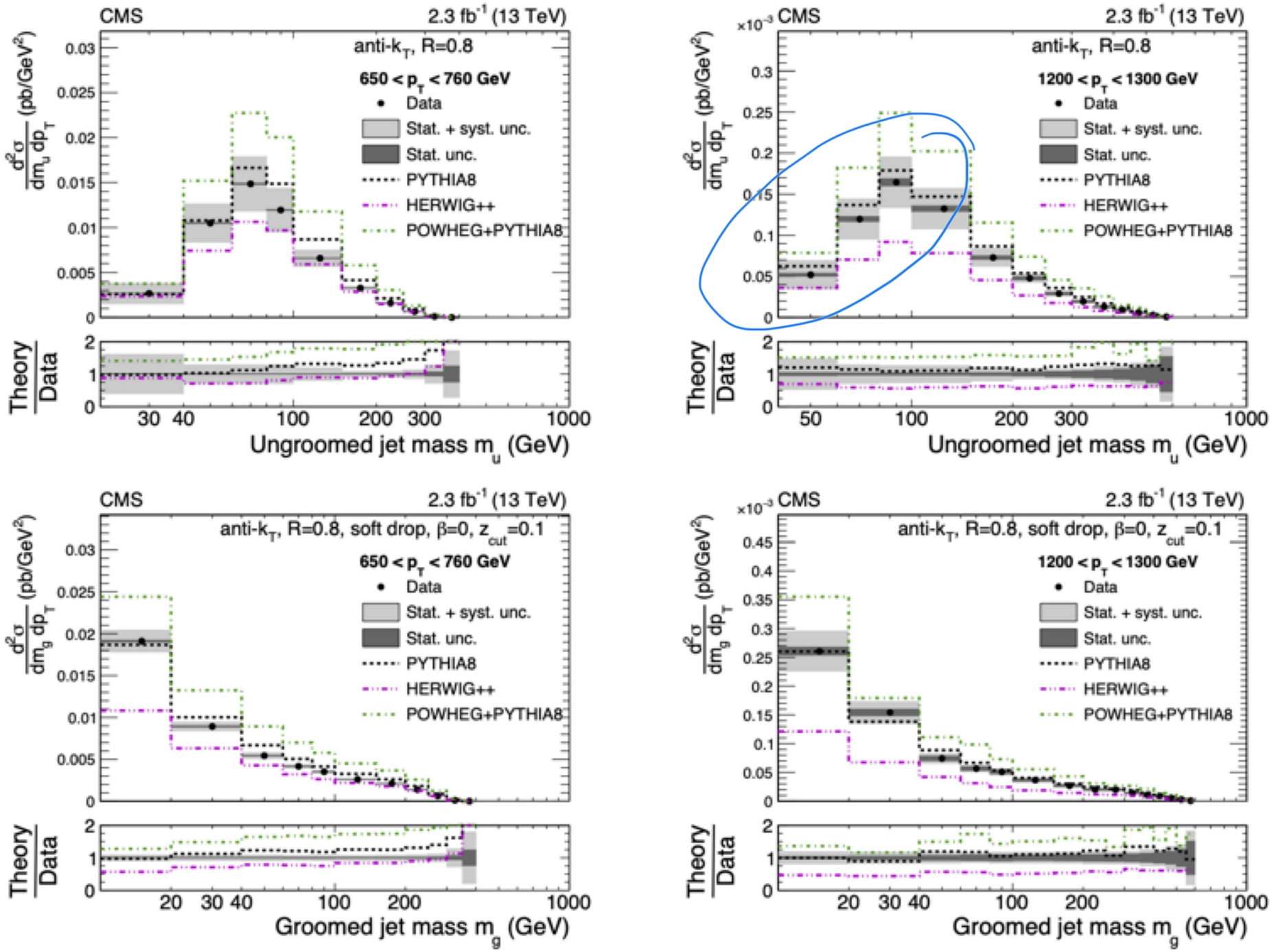
- Jets in final state: time-like evolution of parton radiation
  - follows similar evolution equation as for parton densities
  - evolution from large time-like mass to small scale where hadronization happens
- Evolution according to Sudakov form factors (similar to initial state)



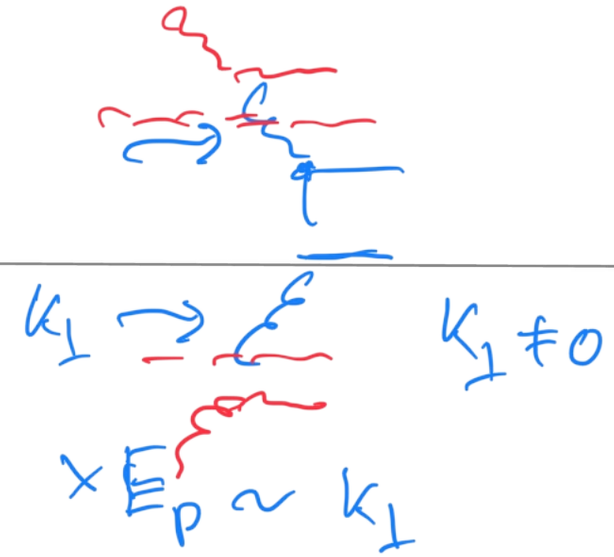
- Jet mass comes from parton radiation:
  - low mass
    - at low transverse momenta, suppression from Sudakov
  - large mass
    - perturbative radiation pattern

Jet Mass – Sudakov peak

Sirunyan, A. M. and others Measurements of the differential jet cross section as a function of the jet mass in dijet events from proton-proton collisions at $ s=13 $ TeV, JHEP, 112018(), 113



k_t - factorization



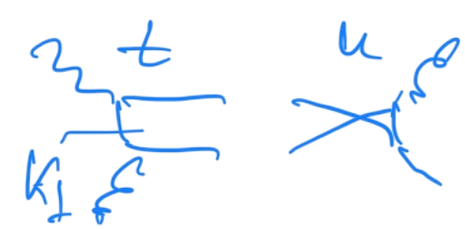
- use high energy (kt -) factorization:

  (Catani, Ciafaloni, Hautmann NPB 366 (1991) 135,
  Gribov, Levin, Ryskin, Phys. Rep.100 ,(1983),1,
  Collins, Ellis, NPB 360 ,(1991) ,3)

- with $\sigma(\mathrm{ep} \to \mathrm{e}'\mathrm{q}\bar{\mathrm{q}}) = \int \frac{dy}{y} d^2Q \frac{dx_g}{x_g} \int d^2k_t \hat{\sigma}(\hat{s}, k_t,) x_g \mathcal{A}(x_g, k_t,)$

$$\int^{Q^2} d^2k_t x_g \mathcal{A}(x_g, k_t, \bar{q}) \simeq x_g G(x_g, Q^2)$$

- *t*-channel gluon with virtuality $k^2 = -k_t^2$ dominates the process in the high energy limit $s \gg \hat{s}$
- collinear limit obtained by: $\hat{\sigma}(\hat{s}, 0, Q) \cdot \Theta(Q - k_T)$



- BUT k_t-factorization is proven only for small x ....

Off-shell matrix element calculation

A.van Hameren Katie, https://arxiv.org/abs/1611.00680

- Automated tool exists for LO calculation: KaTie
  - however k_t factorization only proven for small x
- NLO calculation much more difficult, since virtual corrections are also kt dependent
- First steps toward automated NLO calculation are done
- Where are k_t -dependent matrix elements relevant ?

Why off-shell matrix elements ?

- Behavior of ME as function of k_t:

Baranov, S. and others CASCADE3 A Monte Carlo event generator based on TMDs, Eur. Phys. J. C, 81(2021), 425

  - for small k_t converges to collinear result

  - for large k_t has suppression
  - ➔ suppression appears at "standard factorization scale": $Q^2 + 4\,m^2$

  - collinear factorization:

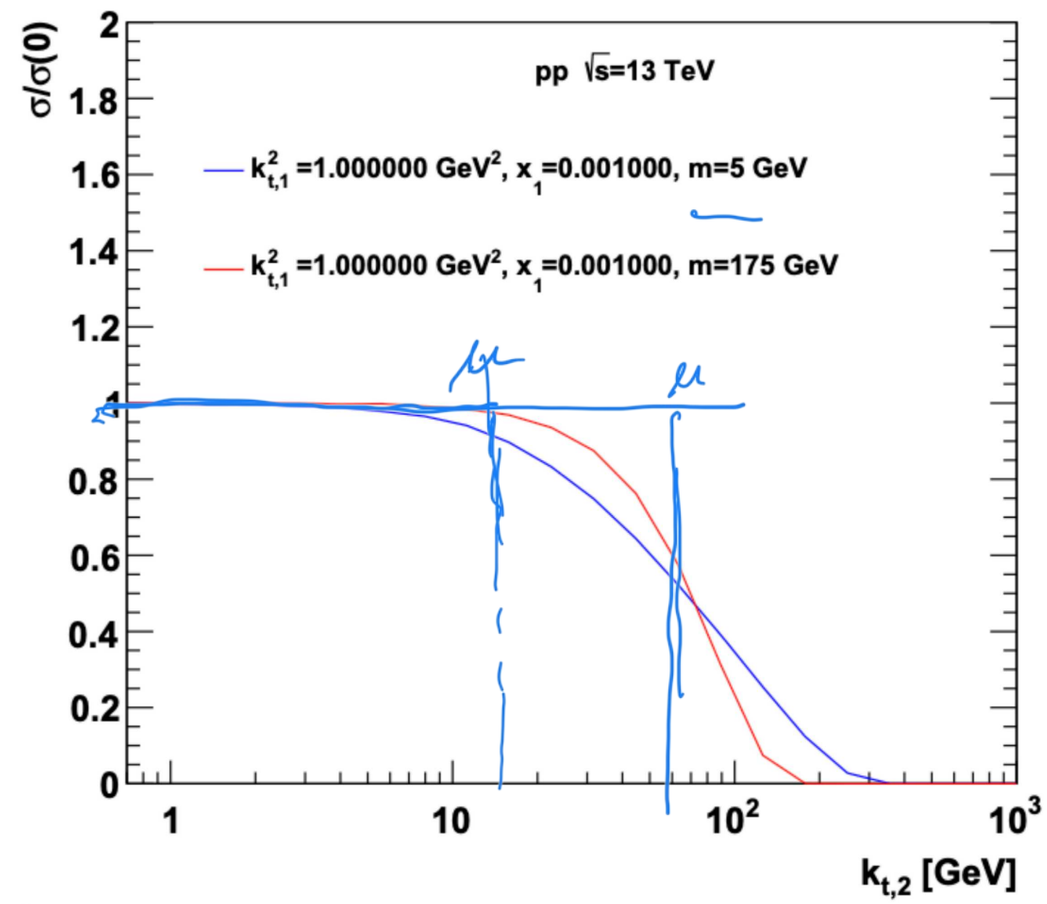
$\mu^2 \sim Q^2 + 4\,m^2$

$$\int_0^{\mu^2} d\hat{\sigma}(, \ldots)$$

$\mu^2 \sim 4\,m^2$

TMD/PS

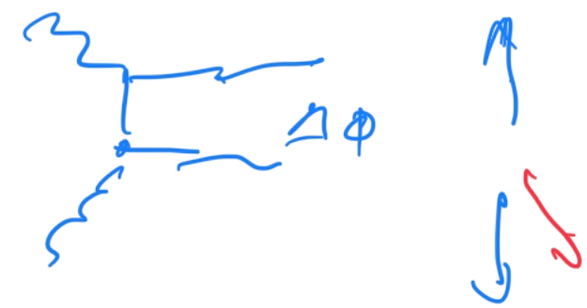
$p_{\perp i} < \mu^2$

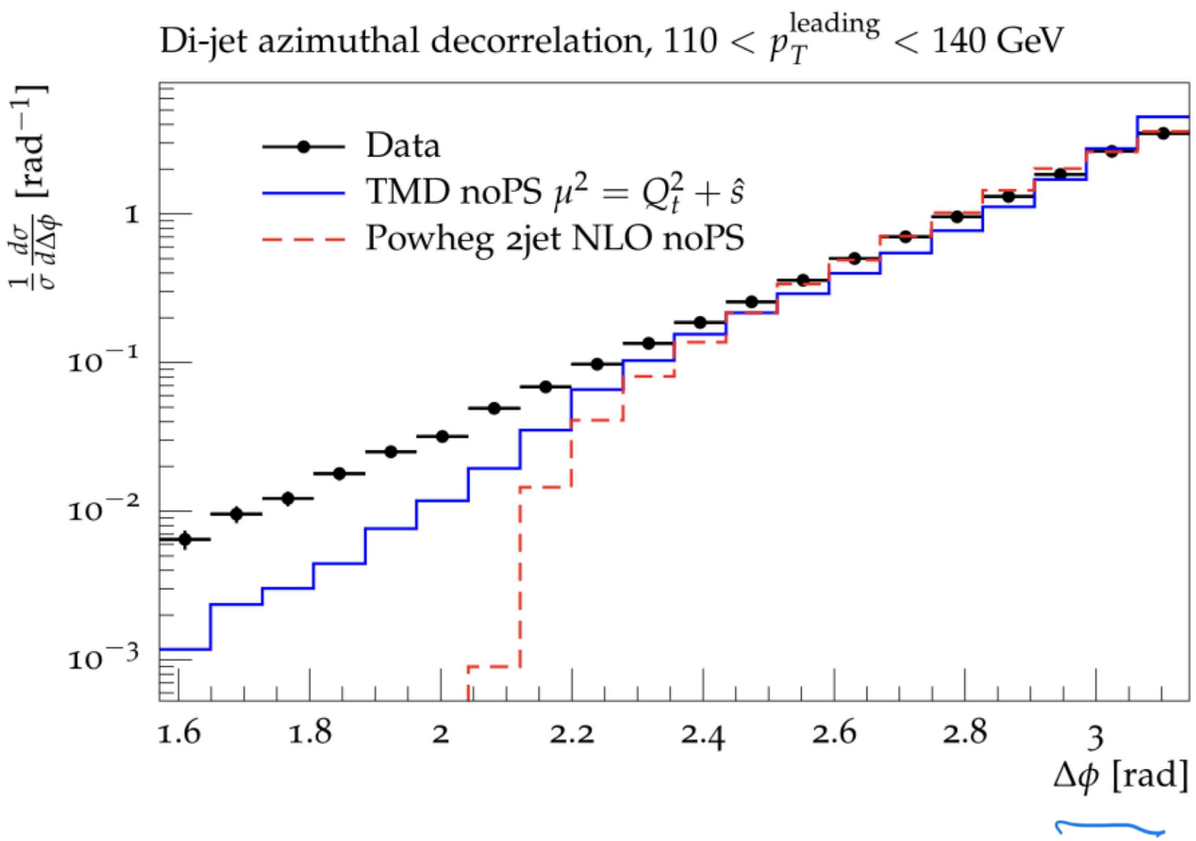


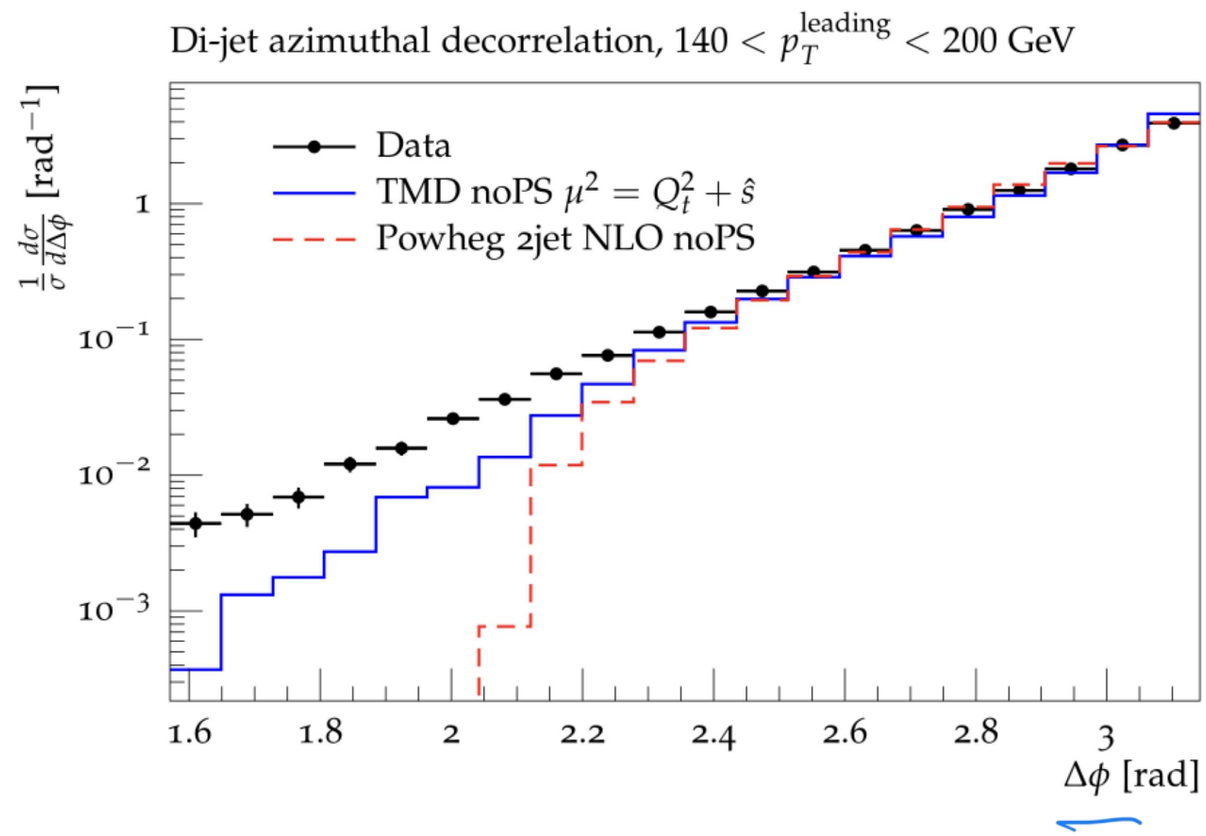
Predictions with TMDs and off-shell ME

- Calculate jet production with KaTie and TMDs
  - no special kinematic reshuffling needed
  - even without explicit parton shower, $\Delta\phi$ well described

