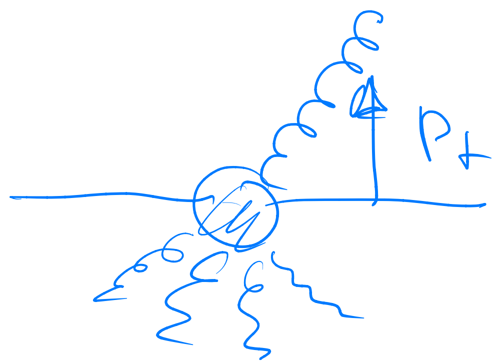
QCD at the Extremes

All order resummation - kinematics

$$\frac{d\sigma}{dM^2\, dy\, dp_\perp} = \left(\frac{d\sigma}{dM^2 dy}\right)_{Born} \cdot \frac{4\alpha_s}{3\pi} \frac{1}{p_\perp^2} \log \frac{s}{p_\perp^2}$$



$$\frac{d\sigma^{(N)}}{dp_\perp^2} = \sigma_0 \prod_{i=1}^{N} \int d^2 k_{\perp i} \cdot M^{(N)} \cdot \delta\left(\sum k_\perp + p_\perp\right)$$

↳ ME for emission of N gluons

$$|M^{(N)}| = \prod_i \frac{1}{k_{\perp i}^2} \ln \frac{s}{k_{\perp i}^2}$$

All order resummation - kinematics

$$\frac{1}{\sigma_0}\frac{d\sigma}{dp_T^2} \sim \prod_{i=1}^{N} \frac{d^2 k_{\perp i}}{k_{\perp i}^2} \ln\frac{s}{k_{\perp i}^2}\, \delta\left(\sum k_{\perp i} + p_\perp\right)$$

$$\hookrightarrow \delta\left(k_{\perp last} + p_\perp\right)$$

strong ordering: $k_{\perp a}^2 \ll k_{\perp b}^2 \ll k_{\perp c}^2 \leq p_\perp^2 \ll s$

$$\sim \frac{1}{p_\perp^2}\ln\frac{s}{p_\perp^2} \int \frac{d^2 k_{\perp N-1}}{k_{\perp N-1}^2} \ln\frac{s}{k_{\perp N-1}^2} \int^{k_{\perp N-1}} \frac{d^2 k_{\perp N-2}}{k_{\perp N-2}^2} \ln\frac{s}{k_{\perp N-2}^2}$$

$\ldots$

$$\frac{d\sigma^{(1)}}{dp_\perp^2} \sim \overbrace{\frac{1}{p_\perp^2}\ln\frac{s}{p_\perp^2}}^{A}$$

$$\frac{d\sigma^{(2)}}{dp_\perp^2} \sim \ldots A \cdot \int \frac{dk_\perp^2}{k_\perp^2}\ln\frac{s}{k_\perp^2} = A \cdot \frac{1}{2}\ln^2\frac{s}{k_\perp^2}$$

All order resummation - kinematics

Ellis, Fleishon,Stirling, PRD 24,1386 (1981)

- impose kinematic constraints ... delta function for k_t

$$\frac{1}{\sigma}\frac{d\sigma^{(N)}}{dp_t^2} \sim \prod_{i=1}^{N}\left[\int d^2k_{ti}dx_iM^{(N)}\right]\delta\left(\sum \vec{k}_{ti}+\vec{p}_t\right)$$

  - with for soft gluons $M^{(N)} \sim \prod \frac{}{k_{ti}^2}$

➔ in limit of strong ordering:

$$\frac{1}{\sigma}\frac{d\sigma^{(N)}}{dp_t^2} \sim \prod_{i=1}^{N}\left[\int \frac{d^2k_{ti}}{k_{ti}^2}\log\frac{s}{k_{ti}^2}\right]\delta\left(\sum \vec{k}_{ti}+\vec{p}_t\right)$$

$$\frac{1}{\sigma}\frac{d\sigma^{(N)}}{dp_t^2} \sim p_t^2\log\frac{s}{p_t^2}\int\frac{d^2k_{t,N-1}}{k_{t,N-1}^2}\log\frac{s}{k_{t,N-1}^2}\int\frac{d^2k_{t,N-2}}{k_{t,N-2}^2}\log\frac{s}{k_{t,N-2}^2}\cdots$$

All order resummation

- Iterative procedure:

$$\frac{d\sigma^{(1)}}{dp_t^2} \simeq \frac{1}{p_t^2}\log\frac{s}{p_t^2} = A$$

$$\frac{d\sigma^{(2)}}{dp_t^2} \simeq A\int\frac{d^2k_t}{k_t^2}\log\frac{s}{k_t^2} = A\frac{1}{2}\log^2\frac{s}{k_t^2}$$

$$\frac{d\sigma^{(3)}}{dp_t^2} \simeq \int\frac{d^2k_t}{k_t^2}\log\frac{s}{k_t^2}\frac{d\sigma^{(2)}}{dp_t^2} = A\frac{1}{2}\int\frac{d^2k_t}{k_t^2}\log^3\frac{s}{k_t^2} = A\frac{1}{2}\left(\frac{1}{2}\log^2\frac{s}{k_t^2}\right)^2$$

$$\frac{d\sigma^{(4)}}{dp_t^2} \simeq \int\frac{d^2k_t}{k_t^2}\log\frac{s}{k_t^2}\frac{d\sigma^{(3)}}{dp_t^2} = A\frac{1}{2}\frac{1}{4}\int\frac{d^2k_t}{k_t^2}\log^5\frac{s}{k_t^2} = A\frac{1}{2}\frac{1}{3}\left(\frac{1}{2}\log^2\frac{s}{k_t^2}\right)^3$$

$$\frac{d\sigma^{(N)}}{dp_t^2} \simeq A\frac{1}{(N-1)!}\left(\frac{1}{2}\log^2\frac{s}{k_t^2}\right)^{N-1}$$

- x-section for up to N gluons:

$$\frac{d\sigma}{dp_t^2} = \sum_i\frac{d\sigma^{(i)}}{dp_t^2} \simeq A\sum_i\frac{1}{(i-1)!}\left(\frac{1}{2}\log^2\frac{s}{p_t^2}\right)^{i-1}$$

$$\simeq A\exp\left[\frac{1}{2}\log^2\frac{s}{p_t^2}\right]$$

→ Sudakov form factor!

All order resummation - kinematics

Ellis, Fleishon,Stirling, PRD 24,1386 (1981)

- impose kinematic constraints ... delta function for k_t

$$\frac{1}{\sigma}\frac{d\sigma^{(N)}}{dp_t^2} \sim \prod_{i=1}^{N}\left[\int d^2k_{ti}dx_iM^{(N)}\right]\delta\left(\sum \vec{k}_{ti}+\vec{p}_t\right)$$

  - with for soft gluons $M^{(N)} \sim \prod \frac{}{k_{ti}^2}$

$$\frac{1}{\sigma}\frac{d\sigma^{(N)}}{dp_t^2} \sim \prod_{i=1}^{N}\int \frac{d^2k_{ti}}{k_{ti}^2}\log\frac{s}{k_{ti}^2}\delta\left(\sum k_{ti}+p_t\right)$$

- relaxing strong ordering constraint → keep delta function...
- instead of nested integrals... integrals are coupled

➔ Collins Soper Sterman (CSS) formalism

➔ implement energy momentum conservation by Fourier transform to b-space

➔ obtain sub-leading corrections from energy-momentum conservation !

