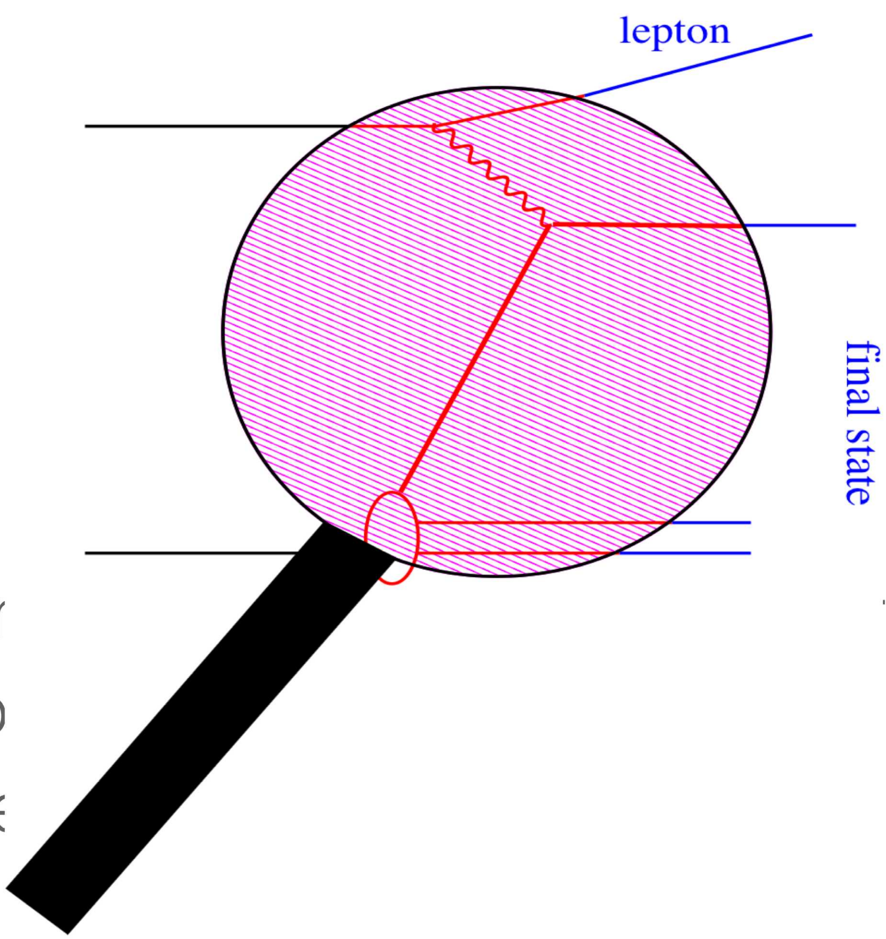
QCD at the Extremes

From ep scattering
to
pp scattering

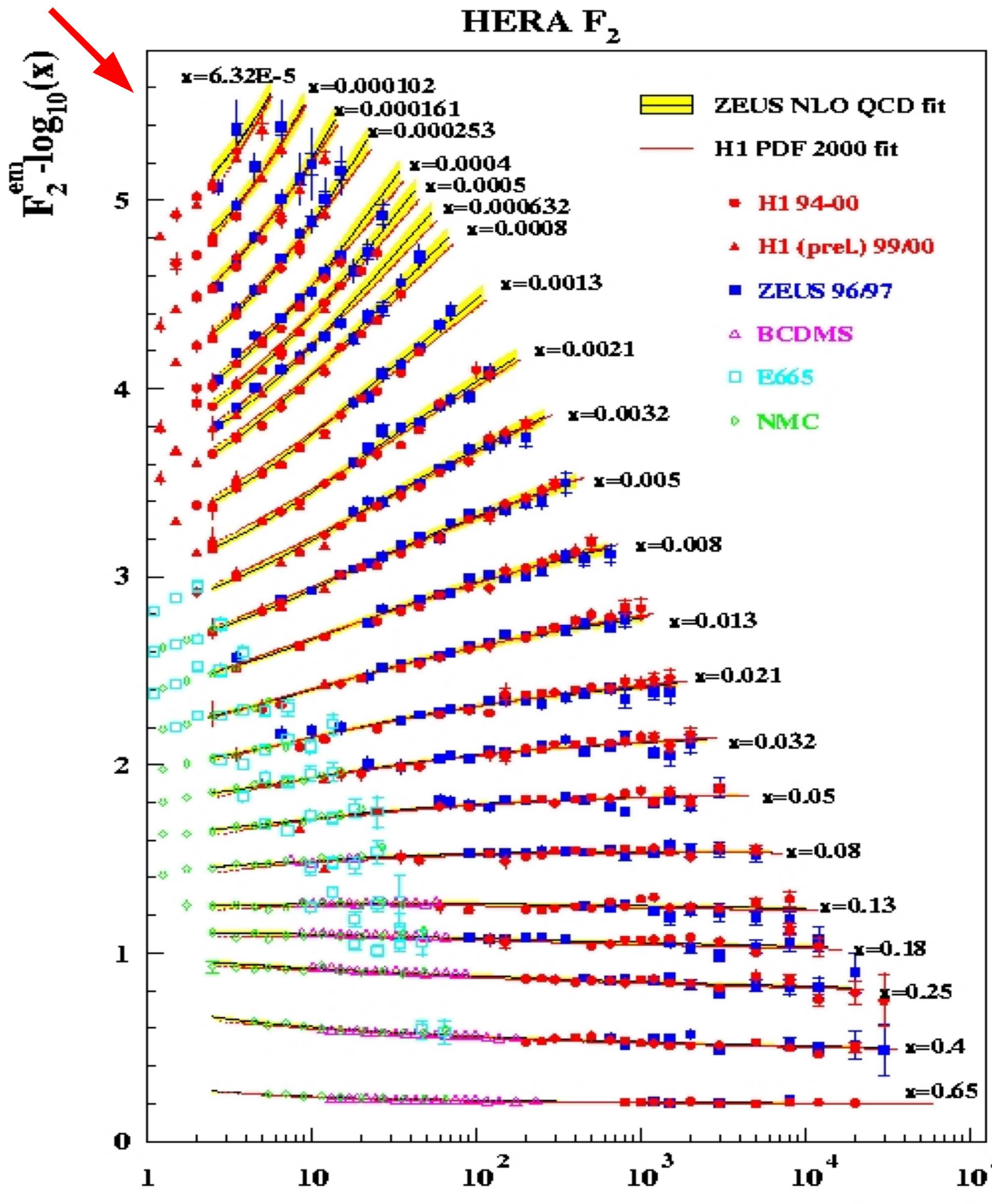
Lepton Hadron scattering

- Deep Inelastic Scattering is a incoherent sum of $e^+ q \to e + q$

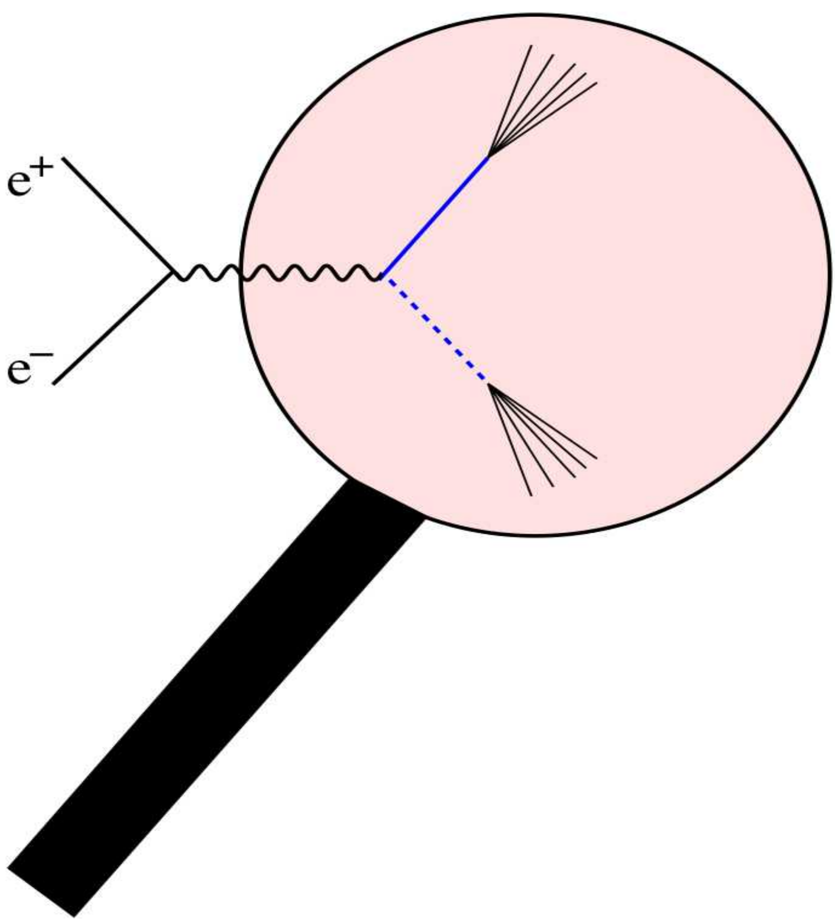
$$\sigma(e^+ p \to e^+ X) = \sum_i f_i(x, Q^2) \sigma(e^+ q_i \to e^+ q_i)$$

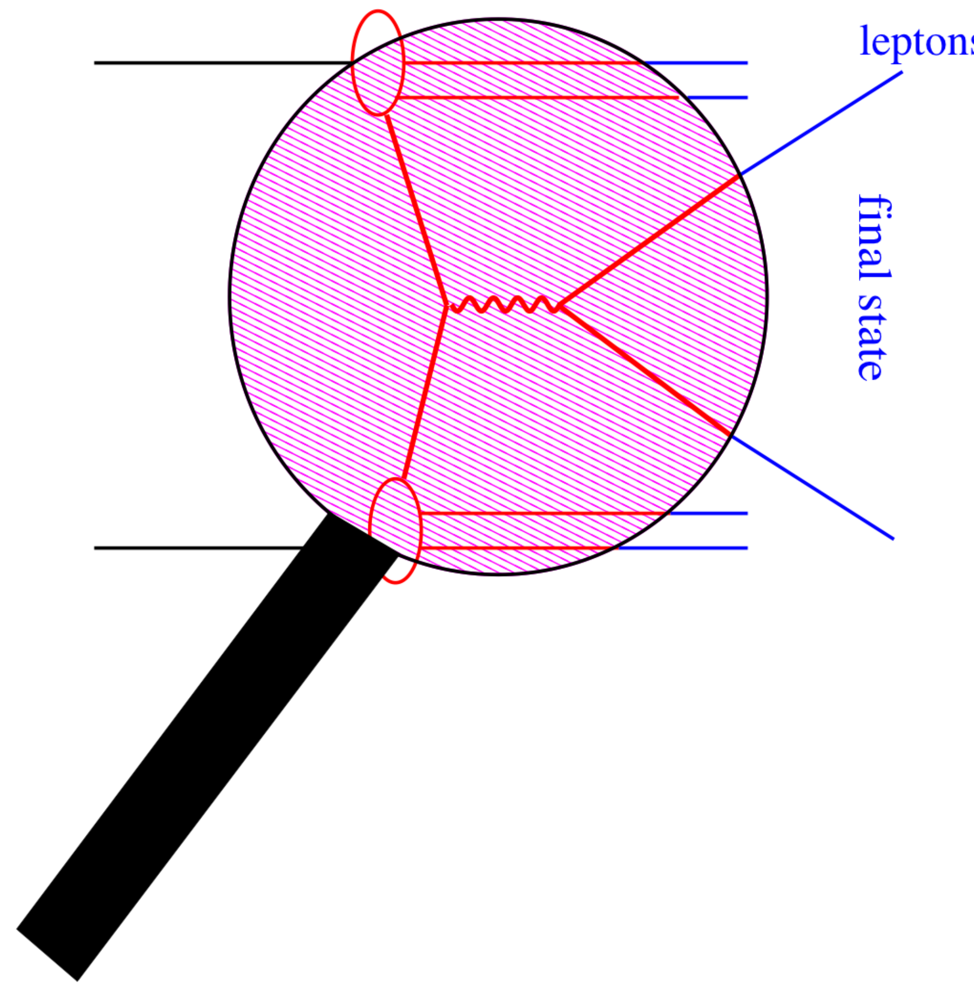


- or ... ied
  b
- ne



Rotating the diagrams





$$\sigma(e^+e^- \to q\bar{q}) = 3\frac{4\pi\alpha^2}{3s}e_q^2$$

$$\sigma(q\bar{q} \to l^+l^-) = \frac{4\pi\alpha^2}{3\times 3s}e_q^2$$

Massive muon pairs

VOLUME 25, NUMBER 21 PHYSICAL REVIEW LETTERS 23 NOVEMBER 1970

Observation of Massive Muon Pairs in Hadron Collisions*

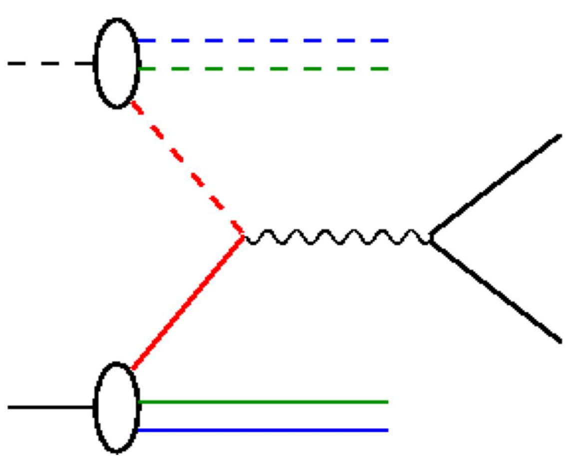
J. H. Christenson, G. S. Hicks, L. M. Lederman, P. J. Limon, and B. G. Pope
Columbia University, New York, New York 10027, and Brookhaven National Laboratory, Upton, New York 11973

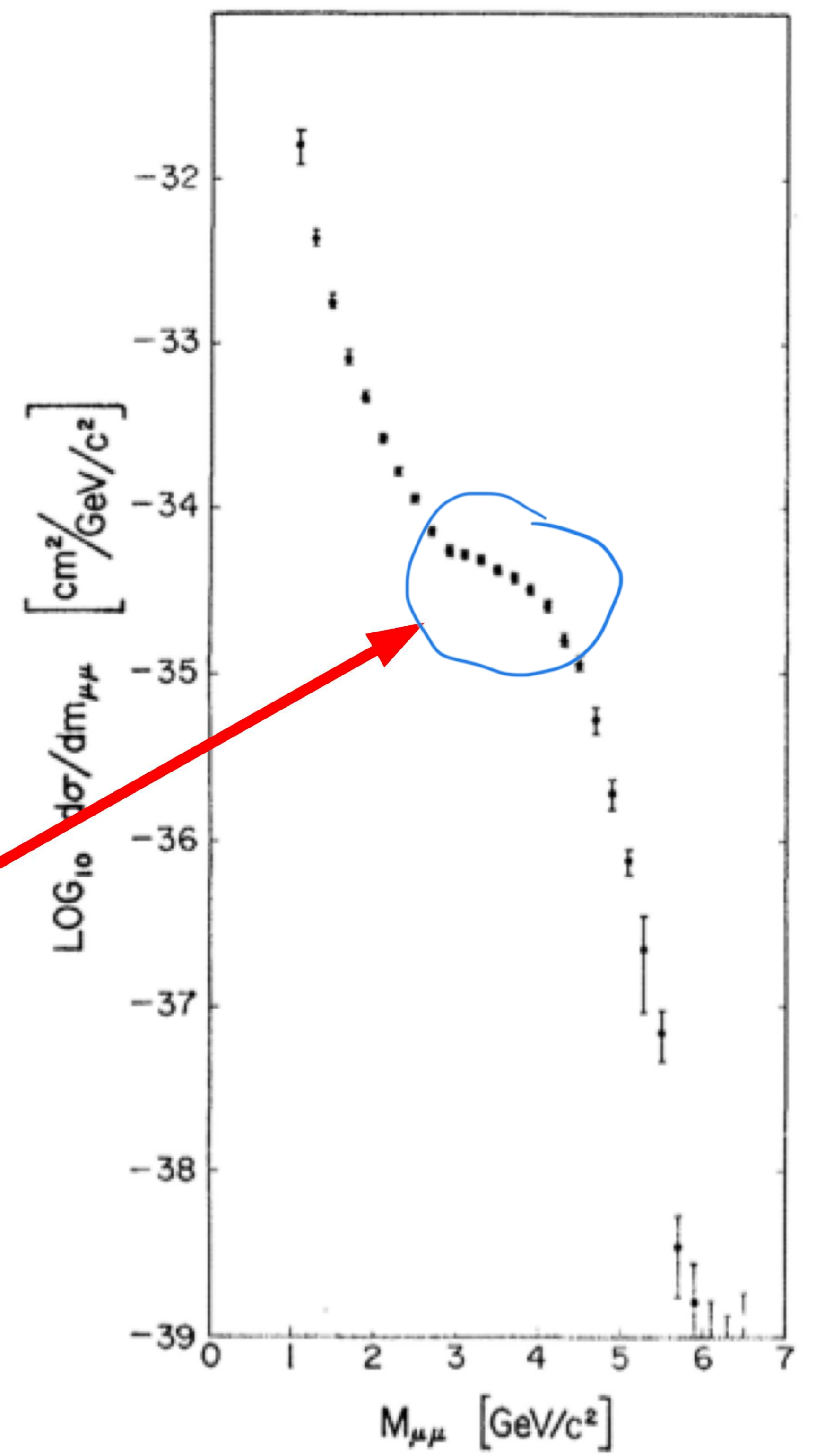
and

E. Zavattini
CERN Laboratory, Geneva, Switzerland

(Received 8 September 1970)

Muon pairs in the mass range $1 < m_{\mu\mu} < 6.7$ GeV/c^2 have been observed in collisions of high-energy protons with uranium nuclei. At an incident energy of 29 GeV, the cross section varies smoothly as $d\sigma/dm_{\mu\mu} \approx 10^{-32}/m_{\mu\mu}{}^5$ cm^2 (GeV/c)$^{-2}$ and exhibits no resonant structure. The total cross section increases by a factor of 5 as the proton energy rises from 22 to 29.5 GeV.





Search for a new weak boson ... not found,

But there were J/psi in the data

Observation of J/psi

VOLUME 33, NUMBER 23 PHYSICAL REVIEW LETTERS 2 DECEMBER 1974

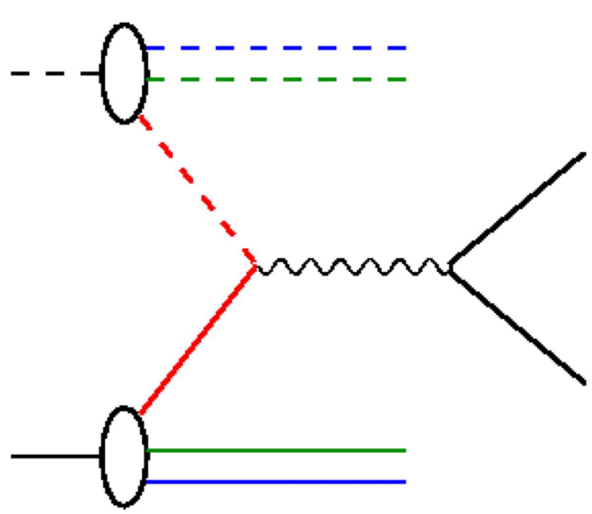
Experimental Observation of a Heavy Particle J†

J. J. Aubert, U. Becker, P. J. Biggs, J. Burger, M. Chen, G. Everhart, P. Goldhagen, J. Leong, T. McCorriston, T. G. Rhoades, M. Rohde, Samuel C. C. Ting, and Sau Lan Wu
Laboratory for Nuclear Science and Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139

and

Y. Y. Lee
Brookhaven National Laboratory, Upton, New York 11973
(Received 12 November 1974)

We report the observation of a heavy particle J, with mass $m = 3.1$ GeV and width approximately zero. The observation was made from the reaction $p + \mathrm{Be} \to e^+ + e^- + x$ by measuring the e^+e^- mass spectrum with a precise pair spectrometer at the Brookhaven National Laboratory's 30-GeV alternating-gradient synchrotron.