https://arxiv.org/abs/1712.05932



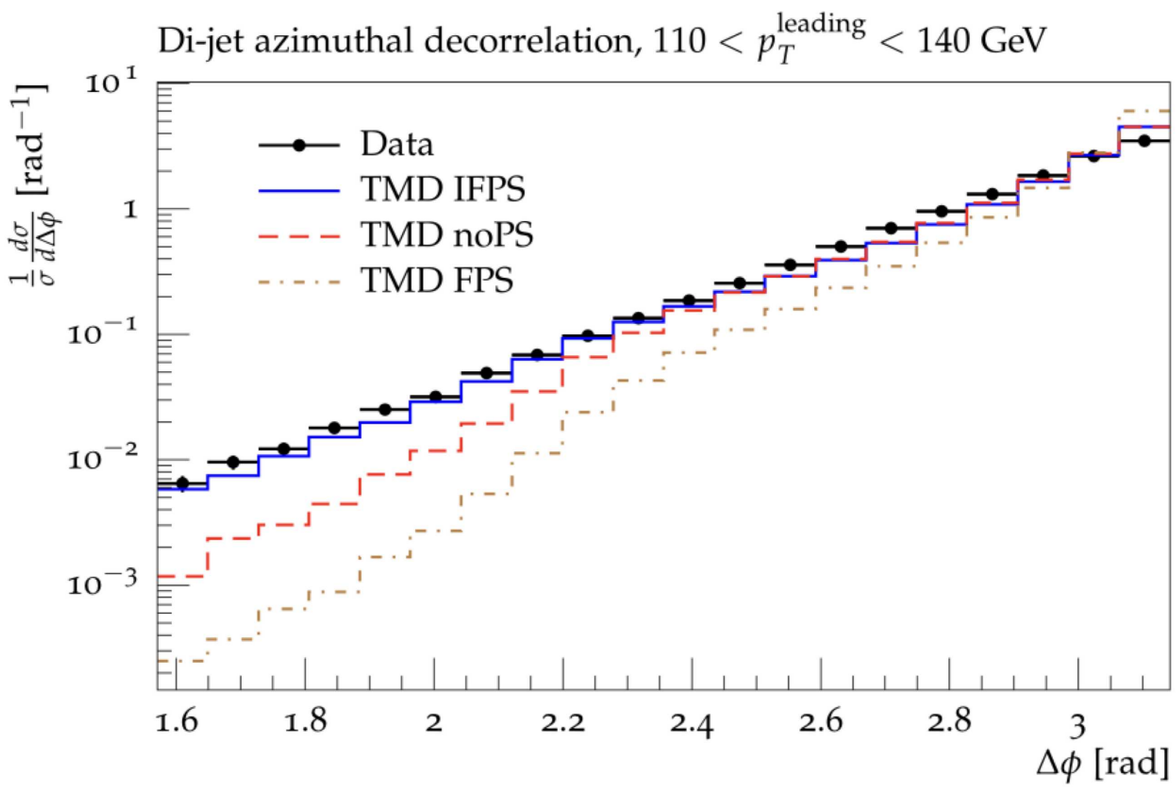


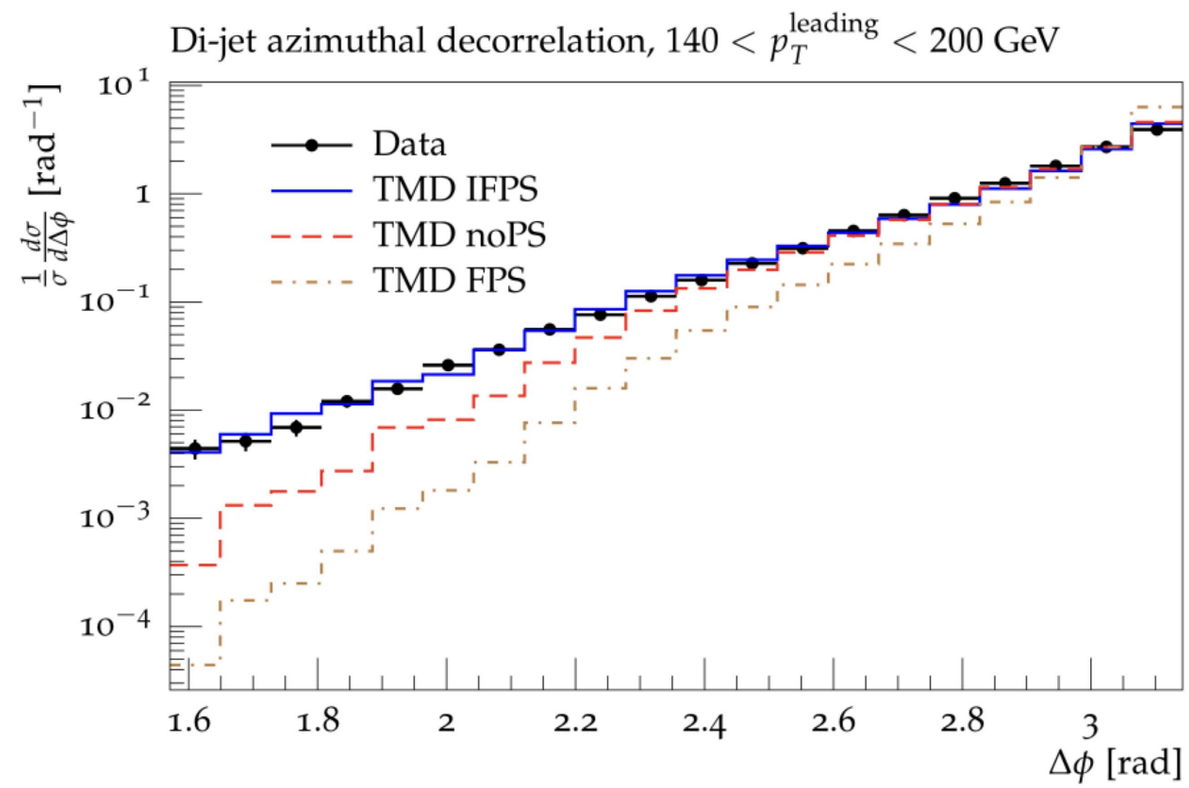


Predictions with TMDs and off-shell ME

- Calculate jet production with KaTie and TMDs
  - no special kinematic reshuffling needed
  - even without explicit parton shower, $\Delta\phi$ well described
    - for small $\Delta\phi$ parton shower is needed – more jets

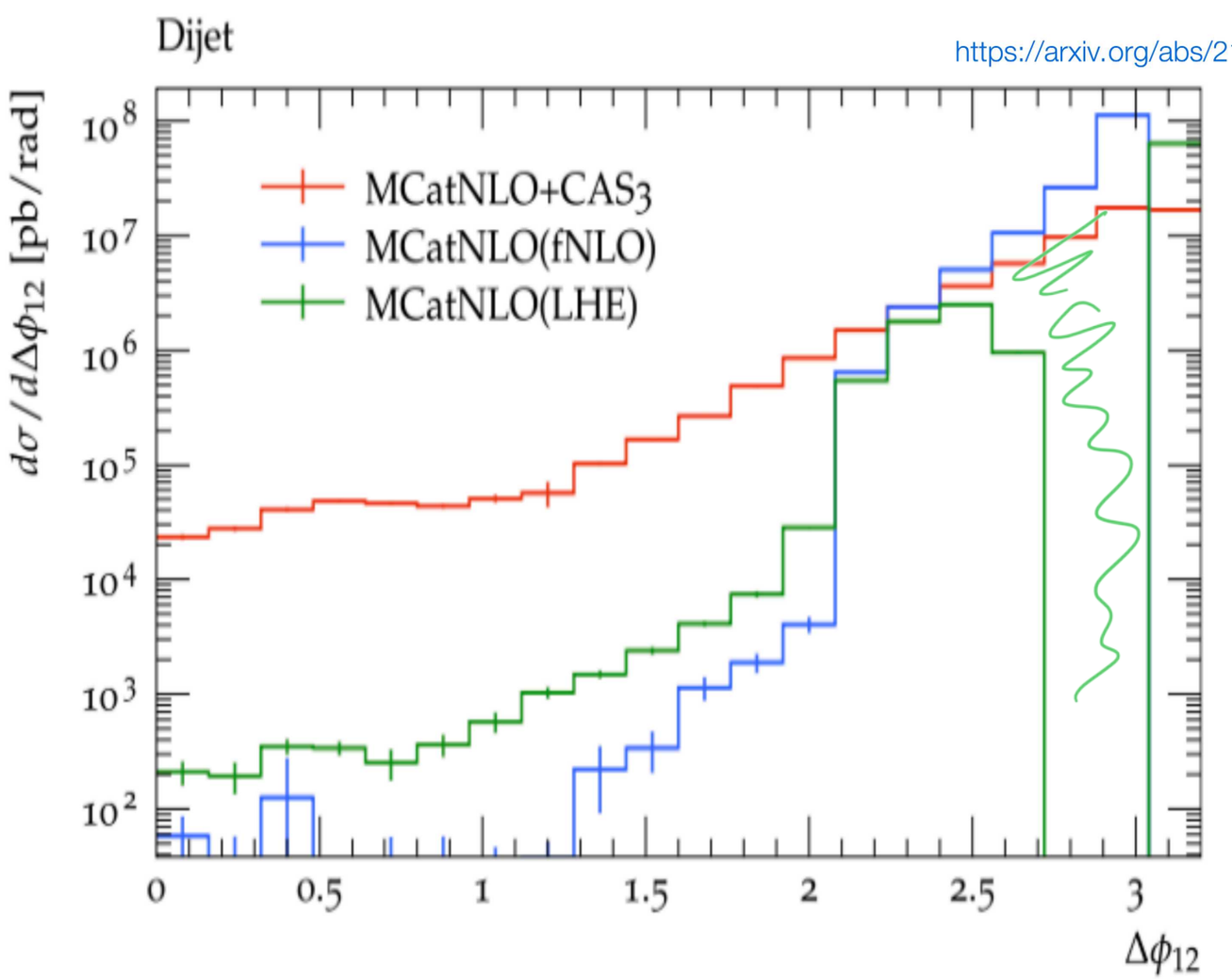
https://arxiv.org/abs/1712.05932



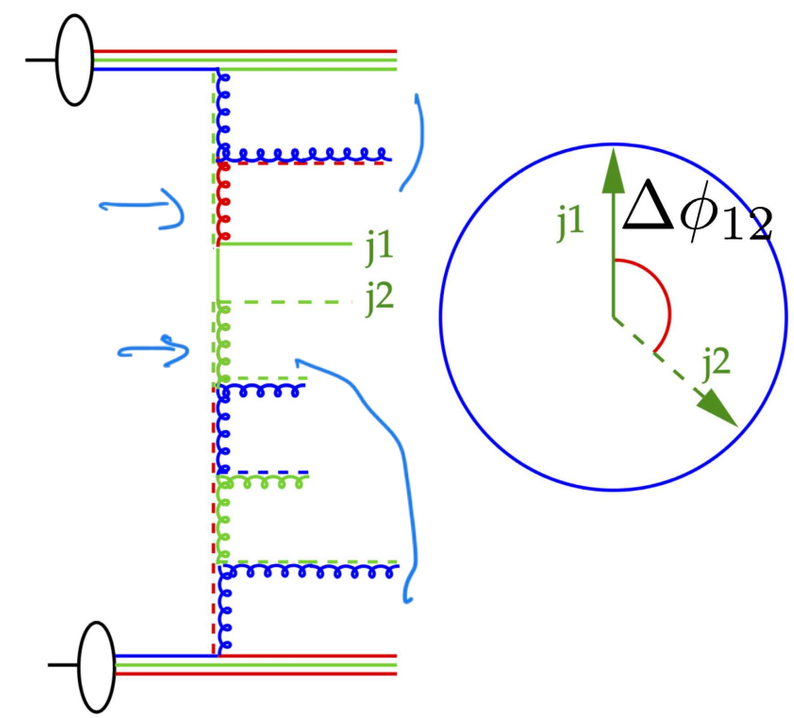


MCatNLO method

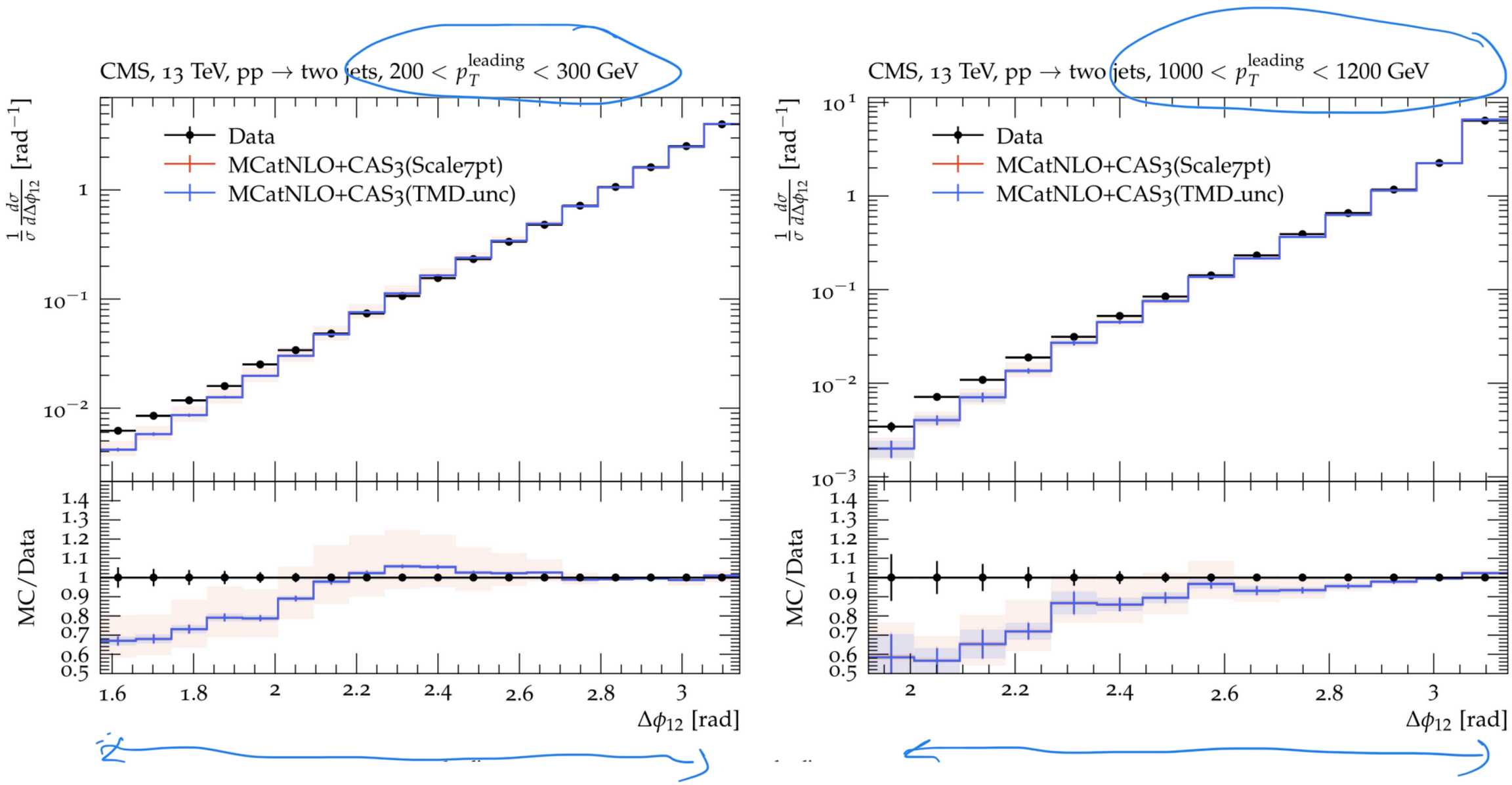
- **MCatNLO** method (*Frixione, S. and Webber, B. R. Matching NLO QCD computations and parton shower simulations, JHEP, 06(2002), 029*)):
  - subtract collinear and soft part
  - add it back by using TMD
- check $\Delta\phi$ distribution