All order resummation – kinematics II

Ellis, Fleishon,Stirling, PRD 24,1386 (1981)

- relaxing strong ordering constraint → keep delta function...
- instead of nested integrals... integrals are coupled

➔ Collins Soper Sterman (CSS) formalism

➔ implement energy momentum conservation by Fourier transform to b-space .... obtain subleading corrections

or alternative formalism in k_t space (Ellis, Veseli NPB511, 649 (1998))

$$\frac{d\sigma}{dM^2 dy dp_t^2} = \sum_q \sigma_0^{qq} \frac{d}{dp_t^2} \left([q(x_a, p_t) q(x_b, p_t) + a \leftrightarrow b] \right.$$

$$\left. \times \exp\left(- \int_{p_t^2}^{M^2} \frac{d\mu^2}{\mu^2} \left[A \log\left(M^2/\mu^2\right) + B \right] \right) \right)$$

➔ with coefficients from energy momentum conservation as in CSS: $A(\alpha_s), B(\alpha_s)$

Correspondence of PB – TMDs with CSS

$$\Delta_S = \exp\left\{ -\frac{\alpha_s}{2\pi} \int_{\mu_0}^{\mu} \frac{d\mu'^2}{\mu'^2} \int dz\, P(z) + \text{virtual terms} \right\}$$

$$P_{qq} = \frac{4}{3} \frac{1+z^2}{1-z} + \frac{3}{2}\delta(1-z) \quad \text{for } m$$

$$\sim \frac{4}{3} \frac{2}{1-z} + \frac{3}{2}\delta(1-z)$$

angular ordering: $z_{cut} = z_{dyn} = 1 - \frac{q_0'}{\mu}$

$\alpha_s(q_t)$

$q_t \leftrightarrow \mu$

$$\Delta_S = \exp\left\{ \ldots \int_{\mu_0}^{\mu^2} \frac{d\mu'^2}{\mu'^2}\, 2\log\frac{\mu}{\mu'} + \frac{3}{2} \right\}$$

$$\Delta_{CSS} = \exp\left\{ \ldots \int \frac{d\mu'^2}{\mu'^2}\, A \log\frac{\mu}{\mu'} + B \right\}$$

$$P_{ab} = \delta_{ab} f_b \delta(1-z) + \delta_{ab} \delta_K \frac{1}{(1-z)_+} + R_{ab}$$

$$\mu^2 \frac{\partial f}{\partial \mu^2} = \sum_q \int^1 \frac{dz}{z} P_{ab} f\left(\frac{x}{z}, \mu^2\right)$$

$$= \sum_q \int_x^1 \frac{dz}{z} \left(\delta_{ab} \delta_b \delta(1-z) + \delta_{ab} \delta_K \frac{1}{(1-z)_+} + R_{ab} \right)$$

$$= \sum_q \int_x^1 \frac{dz}{z} \left(\frac{\delta_{ab} \delta_K}{1-z} + R_{ab} \right) f\left(\frac{x}{z}, \mu^2\right)$$

$$- f(x) \int dz \frac{k_a}{1-z} - d_a \delta(1-z))$$

$z_m = 1 - \varepsilon$, with $\varepsilon \to 0$

$$\Rightarrow \Delta_S = \exp \left\{ - \int \frac{d\mu^2}{\mu^2} \int_0^{z_m} dz \frac{1}{1-z} - d_a \right\}$$

Correspondence of PB – TMDs with CSS

$$P_{ab}(z,\alpha_s) = \delta_{ab} d_a(\alpha_s)\delta(1-z) + \delta_{ab} k_a(\alpha_s)\frac{1}{(1-z)_+} + R_{ab}(z,\alpha_s)$$

$$\begin{aligned}
\mu^2 \frac{\partial f_a(x,\mu^2)}{\partial \mu^2} &= \sum_b \int_x^1 \frac{dz}{z} \; P_{ab}\left(z\right) \; f_b\left(\frac{x}{z},\mu^2\right) \\
&= \sum_b \int_0^1 \frac{dz}{z} \; \left(\delta_{ab}\delta_a \delta(1-z) + \frac{\delta_{ab} k_a}{(1-z)_+} + R_{ab}(z)\right) \; f_b\left(\frac{x}{z}\right) \\
&= \sum_b \int_0^1 \frac{dz}{z} \left(\frac{\delta_{ab} k_a}{1-z} + R_{ab}(z)\right) f_b\left(\frac{x}{z}\right) \\
& \quad -f_a(z) \int_0^1 dz \left(\frac{k_a}{(1-z)} - d_a \delta(1-z)\right)
\end{aligned}$$

$$\Delta_a^S(\mu^2,\mu_0^2) = \exp\left(-\int_{\mu_0^2}^{\mu^2} \frac{\mathrm{d}\mu'^2}{\mu'^2}\left[\int_0^{z_M} k_a(\alpha_s)\frac{1}{1-z}\mathrm{d}z - d_a(\alpha_s)\right]\right)$$

Correspondence of PB – TMDs with CSS

https://arxiv.org/abs/2309.11802

- Essential part of soft gluons is missing: $z_M \leq z \leq 1$
  - non-perturbative region → non-perturbative Sudakov

$$\Delta_a^S(\mu^2, \mu_0^2) = \exp\left(- \int_{\mu_0^2}^{\mu^2} \frac{dq^2}{q^2} \left[\int_0^{z_{\rm dyn}(q)} dz \frac{k_a(\alpha_s)}{1-z} - d_a(\alpha_s) \right] \right)$$

$$\times \exp\left(- \int_{\mu_0^2}^{\mu^2} \frac{dq^2}{q^2} \int_{z_{\rm dyn}(q)}^{z_M} dz \frac{k_a(\alpha_s)}{1-z} \right)$$

$$= \Delta_a^{(\rm P)}\left(\mu^2, \mu_0^2, q_0^2\right) \cdot \Delta_a^{(\rm NP)}\left(\mu^2, \mu_0^2, \epsilon, q_0^2\right)$$

$q_t > q_0$ $\qquad$ $q_t < q_0$

- Non-pert. Sudakov form factor is calculable under certain conditions:
  - coming from soft gluon treatment within evolution equation

Correspondence of PB – TMDs with CSS

- Check correspondence of PB Sudakov form factor with CSS
- use only $P_{qq}(z)$ in large z limit: $P_{qq}(z) \sim \frac{1+z^2}{1-z} + \frac{3}{2}\delta(1-z) \to \frac{2}{1-z} + \frac{3}{2}\delta(1-z)$
- apply angular ordering constraint for z_M $\quad z_{\rm dyn} = 1 - q_0/q$

$$
\begin{aligned}
\Delta_s(z_M, \mu_0, \mu) &= \exp\left(-\frac{\alpha_s}{2\pi}\left[\int_{\mu_0^2}^{\mu^2}\frac{d\mu'^2}{\mu'^2}\int_0^{z_M} dz P(z) + \frac{3}{2}\right]\right) \\
&\sim \exp\left(-\frac{\alpha_s}{2\pi}\left[\int_{\mu_0^2}^{\mu^2}\frac{d\mu'^2}{\mu'^2}\int_0^{1-\frac{\mu'}{\mu}} dz\left[\frac{2}{1-z}\right] + \frac{3}{2}\right]\right) \\
&= \exp\left(-\frac{\alpha_s}{2\pi}\left[\int_{\mu_0^2}^{\mu^2}\frac{d\mu'^2}{\mu'^2} 2\log\frac{\mu^2}{\mu'^2} + \frac{3}{2}\right]\right)
\end{aligned}
$$