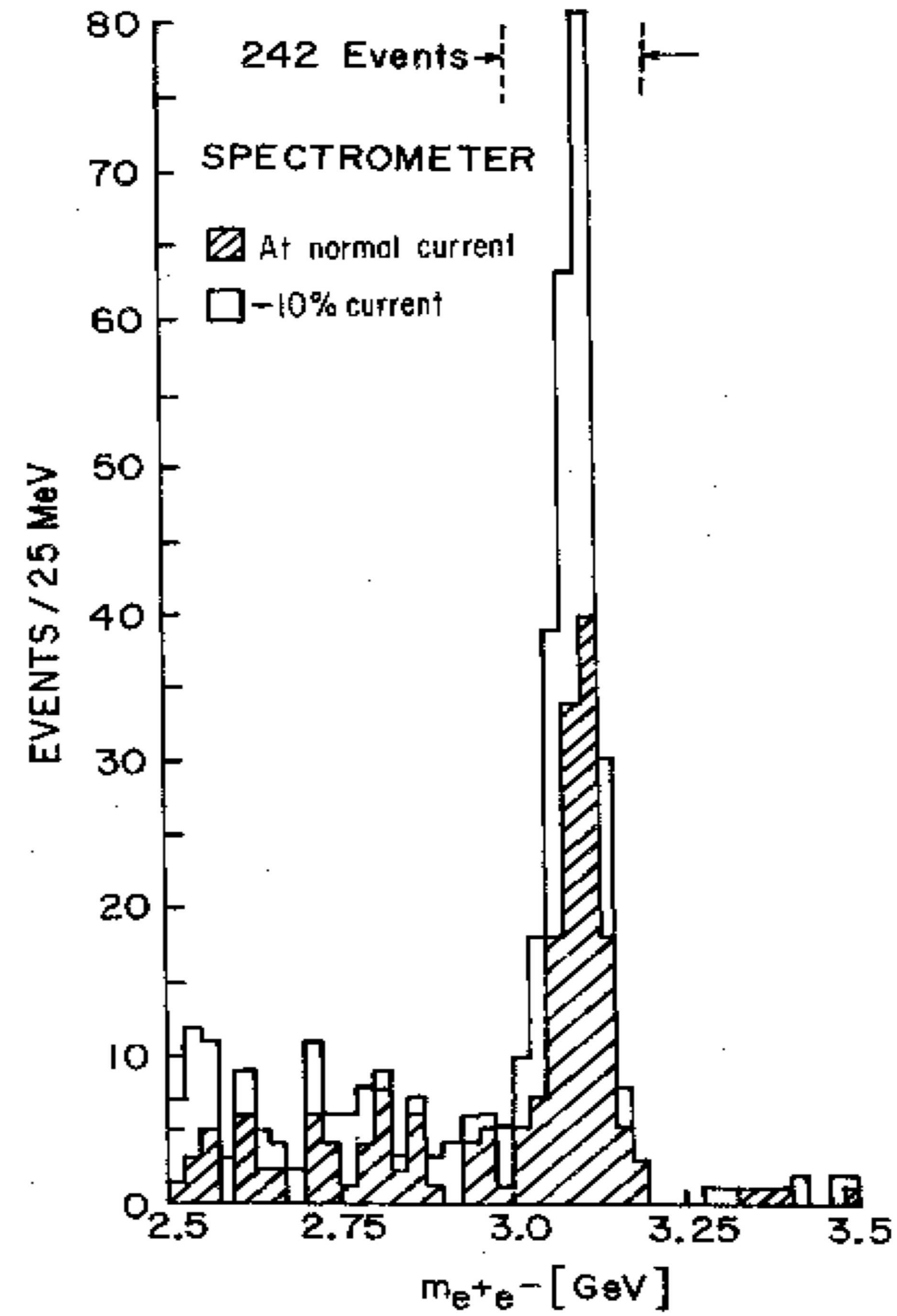
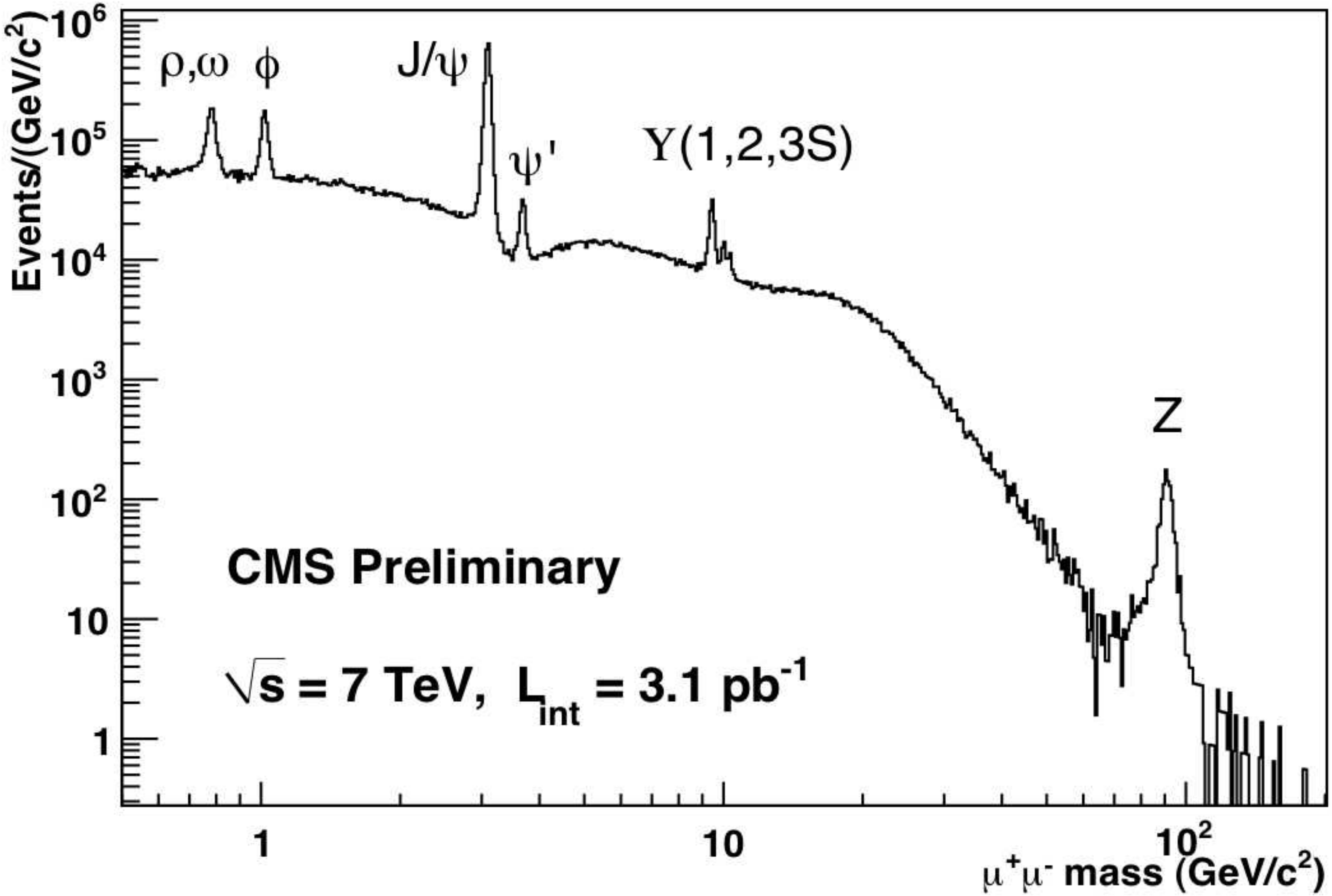


FIG. 2. Mass spectrum showing the existence of J. Results from two spectrometer settings are plotted showing that the peak is independent of spectrometer currents. The run at reduced current was taken two months later than the normal run.

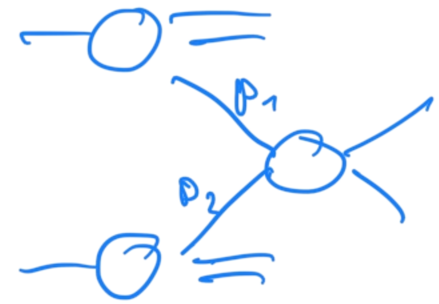
The full mass spectrum



Measurement of the full $\mu^+\mu^-$ mass spectrum at LHC

Rapidities and all that ...

$$y = \frac{1}{2} \ln \frac{E+p_z}{E-p_z}$$



$$p_1 = \frac{\sqrt{s}}{2} (x_1, 0, 0, x_1)$$

$$p_2 = \frac{\sqrt{s}}{2} (x_2, 0, 0, -x_2)$$

$$E = E_1 + E_2 = \frac{\sqrt{s}}{2} (x_1 + x_2)$$

$$P_z = p_{z1} + p_{z2} = \frac{\sqrt{s}}{2} (x_1 - x_2)$$

$$\Rightarrow \quad y = \frac{1}{2} \ln \frac{E+p_z}{E-p_z} = \frac{1}{2} \ln \frac{2x_1}{2x_2} = \frac{1}{2} \ln \frac{x_1}{x_2}$$

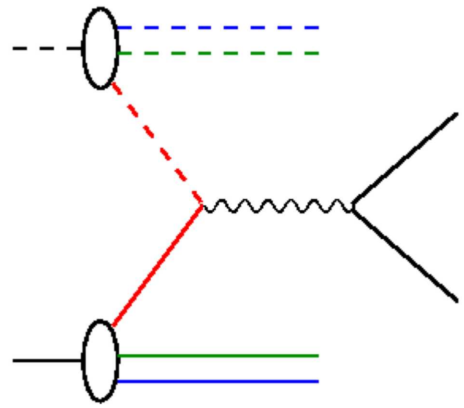
$$\frac{x_1}{x_2} = \exp\{-2y\} \qquad \tau = \frac{M^2}{s} = \frac{x_1 \cdot x_2 \cdot s}{s} = x_1 \cdot x_2$$

$$x_1 = \sqrt{\tau} \exp\{y\}$$

Rapidities and all that ...

- define rapidity:

$$y = \frac{1}{2} \log \frac{E + p_z}{E - p_z}$$



$$y(l^+ \, l^-) \text{ pair} = y(q\bar{q}) \text{ pair}$$

- gives:

$$y = \frac{1}{2} \log \frac{x_1}{x_2}$$

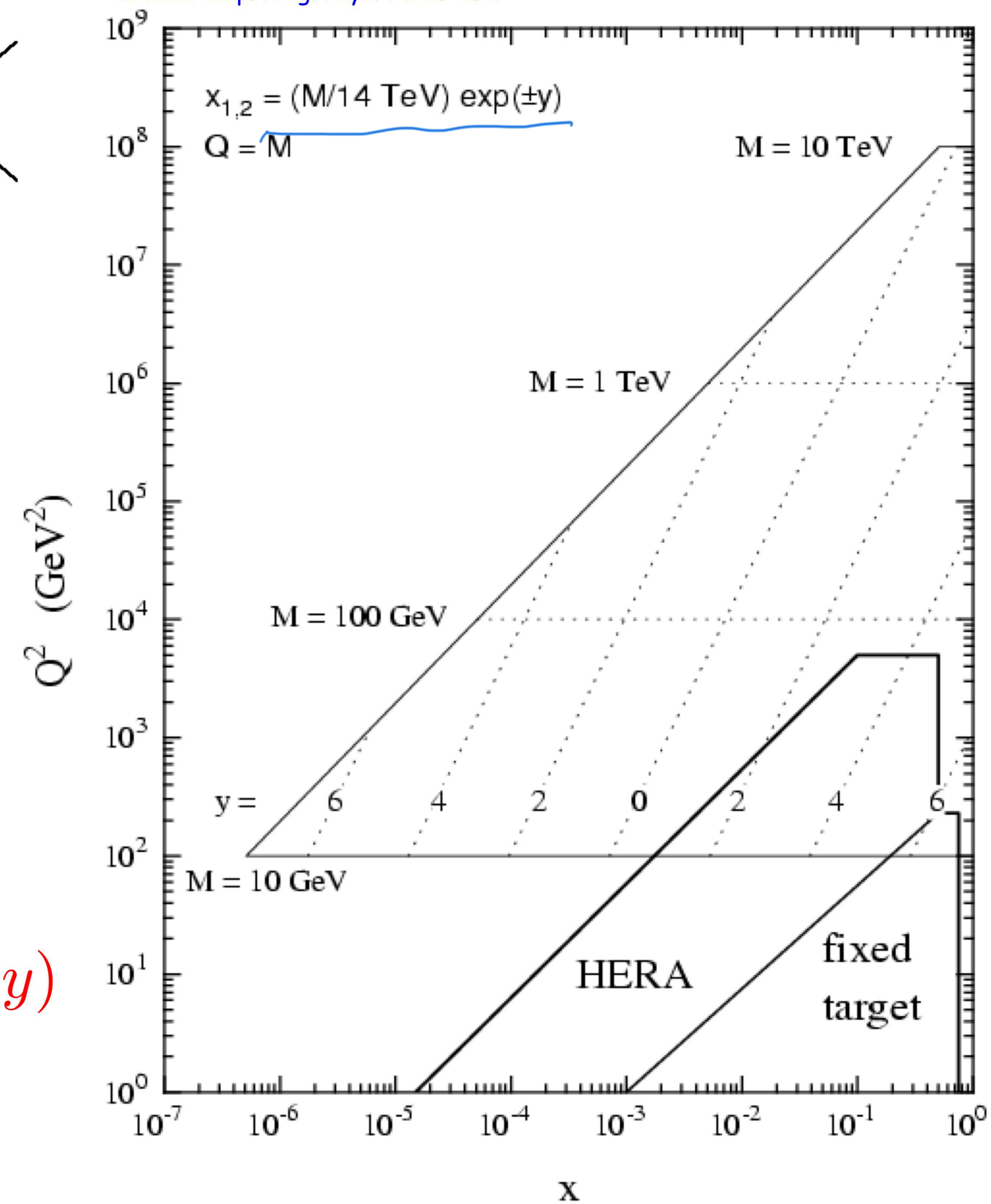
- using:

$$\tau = \frac{M^2}{s} = x_1 x_2$$

- gives:

$$x_{1,2} = \sqrt{\tau} \exp\left(\pm y\right) = \frac{M}{\sqrt{s}} \exp\left(\pm y\right)$$

Hard interactions of quarks and gluons: a primer for LHC physics J M Campbell et al 2007 Rep. Prog. Phys. 70 89-193



Drell – Yan in lowest order

$$|M|^2 = 2e^4 \frac{s^2+t^2}{u^2} \qquad e\mu \rightarrow e\mu$$

$$|M|^2 = 2e^4 \frac{t^2+u^2}{s^2} \qquad e^-e^+ \rightarrow \mu\mu$$

$$s = (\ell_1+\ell_2)^2 = 4E_b^2$$

$$t = (\ell_1-\mu_2)^2 = -2E_b(1-\cos\theta)$$

$$u = \quad = -2E_b(1+\cos\theta)$$

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s}|M|^2$$

$$= \frac{4E_b^2(1-\cos)^2 + 4E_b(1+\cos)^2}{16E_b^2}$$

$$= \frac{\alpha^2}{4s}(1+\cos^2\theta)$$

$$\boxed{\sigma = \int_0^{2\pi} d\varphi \int_{-1}^{+1} d\cos\theta \ldots\ldots = \frac{4}{3}\pi\frac{\alpha^2}{s}}$$

Drell – Yan in lowest order

$$\sigma(q\bar{q} \to \ell\ell) = \frac{4}{3}\pi\frac{\alpha^2}{s}\cdot e_q^2$$

$$\frac{d\sigma}{dM^2} = \frac{4}{3}\pi\frac{\alpha^2}{M^2}e_q^2\,\delta(\hat{s}-M^2) \qquad \hat{s} = x_1 x_2 \cdot s$$

$$\frac{d\sigma}{dM^2} = \frac{1}{3}\frac{1}{3}\cdot 3\sum_q \int dx_1\, dx_2\; f(x_1) f(x_2)\, \frac{d\hat{\sigma}}{dM^2}$$

$$= \frac{4\pi}{9}\frac{\alpha^2}{M^2}\sum e_q^2 \int dx_1\, dx_2 \left(f_1(x_1)\, f_2(x_2) + f_1(x_1)\, f_2(x_2)\right)\cdot \delta(x_1 x_2 \cdot s - M^2)$$

$$\tau = M^2/s$$

Drell – Yan in lowest order

$$= \quad \ldots \quad \frac{1}{s}\int dx_1 dx_2 \; \delta(x_1 x_2 - \tau)$$

$$= \int \frac{dx_1}{x_1}\left(f_1(x_1)\cdot f_2\left(x_2 = \frac{\tau}{x_1}\right) + \longleftrightarrow \right)$$

$$x_1 = \sqrt{\tau}\,\exp\{y\} \qquad dx_1/dy = x_1$$

$$\frac{d\sigma}{dM\,dy} = \frac{d\sigma}{s\cdot d\tau\,dy}$$

$$\frac{d\sigma}{d\tau\,dy} = \frac{4\pi\alpha^2}{9}\frac{1}{M^2}\sum e_q \left\{ f(x_1)\cdot f(x_2) + \longleftrightarrow \right\}$$

$$\Rightarrow \quad s\cdot\frac{d\sigma}{d\tau\,dy} = \quad \ldots \quad \frac{1}{\tau} \ldots$$

Drell – Yan in lowest order

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s}\frac{p_f}{p_i}|M|^2$$

$$|M_{e^+e^-\to l^+l^-}|^2 = 2\,(4\pi\alpha)^2\,\frac{t^2+u^2}{s^2}$$

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4s}(1+\cos\theta)^2$$

color average and sum

$$\sigma(q\bar{q}\to l^+l^-) = \frac{4\pi\alpha^2}{3\times 3s}e_q^2$$

$$s\frac{d^2\sigma}{d\tau\,dy} = \frac{4\pi\alpha^2}{9}\frac{1}{\tau}\sum_q e_q^2 \times \left[f_q(x_1)f_{\bar{q}}(x_2) + (q\leftrightarrow\bar{q})\right]$$

$$\tau = z = \frac{M^2}{s},\, M^2 = m^2_{l^+l^-}$$

Study Of Scaling In Hadronic Production Of Dimuons.
J.K.Yoh et al. Phys.Rev.Lett.41:684,1978, Erratum-ibid.41:1083,1978.