https://arxiv.org/abs/2112.10465

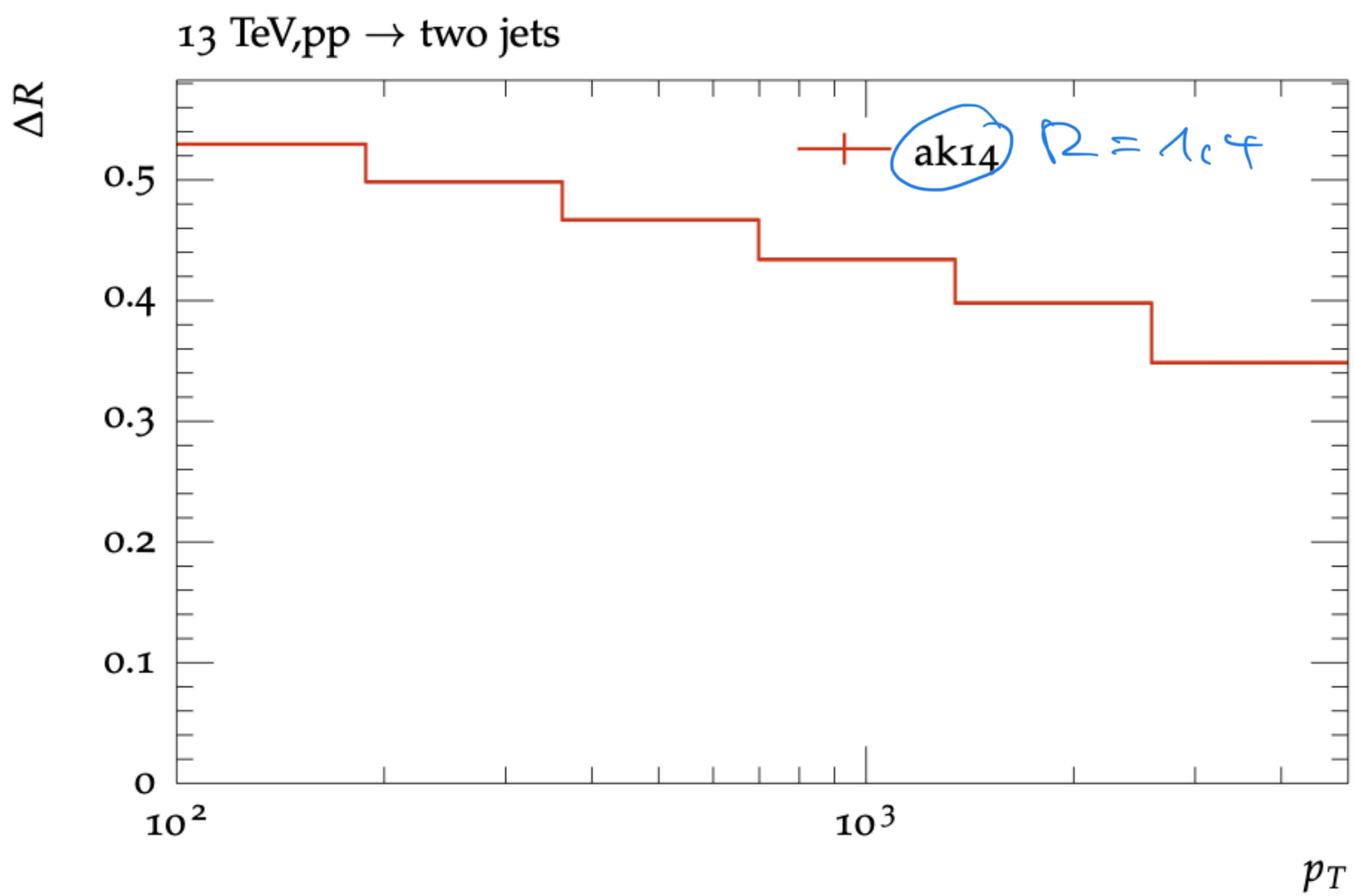


Measurements at large p_T. $\Delta\phi$

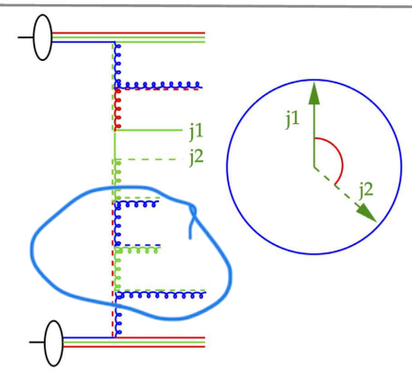


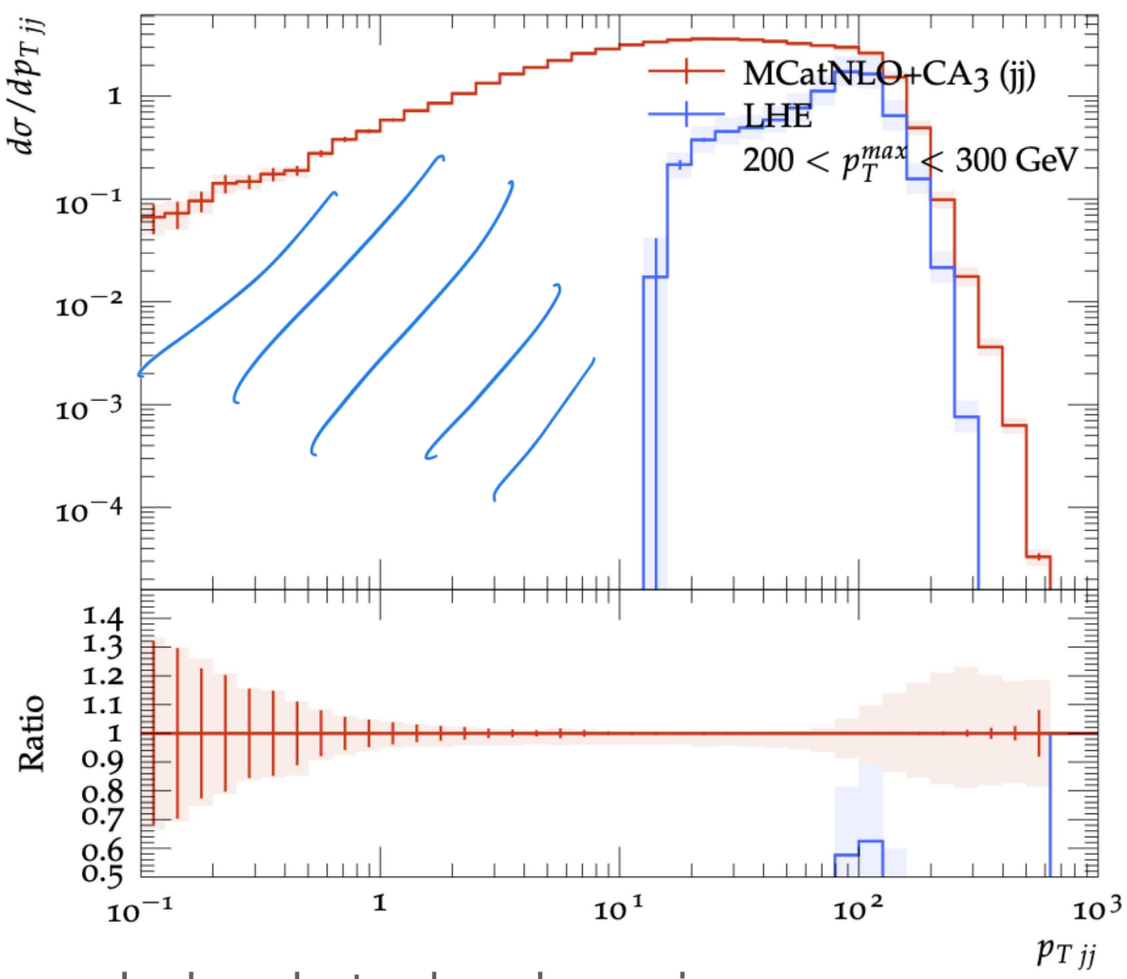
Jets at large p_T

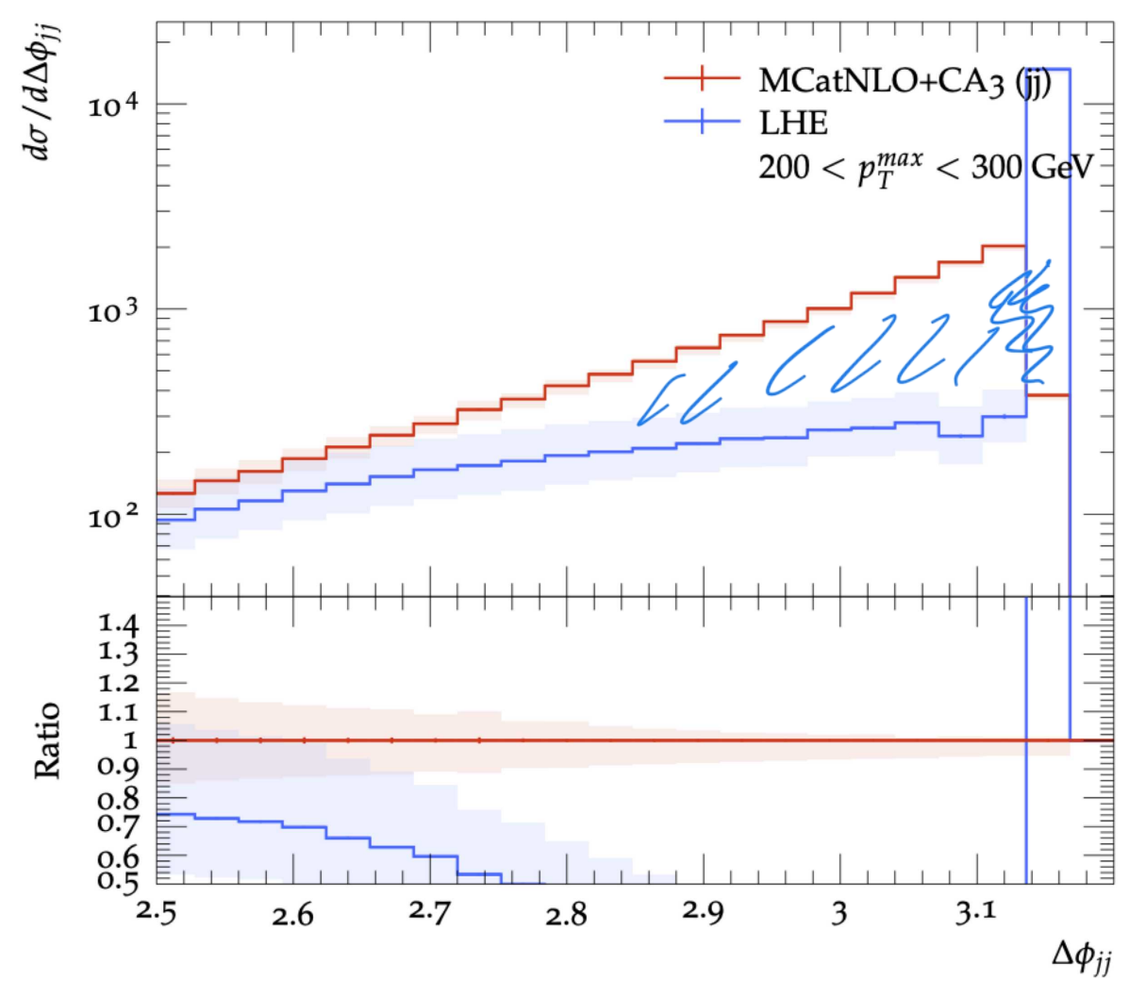
- At large p_T , jets become smaller, pencil-like
- much better experimental resolution
  - enables studies in new environment