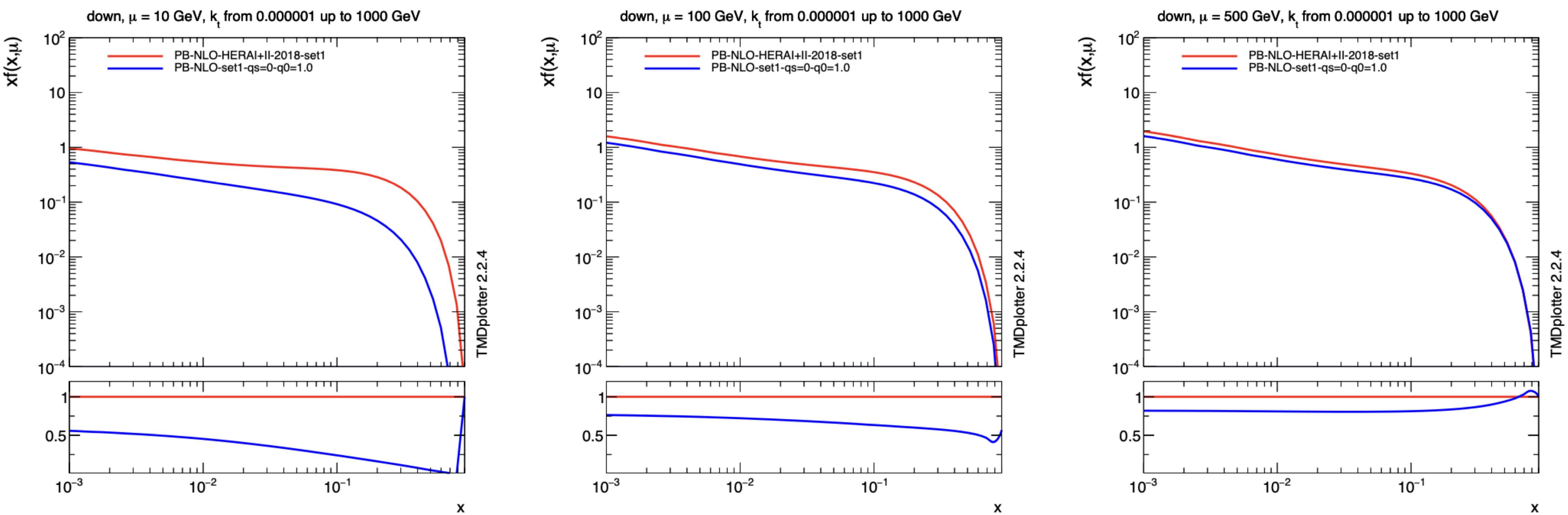
- Sudakov from PB formalism follows exactly to form of Sudakov from CSS:

$$
\Delta^{CSS} \sim \exp\left(-\int_{p_t^2}^{M^2}\frac{d\mu^2}{\mu^2}\left[A\log\left(M^2/\mu^2\right) + B\right]\right)
$$

Role of soft gluons in inclusive distributions

https://arxiv.org/abs/2309.11802

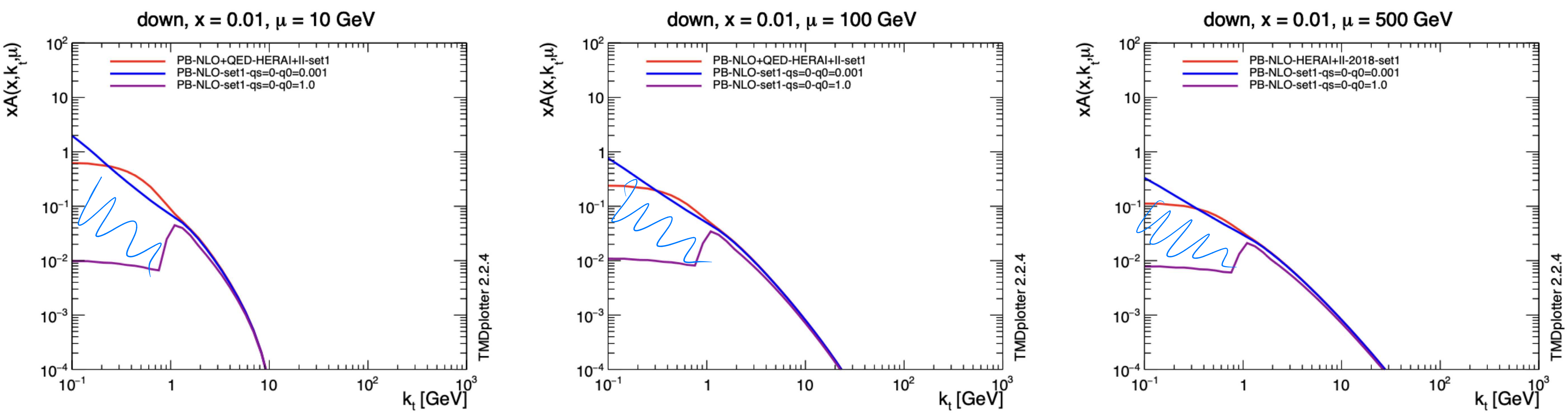
- Perform evolution with PB method with and without cut $z_{\mathrm{dyn}} = 1 - \frac{q_{t\ \mathrm{cut}}}{\mu'}$



Role of soft gluons in TMD distributions

https://arxiv.org/abs/2309.11802

- Perform evolution with PB method with and without cut $z_{\text{dyn}} = 1 - \frac{q_{t\ \text{cut}}}{\mu'}$