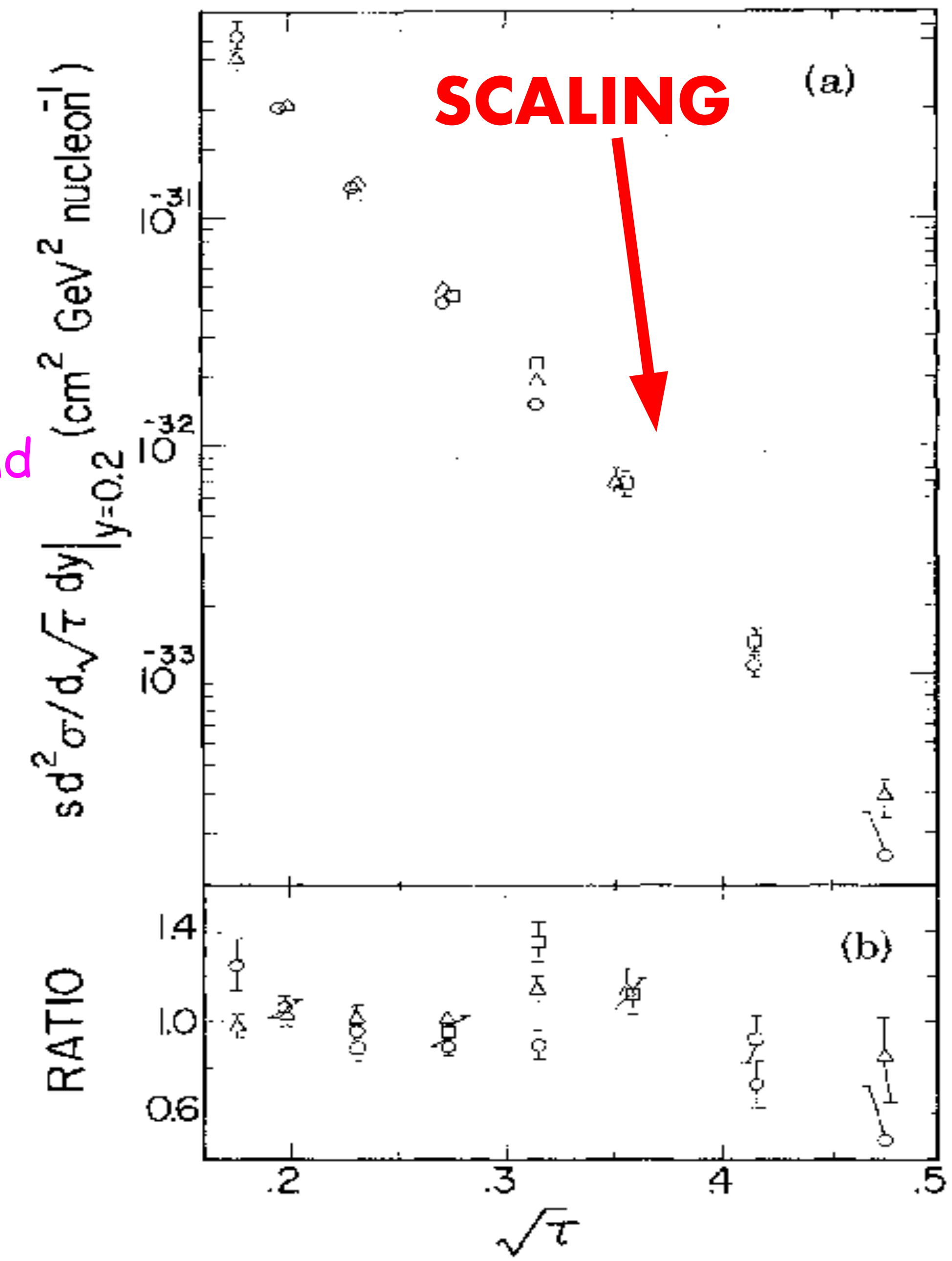
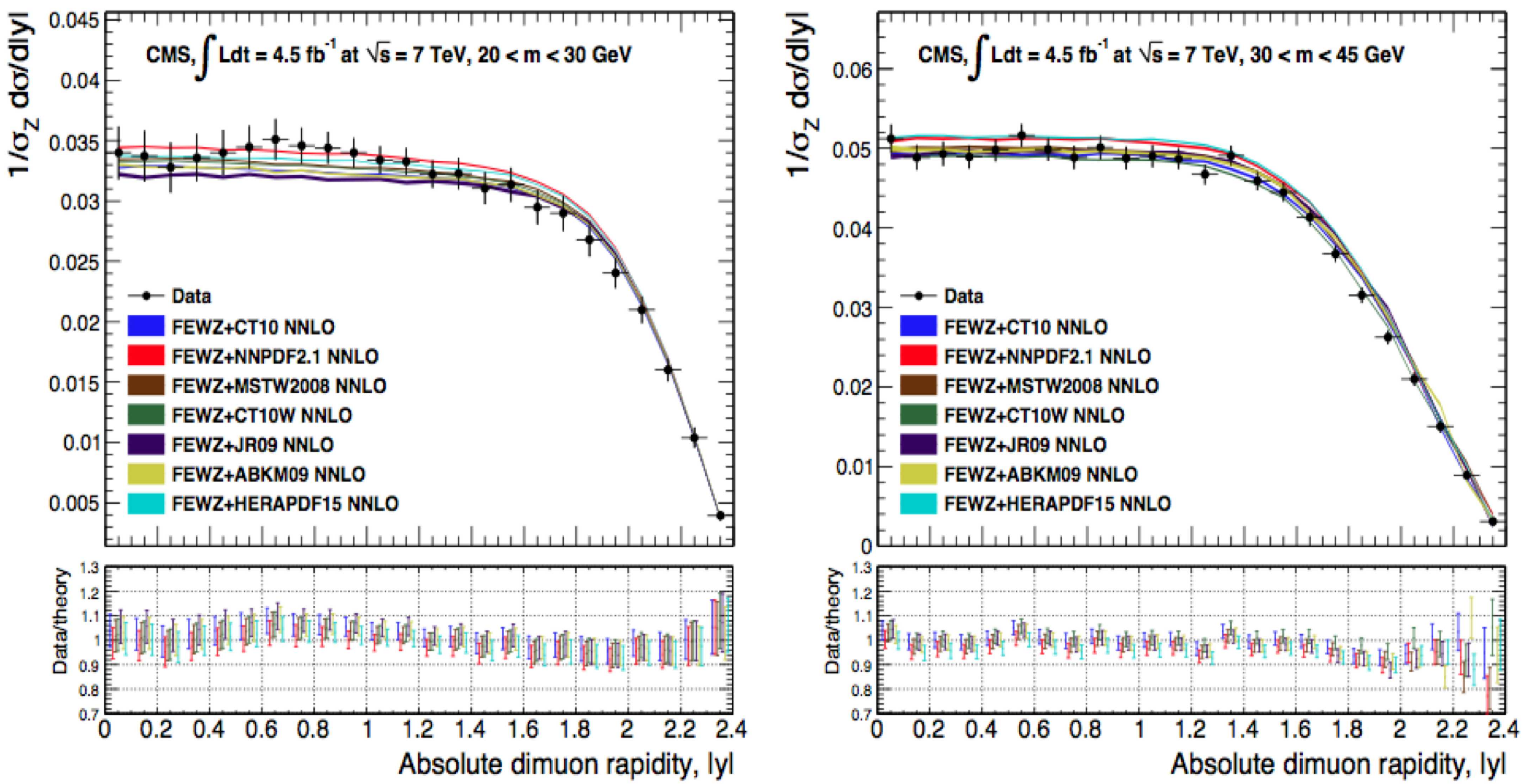


FIG. 3. (a) $s\,d^2\sigma/d\sqrt{\tau}dy|_{y=0.2}$ vs $\sqrt{\tau}$. Circles, triangles, and squares correspond to 400-, 300-, and 200-GeV beam energy, respectively. (b) Above data divided by the overall fit $Ae^{-b\sqrt{\tau}}$.

PDFs from Drell – Yan

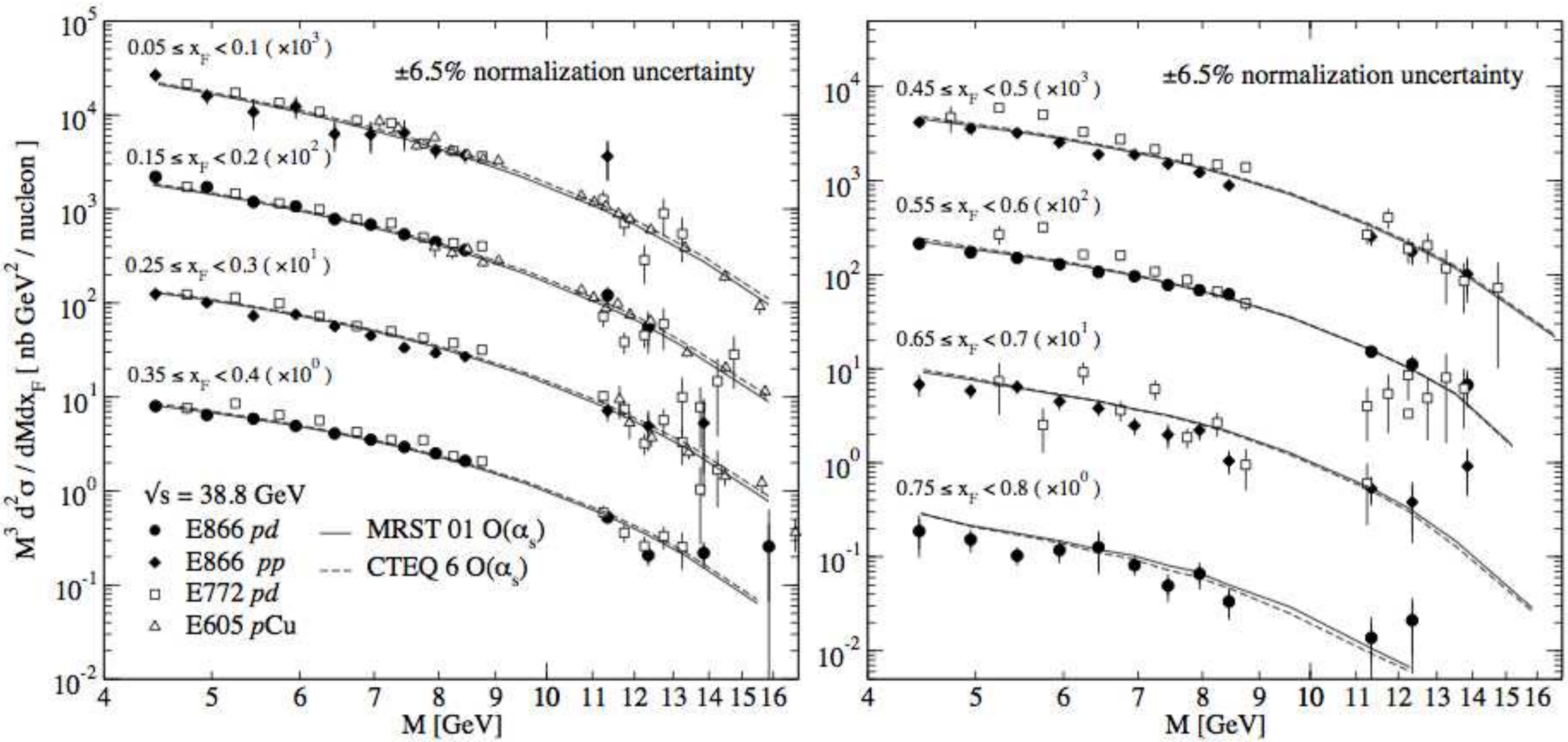
Measurement of the differential and double-differential Drell-Yan cross sections in proton-proton collisions at s√ = 7 TeV
CMS Collaboration arXiv:1310.7291



- at low masses → sensitive to low x sea quark and gluon density
- different parton-density-functions result in different x-sections

PDFs from Drell – Yan

from : J. C. Webb et al.
Absolute Drell-Yan dimuon cross sections in 800-GeV/c p p and p d collisions. hep-ex/0302019



PDFs from Drell – Yan

Measurement of the differential and double-differential Drell-Yan cross sections in proton-proton collisions at s√ = 7 TeV
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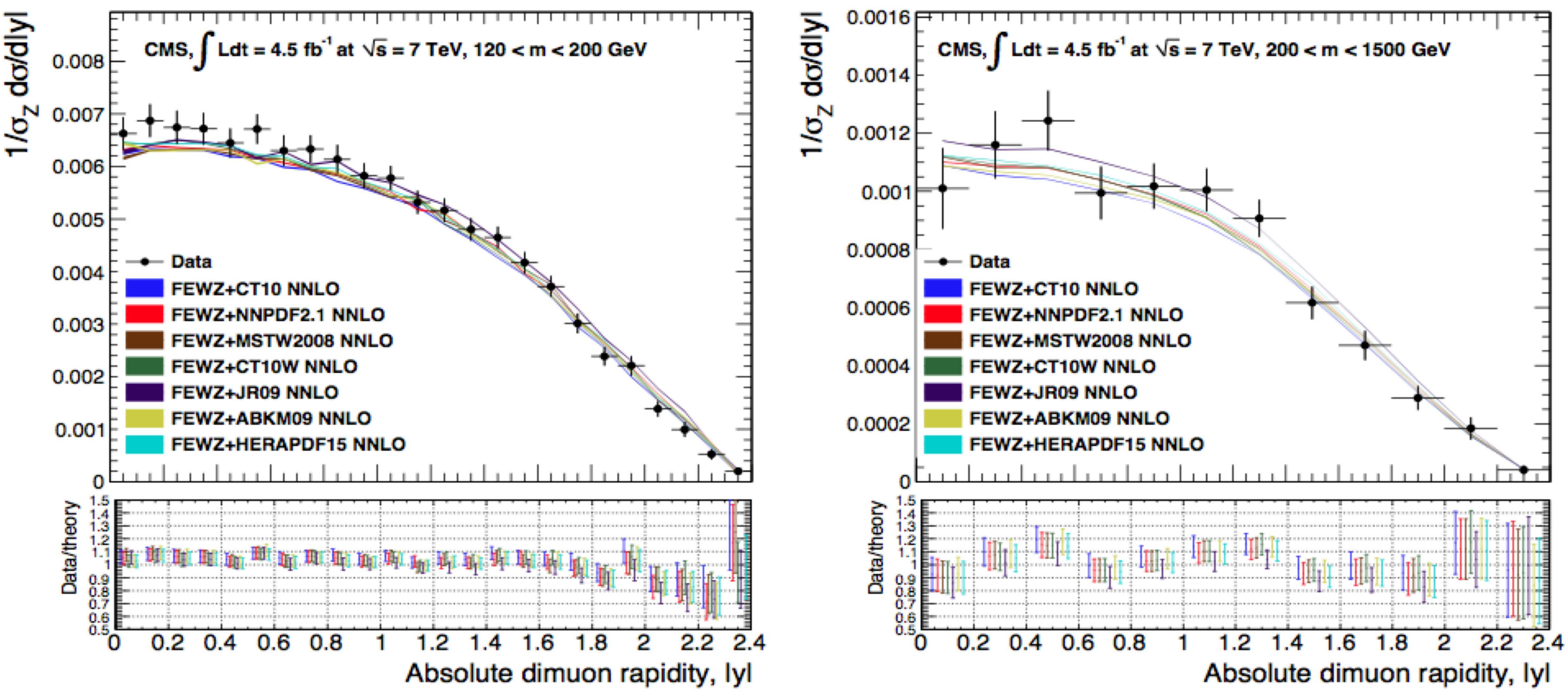


- at high masses → sensitive to higher x sea quark and gluon density
- different parton-density-functions result in different x-sections

Trying to do things easier ...

$$d\sigma(q+\bar{q}\to l^+ + l^-) \quad = \; d\sigma(q+\bar{q}\to\gamma^*) \otimes \frac{1}{Q^4} \otimes d\sigma(\gamma^*\to l^+ + l^-)$$

$q q \to \gamma^*$

$$d\sigma(qq\to ll) = \frac{1}{2M^2}\, |M(qq\to\gamma^*)|^2\, d^4q\, \delta^4(p_1+p_2-q) \cdot \frac{1}{Q^4}$$

$$\cdot\, |M(\gamma^*\to qq)|^2\, \frac{d^4k_1}{(2\pi)^3}\, \frac{d^4k_2}{(2\pi)^3}\, (2\pi)^4\, \delta(q-k_1-k_2)$$

$$= \frac{1}{2s}\, |M(qq\to\gamma^*)|^2\, d^4q\, \delta^4(\quad) \cdot \frac{1}{Q^4}\, |M|^2\, \frac{d\Omega}{32\pi^2}$$

Trying to do things easier ...

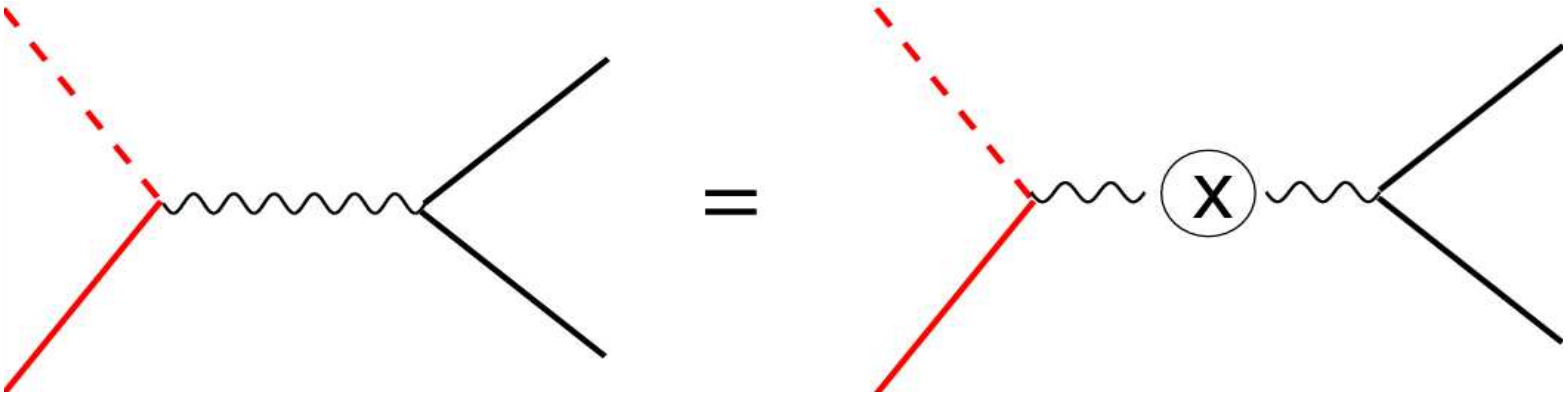
$$\gamma^* \to l^+ l^- \quad \Longrightarrow \quad |M|^2 = \frac{16\pi\alpha M^2}{3}$$

$$\frac{d\sigma}{dM^2} = \frac{1}{2M^2}\,\frac{4}{3}\pi\alpha M^2 e_q^2\,\delta(q^2 - M^2)$$

$$= \frac{4\pi^2}{3}\alpha e_q^2\,\delta(q^2 - M^2)$$

Trying to do things easier ...

$$d\sigma(q+\bar{q}\to l^+ + l^-) \;=\; d\sigma(q+\bar{q}\to\gamma^*) \otimes \frac{1}{Q^4} \otimes d\sigma(\gamma^* \to l^+ + l^-)$$



For example:

$$\frac{d\sigma(q\bar{q}\to l^+l^-)}{dM^2} = \frac{d\sigma(q\bar{q}\to\gamma^*)}{dM^2} \times \frac{\alpha}{3\pi M^2}$$

$$\frac{d\sigma(q\bar{q}\to l^+l^-)}{dM^2} = \frac{4\pi^2\alpha e_q^2}{3}\delta(M^2-s)\frac{\alpha}{3\pi M^2} = \frac{4\pi\alpha^2 e_q^2}{9M^2}\delta(M^2-s)$$

W & Z cross sections

- Basic process: Drell – Yan

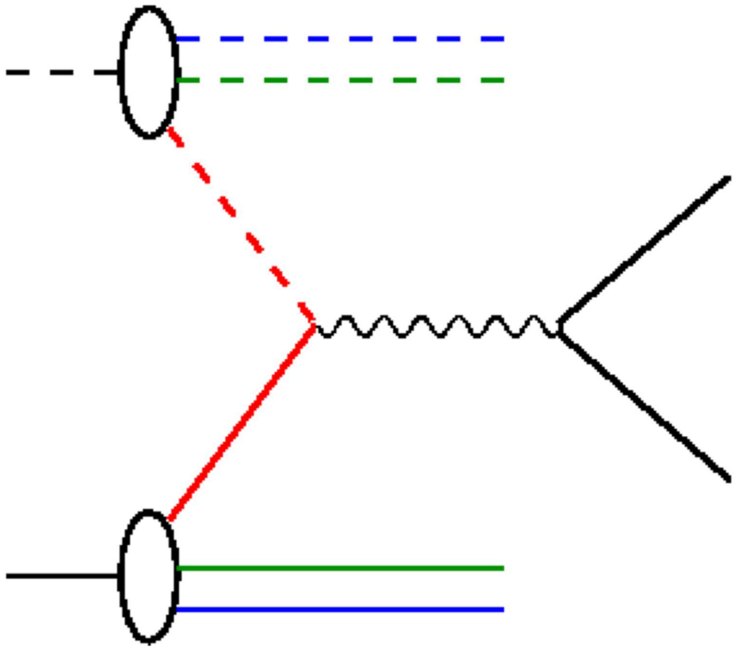
$$q + \bar{q} \to \gamma^* \to l^+ + l^-$$

- Factorize process:

$$q + \bar{q} \to \gamma^*$$

$$q + \bar{q} \to Z_0$$

$$q + \bar{q'} \to W^\pm$$



- Include decay of

$$Z_0 \to l^+ + l^-$$

$$W^\pm \to l + \nu$$

$$\gamma^* \to l^+ + l^-$$

➔Not considered further...