Measurements at large p_T. $\Delta\phi$ back-to-back

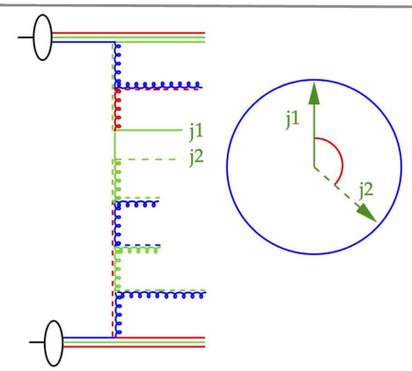


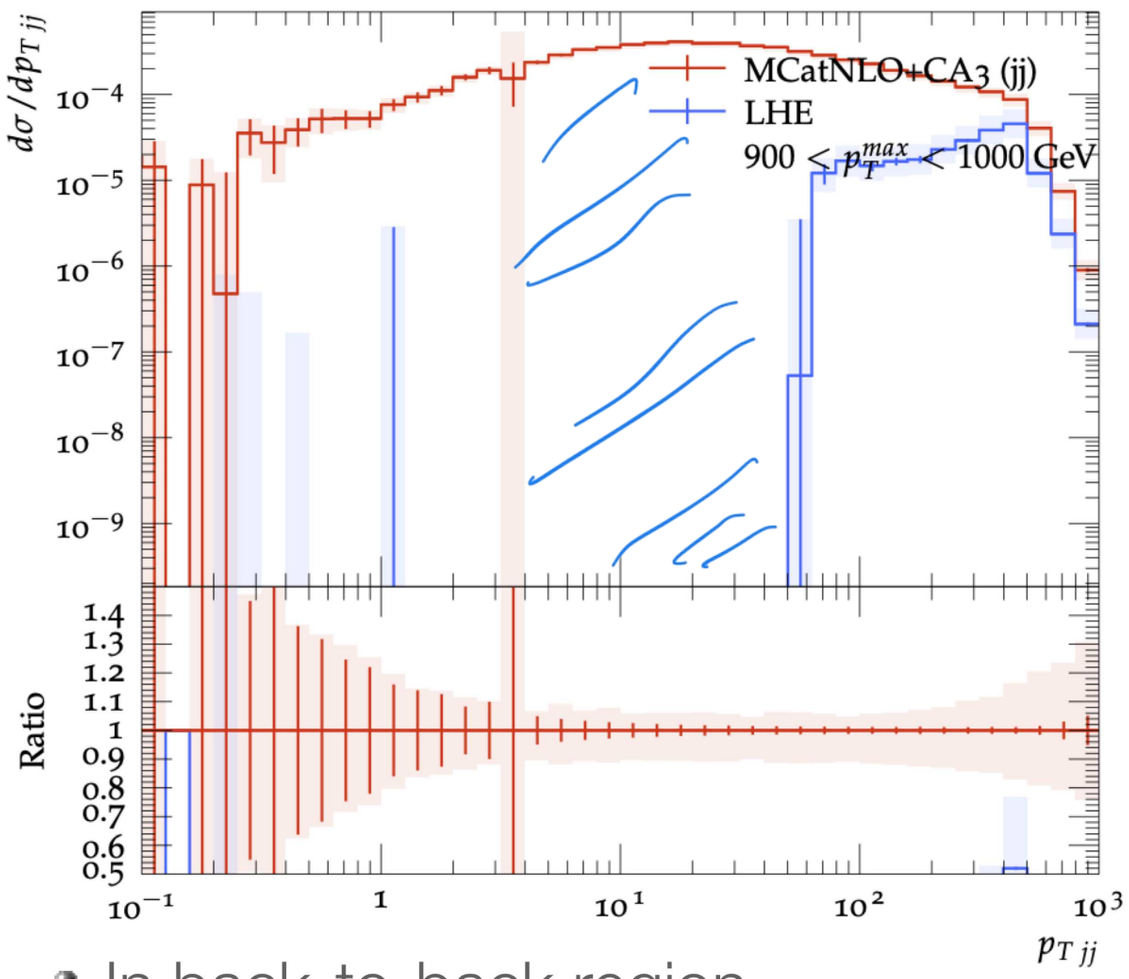


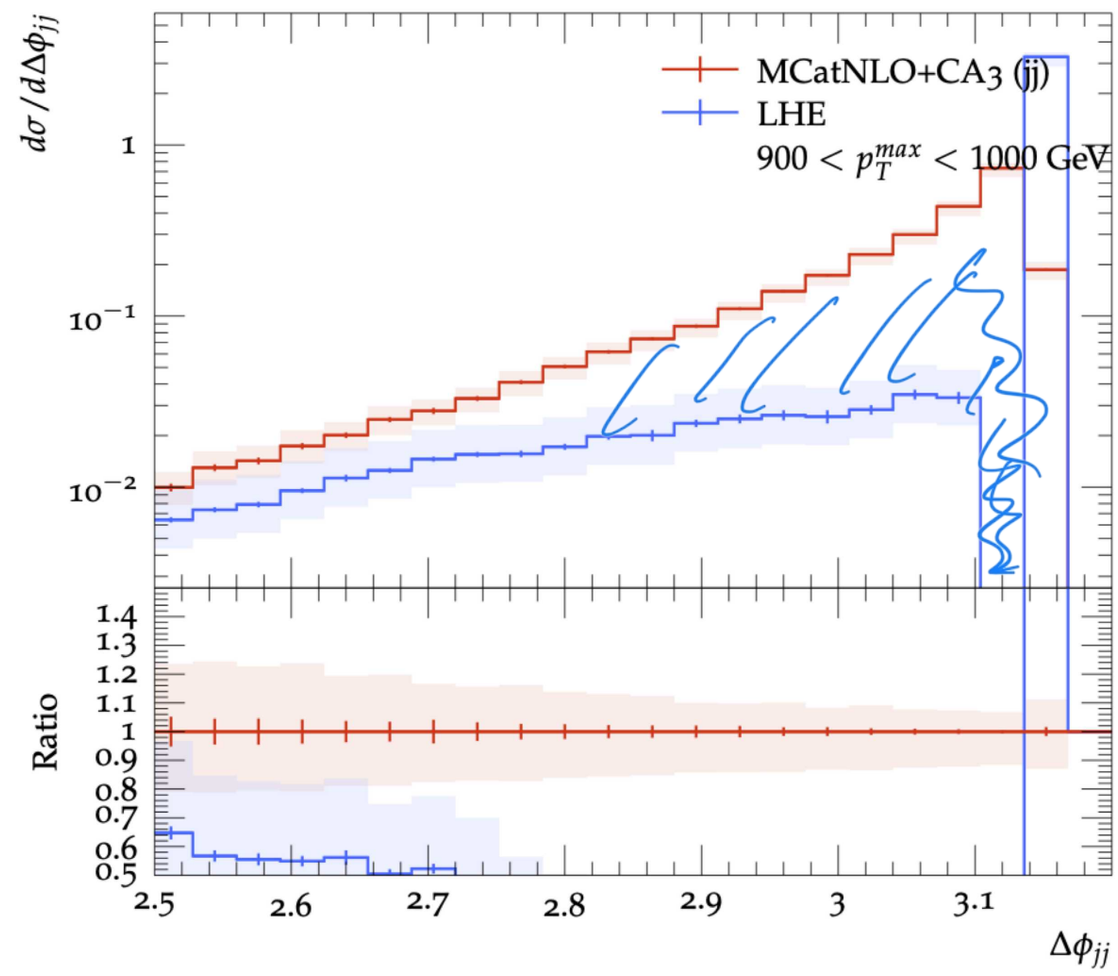


- In back-to-back region
  - for $p_T^{leading} > 200GeV$ sensitive to "soft gluon" k_T kick of ~ 10 GeV

Measurements at large p_T. $\Delta\phi$ back-to-back

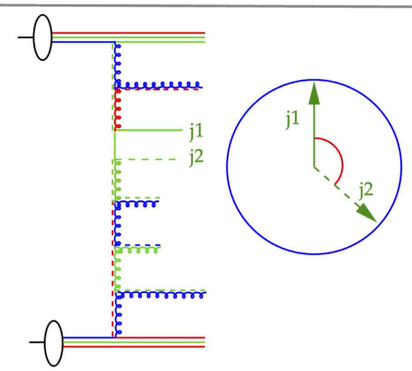


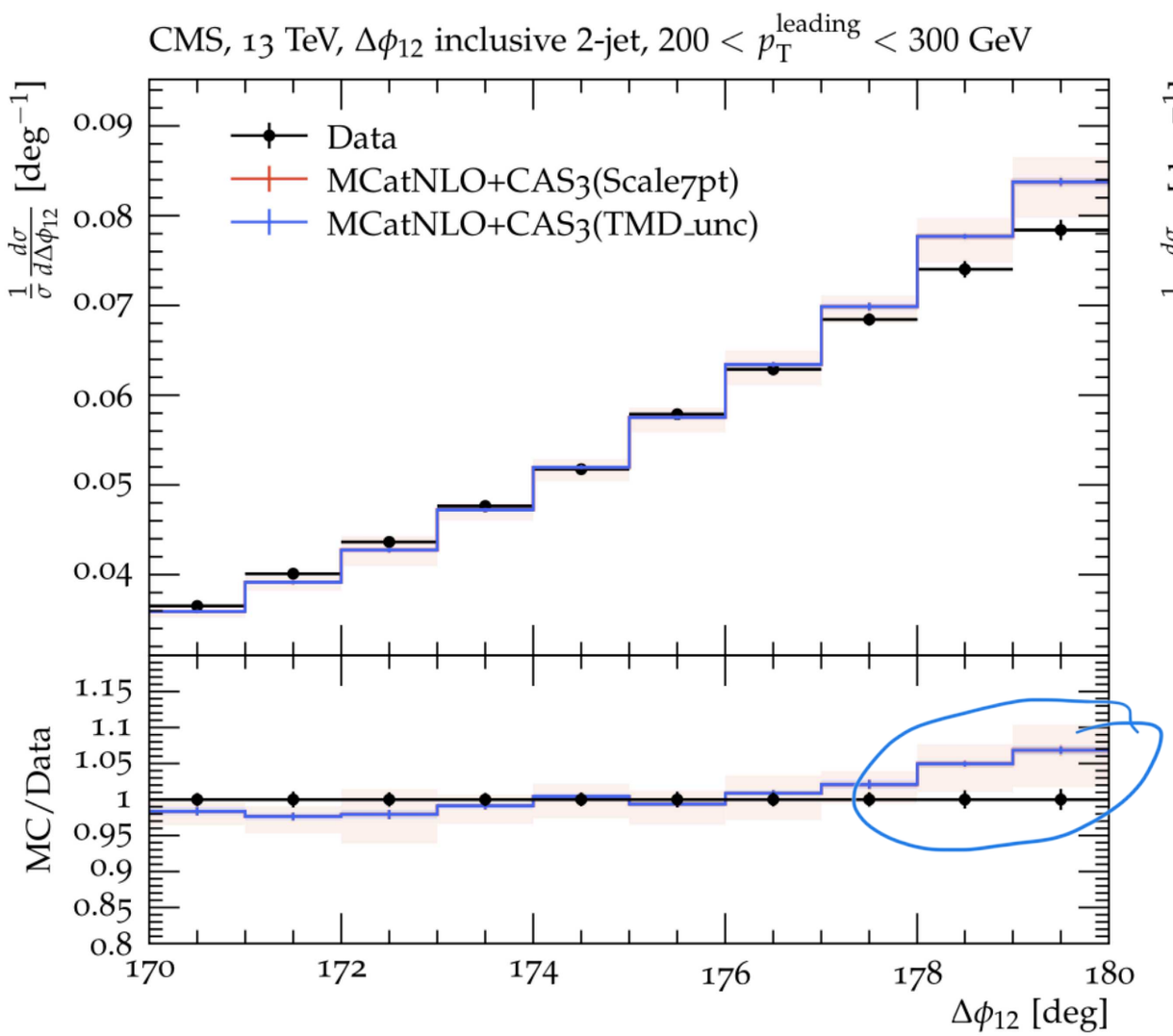


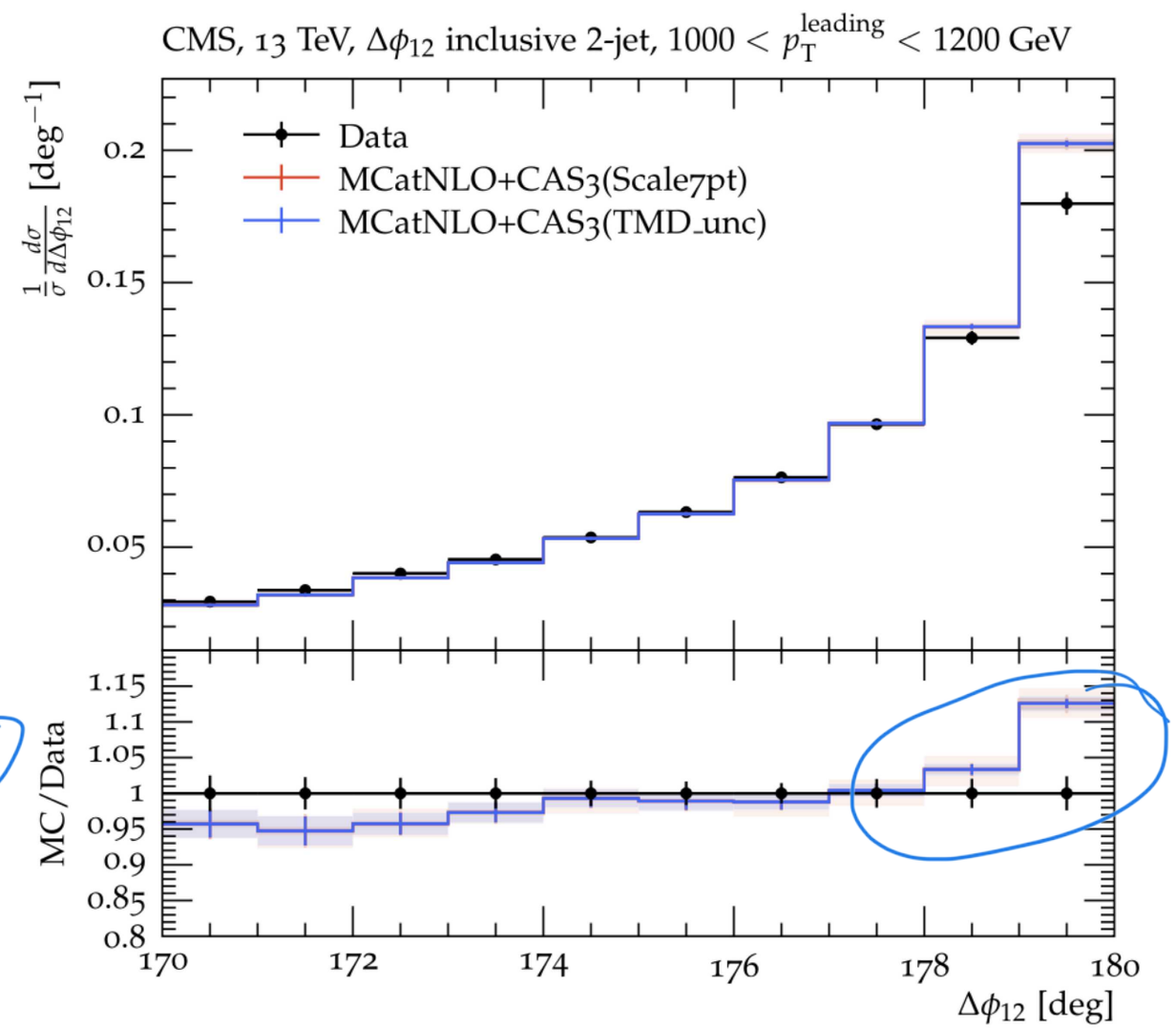


- In back-to-back region
  - for $p_T^{leading} > 900GeV$ sensitive to "soft gluon" k_T kick of ~ 100 GeV

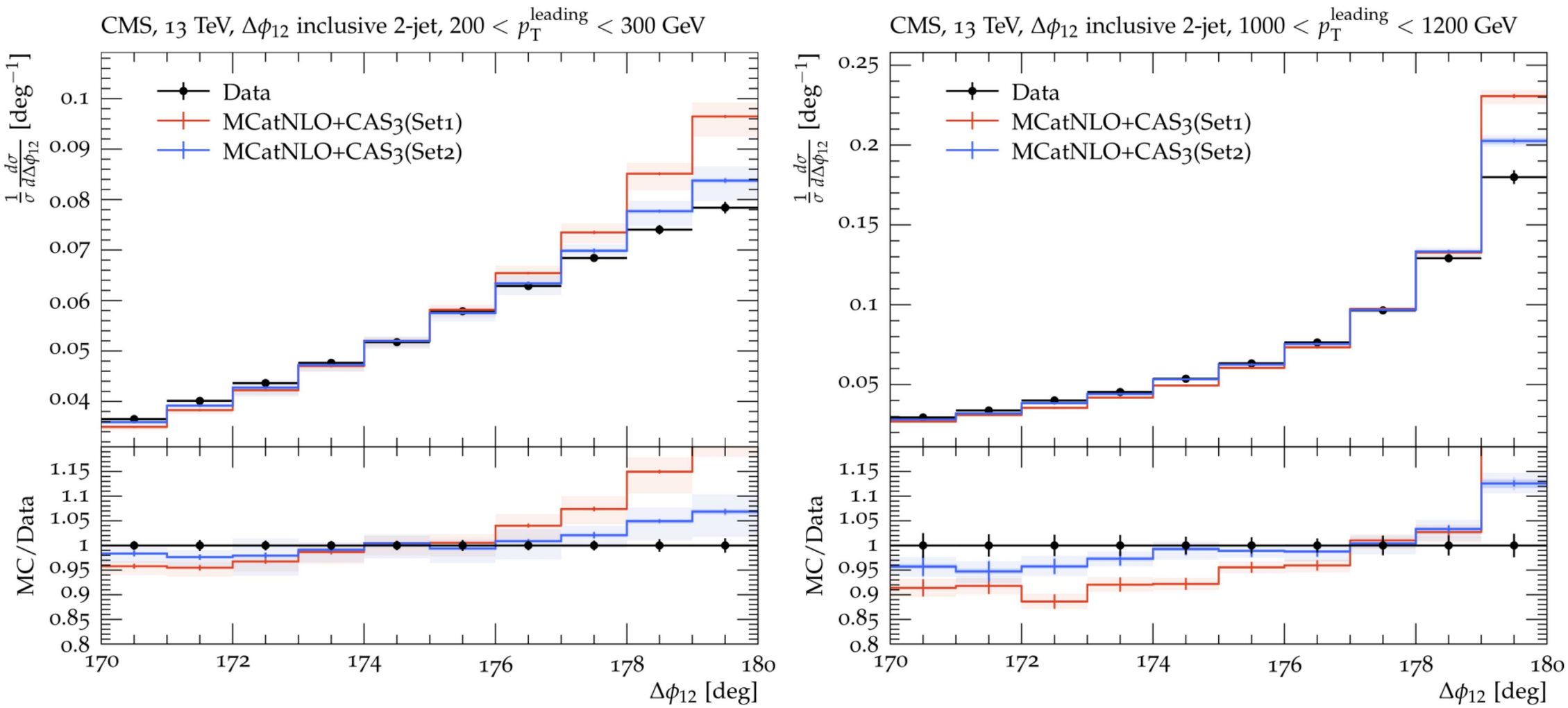
Measurements at large p_T. $\Delta\phi$ back-to-back