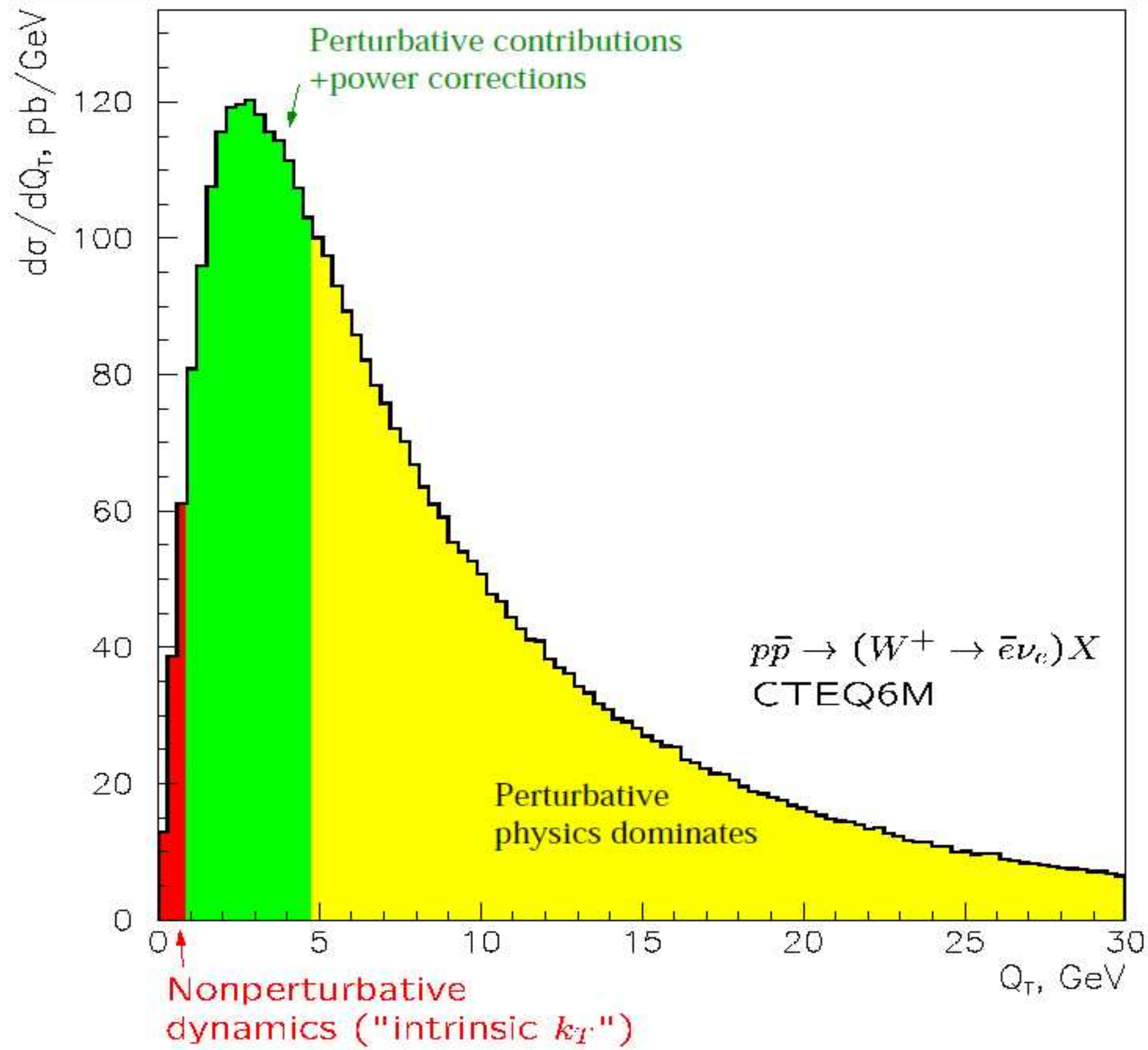


Transverse Momentum of W/Z

Fred Olness, CTEQ summerschool 2003

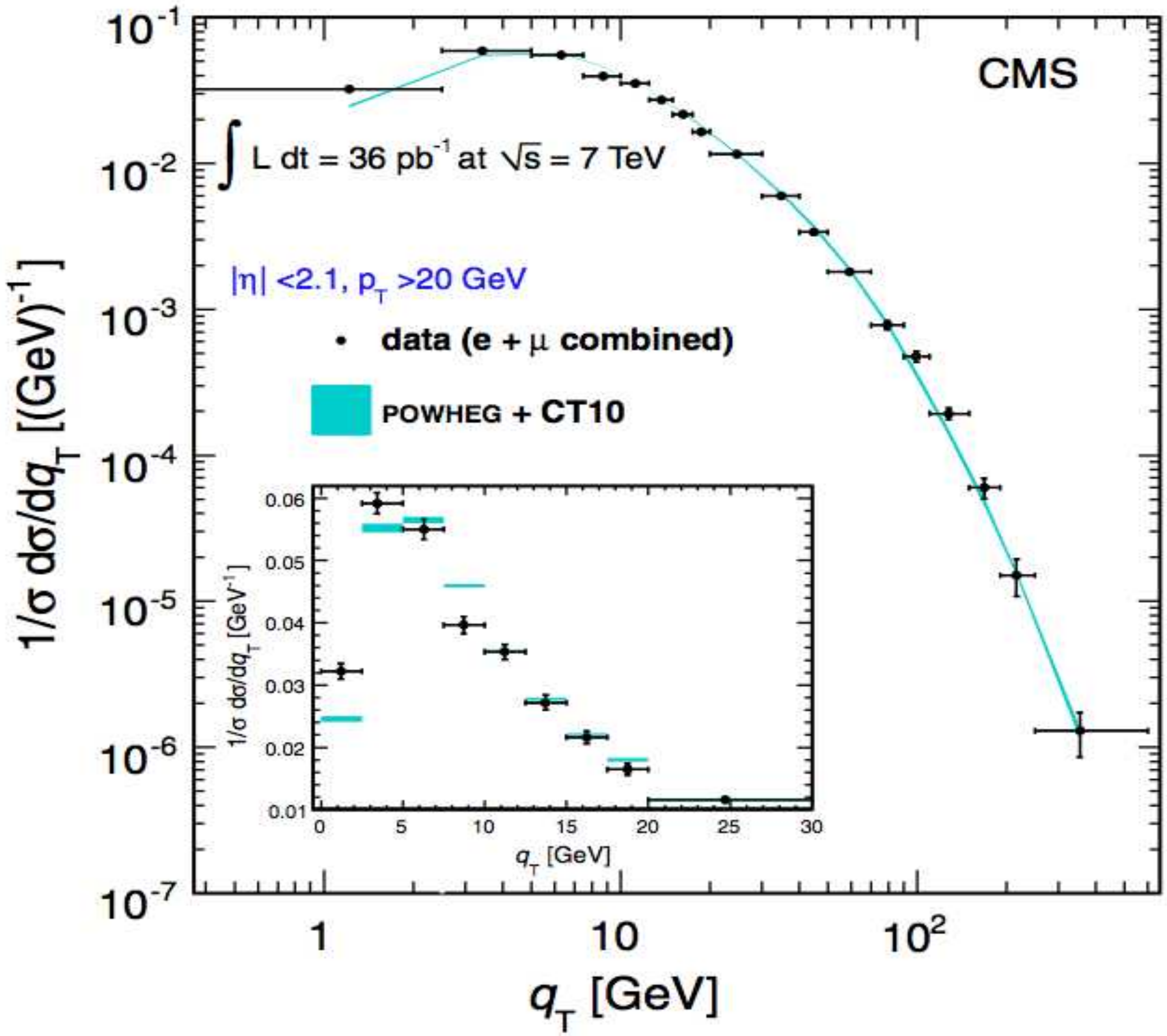
The complete P_T spectrum for the W boson

The full P_T spectrum for the W-boson showing the different theoretical regions



Transverse Momentum of Z

CMS Coll. Measurement of the Rapidity and Transverse Momentum Distributions of Z Bosons in pp Collisions at√s = 7 TeV. Phys.Rev., D85:032002, 2012.



Q_t - Resummation

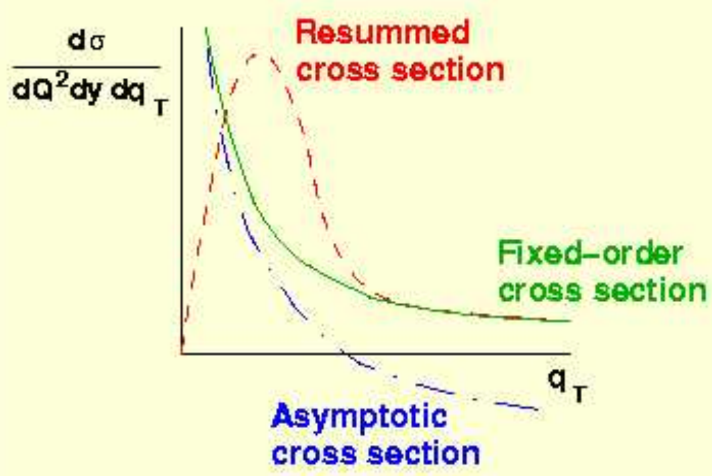
RESUMMED TRANSVERSE MOMENTUM DISTRIBUTIONS

go directly to the plotter

On this website, you can plot transverse momentum distributions for cross sections of several particle reactions. Currently, the following processes are implemented (p corresponds both to protons and antiprotons):