Z/γ^* production in LO

Z_0/γ^* production $\quad q(p_1) + \bar{q}(p_2) \to Z^0(p)(\gamma^*(p))$

- Matrix element: Z_0 $\qquad\qquad \gamma^*$

$$\mathcal{M} = -ig\epsilon_\mu \bar{v}(p_2)\gamma^\mu(g_v - g_a\gamma_5)u(p_1) \text{ or } \mathcal{M} = -ig\epsilon_\mu \bar{v}(p_2)\gamma^\mu u(p_1)$$

$$|\mathcal{M}|^2 = \frac{2}{3}\frac{G_F M_Z^4}{\sqrt{2}}(g_a^2 + g_v^2) \qquad |\mathcal{M}|^2 = \frac{4\pi\alpha}{3}e_q^2 M_\gamma^2$$

with

$$g_a^2 + g_v^2 = \frac{1}{8}(1 - 4|e_q|\sin^2\Theta_W + 8e_q^2\sin^4\Theta_W)$$

- partonic x-section:

$$d\sigma = \frac{1}{F} d\text{Lips}\,|\mathcal{M}|^2$$

$$\text{dLips} = (2\pi)^4\delta^4(-p_1 - p_2 + \sum_i p_i)\sum_i \frac{d^4 p_i}{(2\pi)^3}\delta(p_i^2 - m_i^2)$$

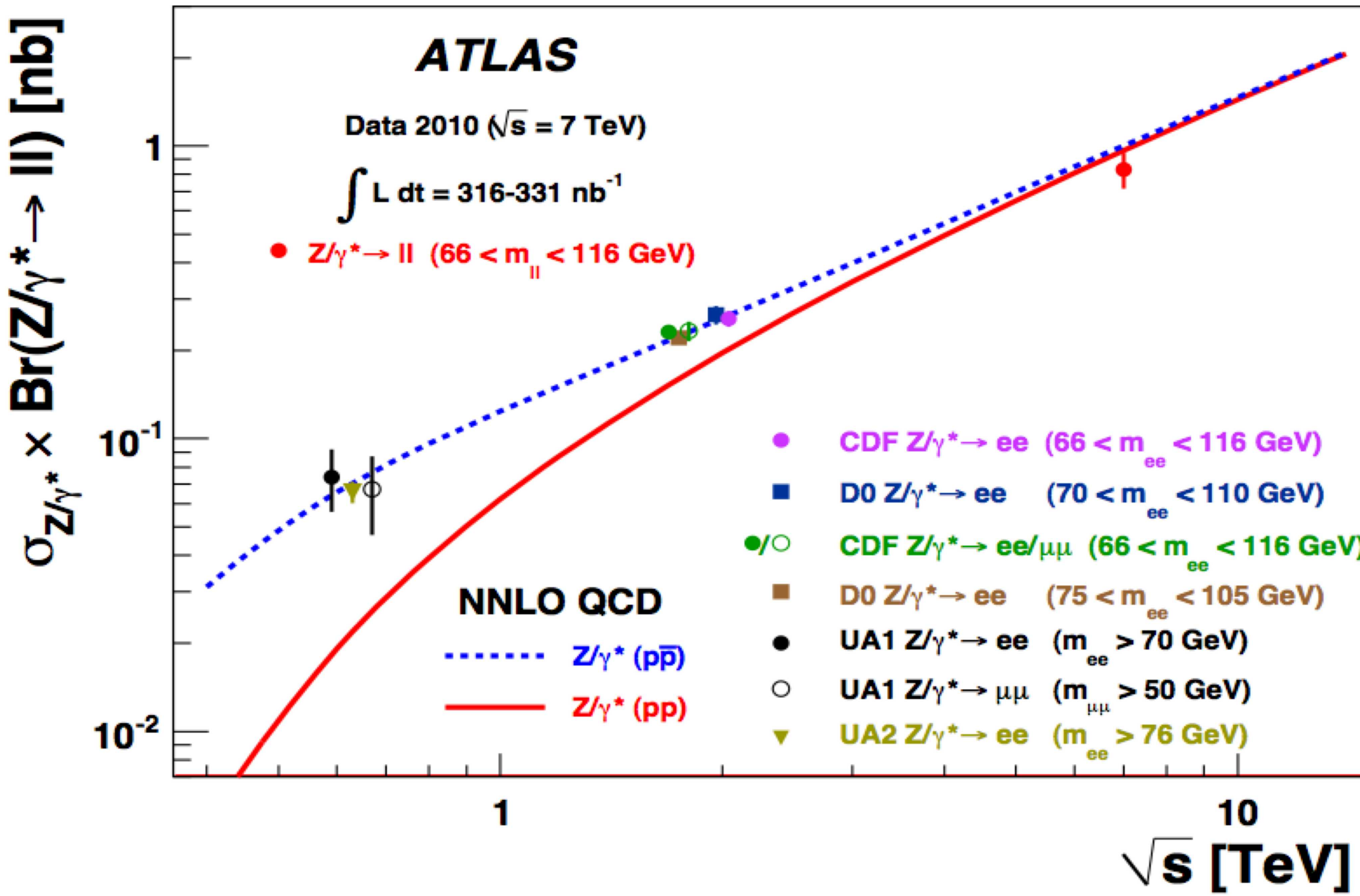
$$\hat{\sigma}(Z^0) = \frac{\pi}{3}\sqrt{2}G_F M_Z^2(g_a^2 + g_v^2)\delta(\hat{s} - M_Z^2)$$

$$\hat{\sigma}(\gamma^*) = \frac{4\pi^2\alpha}{3}e_q^2\delta(\hat{s} - M_\gamma^2)$$

Z cross section

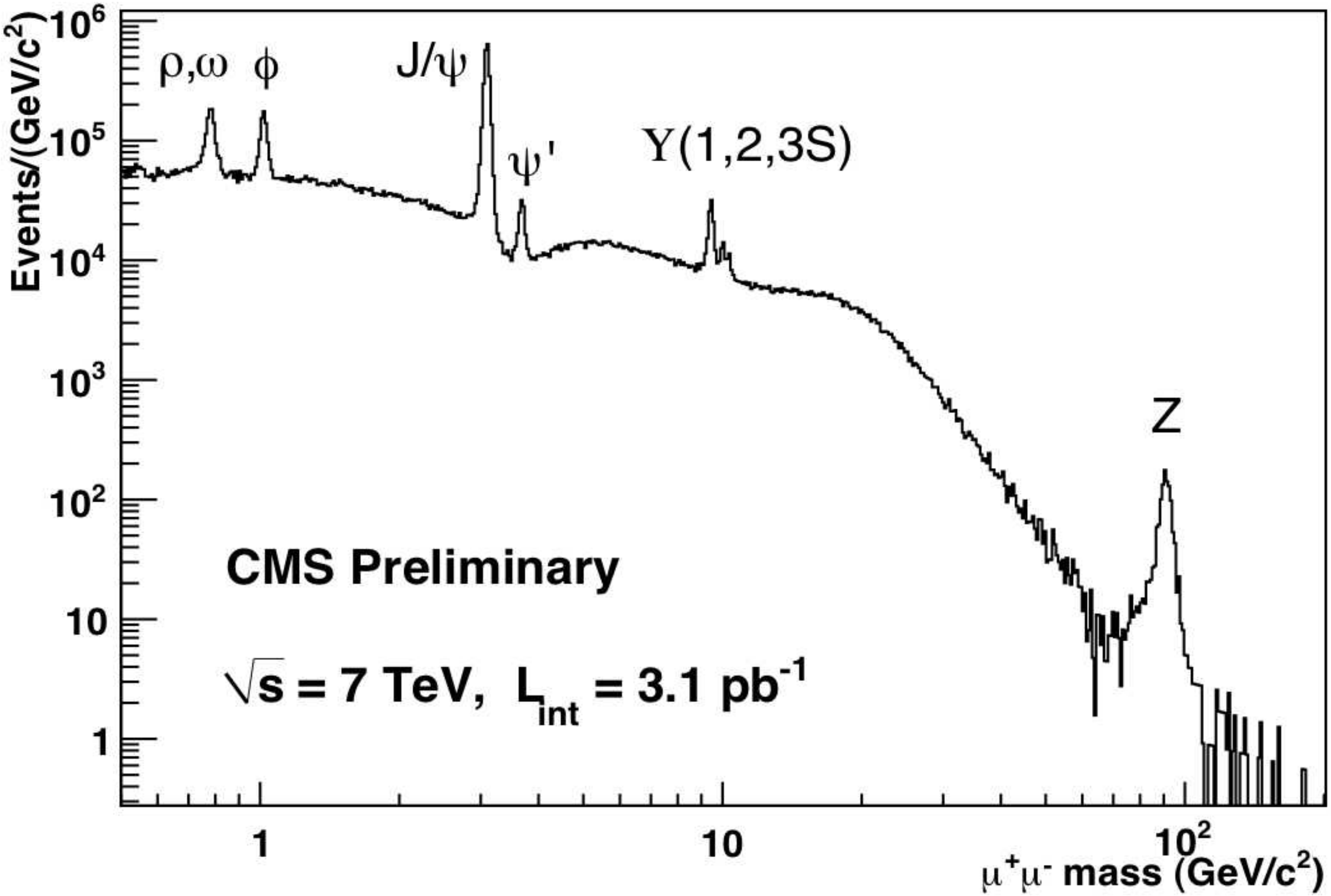
- new measurements from LHC

Atlas Collaboration.
Measurement of the W -> lnu and Z/gamma* -> ll production cross sections in proton-proton collisions at sqrt(s) = 7 TeV with the ATLAS detector. arXiv 1010.2130



➔ perfect description of measurements ... for total x-section

The full mass spectrum



Measurement of the full $\mu^+\mu^-$mass spectrum at LHC

Drell – Yan at high energies: Z, W

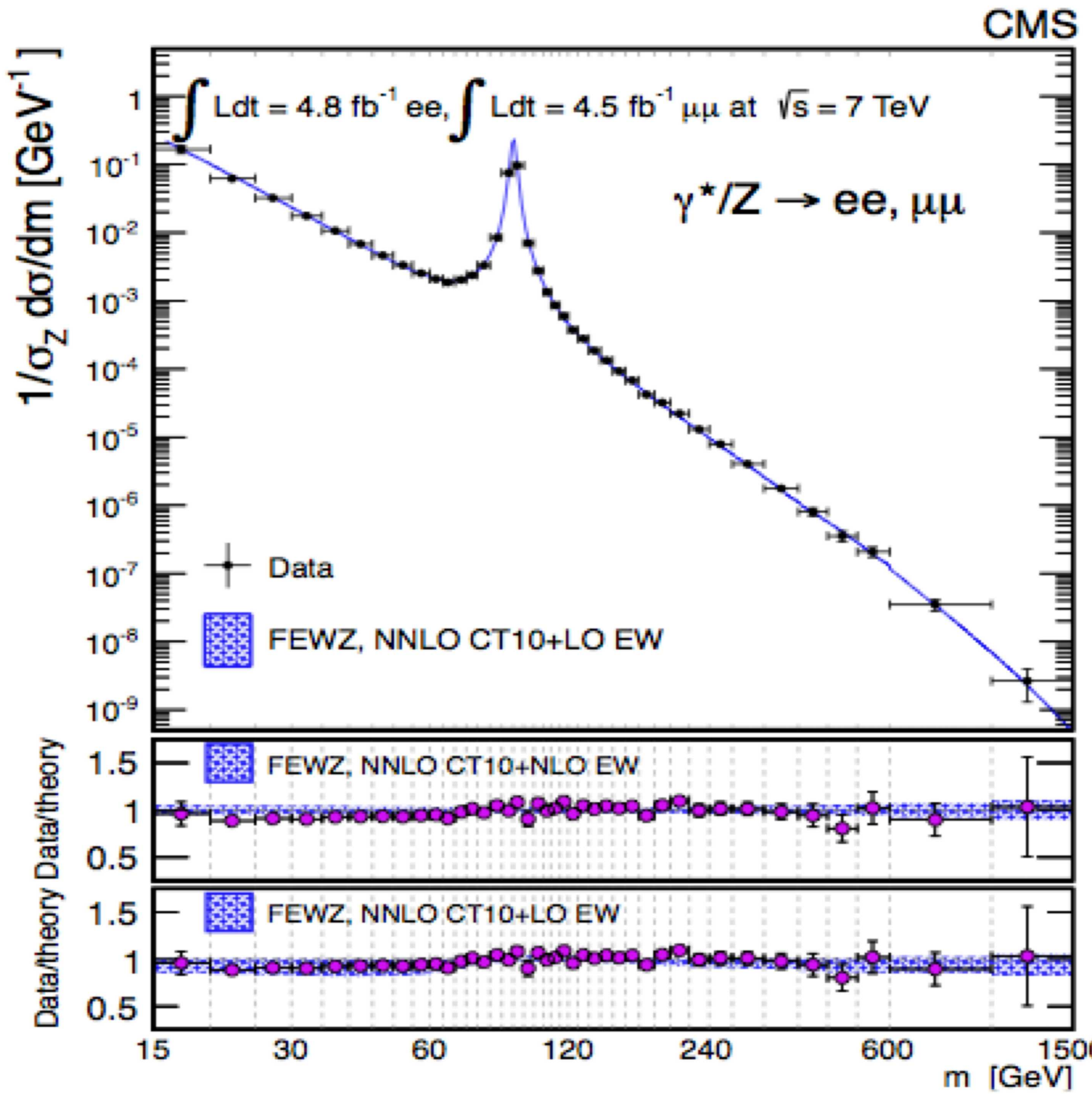
$$\sigma(q\bar{q} \to l^+l^-) = \frac{4\pi\alpha^2}{3s}\frac{1}{N}\Big(e_q^2 - 2e_q V_l V_q \chi_1(s) + (A_l^2 + V_l^2)(A_q^2 + V_q^2)\chi_2(s)\Big)$$

Measurement of the differential and double-differential Drell-Yan cross sections in proton-proton collisions at s√ = 7 TeV
CMS Collaboration arXiv:1310.7291

$$\chi_1(s) = \xi\frac{s(s-M_Z^2)}{(s-M_Z^2)^2+\Gamma_Z^2 M_Z^2}$$

$$\chi_2(s) = \xi^2\frac{s^2}{(s-M_Z^2)^2+\Gamma_Z^2 M_Z^2}$$

$$\xi = \frac{\sqrt{2}G_F M_Z^2}{16\pi\alpha}$$



- Z exchange visible !!!

W production in LO

- W production $q(p_1) + \bar{q}'(p_2) \to W^{\pm}(p)$
- Matrix element:

$$\mathcal{M} = -iV_{qq'} \frac{g}{\sqrt{2}} \epsilon \bar{v}(p_2)\gamma^{\mu} \frac{1}{2}(1-\gamma_5)u(p_1)$$

$$|\mathcal{M}|^2 = |V_{qq'}|^2 \frac{G_F M_W^4}{\sqrt{2}} \frac{2}{3} \quad \text{with} \quad g^2 = \frac{8 G_F M_W^2}{\sqrt{2}}$$

- partonic x-section:

$$d\sigma = \frac{1}{F} d\text{Lips}\, |\mathcal{M}|^2 \quad \text{with} \quad \text{dLips} = (2\pi)^4 \delta^4(-p_1 - p_2 + \sum_i p_i) \sum_i \frac{d^4 p_i}{(2\pi)^3} \delta(p_i^2 - m_i^2)$$

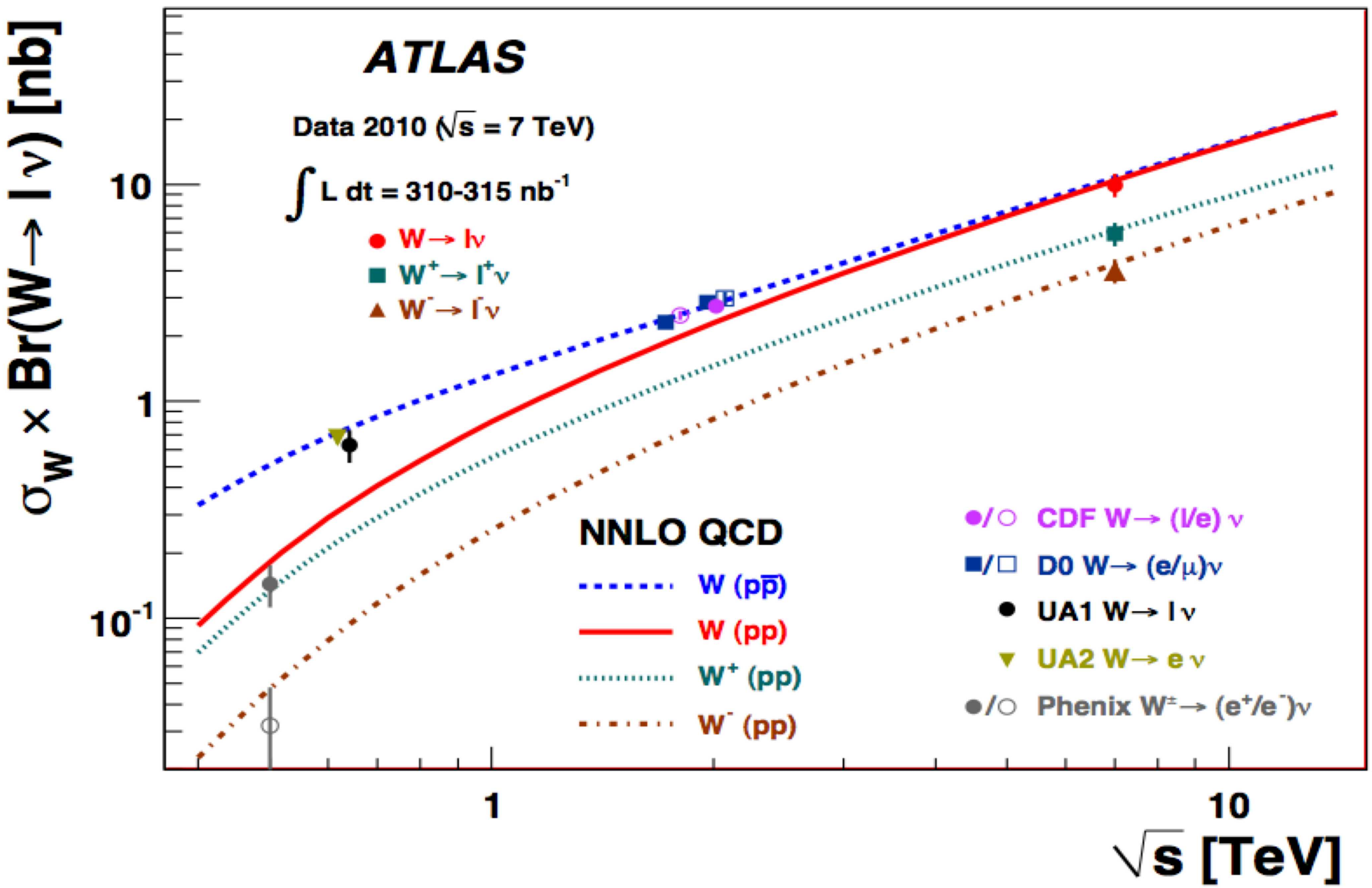
➔gives:

$$\hat{\sigma} = \frac{2\pi}{3} \frac{G_F M_W^2}{\sqrt{2}} |V_{qq'}|^2 \delta(\hat{s} - M_W^2)$$

W cross section

- new measurements from LHC

Atlas Collaboration.
Measurement of the W -> lnu and Z/gamma* -> ll production cross sections in proton-proton collisions at sqrt(s) = 7 TeV with the ATLAS detector. arXiv 1010.2130

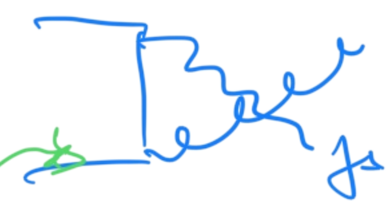


➔ perfect description of measurements ... for total x-section

Do we know the p_t spectrum ?