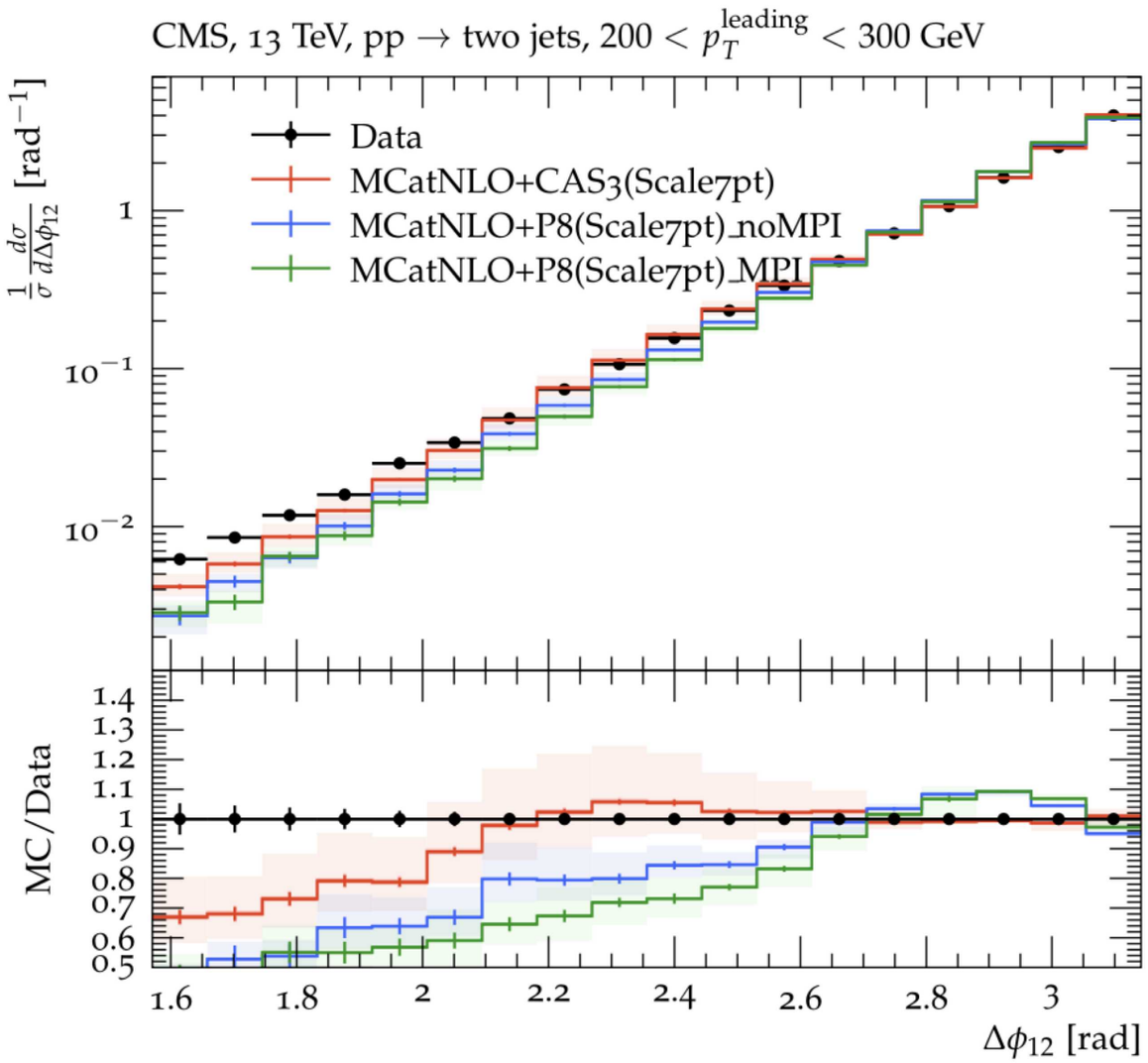


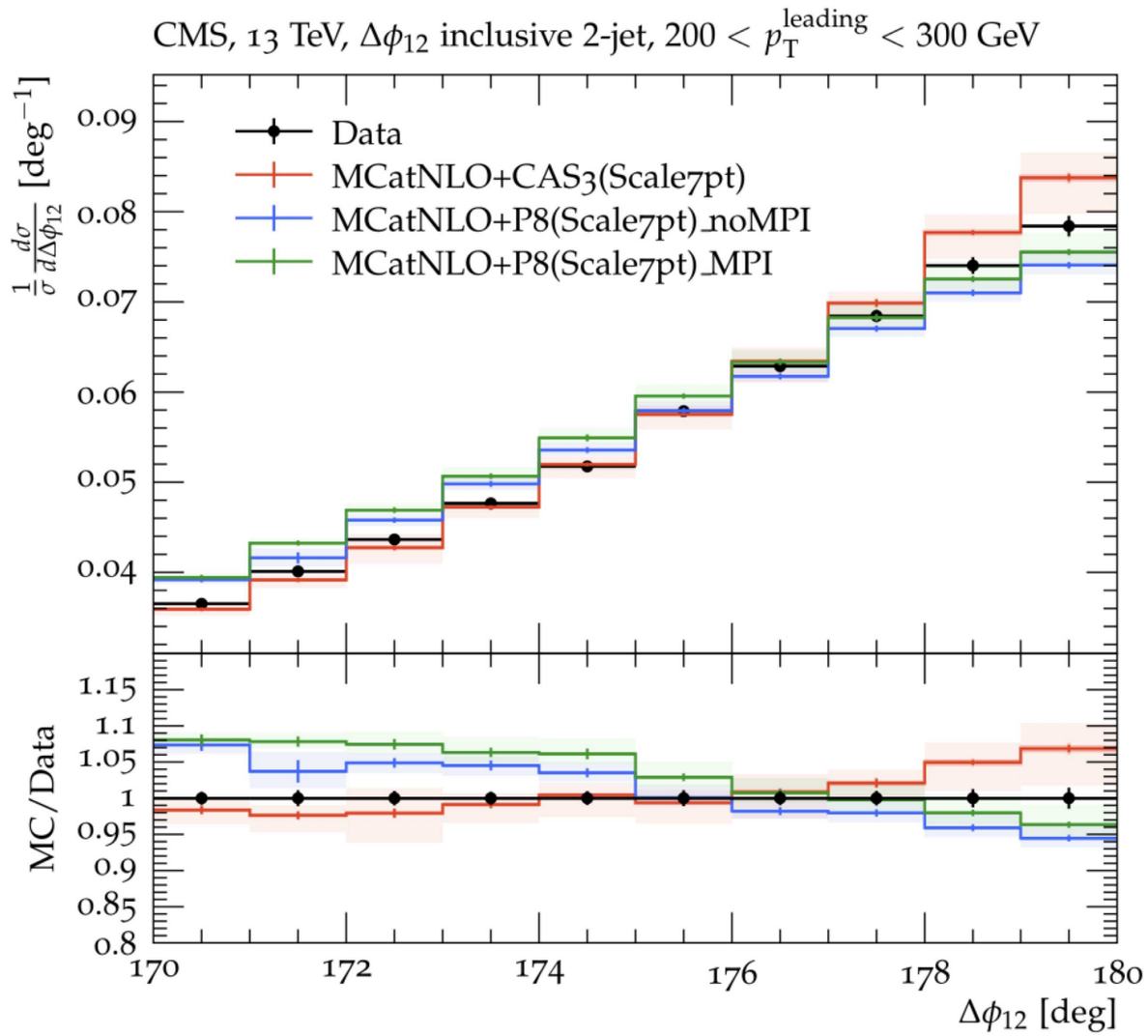
Measurements at large p_T. $\Delta\phi$ back-to-back



- Sensitivity in back-to-back region on differences in “small kt” region

Measurements at large p_T





- differences to standard parton shower approaches are observed
  - details matter

But even this is not the full story...

- factorization breaking in pp → $j_1 j_2 X$

  J. Collins, J.W. Qiu hep-ph 0705.2141

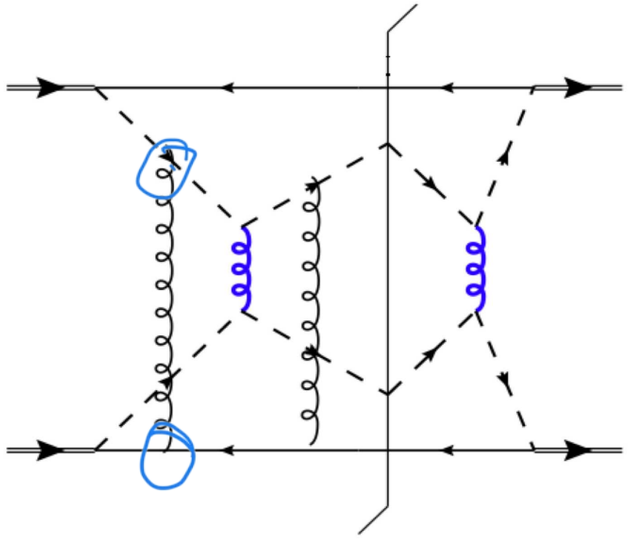
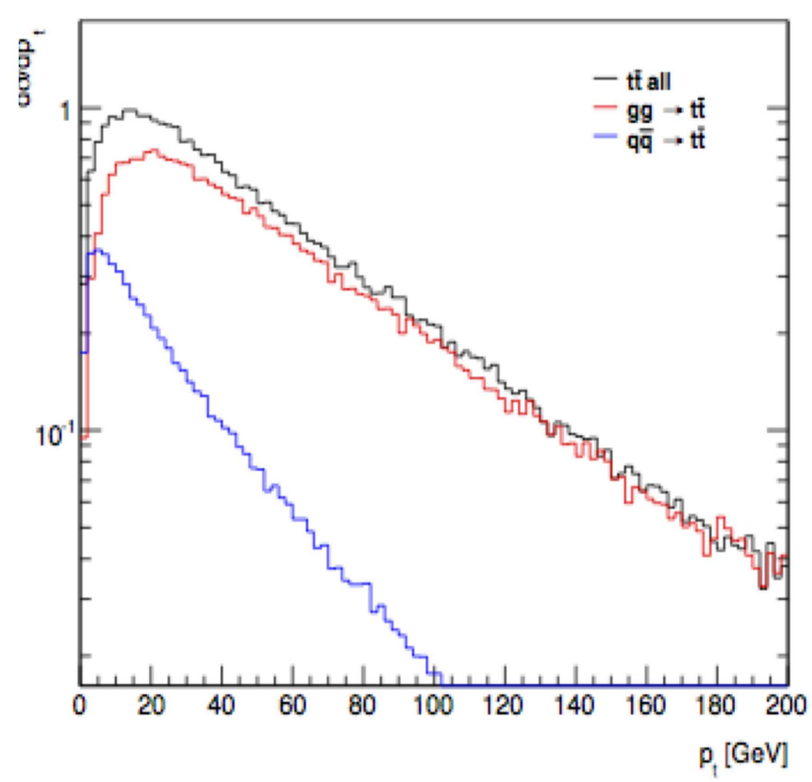


FIG. 8 (color online). The exchange of two extra gluons, as in this graph, will tend to give nonfactorization in unpolarized cross sections.

- factorization breaking also in tt -production at large p_t^{top}

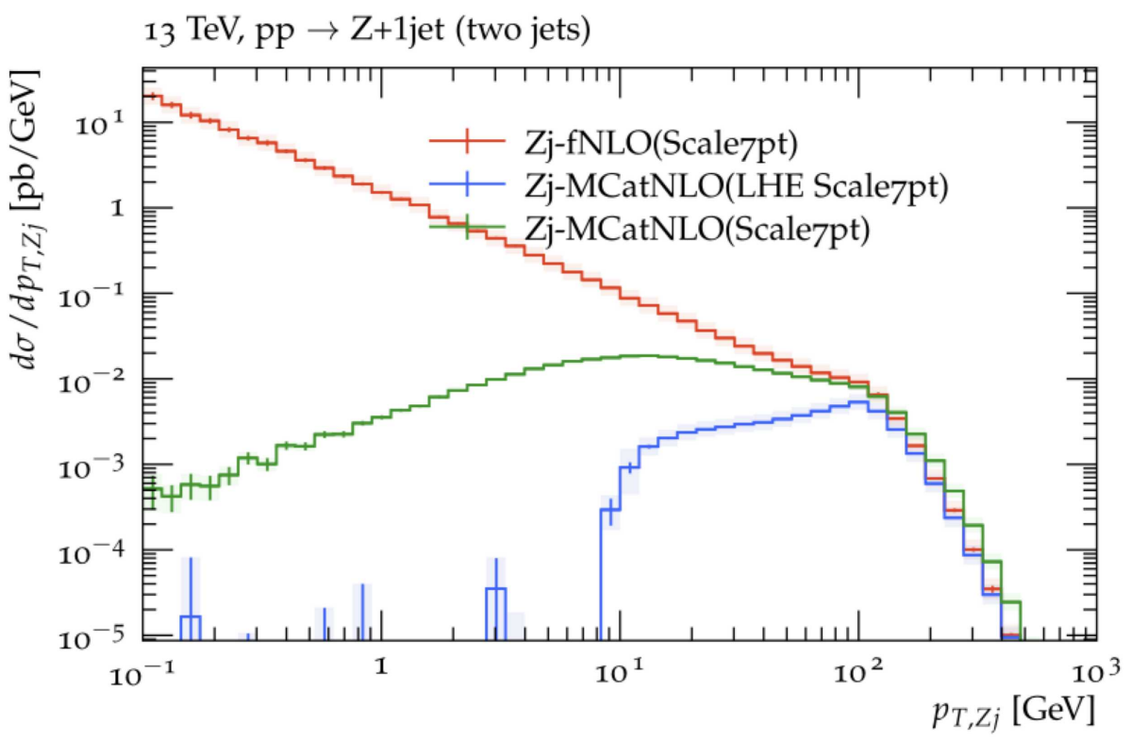
  S. Catani, M. Grazzini, and A. Torre. Transverse-momentum resummation for heavy-quark hadropro- duction. arXiv 1408.4564

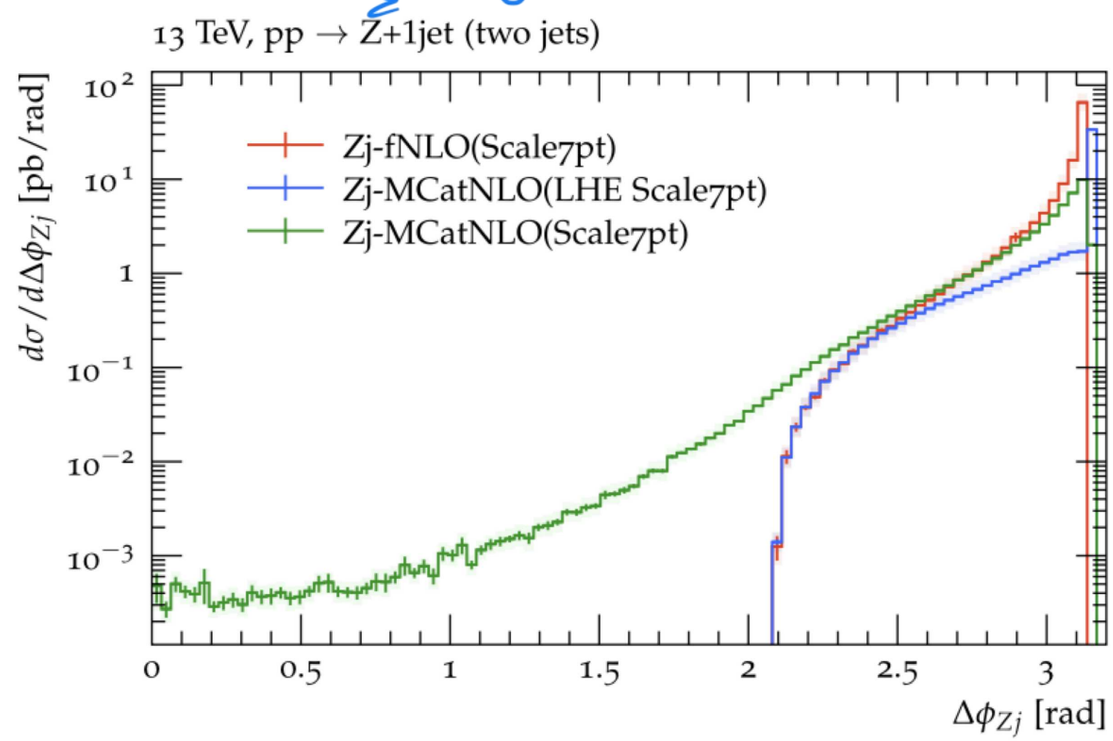


Testing possible factorization breaking

- No real calculations are available
- Compare with experimental measurements
  - and test with scenario, where factorization effects are absent or at least much smaller:
    - compare jj with Zj (in Zj color connections to of initial to final state are reduced, since Z is colorless).