- Massive vector boson production: $pp \to W^{\pm}X$, $pp \to Z^0 X$
- Photon pair production: $pp \to \gamma\gamma X$
- Z-boson pair production: $pp \to Z^0 Z^0 X$
- SM Higgs boson production $pp \to H^0 X$

The output figure shows distributions $d\sigma/dQ^2 dy dq_T$ for the production of *on-shell* particles (or pairs of *on-shell* particles in the case of the $\gamma\gamma$ and ZZ production) with specified invariant mass Q, rapidity y and transverse momentum q_T in the lab frame (the center-of-mass frame of the hadron beams). You can plot resummed, fixed-order and asymptotic cross sections. For a short explanation of these quantities, visit **this page** (for a detailed explanation see, for instance, a paper by J.C. Collins, D.E. Soper and G. Sterman in Nucl. *Phys. B250, 199 (1985)).*

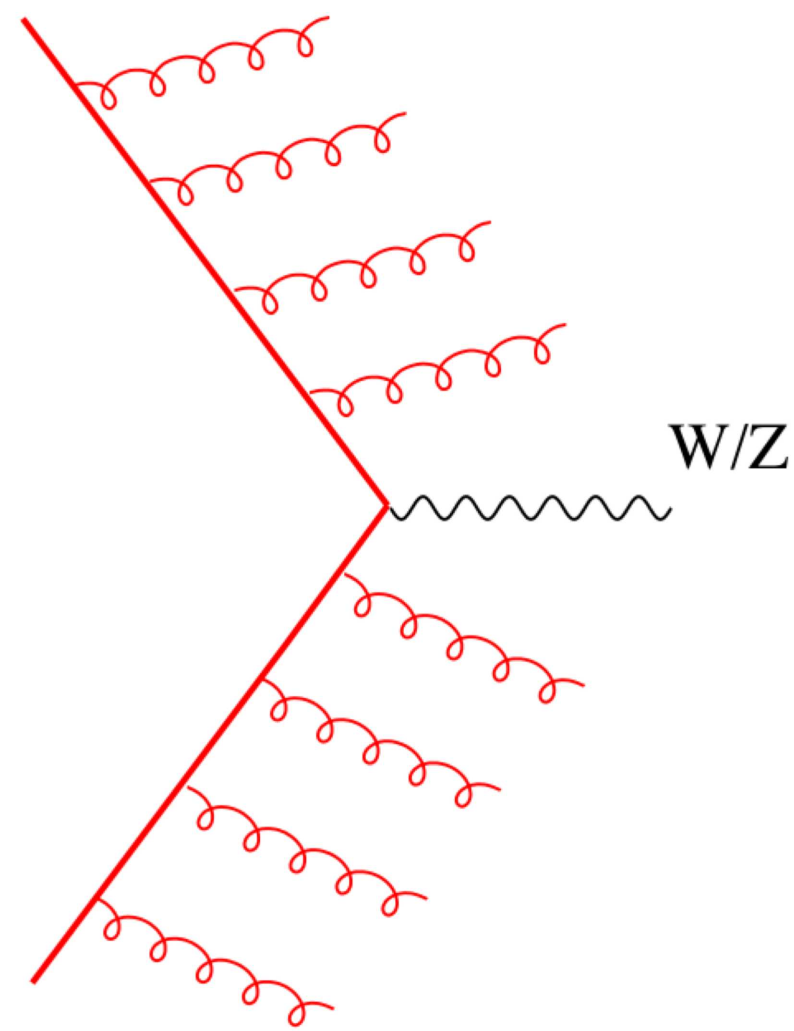


http://hep.pa.msu.edu/wwwlegacy/

Monte Carlo approach

- simulate explicitly parton radiation with evolution of parton densities
- advantage to include properly energy momentum conservation in each step
- perform resummation numerically

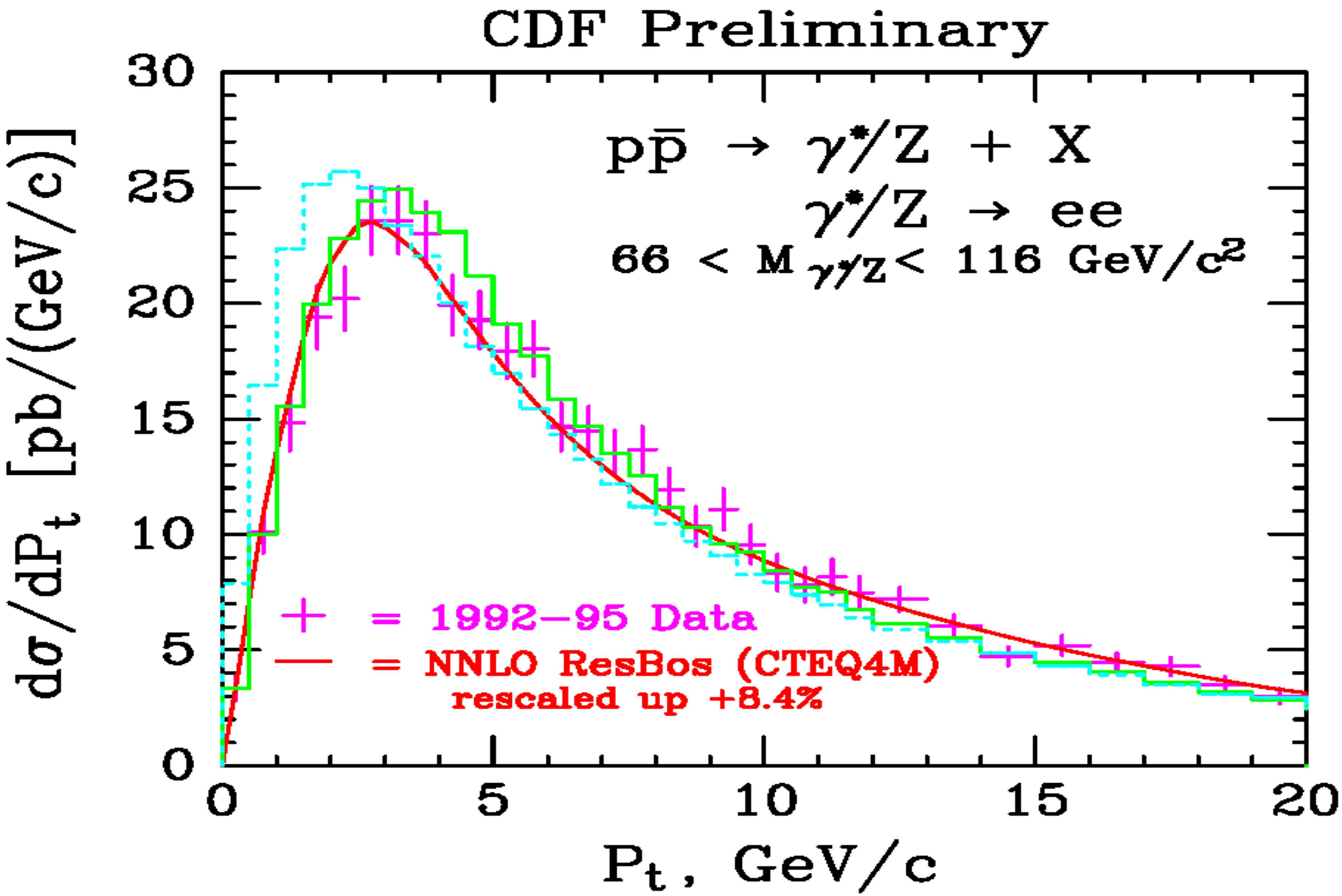
- will be done in exercise !!!