Factorization and transverse momenta

$q\bar{q} \to \gamma^* g$ $\Big|$ 

$+$ 

$\Big|^2$

$$|M|^2 = \ldots \frac{4}{9}\,\frac{1}{4}\cdot 8\left(\frac{u}{t} + \frac{t}{u} + \frac{2M^2\cdot s}{t\cdot u}\right) \qquad z = \frac{M^2}{s}$$

$$u + t + s = M^2$$

$$\Rightarrow u + t = s\left(\frac{M^2}{s} - 1\right) = -s(1-z)$$

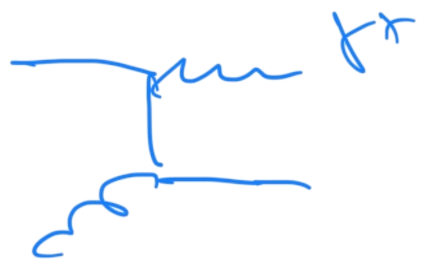
$$= \frac{1}{t\cdot u}\left(s^2(1+z^2) - 2ut\right) \qquad \frac{1}{t\cdot u} = \frac{-1}{s(1-z)}\left(\frac{1}{t} + \frac{1}{u}\right)$$

$$= \ldots \frac{-s^2(1+z^2)}{s(1-z)}\left(\frac{1}{t} + \frac{1}{u}\right) - 2$$

$$\simeq P_{qq}(z)\cdot\frac{1}{t}$$

$$\boxed{P_{qq} = \frac{4}{3}\,\frac{1+z^2}{1-z}}$$

Factorization and transverse momenta



$$P_{qg} = \frac{1}{2}\left(z^2 + (1-z)^2\right)$$

$$d\xi\, f(\xi) \Rightarrow d^2k_\perp\, d\xi \quad P(\xi, k_\perp)$$

$$P(\xi, k_\perp) = \delta(k_\perp) \cdot f(\xi)$$

$$\Rightarrow P(\xi, k_\perp) = h(\vec{k}_\perp) \cdot f(\xi)$$

$$h(\vec{k}_\perp) = \frac{b}{\pi} \exp\left\{-b\, k_\perp^2\right\} \quad (\text{intrinsic } k_\perp)$$

$$\frac{1}{\sigma}\frac{d\sigma}{d^2p_\perp} = \int d^2k_{\perp 1}\, d^2k_{\perp 2}\, \delta^{(2)}(\vec{k}_{\perp 1} + \vec{k}_{\perp 2} - \vec{p}_\perp)\, h(\vec{k}_{\perp 1})\, h(\vec{k}_{\perp 2})$$

$$= \int d^2k_{\perp 1}\, h(\vec{k}_{\perp 1})\, h(\vec{p}_\perp - \vec{k}_{\perp 1})$$

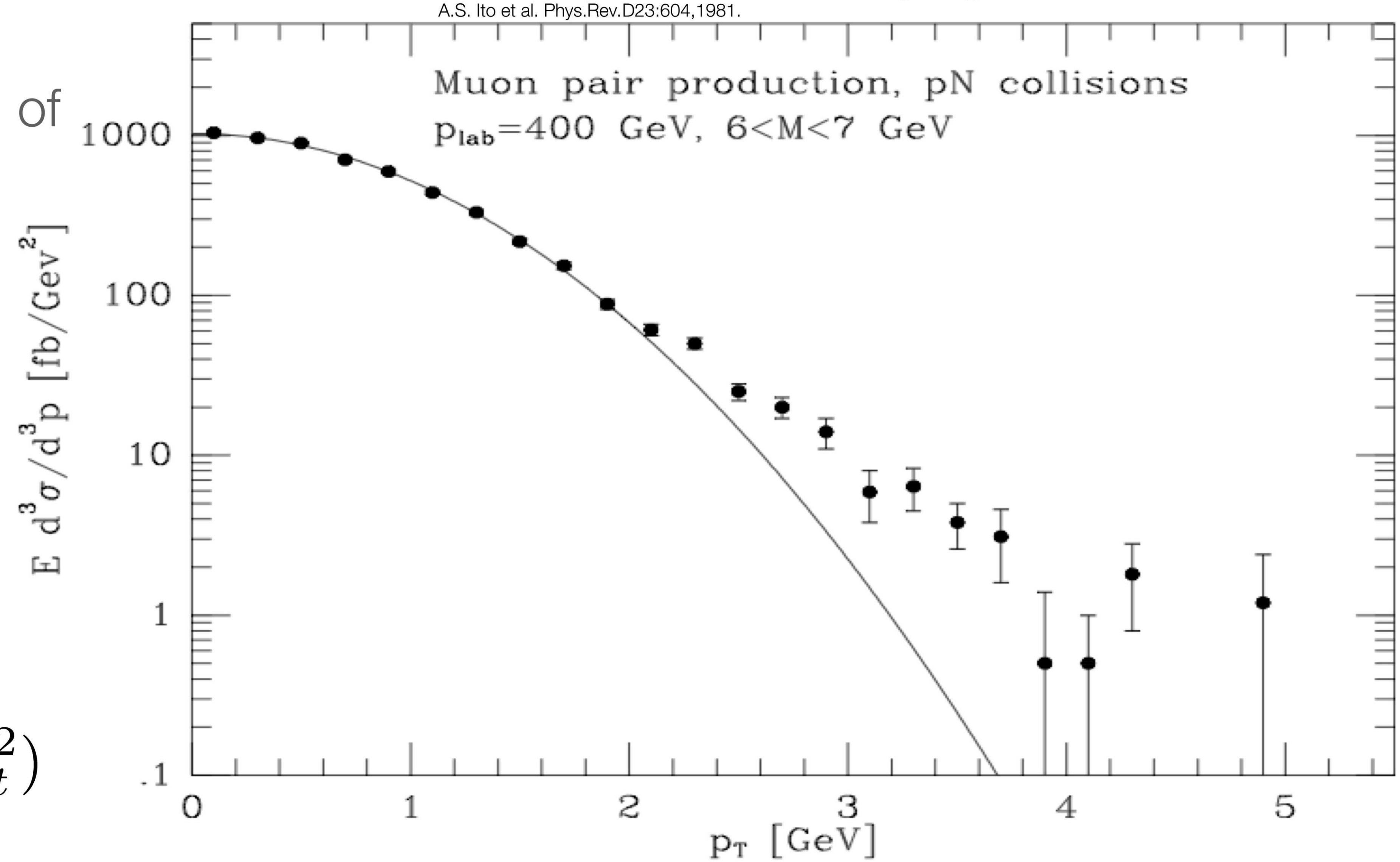
$$= \dots \int dk_\perp^2\, d\varphi \dots \left[e^{-2bk_{\perp 1}^2}\, e^{-bp_\perp^2}\, e^{+2b|p_\perp||k_\perp|\cdot\cos\varphi} \right]$$

$$\boxed{= \frac{b}{2\pi}\, e^{-\frac{1}{2}bp_\perp^2}}$$

Factorization and transverse momenta

- Observation of transverse momentum of DY pair

Measurement of the Continuum of Dimuons Produced in High-Energy Proton - Nucleus Collisions.
A.S. Ito et al. Phys.Rev.D23:604,1981.



- Using:

$$h(k_t) = \frac{b}{\pi} \exp\left(-b k_t^2\right)$$

$$\frac{1}{\sigma}\frac{d^2\sigma}{d^2p_t} = \int d^2k_{t1} d^2k_{t2} \delta^{(2)}(\vec{k}_t + \vec{k}_{t2} - \vec{p}_t) h(k_{t1}) h(k_{t2})$$

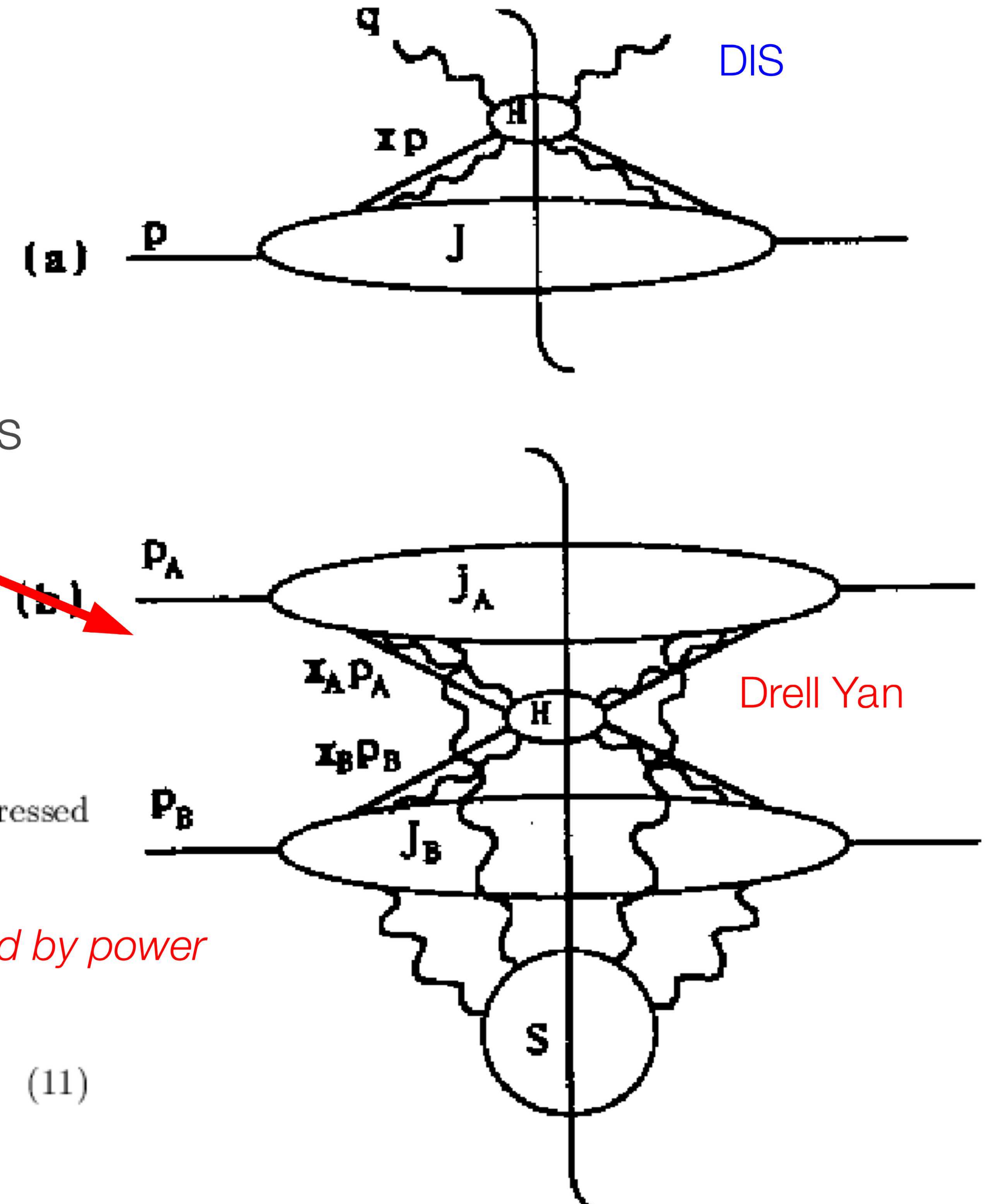
$$\frac{1}{\sigma}\frac{d^2\sigma}{d^2p_t} = \frac{b}{2\pi} \exp\left(-\frac{bp_t^2}{2}\right)$$

with $< k_t >= \sqrt{\frac{\pi}{4b}} = 760 \text{ MeV}$

Factorization in Drell – Yan

- problem are soft gluon fields of 2 incoming hadrons
  - consider A-jet passing through soft color field of B-jet
  - tricky technical proof of factorization
  - factorization holds, but not on a graph by graph basis
  - cancellation between different graphs connected by soft gluons

Factorization Of Hard Processes in QCD
J C. Collins, D. E. Soper, George Sterman
'Perturbative QCD' (A.H. Mueller, ed.) 1999
Adv.Ser.Direct.High Energy Phys.5:1-91,1988., hep-ph/0409313



From proof of factorization Collins et al:

The relevant factorization theorem, accurate up to corrections suppressed by a power of Q^2, is

$$\frac{d\sigma}{dQ^2 dy} \sim \sum_{a,b} \int_{x_A}^{1} d\xi_A \int_{x_B}^{1} d\xi_B \times$$
$$\times f_{a/A}(\xi_A,\mu)\; H_{ab}\left(\frac{x_A}{\xi_A}, \frac{x_B}{\xi_B}, Q; \frac{\mu}{Q}, \alpha_s(\mu)\right)\; f_{b/B}(\xi_B,\mu). \quad (11)$$

violation suppressed by power of Q^2

Drell-Yan: comparison with experiment

- K – factors at low energies

$$K = \frac{\sigma^{measured}}{\sigma^{calc}(LO)}$$

➔ Need for higher order calculations......

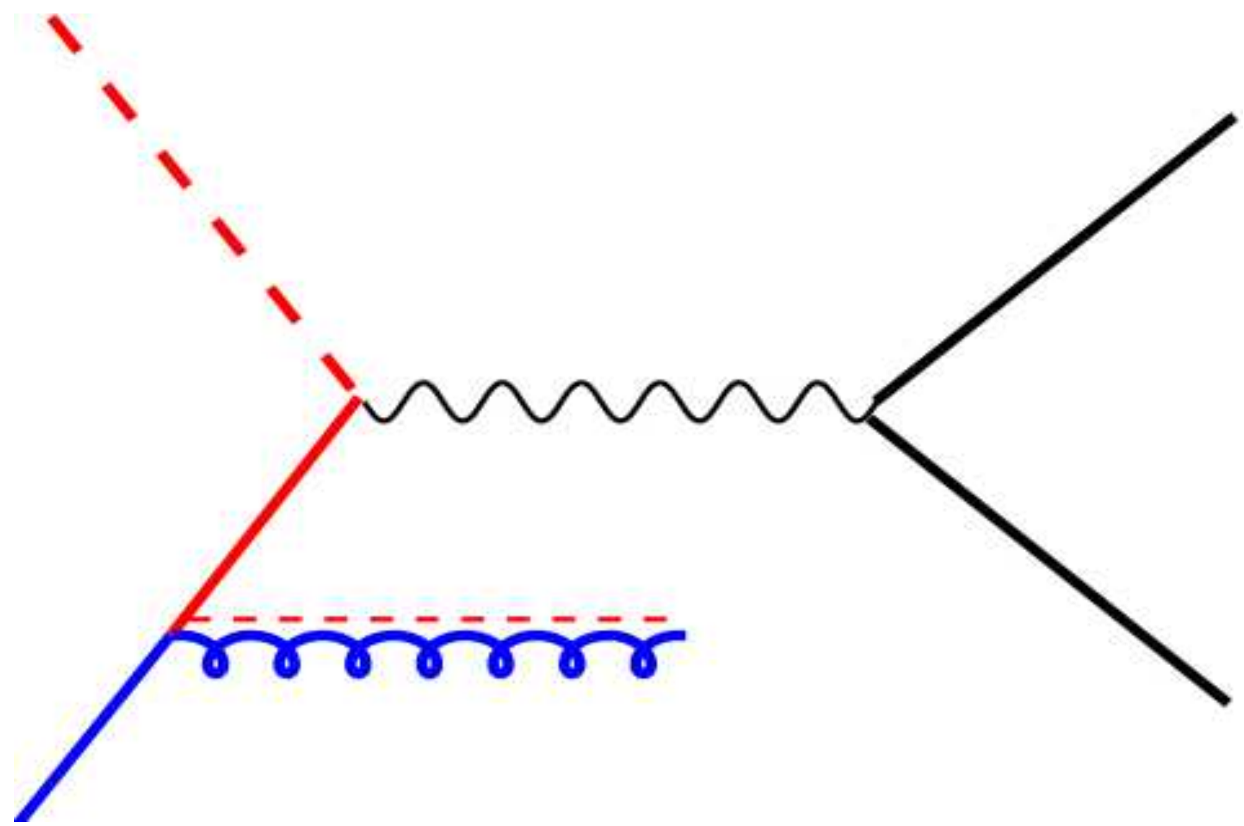
Measurement Of Continuum Dimuon Production In 800-GeV/C Proton-Nucleon Collisions Jason C. Webb, hep-ex/0301031

Table 1.2: Experimental K-factors.

Experiment	Interaction	Beam Momentum	$K = \sigma_{\text{meas.}}/\sigma_{\text{DY}}$
E288 [Kap 78]	$p\ Pt$	300/400 GeV	~ 1.7
WA39 [Cor 80]	$\pi^\pm\ W$	39.5 GeV	~ 2.5
E439 [Smi 81]	$p\ W$	400 GeV	1.6 ± 0.3
NA3 [Bad 83]	$(\bar{p}\text{ - }p)Pt$	150 GeV	2.3 ± 0.4
	$p\ Pt$	400 GeV	$3.1 \pm 0.5 \pm 0.3$
	$\pi^\pm\ Pt$	200 GeV	2.3 ± 0.5
	$\pi^-\ Pt$	150 GeV	2.49 ± 0.37
	$\pi^-\ Pt$	280 GeV	2.22 ± 0.33
NA10 [Bet 85]	$\pi^-\ W$	194 GeV	$\sim 2.77 \pm 0.12$
E326 [Gre 85]	$\pi^-\ W$	225 GeV	$2.70 \pm 0.08 \pm 0.40$
E537 [Ana 88]	$\bar{p}\ W$	125 GeV	$2.45 \pm 0.12 \pm 0.20$
E615 [Con 89]	$\pi^-\ W$	252 GeV	1.78 ± 0.06

What about higher orders ?