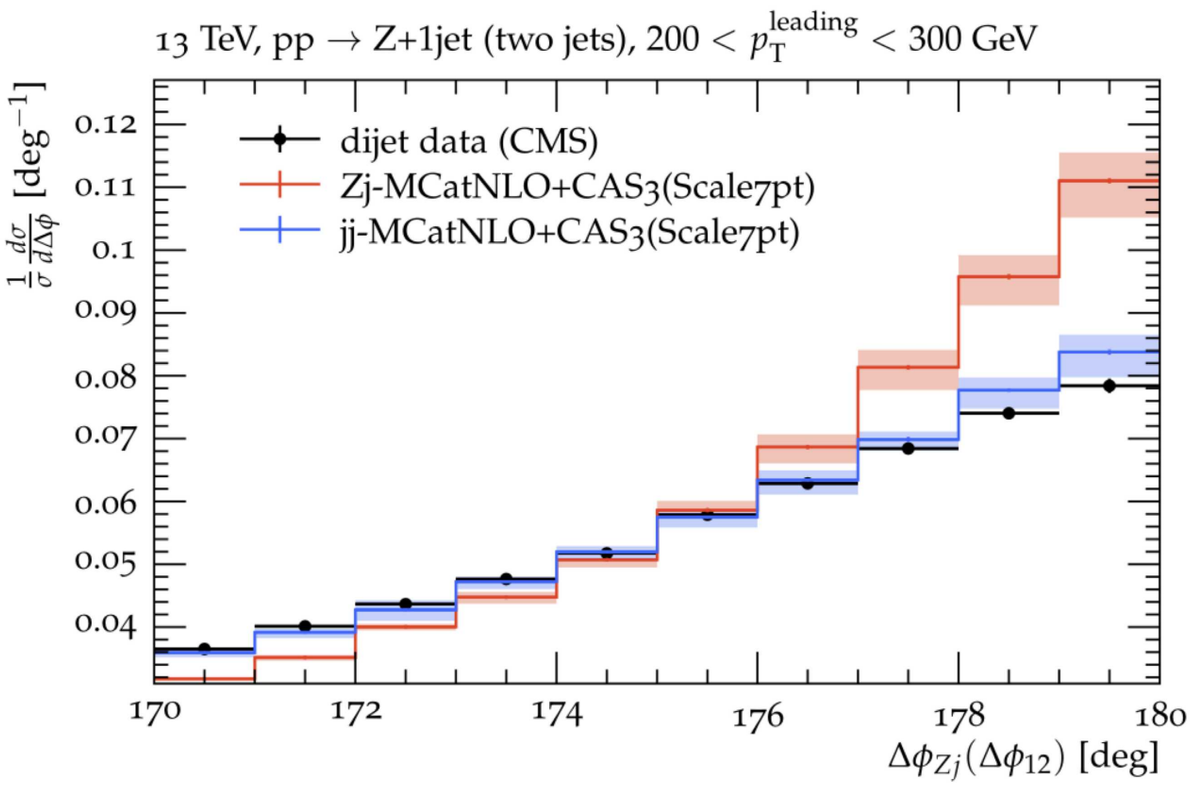
https://arxiv.org/abs/2204.01528

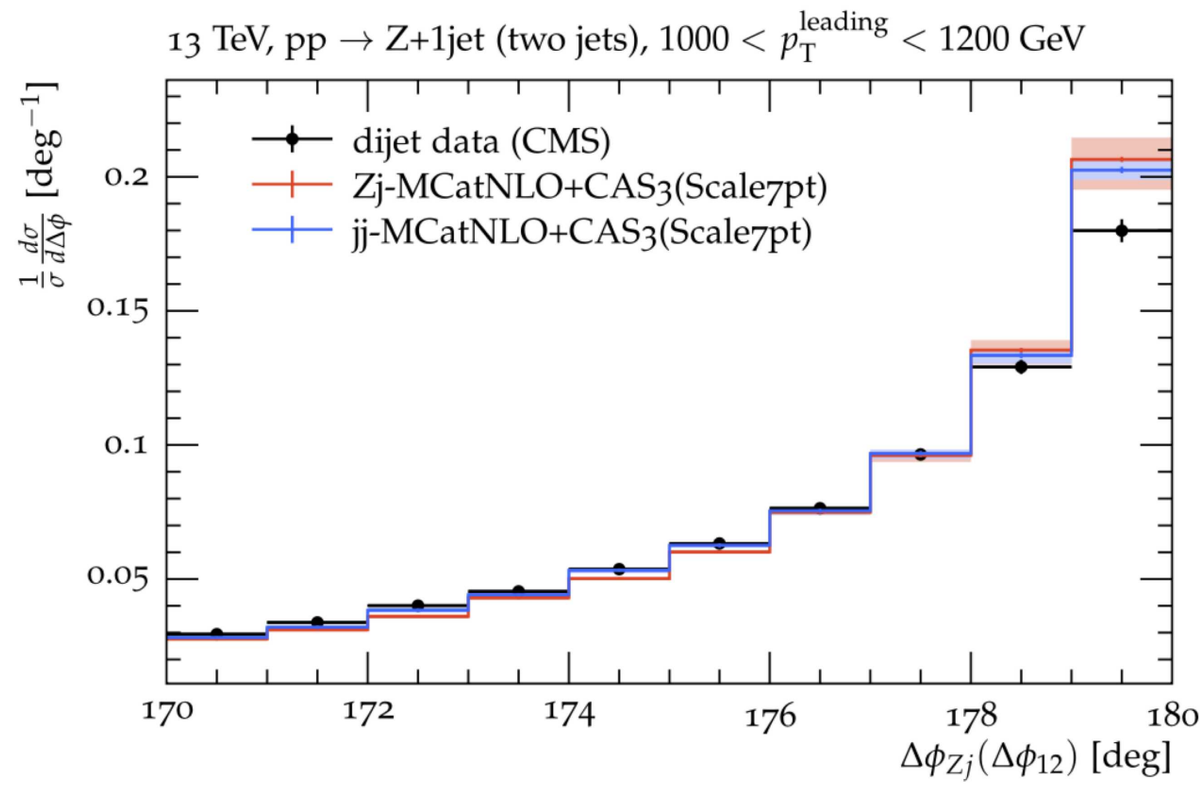




Dijet and Z+j comparisons I

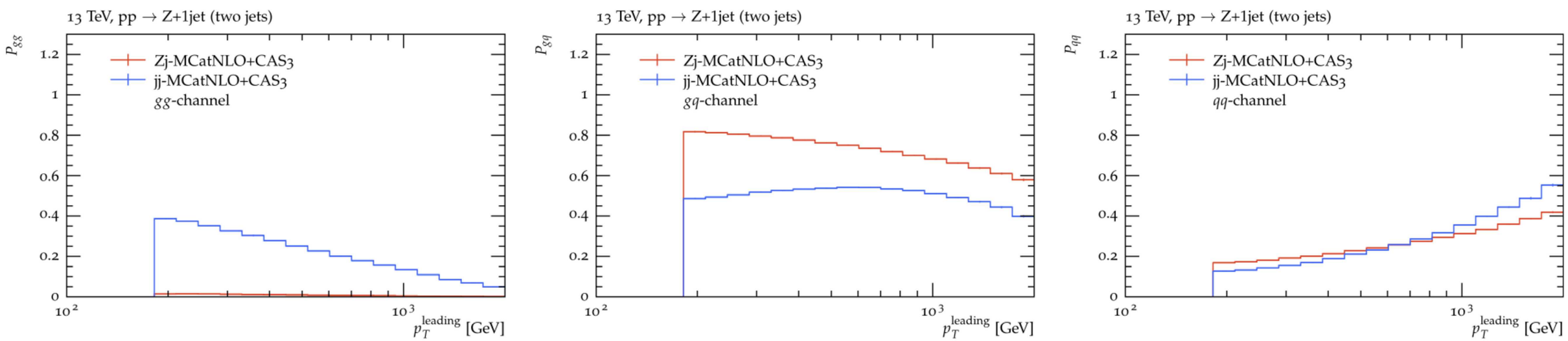
https://arxiv.org/abs/2204.01528





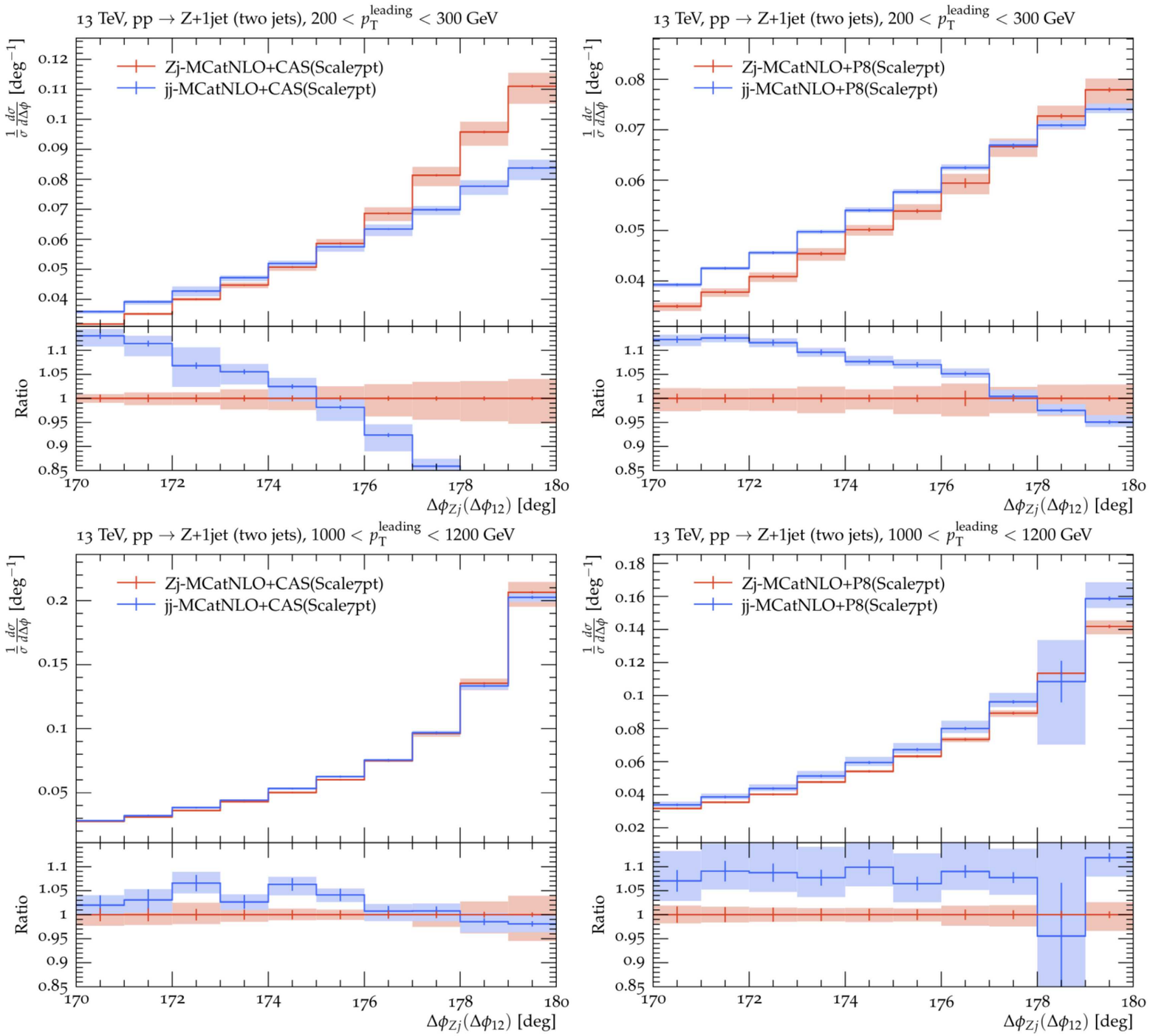
Dijet and Z+j comparisons II

https://arxiv.org/abs/2204.01528



Dijet and Z+j comparisons III

https://arxiv.org/abs/2204.01528



The role of soft gluons

Role of soft gluons in inclusive distributions

- Does evolution equation work with cut z_M ?

$$\int_x^1 dz\, P(z) \to \int_x^{z_m} dz\, P(z)$$

$$\mu^2 \frac{df_a}{d\mu^2} = \int \frac{dz}{z} \frac{\alpha_s}{2\pi} P_{ab}(z)\, f_b\left(\frac{x}{z}, \mu^2\right)$$

$$P_{ab} = \underline{R_{ab}} + \frac{\bar{K}_{ab}}{(1-z)_+} + \underline{D_{ab}\,\delta(1-z)}$$

$$P^{(R)} = R_{ab} + \frac{K_{ab}}{(1-z)}$$

$$\mu^2 \frac{df}{d\mu^2} = \int \frac{dz}{z} \frac{\alpha_s}{2\pi} P \cdot f\left(\frac{x}{z}\right) = \int \frac{dz}{z} \frac{\alpha_s}{2\pi} \left(R_{ab} + \frac{K_{ab}}{(1-z)_+} + D_{ab}\,\delta(1-z) \right) f$$

$$= \int_x^1 \frac{dz}{z} \alpha_s/2\pi \left(R_{ab} f + \frac{f(x/z) - f(x)}{1-z} \right) + D_{ab} f(x)$$

$$= \int_x^{z_m} \frac{dz}{z} \left(R_{ab} \cdot f + \frac{1}{1-z} \cdot f \right)$$

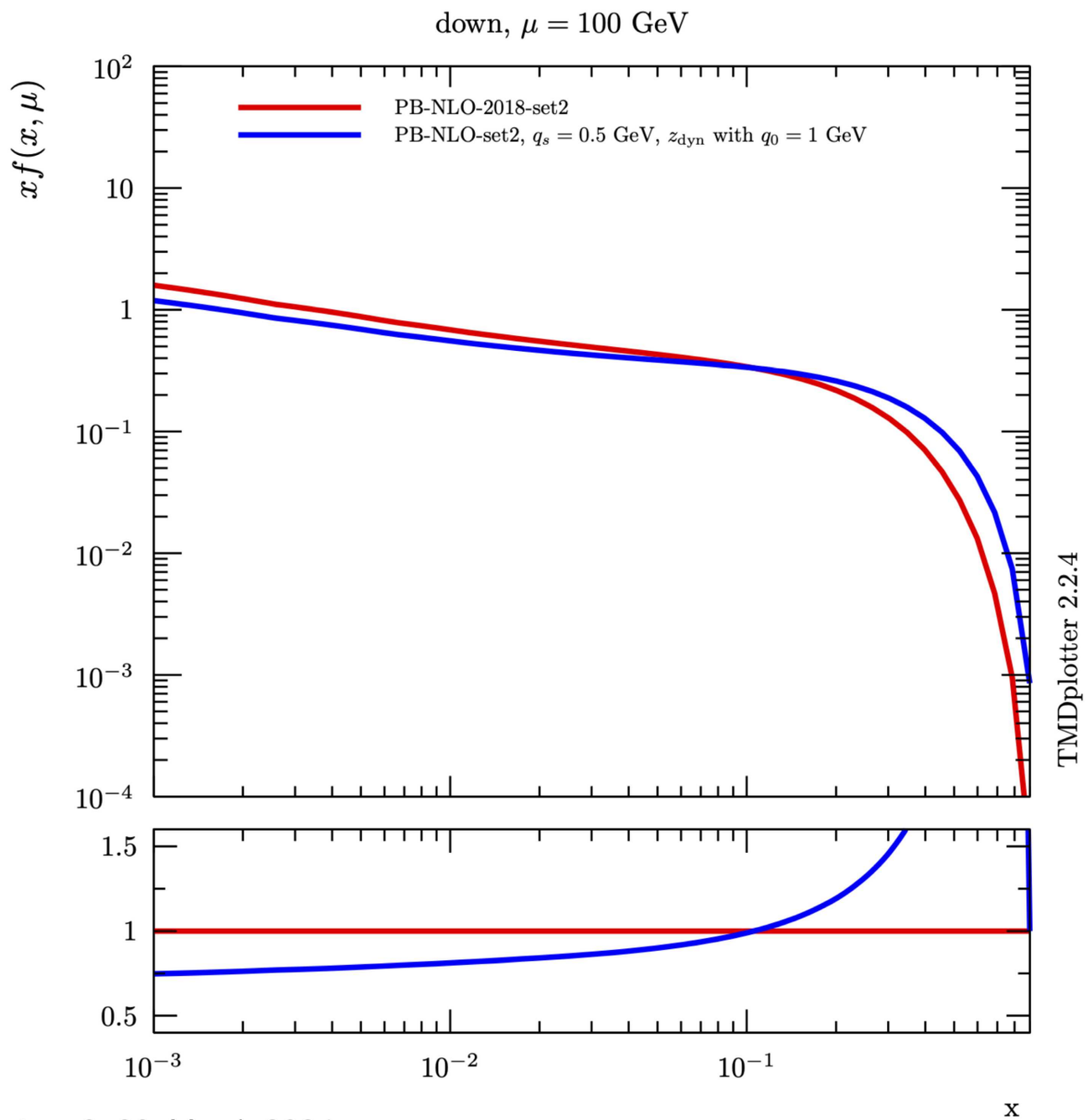
$$+ \int_{z_m}^1 \frac{dz}{z} \left(\cancel{R_{ab} f} + \cancel{\frac{f(\frac{x}{z})}{1-z}} - \cancel{\frac{f(x)}{1-z}} \right) + D_{ab} f(x)$$

$$\boxed{z_m \to 1}$$

Role of soft gluons in inclusive distributions

https://arxiv.org/abs/2312.08655

- Perform evolution with PB method with and without cut $z_{\rm dyn} = 1 - \frac{q_{t\ \rm cut}}{\mu'}$