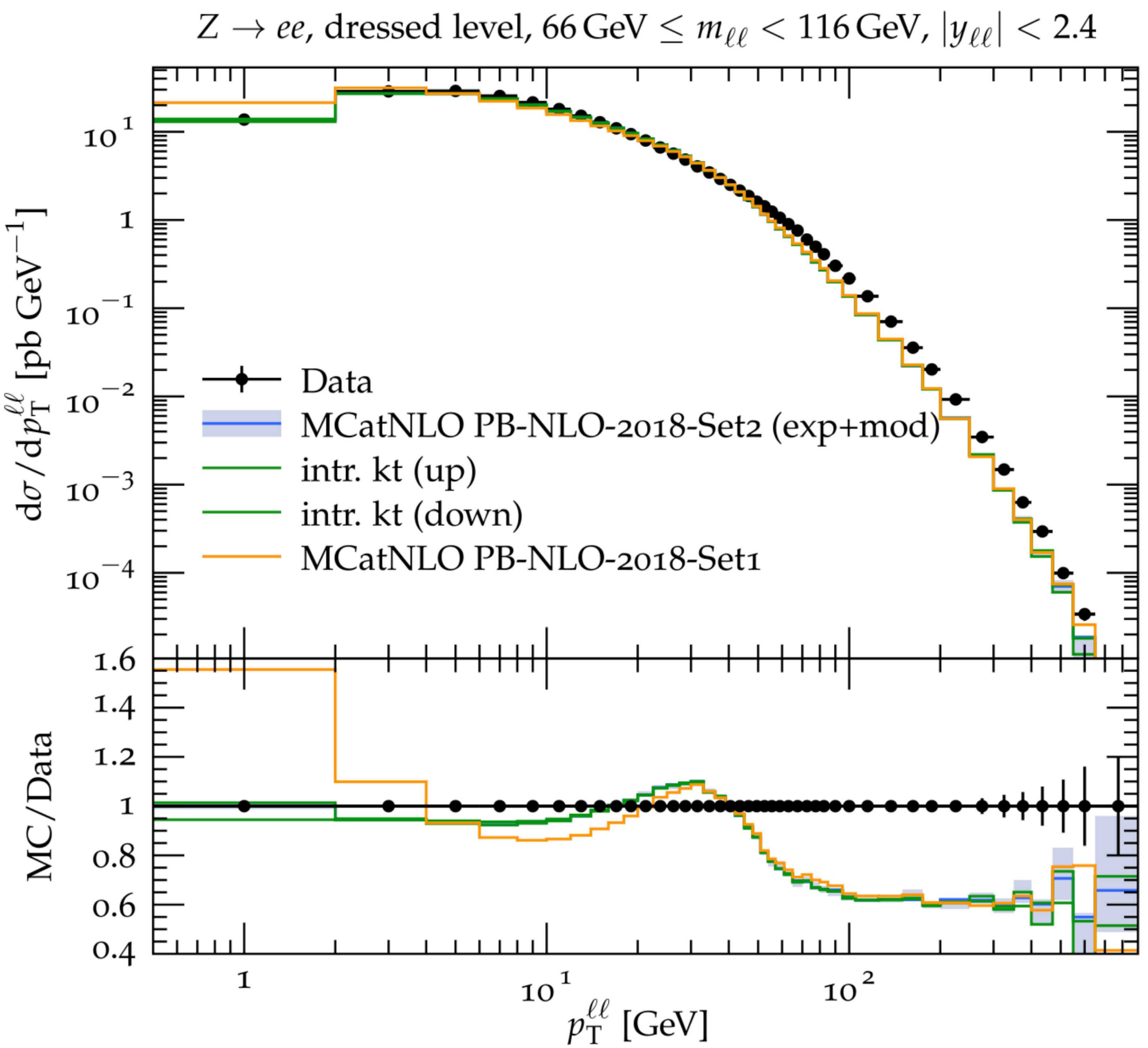
Monte Carlo vrs ResBos

- Comparison of pt spectrum from ResBos and PYTHIA

Campbell, Huston Stirling
Rep.Prog.Phys 70 (2007) 89



Transverse Momentum of Z - bosons



Bermudez Martinez, A. and others. Production of Z-bosons in the parton branching method, Phys. Rev. D, 100(2019), 074027

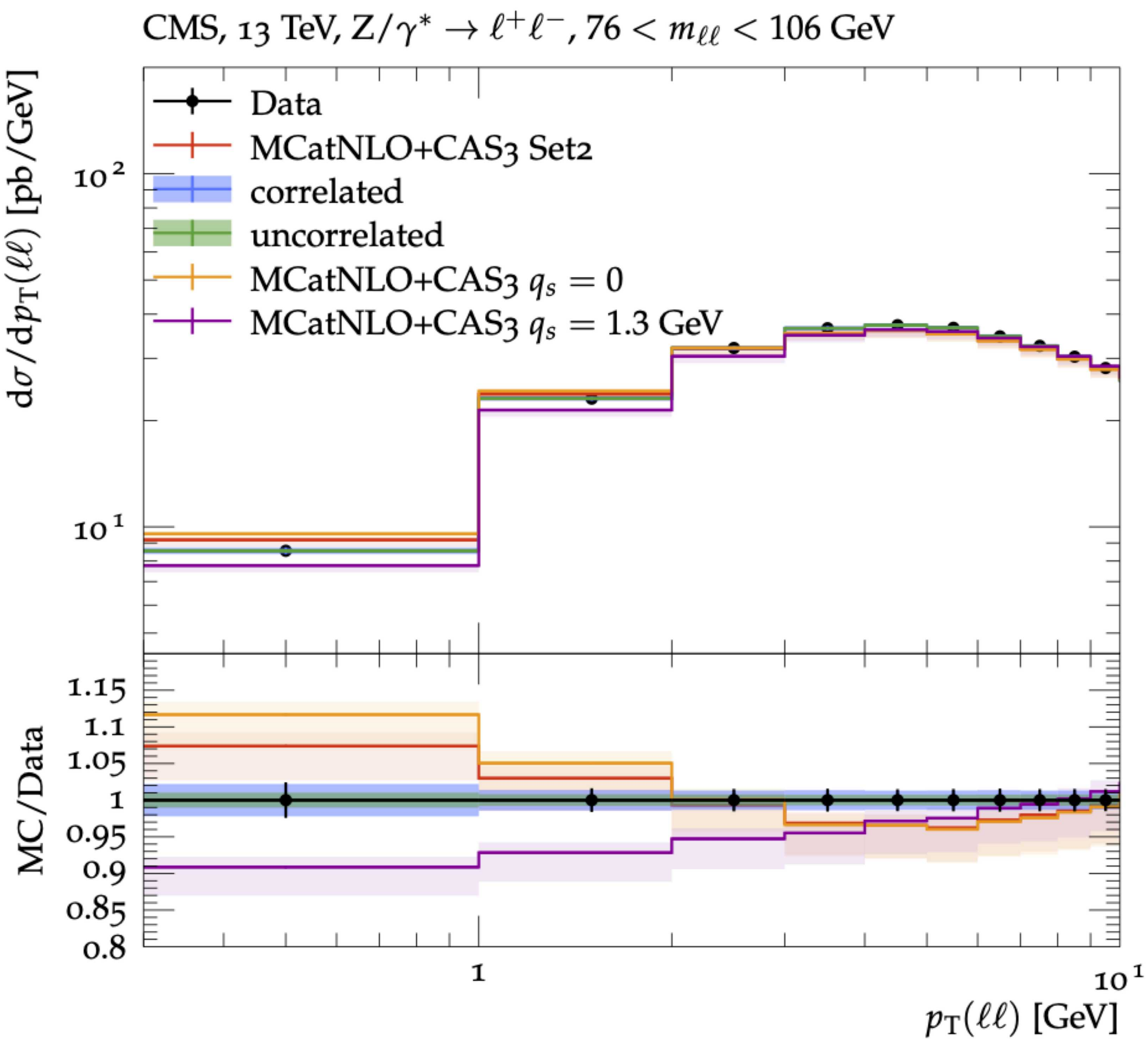
- Sensitivity to ordering condition: Set1 vrs Set2