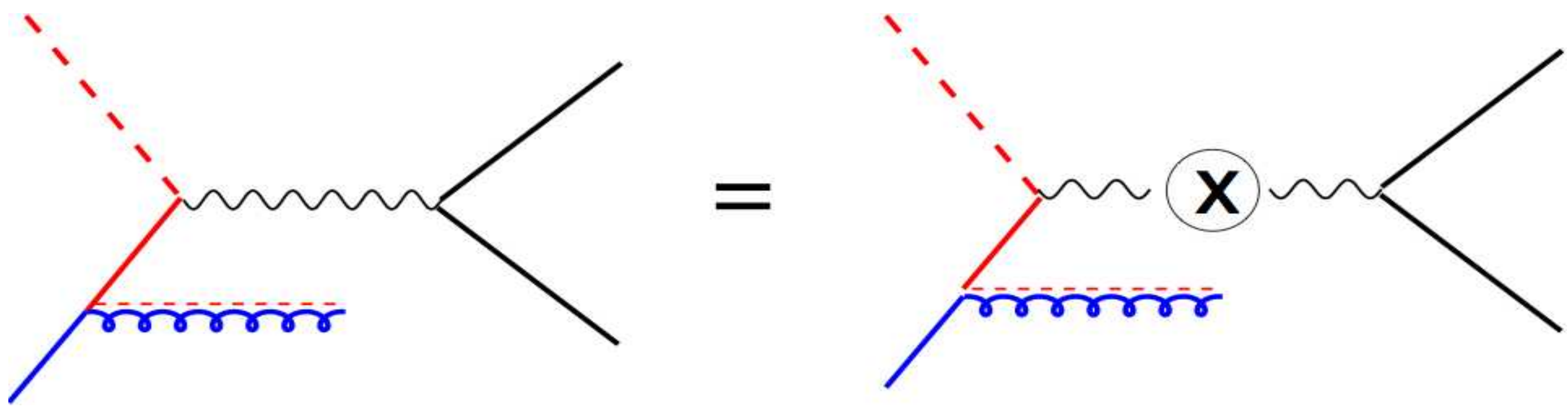
Calculating higher order contributions



Complicated: 2 → 3 process

Calculating higher order contributions

$$d\sigma(q+\bar{q}\to l^+ + l^-) \quad = \; d\sigma(q+\bar{q}\to\gamma^* + g) \otimes \frac{1}{Q^4} \otimes d\sigma(\gamma^*\to l^+ + l^-)$$



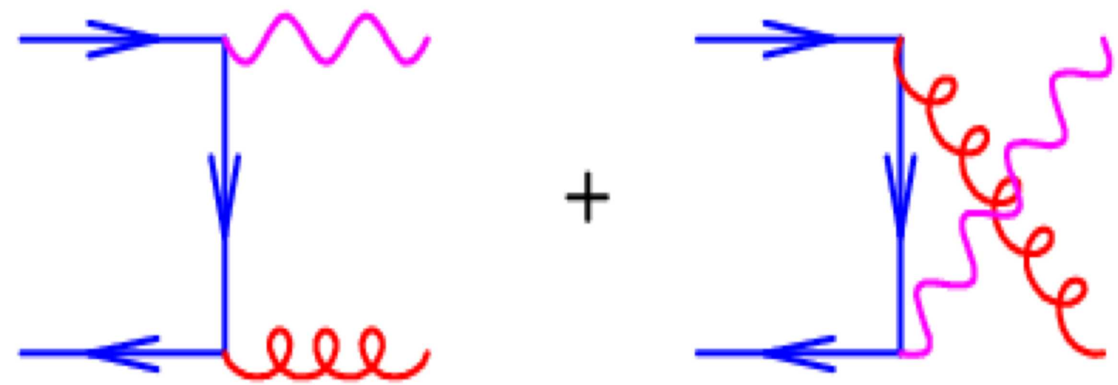
For example:

$$\frac{d^2\sigma(q+\bar{q}\to g + l^+ + l^-)}{dM^2 dp_t^2} = \frac{d^2\sigma(q+\bar{q}\to\gamma^* + g)}{dM^2 dp_t^2} \times \frac{\alpha}{3\pi M^2}$$

QCD corrections for Drell-Yan

- annihilation process

$$q + \bar{q} \to \gamma^* + g$$



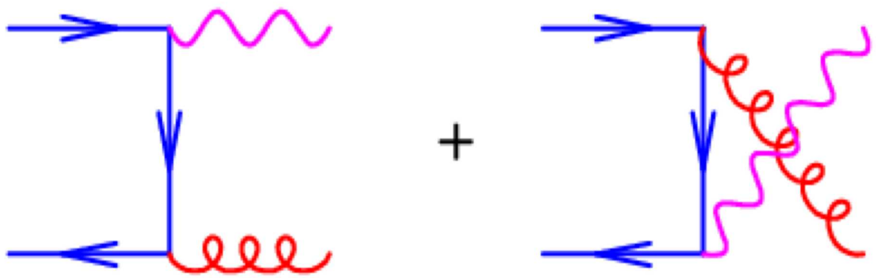
QCD corrections for Drell-Yan

QCD corrections for Drell-Yan

K. Ellis, LHC lecture, http://theory.fnal.gov/people/ellis/Talks

- annihilation process

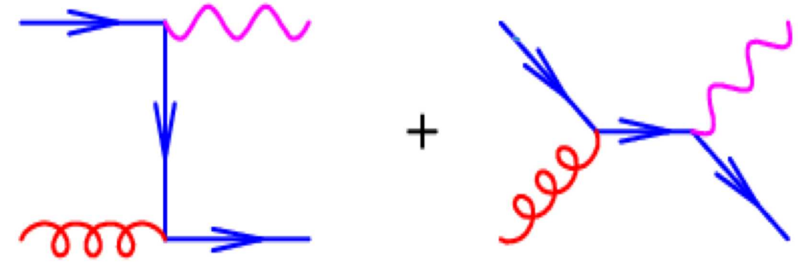
$$q + \bar{q} \to \gamma^* + g$$



$$
\begin{aligned}
|M|^2 &= 16\pi^2 \alpha_s \alpha \frac{8}{9} \left[\frac{\hat{u}}{\hat{t}} + \frac{\hat{t}}{\hat{u}} + \frac{2(M^2 \hat{s})}{\hat{u}\hat{t}} \right] \\
&= 16\pi^2 \alpha_s \alpha \frac{8}{9} \left[\left(\frac{1+z^2}{1-z} \right) \right. \\
&\quad \times \left. \left(\frac{-\hat{s}}{\hat{t}} + \frac{-\hat{s}}{\hat{u}} \right) - 2 \right] \\
&= 16\pi^2 \alpha_s \alpha \frac{8}{9} \left[P_{qq}(z) \right. \\
&\quad \times \left. \left(\frac{-\hat{s}}{\hat{t}} + \frac{-\hat{s}}{\hat{u}} \right) - 2 \right]
\end{aligned}
$$

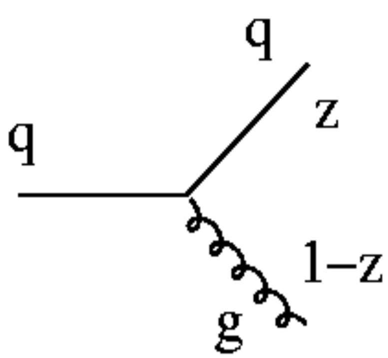
- QCDC process

$$q + g \to \gamma^* + q$$

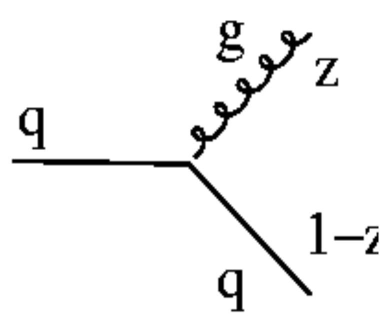


$$
\begin{aligned}
|M|^2 &= 16\pi^2 \alpha_s \alpha \frac{1}{3} \left[-\frac{\hat{t}}{\hat{s}} - \frac{\hat{s}}{\hat{t}} - \frac{2(M^2 \hat{u})}{\hat{s}\hat{t}} \right] \\
&= 16\pi^2 \alpha_s \alpha \frac{1}{3} \left[\left(z^2 + (1-z)^2 \right) \times \cdots \right) \\
&= 16\pi^2 \alpha_s \alpha \frac{1}{3} \left[P_{qg}(z) \times \cdots \right)
\end{aligned}
$$

Splitting functions in lowest order

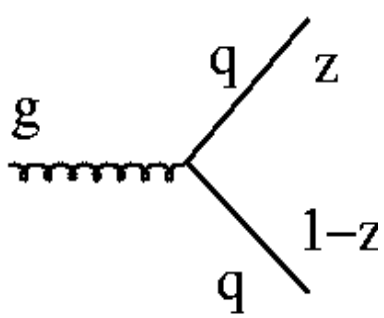


$$P_{qq} = \frac{4}{3}\left(\frac{1+z^2}{1-z}\right)$$

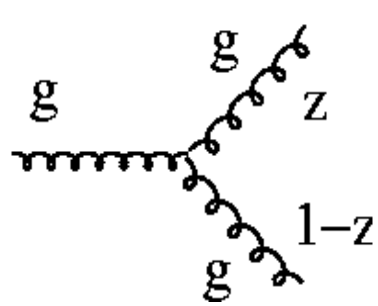


$$P_{gq} = \frac{4}{3}\left(\frac{1+(1-z)^2}{z}\right)$$

similarity to EPA...



$$P_{qg} = \frac{1}{2}\left(z^2+(1-z)^2\right)$$



$$P_{gg} = 6\left(\frac{1-z}{z}+\frac{z}{1-z}+z(1-z)\right)$$

Drell–Yan + 1-jet production

$$\frac{d\sigma}{dM^2 dy dp_t^2} = \frac{8}{27}\frac{\alpha^2}{M^2}\frac{1}{p_T^2}\int_{x_a^{min}}^{1} dx_a H_q(x_a, x_b, M^2) \frac{x_a\, x_b}{x_a - x_1}\left(1 + \frac{\tau^2}{(x_a x_b)^2} - \frac{x_T^2}{2 x_a x_b}\right)$$

- with
$$x_a^{min} = \frac{x_a x_2 - \tau}{x_a - x_1}$$

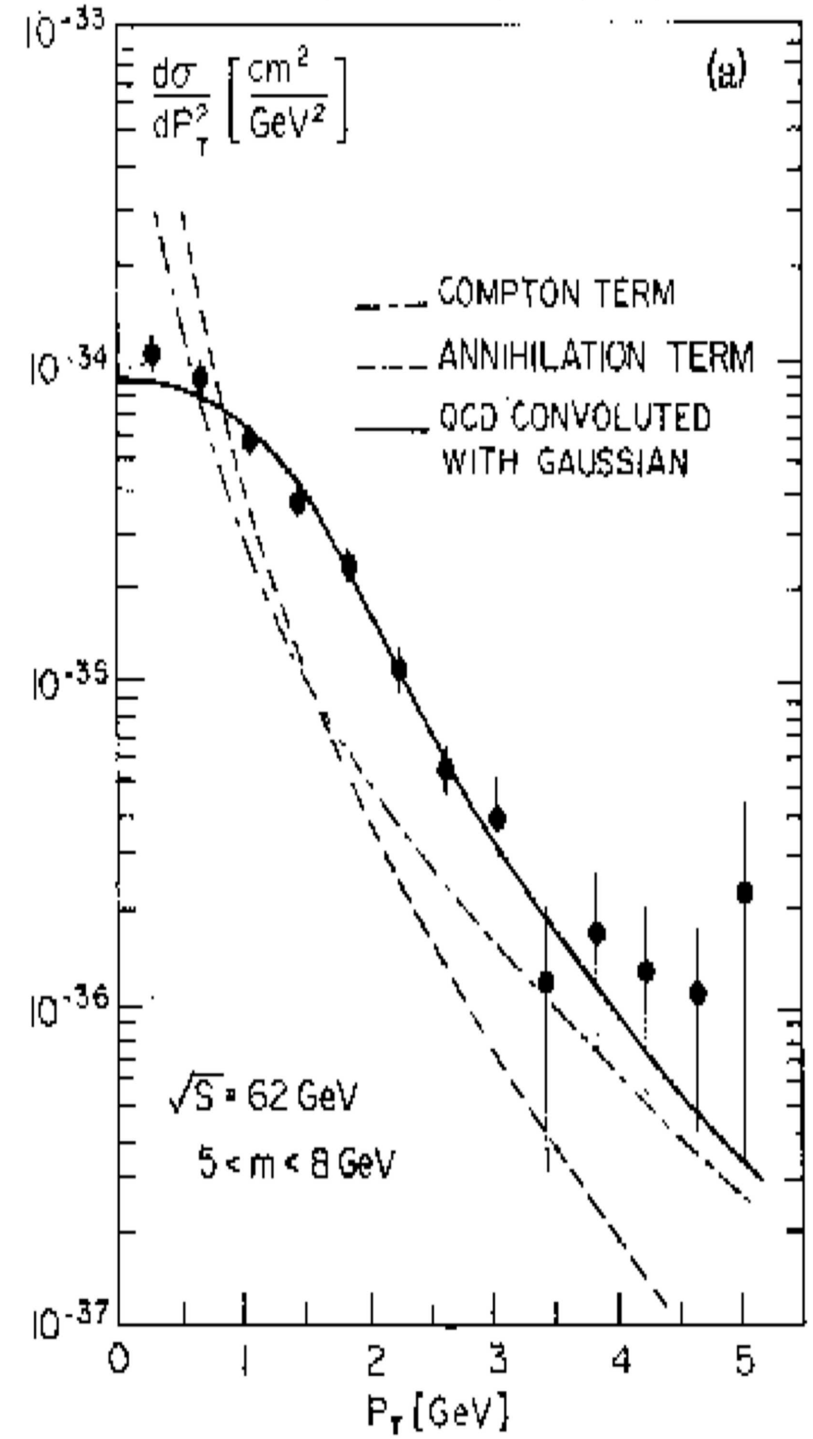
$$p_t^2 = \frac{\hat{t}\hat{u}}{\hat{s}}$$

$$x_t = \frac{2p_t}{\sqrt{s}}, \quad x_1 = \frac{1}{2}(x_t^2 + 4\tau)^{1/2}\exp{(y)}$$

$$H_q(x_a, x_b, Q^2) = \sum e_q^2 \left(q_i(x_a, Q^2)\bar{q}_i(x_b, Q^2) + \bar{q}_i(x_a, Q^2) q_i(x_b, Q^2)\right)$$

R. Field, Appl. of pQCD, p195f ff

Antreasyan PRL 48 p302 (1982)



Drell–Yan + 1-jet production

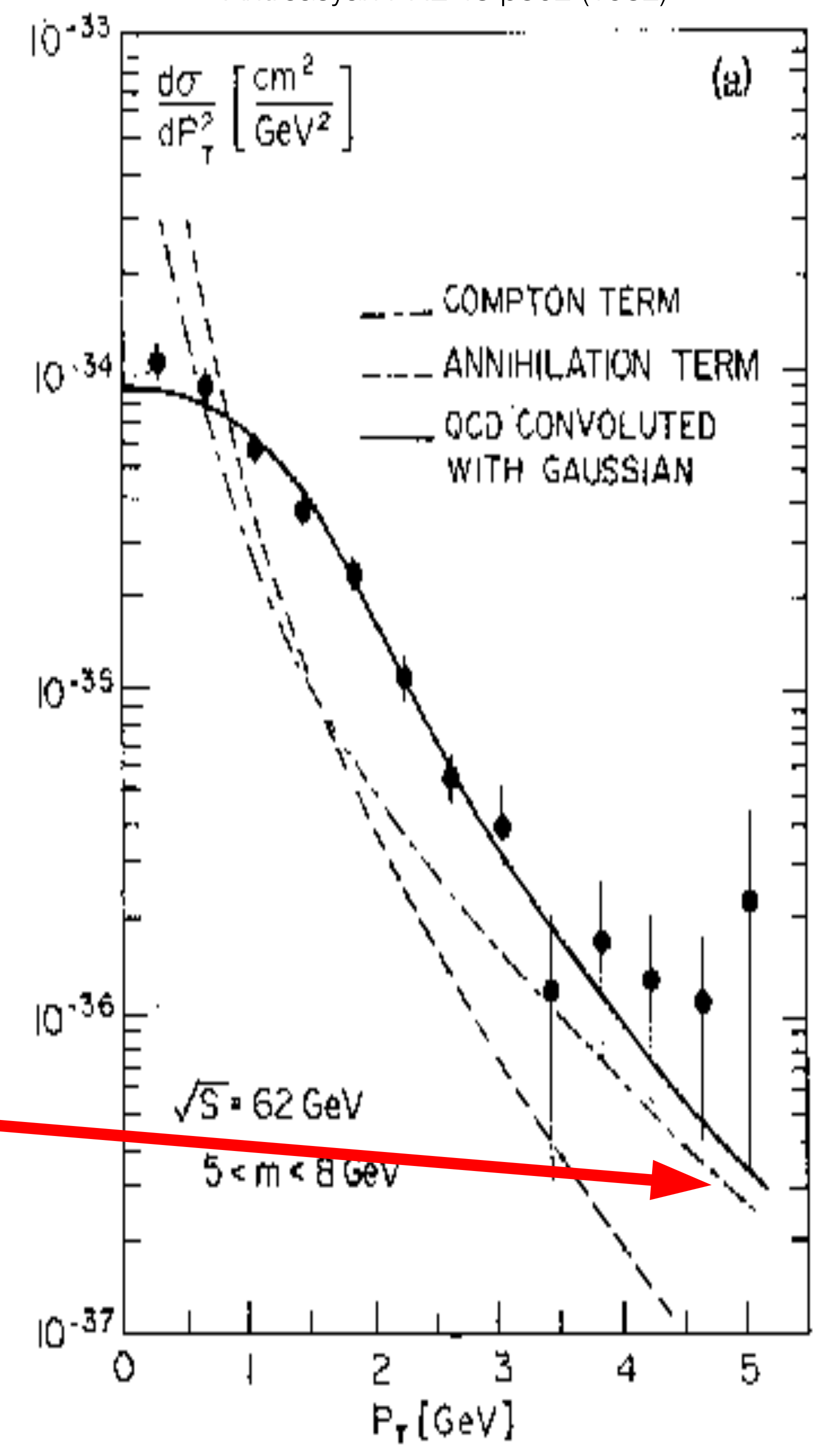
$$\frac{d\sigma}{dM^2 dy dp_t^2} = \frac{8}{27}\frac{\alpha^2}{M^2}\frac{1}{p_T^2}\int_{x_a^{min}}^1 dx_a H_q(x_a, x_b, M^2)$$
$$\frac{x_a\, x_b}{x_a - x_1}\left(1 + \frac{\tau^2}{(x_a x_b)^2} - \frac{x_T^2}{2 x_a x_b}\right)$$

annihilation term gives a p_t^{-2} tail to the p_t distribution (this falls off more slowly than a gaussian)

➔ **Tail of pt distribution can be calculated in QCD**

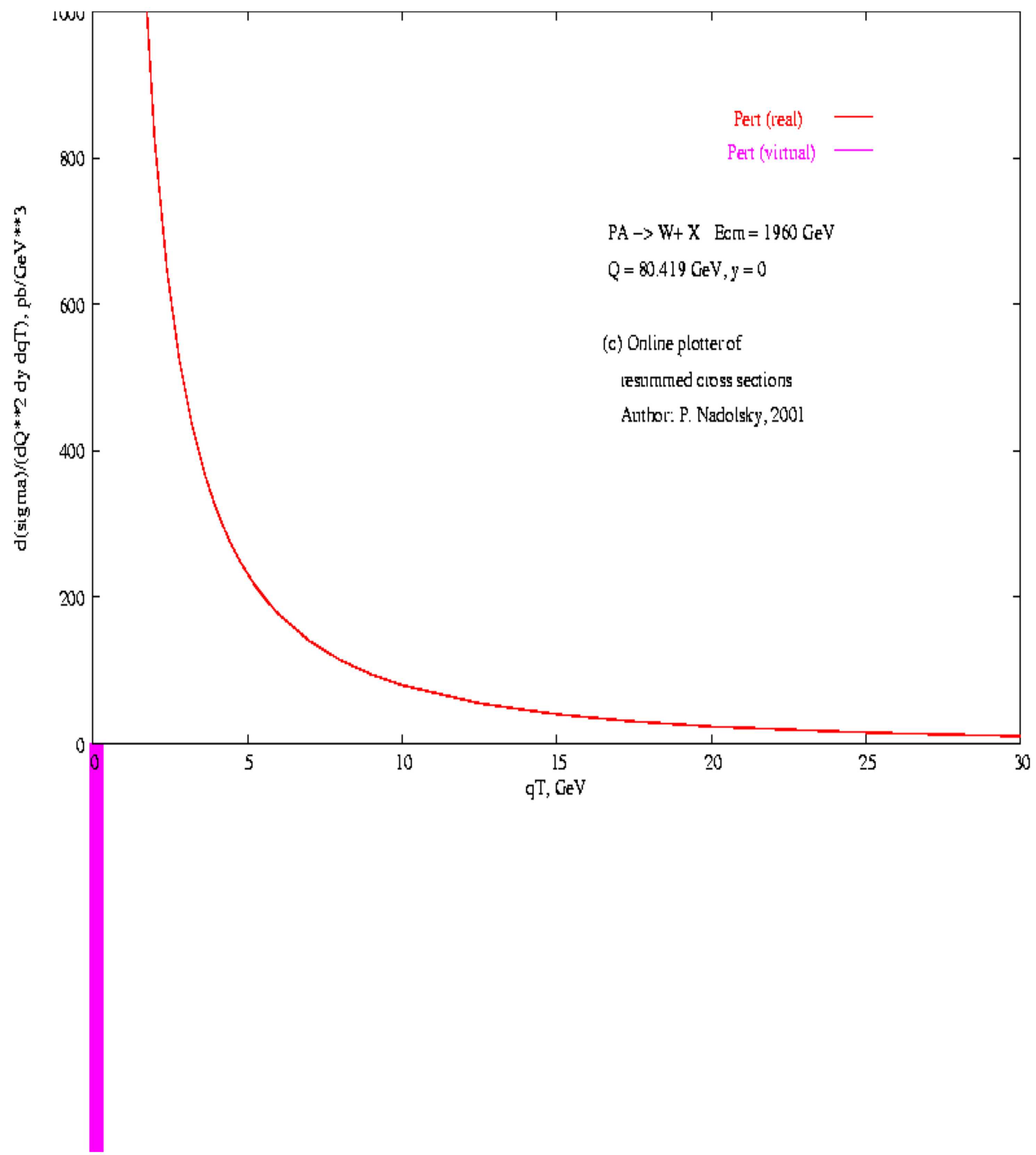
R. Field, Appl. of pQCD, p195 ff

Antreasyan PRL 48 p302 (1982)



How to obtain a finite x-section ?