q_t

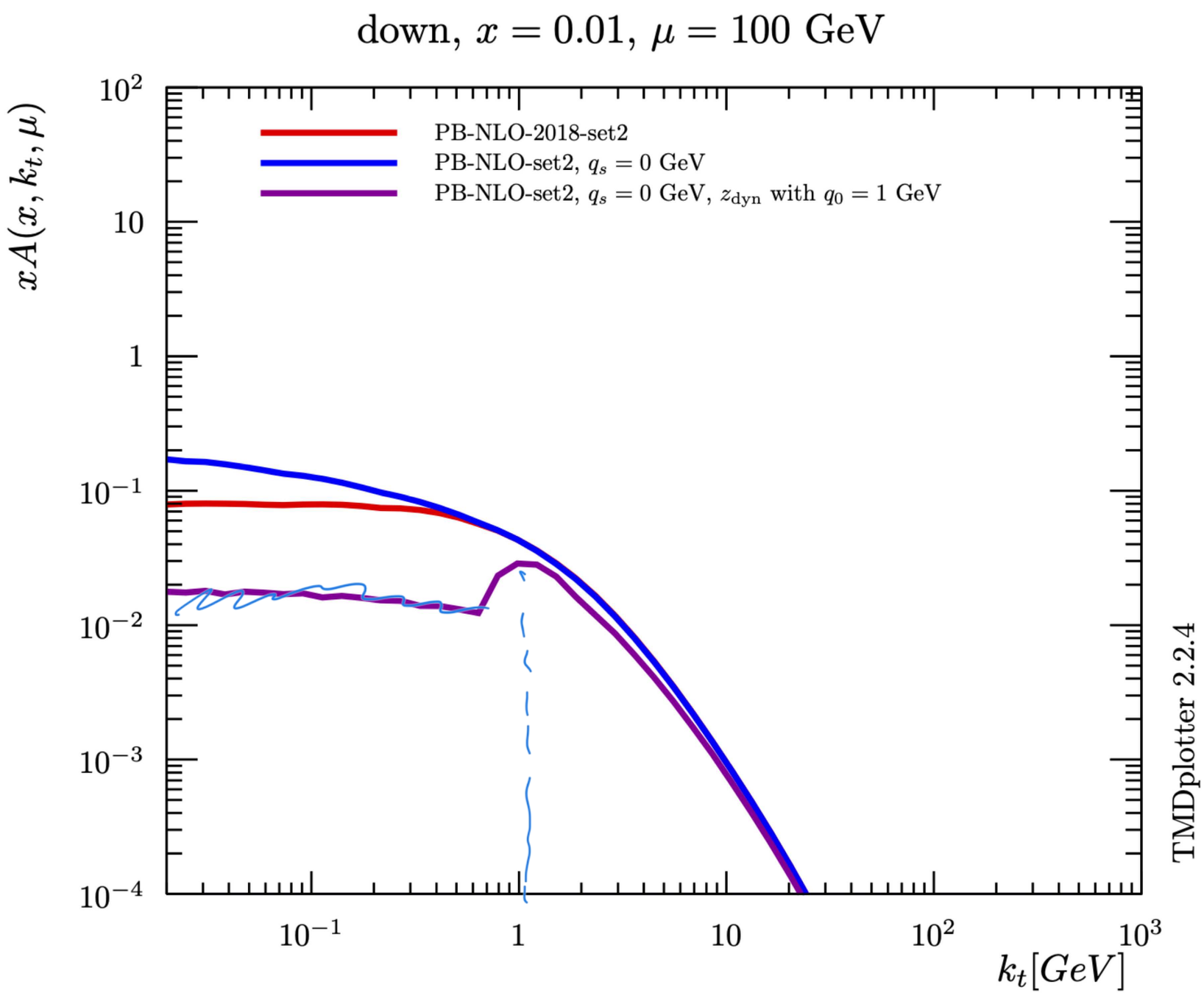
$q_t > 1$

$z < 1 - \frac{q_t}{\mu'}$

Role of soft gluons in TMD distributions

https://arxiv.org/abs/2312.08655

- Perform evolution with PB method with and without cut $z_{\rm dyn} = 1 - \frac{q_{t\ \rm cut}}{\mu'}$



Cutoff and connection to cross section

https://arxiv.org/abs/2309.11802

- For simplicity consider again DIS cross section (at NLO)

$$F_2(x,Q^2) = x\sum_{q,\bar{q}} e_q^2 \int_x^1 \frac{d\xi}{\xi} q(\xi,Q^2)\left[\delta\left(1-\frac{x}{\xi}\right) + \frac{\alpha_s}{2\pi} C_{\overline{\mathrm{MS}}}\left(\frac{x}{\xi}\right) + \cdots\right]$$

- with:

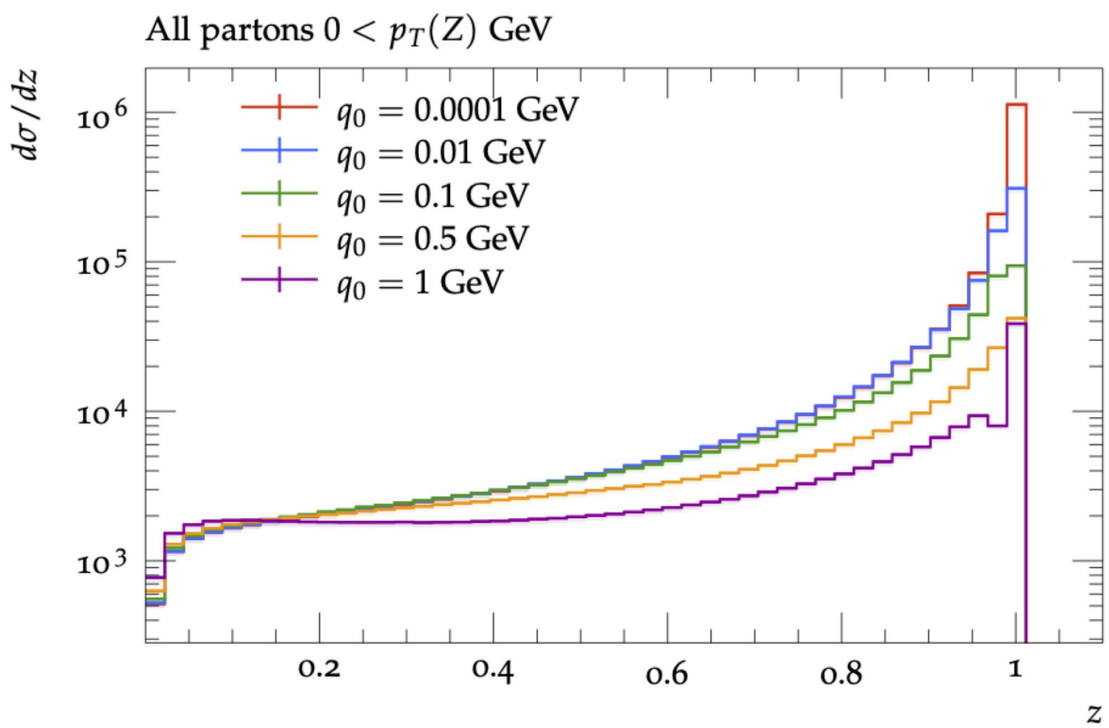
$$\begin{aligned} C_q^{\overline{\mathrm{MS}}}(z) &= C_F\left[2\left(\frac{\log(1-z)}{1-z}\right)_+ - \frac{3}{2}\left(\frac{1}{1-z}\right)_+ - (1+z)\log(1-z)\right. \\ &\quad \left. -\frac{1+z^2}{1-z}\log z + 3 + 2z - \left(\frac{\pi^2}{3}+\frac{9}{2}\right)\delta(1-z)\right] \end{aligned}$$

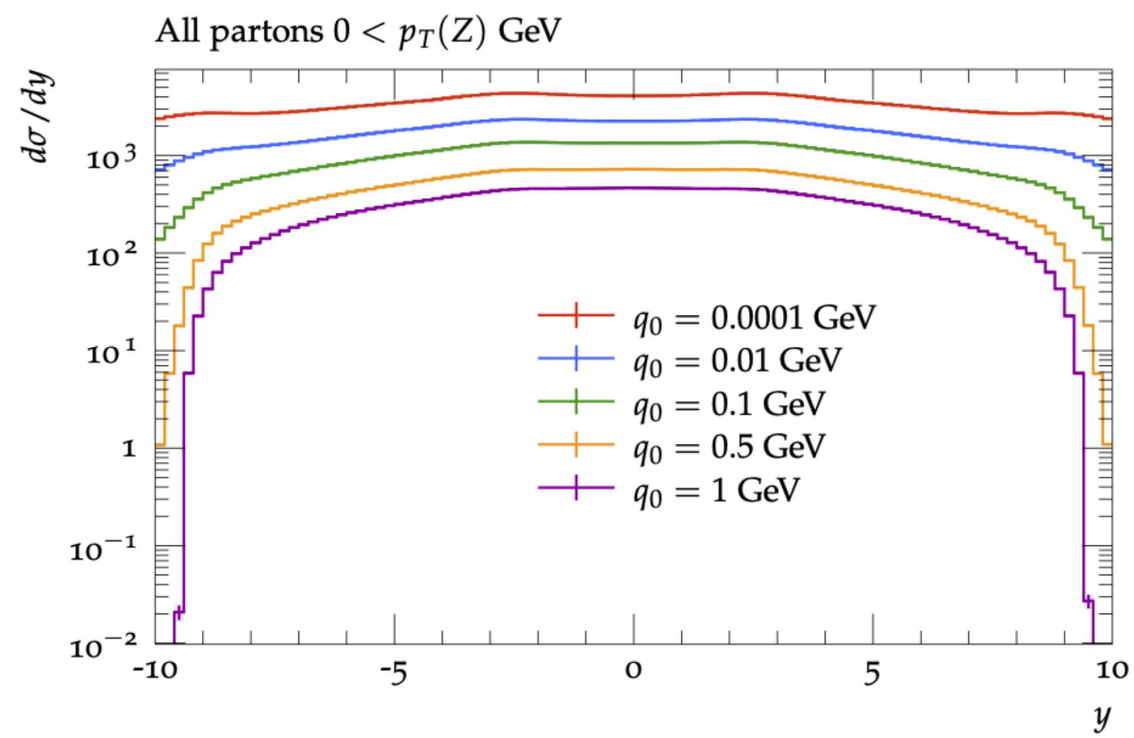
- to ensure proper cancellation of terms in parton density as well as in cross section, $z_M \to 1$
- see similar study in Frixione, Webber https://arxiv.org/abs/2309.15587

Soft gluons in Parton Shower

https://arxiv.org/abs/2309.11802

- With PB-TMD parton shower study effect of $z_{\rm dyn} = 1 - \frac{q_{t\ \rm cut}}{\mu'}$



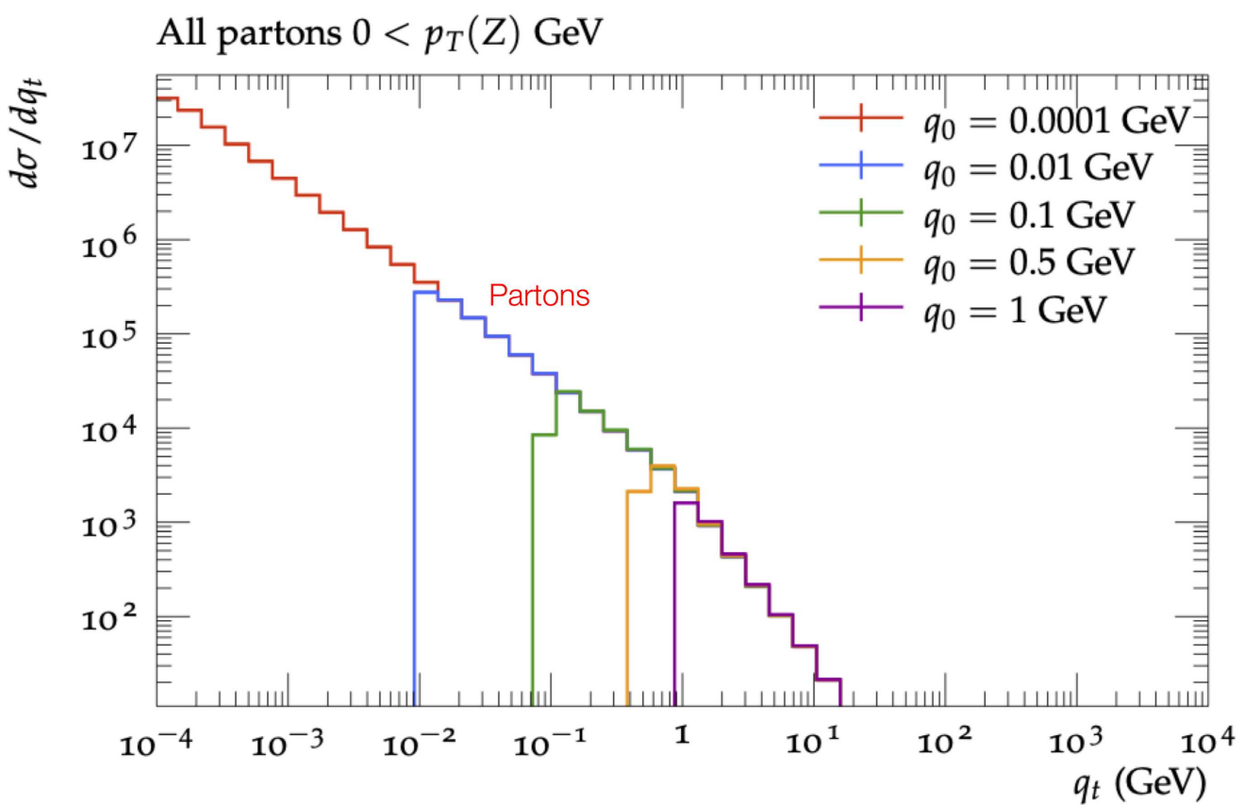


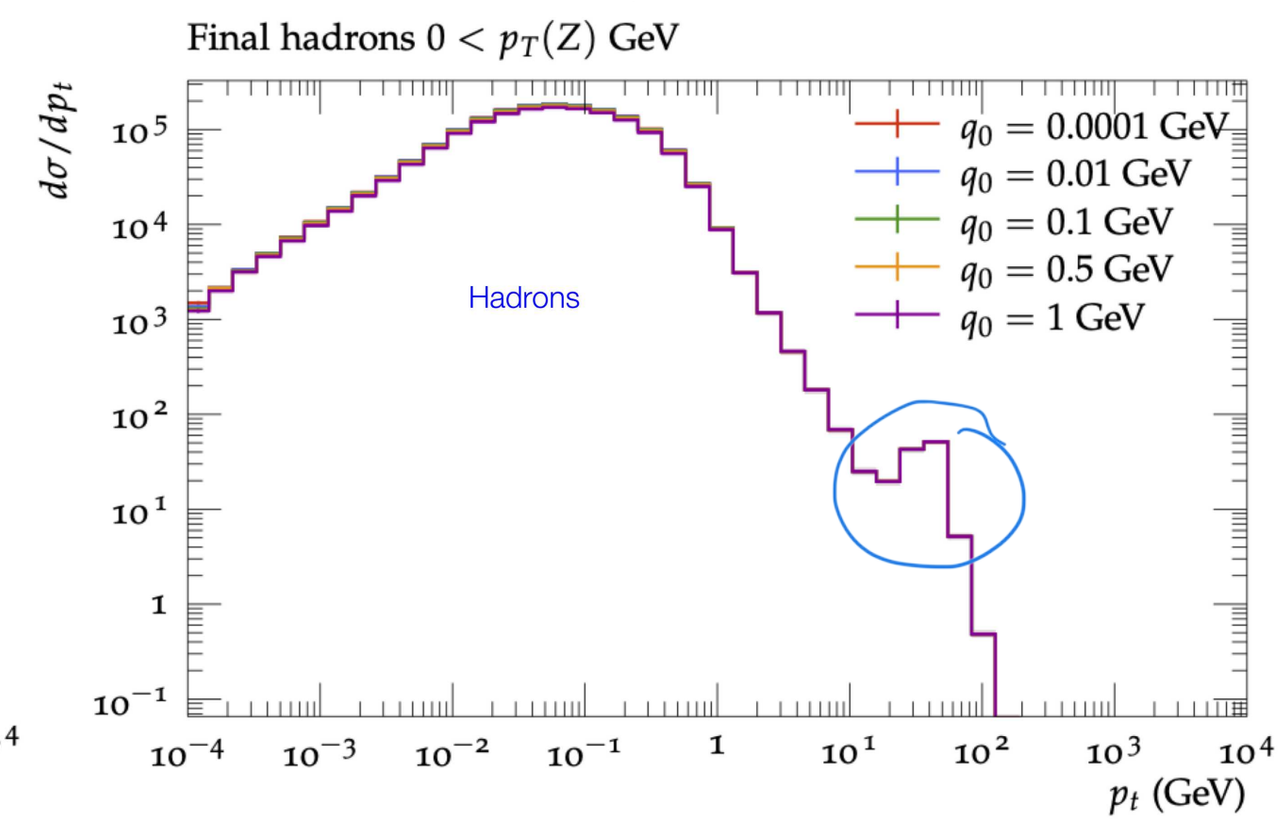
- Huge effect on soft partons
  - they are important for inclusive distributions: pdfs and TMDs