- Comparison with measurements from ATLAS (Aad, G. and others Measurement of the transverse momentum and $\phi ^*_\eta $ distributions of Drell--Yan lepton pairs in proton--proton collisions at $s=8$ TeV with the ATLAS detector, Eur. Phys. J., C76(2016), 291)

Determination of intrinsic k_T

Intrinsic k_T in DY - production at 13 TeV (CMS)

https://arxiv.org/abs/2312.08655

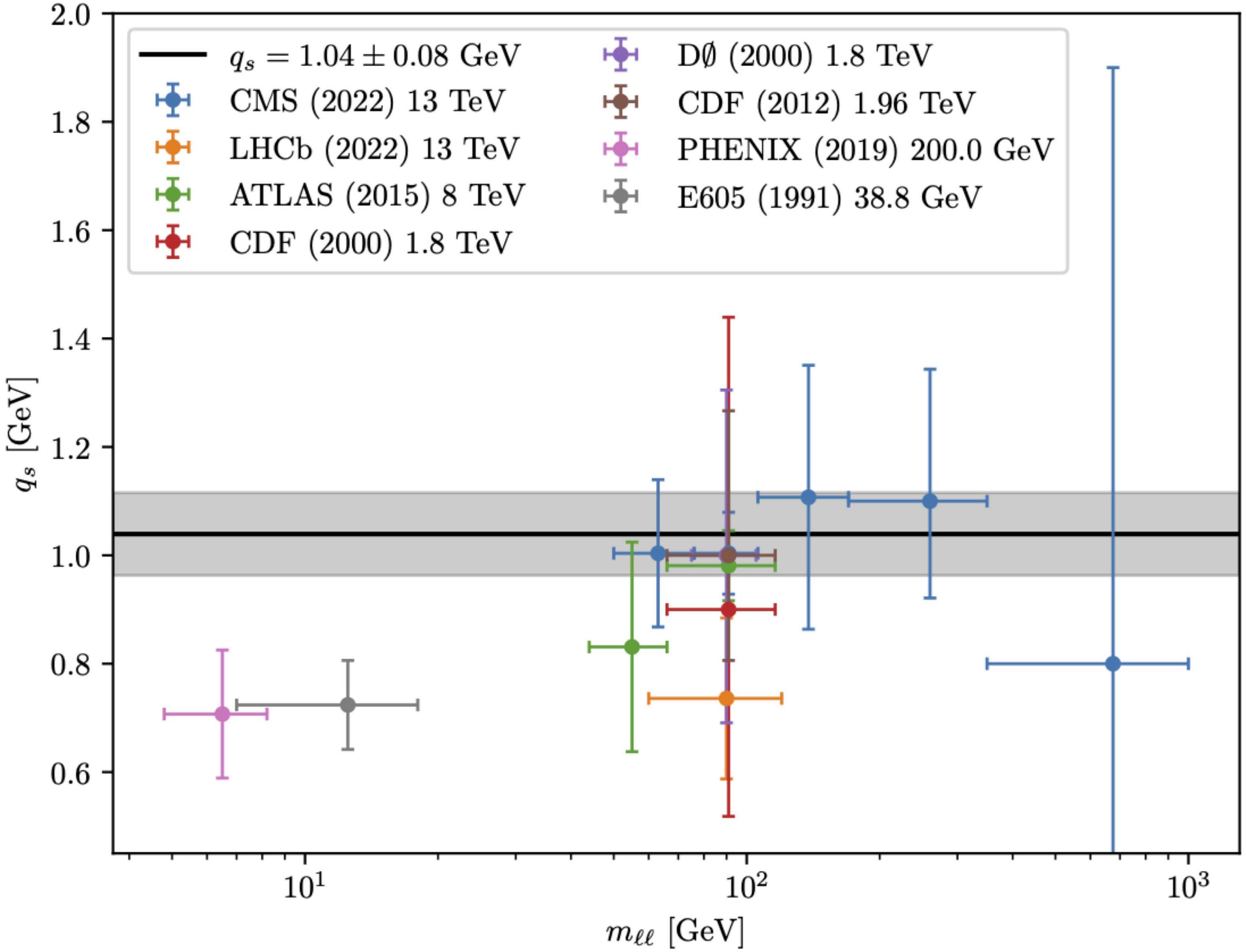


- in TMD, intrinsic k_T distribution:
  - Gauss with zero mean, width q_s

$$\sim \exp\left(-|k_T^2|/q_s^2\right)$$

- Focus on small k_T region:
  - in lowest p_T bin, sensitivity to intrinsic k_T
- Use DY production at different m_{DY} and $\sqrt{s}$ to determine q_s
- Is intrinsic k_T dependent on m_{DY} and $\sqrt{s}$?