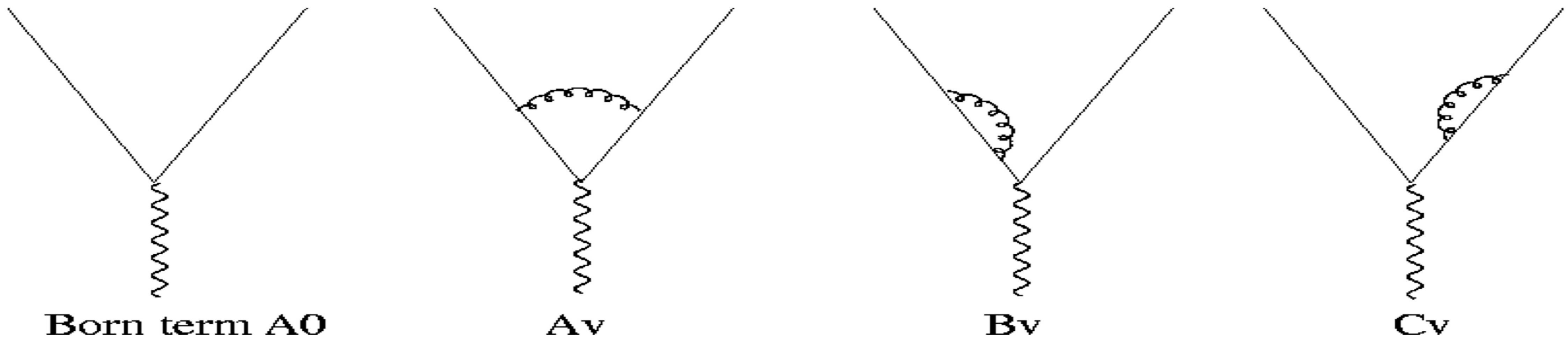
- Perturbative calculations of $\mathcal{O}(\alpha_S),\ \ \mathcal{O}(\alpha_S^2)$ diverge for small p_t
- virtual corrections are expected to cancel small p_t divergency



http://hep.pa.msu.edu/wwwlegacy/

… virtual corrections

R. Field, App pQCD, p31 ff



- amplitudes must be added:

$$|A_0 + A_v + B_v + C_v|^2 = |A_0|^2 + 2Re(A_0 A_v^* + A_0 A_v^* + A_0 C_v^*) + |A_v + B_v + C_v|^2$$

enter loop integrals which are divergent for $\cdot \to \infty$ and $\cdot \to 0$

- Adding vertex + self-energy diagrams
- UV divergencies cancel (similar to that in calc of α_{em})

➔ only IR divergencies stay…. and can cancel real emissions

LO/NLO K-factors

- K-factor: $$K = \frac{\sigma(LO)}{\sigma(NLO)}$$

Hard interactions of quarks and gluons: a primer for LHC physics J M Campbell et al 2007 Rep. Prog. Phys. 70 89-193

Hard interactions of quarks and gluons: a primer for LHC physics 113

Table 1. K-factors for various processes at the Tevatron and the LHC calculated using a selection of input parameters. In all cases, the CTEQ6M pdf set is used at NLO. $\mathcal{K}$ uses the CTEQ6L1 set at leading order, whilst $\mathcal{K}'$ uses the same set, CTEQ6M, as at NLO. Jets satisfy the requirements $p_T > 15$ GeV and $|\eta| < 2.5$ (5.0) at the Tevatron (LHC). In the $W + 2$ jet process the jets are separated by $\Delta R > 0.52$, whilst the weak boson fusion (WBF) calculations are performed for a Higgs boson of mass 120 GeV. Both renormalization and factorization scales are equal to the scale indicated.

	Typical scales		Tevatron K-factor			LHC K-factor		
Process	μ_0	μ_1	$\mathcal{K}(\mu_0)$	$\mathcal{K}(\mu_1)$	$\mathcal{K}'(\mu_0)$	$\mathcal{K}(\mu_0)$	$\mathcal{K}(\mu_1)$	$\mathcal{K}'(\mu_0)$
W	m_W	$2m_W$	1.33	1.31	1.21	1.15	1.05	1.15
$W + 1$ jet	m_W	$\langle p_T^{\text{jet}} \rangle$	1.42	1.20	1.43	1.21	1.32	1.42
$W + 2$ jets	m_W	$\langle p_T^{\text{jet}} \rangle$	1.16	0.91	1.29	0.89	0.88	1.10
$t\bar{t}$	m_t	$2m_t$	1.08	1.31	1.24	1.40	1.59	1.48
$b\bar{b}$	m_b	$2m_b$	1.20	1.21	2.10	0.98	0.84	2.51
Higgs via WBF	m_H	$\langle p_T^{\text{jet}} \rangle$	1.07	0.97	1.07	1.23	1.34	1.09

Even higher orders are calculated !

W.L. van Neerven and E.B. Zijistra
NPB 382 (1992) 11

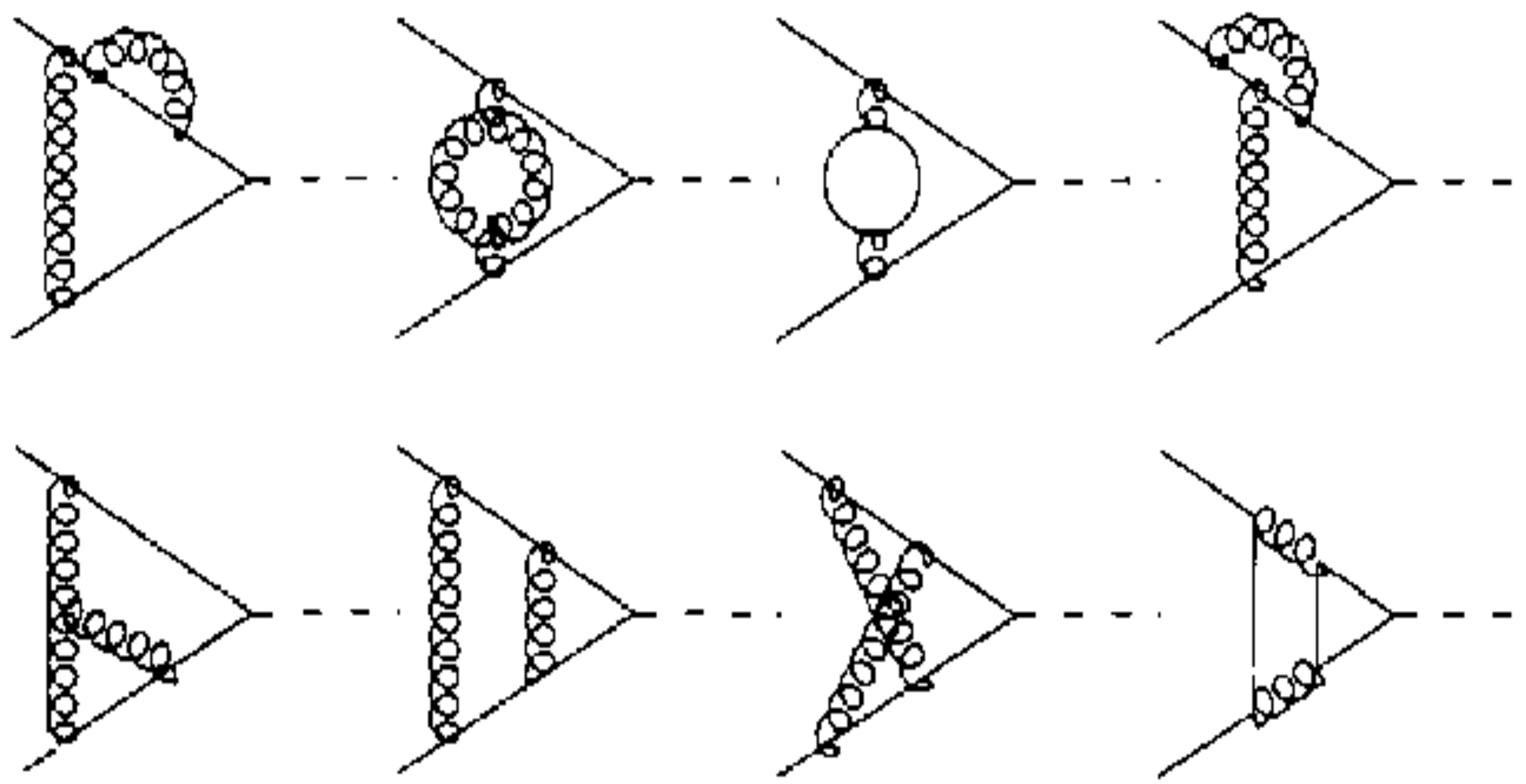


Fig. 4. The two-loop corrections to the process $q + \overline{q} \rightarrow V$.

Even higher orders are calculated !

W.L. van Neerven and E.B. Zijistra
NPB 382 (1992) 11

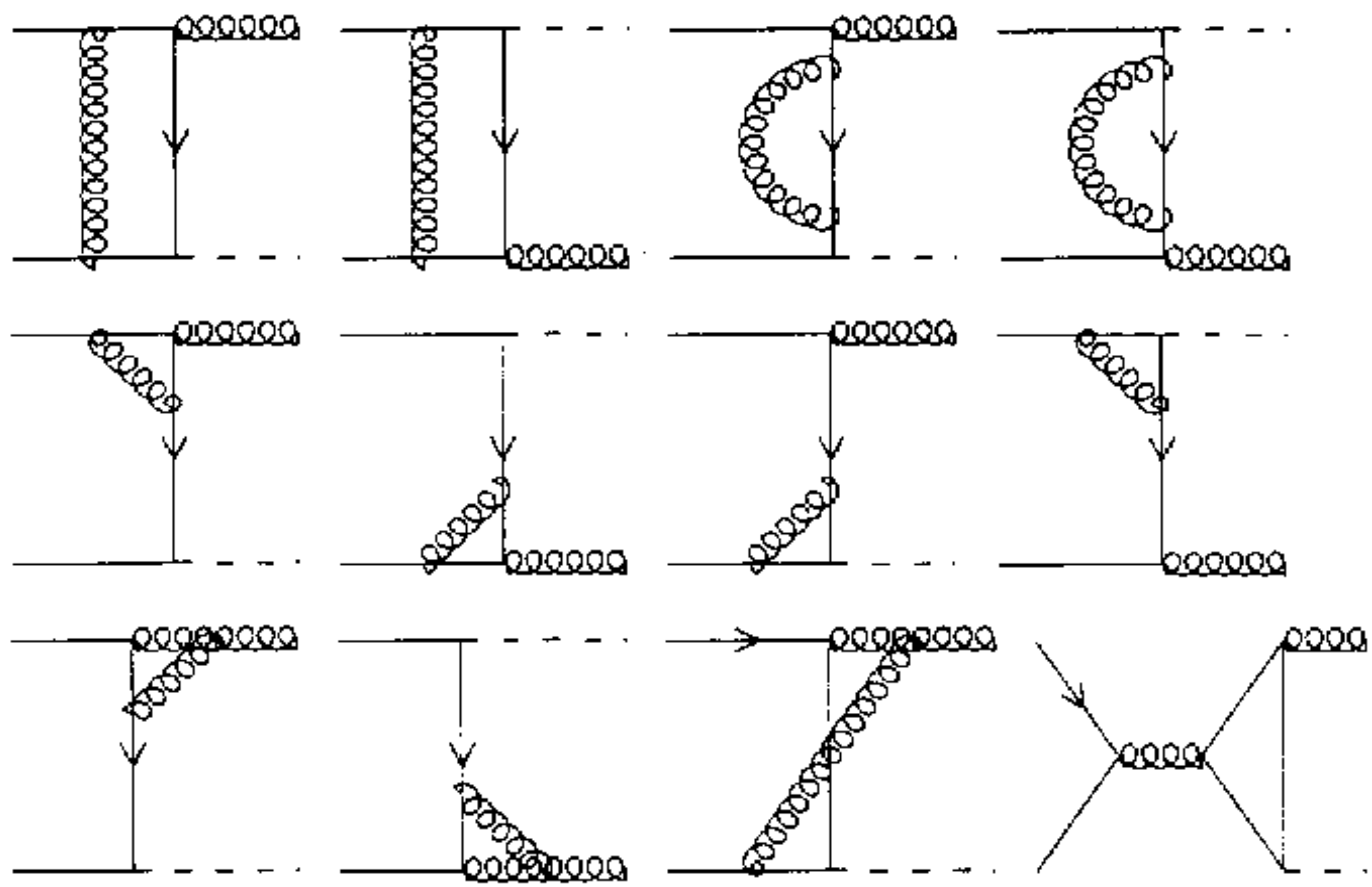


Fig. 5. The one-loop corrections to the process $q + \overline{q} \to V + g$. The diagrams corresponding to the one-loop correction to the subprocess $q(\overline{q}) + g \to V + q(\overline{q})$ can be obtained via crossing.

Even higher orders are calculated !

W.L. van Neerven and E.B. Zijistra
NPB 382 (1992) 11

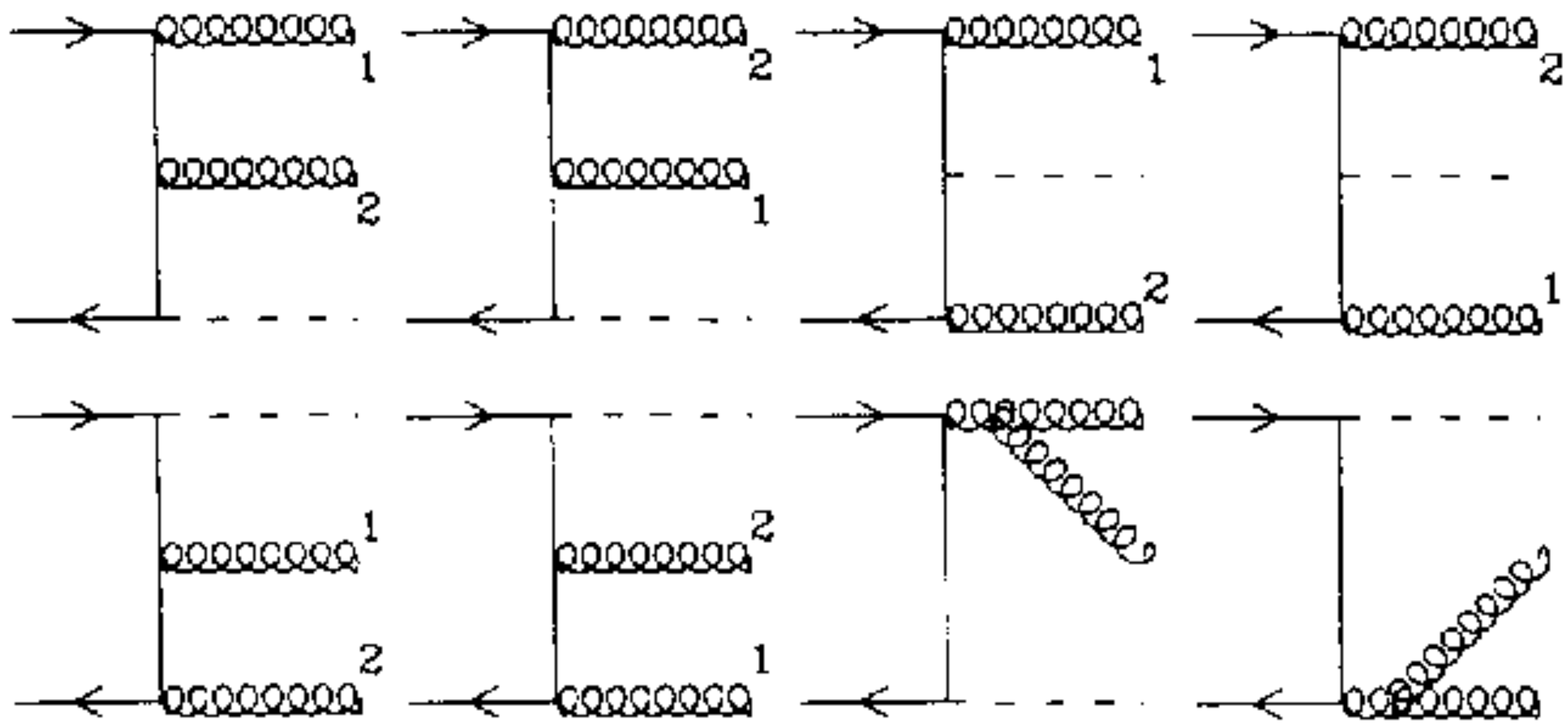
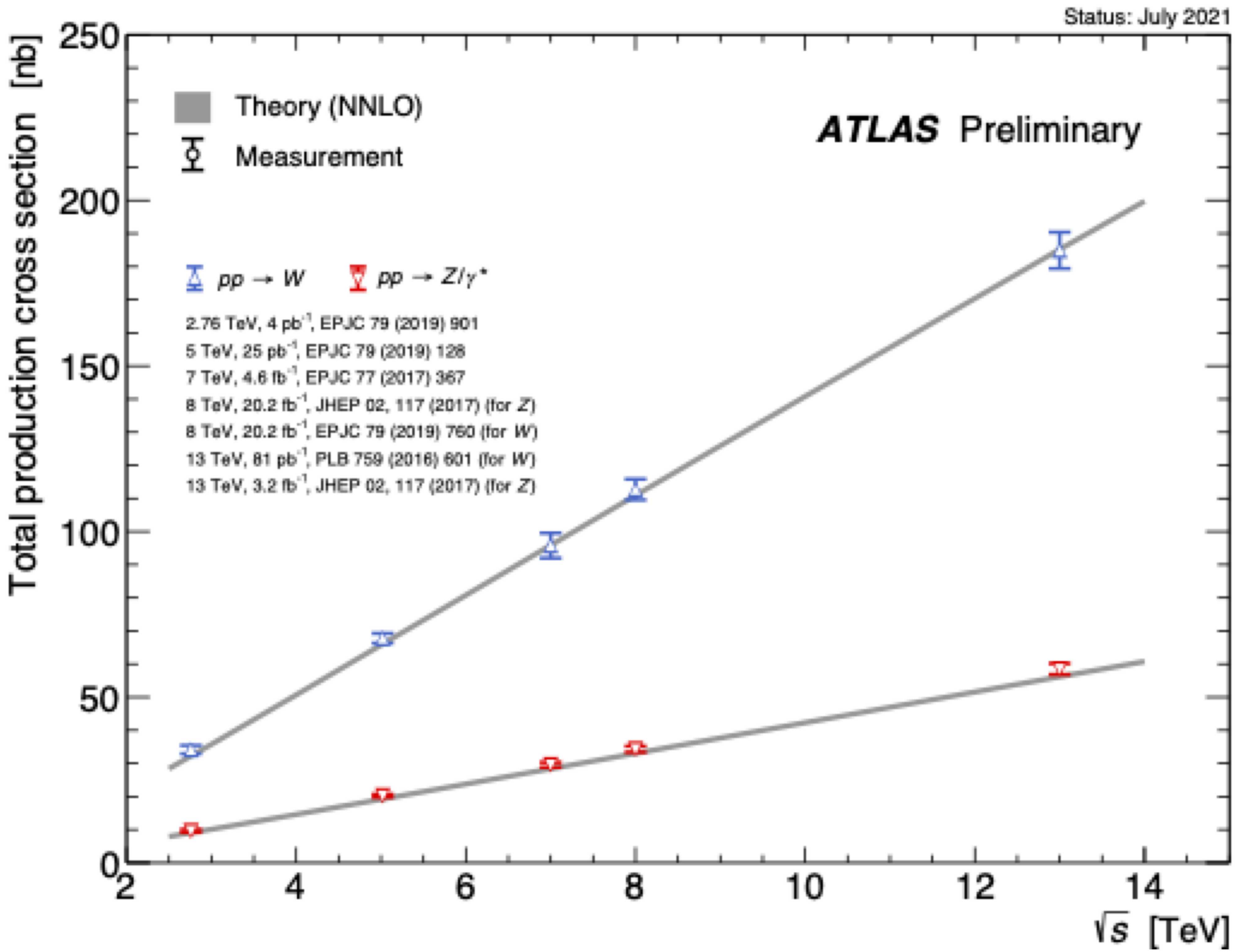
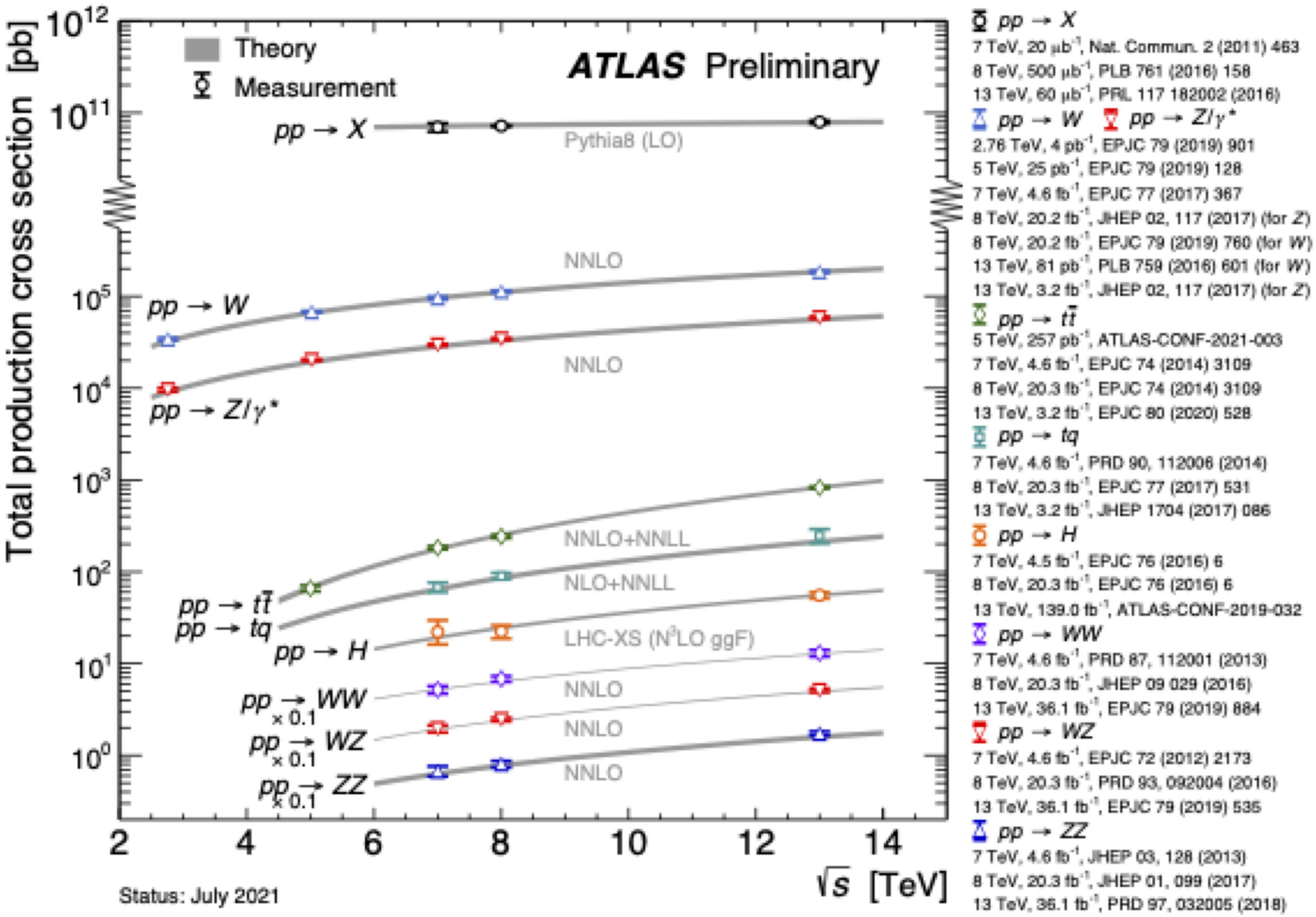