Soft gluons in Parton Shower

https://arxiv.org/abs/2309.11802

- With PB-TMD parton shower study effect of $z_{\rm dyn} = 1 - \frac{q_{t\ \rm cut}}{\mu'}$



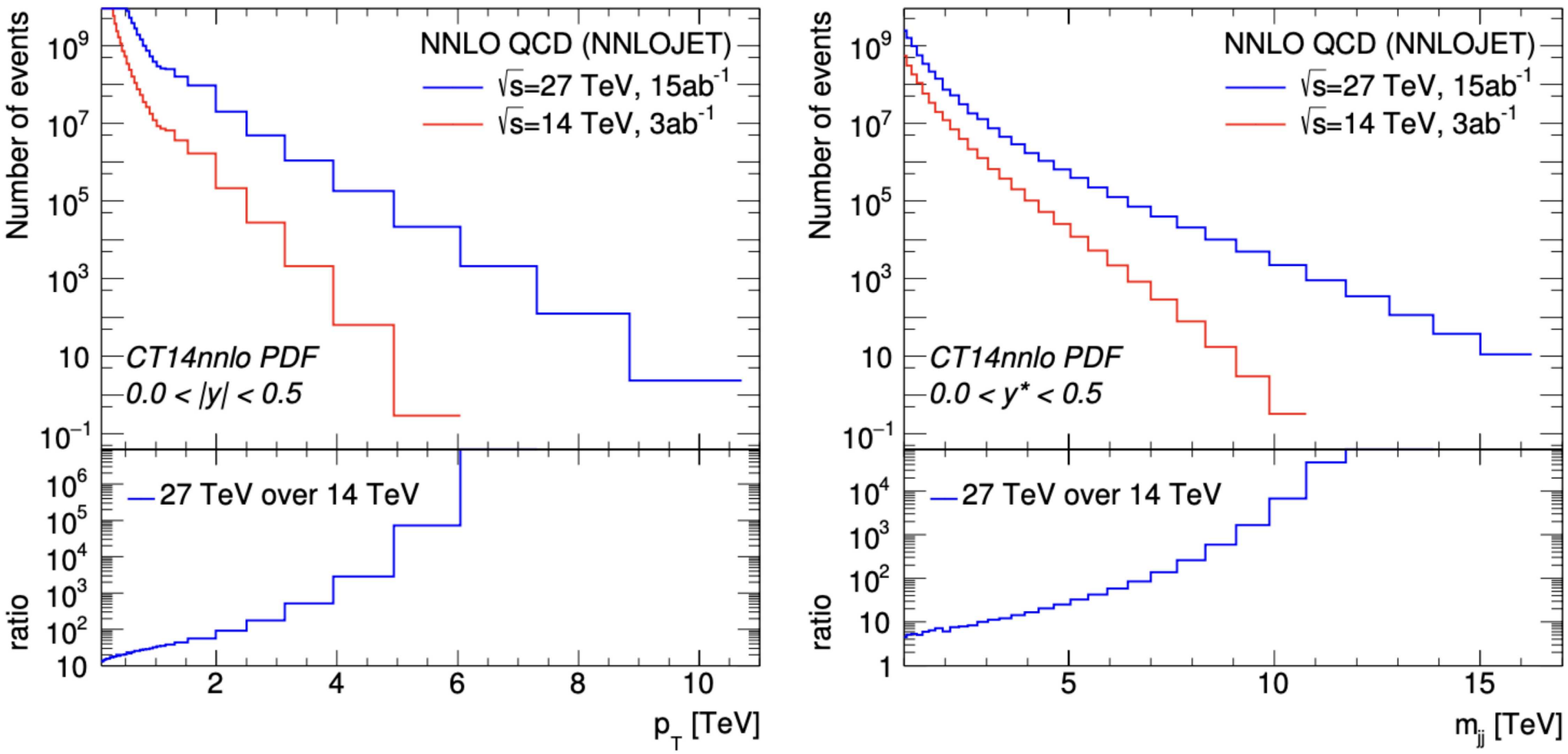


- Huge effect on soft partons
  - they are important for inclusive distributions: pdfs and TMDs
  - see DY q_T spectra
- in Lund string fragmentation these soft partons do not change hadron spectra
  - no issue for hadrons or jets from parton shower !

Expectations at high luminosity

Expectations at highest luminosity: jets I

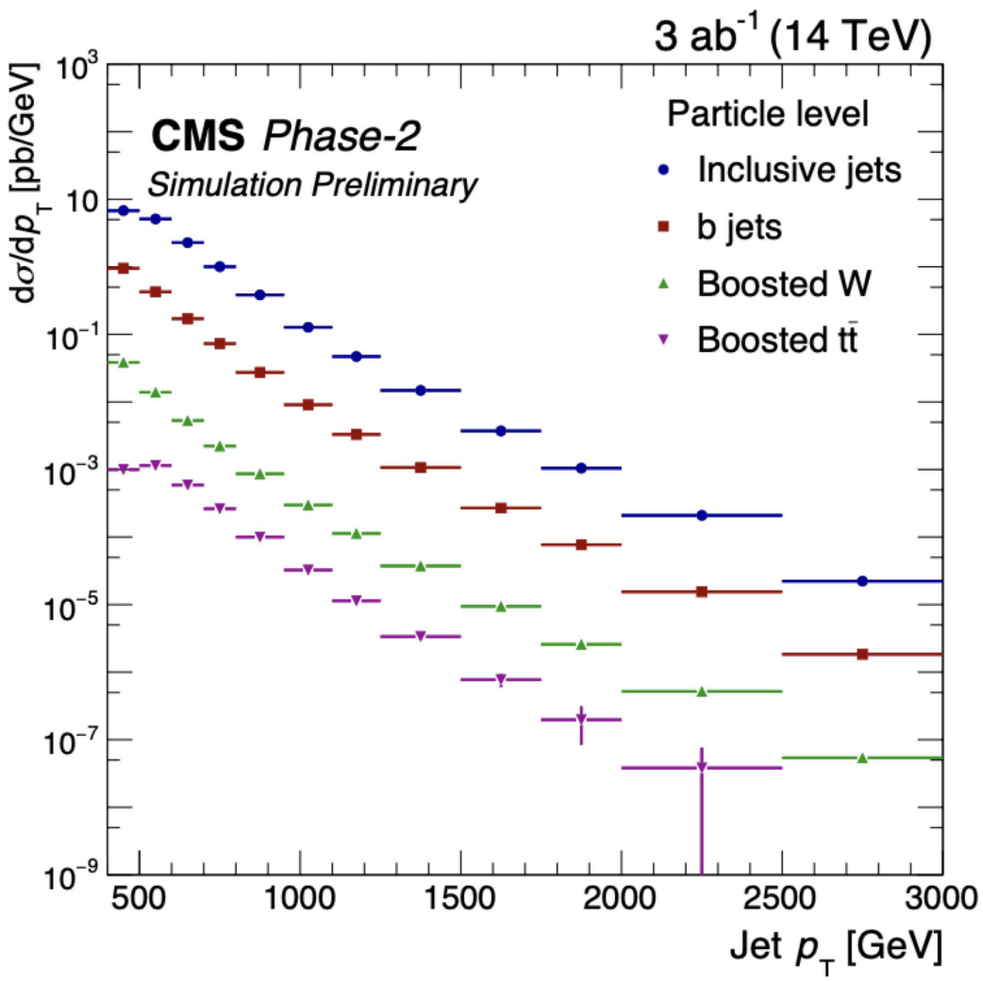
https://arxiv.org/abs/1902.04070

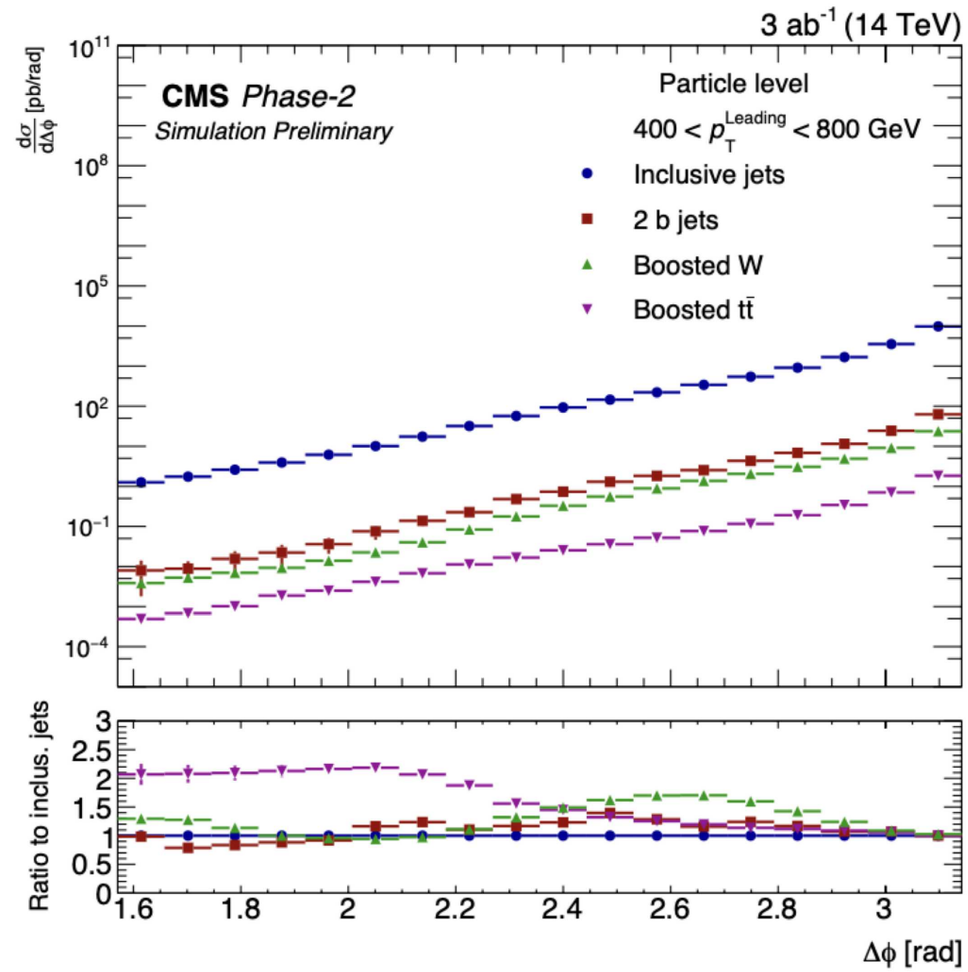


- At highest luminosity (13 TeV, L=3ab)
  - jets with p_T up to 4 TeV can be observed
  - djijet masses up to 6 TeV (~ ½ of $\sqrt{s}$)

Expectations at highest luminosity: jets II

https://arxiv.org/abs/1902.04070

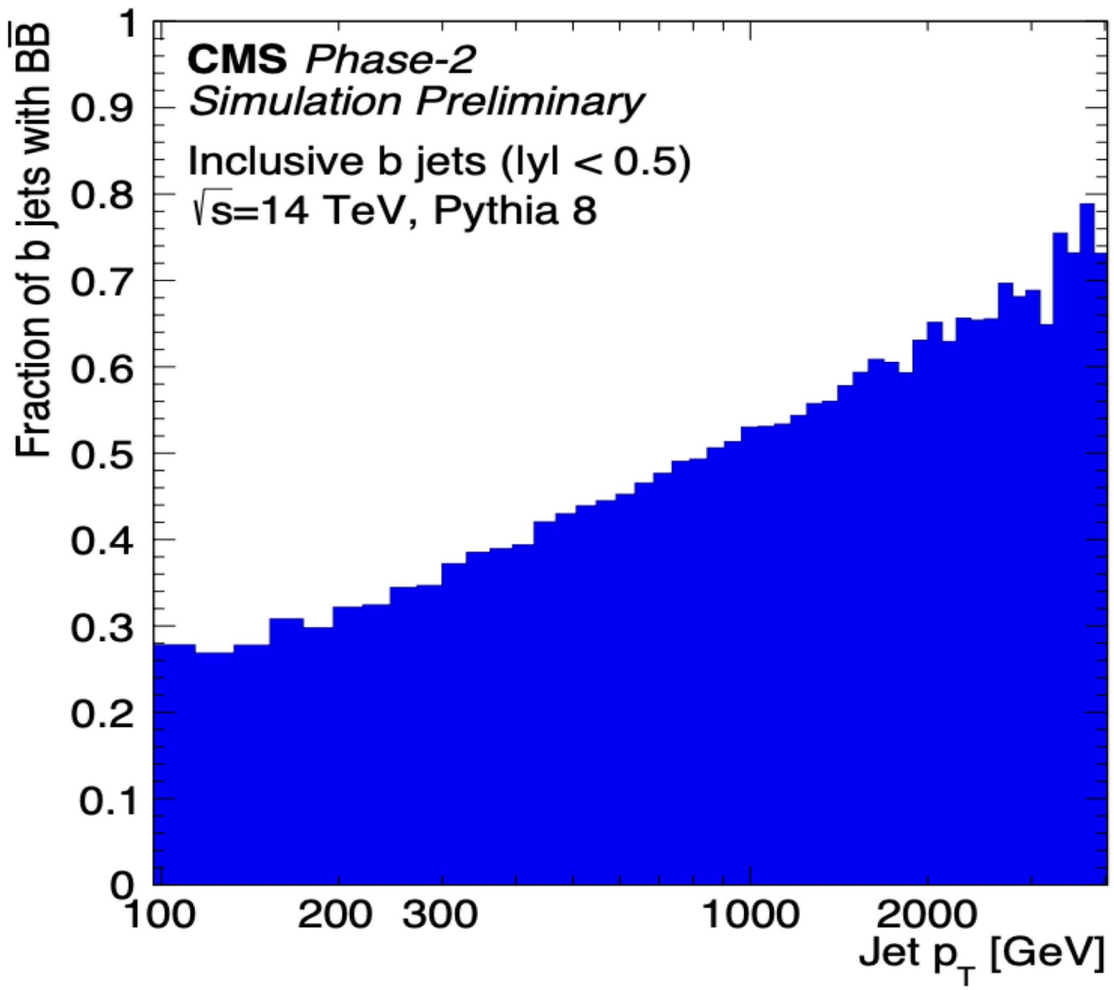


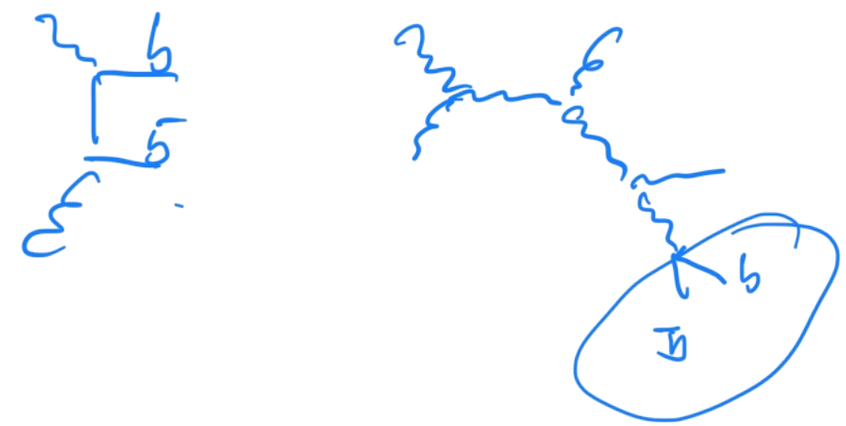


- At highest luminosity (13 TeV, L=3ab)
  - p_T distribution for different flavors (even for boosted W and top) are similar
    - masses play no role
    - $\Delta\phi$ shows differences at smaller angles, due to different production mechanism

Expectations at highest luminosity: jets III

https://arxiv.org/abs/1902.04070





- At highest luminosity (13 TeV, L=3ab)
  - b-jets come mainly from $g \to b\bar{b}$
    - look for pairs of B-mesons inside jet:
  - at high p_T fraction of pairs of B-mesons is large > 70 %
    - b-jet comes mainly from gluon splitting