Fig. 6. Diagrams contributing to the subprocess $q + \bar{q} \to V + g + g$. The graphs corresponding to the subprocess $q(\bar{q}) + g \to V + q(\bar{q}) + g$ can be obtained from those presented in this figure via crossing. By crossing two pairs of lines one can obtain the diagrams corresponding to the subprocess $g + g \to V + q + \bar{q}$.

W/Z cross section summary (LHC)

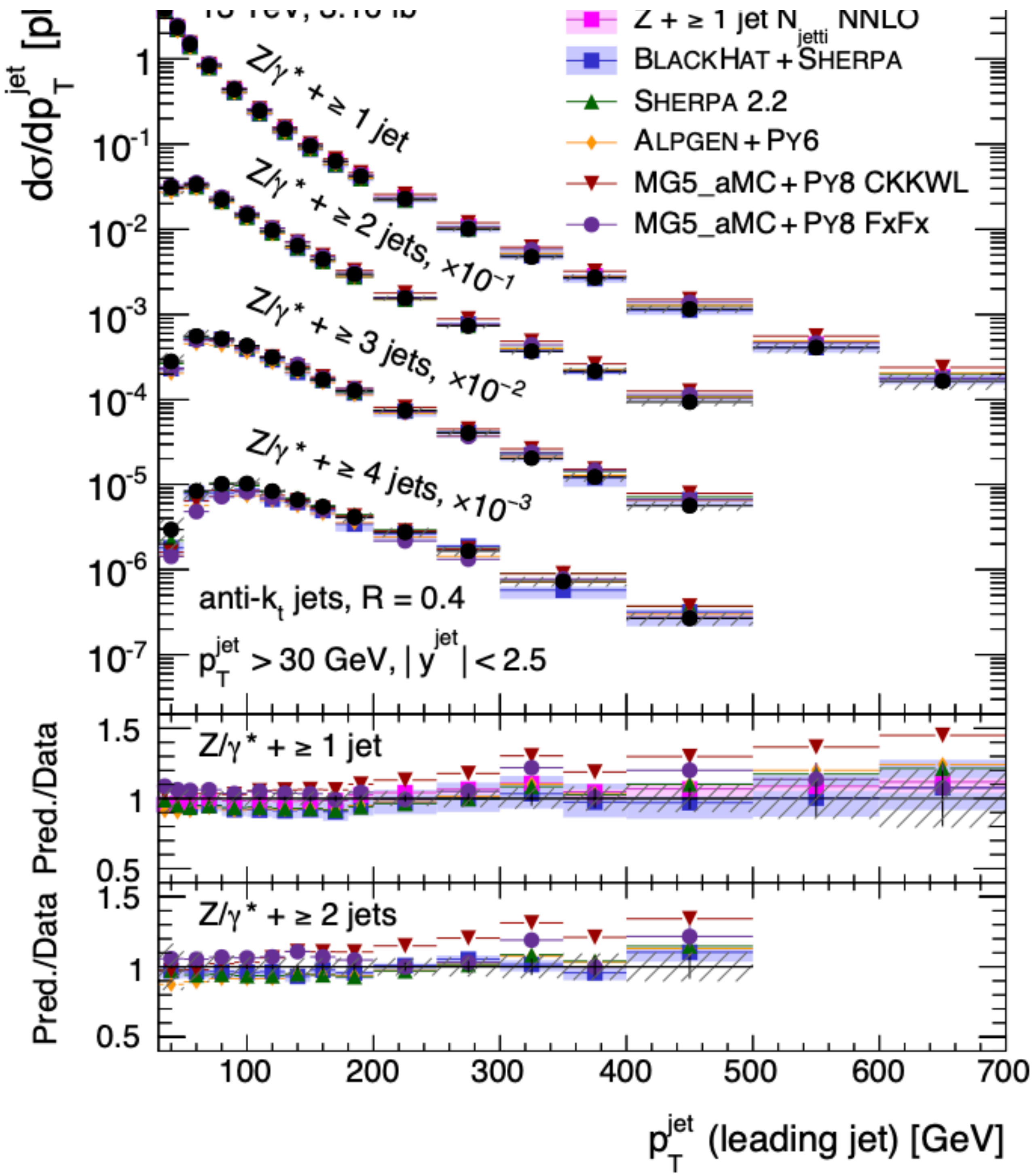


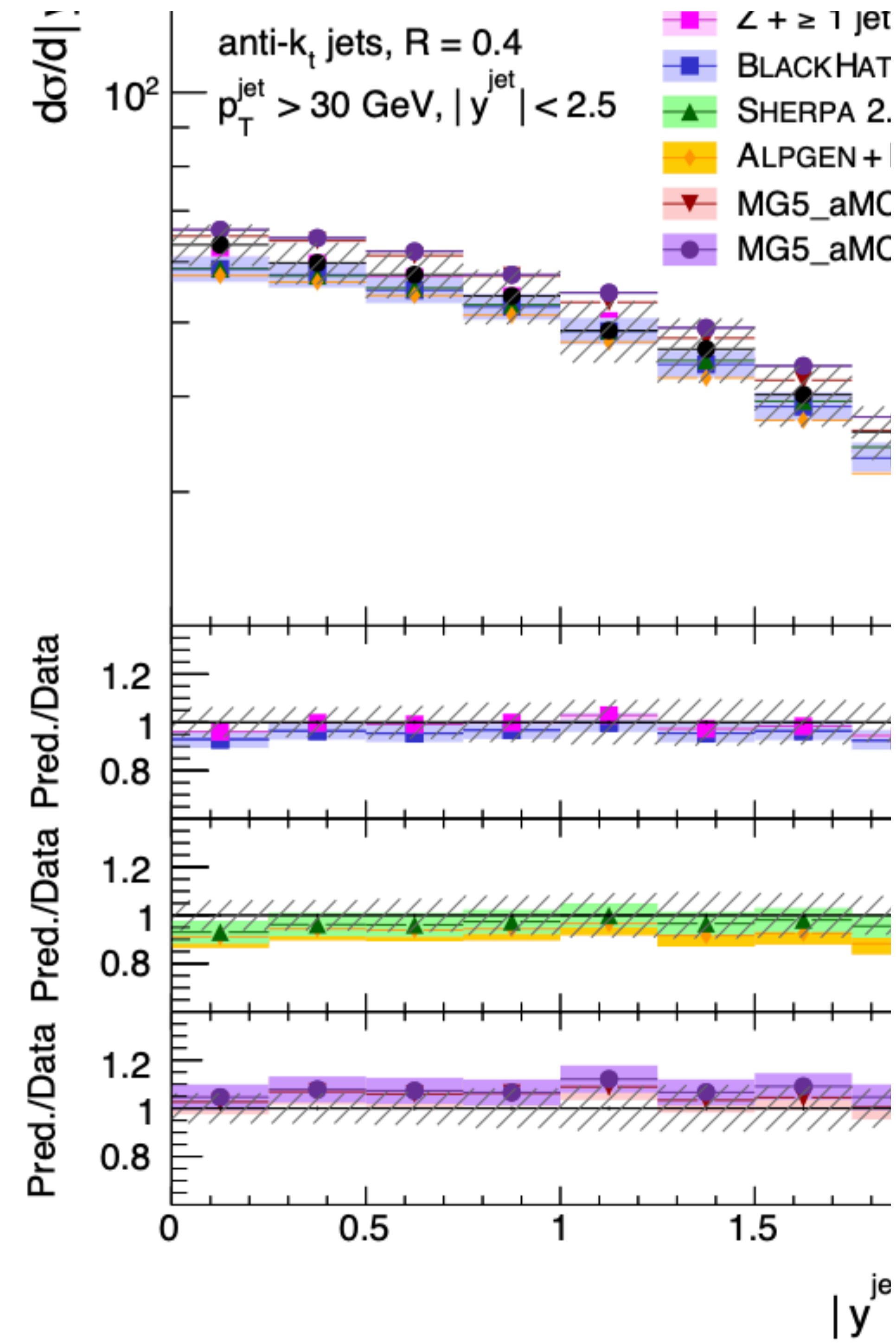
Total cross section at LHC



Z+jet measurements

Aaboud, M. and others Measurements of the production cross section of a Z boson in association with jets in pp collisions at $s = 13$ TeV with the ATLAS detector, Eur. Phys. J., C77(2017), 361





What happens at small p_t ?

What happens at small p_T ?

- taking the limit of small p_t:

$$\frac{d\sigma}{dM^2 dy dp_t^2} = \frac{8}{27}\frac{\alpha^2}{sM^2}\frac{1}{p_T^2}\int_{x_a^{min}}^{1} dx_a H_q(x_a, x_b, M^2)$$

$$\frac{x_a\, x_b}{x_a - x_1}\left(1 + \frac{\tau^2}{(x_a x_b)^2} - \frac{x_T^2}{2x_a x_b}\right)$$

$$\sim \frac{8}{27}\frac{\alpha^2}{sM^2}\frac{2}{p_T^2} H_q(x_a, x_b, M^2)\log\frac{s}{p_t^2}$$

$$= \left(\frac{d\sigma}{dM^2 dy}\right)_{Born} \times \left(\frac{4\alpha_s}{3\pi}\frac{1}{p_t^2}\log\frac{s}{p_t^2}\right)$$

$P_{qq} = \frac{4}{3}\alpha\frac{1+z^2}{1-z}$

$\frac{1}{t} \to \log\frac{t}{t_0}$

- with $$\left(\frac{d\sigma}{dM^2 dy}\right)_{Born} = \frac{4\pi\alpha^2}{9sM^2} H_q(x_a, x_b, M^2)$$

- cross section diverges as for $p_t \rightarrow 0$: $$\frac{\log\frac{s}{p_t^2}}{p_t^2}$$

Original References...

PHYSICS REPORTS (Review Section of Physics Letters) 58, No. 5 (1980) 269–395. North-Holland Publishing Company

HARD PROCESSES IN QUANTUM CHROMODYNAMICS

Yu.L. DOKSHITZER, D.I. DYAKONOV and S.I. TROYAN

Leningrad Nuclear Physics Institute, Gatchina,
Leningrad 188350, U.S.S.R.

Received 28 May 1979

Nuclear Physics B154 (1979) 427–440

SMALL TRANSVERSE MOMENTUM DISTRIBUTIONS IN HARD PROCESSES

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CERN, Geneva, Switzerland

Received 8 February 1979

Small p_T x-section

- x-section at small pt

$$\frac{d\sigma}{dM^2 dy dp_t^2} \sim \frac{8}{27}\frac{\alpha^2}{sM^2}\frac{2}{p_T^2} H_q(x_a, x_b, M^2) \log\frac{s}{p_t^2}$$
$$= \left(\frac{d\sigma}{dM^2 dy}\right)_{Born} \times \left(\frac{4\alpha_s}{3\pi}\frac{1}{p_t^2}\log\frac{s}{p_t^2}\right)$$

- with $\left(\frac{d\sigma}{dM^2 dy}\right)_{Born} = \frac{4\pi\alpha^2}{9sM^2} H_q(x_a, x_b, M^2)$

$$\int_0^s \frac{d\sigma}{dM^2 dy dp_t} = \int_0^{p_\perp^2} \cdots + \int_{p_t}^s \cdots$$

$$\int_0^{p_t} \cdots = \int_0^s - \int_{p_t}^s$$

$$\int_0^{p_t} \cdots = \left.\frac{d\sigma}{dp_t^2 dy}\right|_{Born} \left(1 + \delta_{K\,factor}\right) - \int_{p_\perp}^{s} \frac{d\sigma}{dp_t^2 dy\, dp_f'^2}$$

$$= \left.\frac{d\sigma}{dp_t^2 dy}\right|_{Born} \left(1 - \frac{4\alpha_s}{3\pi} \int_{p_\perp^2}^{s} \frac{\ln s/p_f'^2}{p_\perp'^2}\, dp_f'^2 \right)$$

$$= \left(1 - \frac{4\alpha_s}{3\pi} \left(-\frac{1}{2}\right) \left\{ - \left(\ln \frac{s}{p_f^2}\right)^2 \right)$$

$$= \left(1 - \frac{2\alpha_s}{3\pi} \left(\ln \frac{s}{p_f^2}\right)^2 \right)$$

$$= \left.\frac{d\sigma}{dp_t^2 dy}\right|_{Born} \exp\left\{ - \frac{2\alpha_s}{3\pi} \ln^2 \frac{s}{p_f^2} \right)$$

$$\frac{d\sigma}{dM^2\,dy\,dp_t^2} = \left.\frac{d\sigma}{dM^2\,dy}\right|_{Born} \cdot \frac{4\alpha_s}{3\pi}\,\frac{s}{p_t}\,\ln\frac{s}{p_t^2} \cdot$$

$$\exp\left\{-\frac{2\alpha_s}{3\pi}\,\ln^2\frac{s}{p_t^2}\right\}$$

Small p_T x-section

- x-section at small pt

$$\frac{d\sigma}{dM^2 dy dp_t^2} \sim \frac{8}{27}\frac{\alpha^2}{sM^2}\frac{2}{p_T^2} H_q(x_a, x_b, M^2) \log\frac{s}{p_t^2}$$
$$= \left(\frac{d\sigma}{dM^2 dy}\right)_{Born} \times \left(\frac{4\alpha_s}{3\pi}\frac{1}{p_t^2}\log\frac{s}{p_t^2}\right)$$

- with $\left(\frac{d\sigma}{dM^2 dy}\right)_{Born} = \frac{4\pi\alpha^2}{9sM^2} H_q(x_a, x_b, M^2)$

- from previous we know, that integral over p_t^2 is finite:

$$\int_0^s \frac{d\sigma}{dM^2 dy dp_t^2} = \left(\frac{d\sigma}{dM^2 dy}\right) + \mathcal{O}()$$

➔ which gives

$$\int_0^{p_t^2} \frac{d\sigma}{dM^2 dy dp_t^2} = \left(\frac{d\sigma}{dM^2 dy}\right)_{Born} \left[1 - \int_{p_t^2}^s \frac{4}{3\pi}\frac{\log s/p_t^2}{p_t^2} dp_t^2\right]$$
$$= \left(\frac{d\sigma}{dM^2 dy}\right)_{Born} \left[1 - \frac{2}{3\pi}\log^2 s/p_t^2\right]$$

small p_T -resummation to all orders

- Result suggest series of logs...: $1 - a + \frac{a^2}{2!} - \frac{a^3}{3!} ... = \exp(a)$

$$\int_0^{p_t^2} \frac{d\sigma}{dM^2 dy dp_t^2} = \left(\frac{d\sigma}{dM^2 dy}\right)_{Born} \exp\left(-\frac{2}{3\pi}\log^2 s/p_t^2\right)$$

differentiate wrt $p_t{}^2$:

$$\frac{d\sigma}{dM^2 dy dp_t^2} = \left(\frac{d\sigma}{dM^2 dy}\right)_{Born} \left(\frac{s}{p_t^2}\frac{4}{3\pi}\log s/p_t^2\right) \exp\left(-\frac{2}{3\pi}\log^2 s/p_t^2\right)$$

➔ Sudakov form factor appears

➔ expresses resummation of leading double logs

exponential cancels singularity at $p_T \rightarrow 0$

➔ Probability to produce massive lepton pair (or Z_0, W etc) without additional soft gluon radiation is ZERO