Fit of Intrinsic k_T in DY – production vers m_{DY}



https://arxiv.org/abs/2312.08655

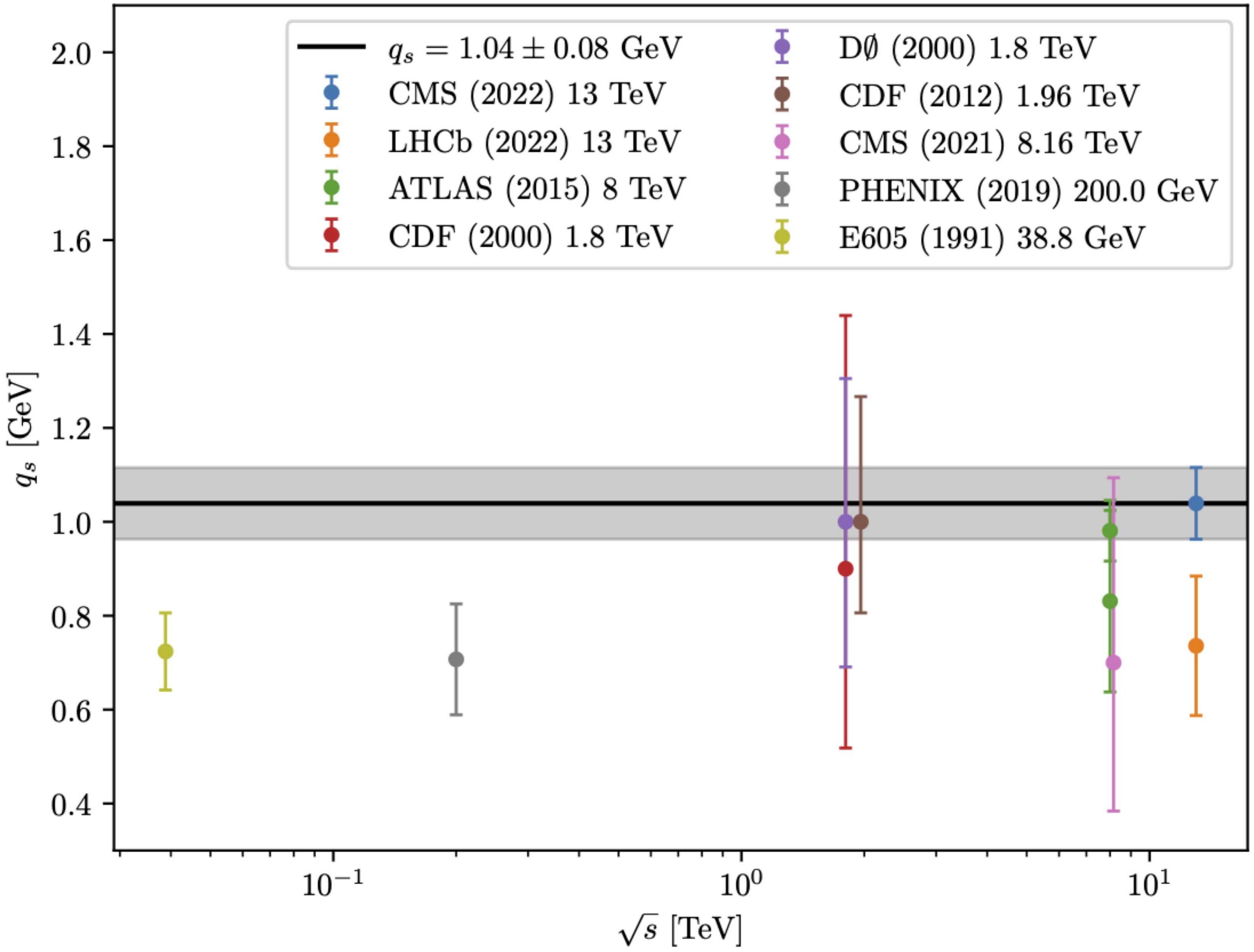
- Gauss with zero mean, width q_s

$$\sim \exp\left(-|k_T^2|/q_s^2\right)$$

Fit to determine q_s of intrinsic k_T distribution from DY production as a function of m_{DY}

- obtain q_s rather independent on m_{DY}

Fit of Intrinsic k_T in DY – production vers $\sqrt{s}$



https://arxiv.org/abs/2312.08655

- Gauss with zero mean, width q_s

  $\sim \exp\left(-|k_T^2|/q_s^2\right)$

Fit to determine q_s of intrinsic k_T distribution from DY production as a function of $\sqrt{s}$

- obtain q_s rather independent on $\sqrt{s}$

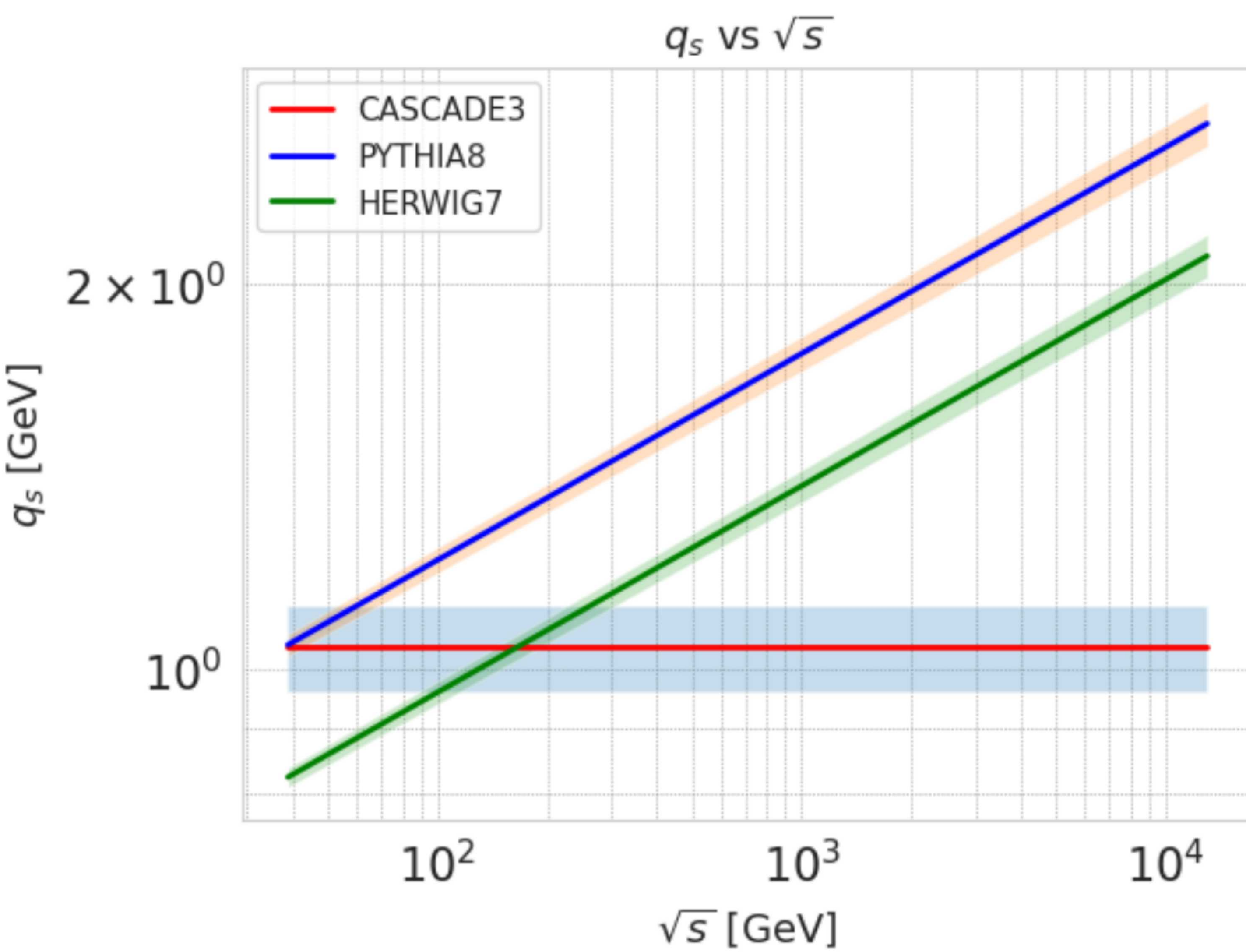
Comparison to MC event generators

Non-perturbative effects: lessons from fixed target,Tevatron, and LHC data,
Weijie Jin, Armando Bermudez Martinez, Sara Taheri Monfared, Mikel Mendizabal Morentin, Kyle Cormier, Saptaparna Bhattacharya
(paper in preparation)

S. Taheri Monfared at
Physics In Collision 2023

- Gauss with zero mean, width q_s

$$\sim \exp\left(-|k_T^2|/q_s^2\right)$$



- MC generators need q_s dependent on $\sqrt{s}$
- PB TMDs work with constant q_s !

- Why ?

Parton Shower MC event generators

- Parton shower follows backward evolution:

$$\Pi = \exp\left[- \int_{\mu_l^2}^{\mu_h^2} \frac{d\mu'^2}{\mu'^2} \int^{z_{\rm dyn}} \frac{dz}{z} \hat{P}(z) \frac{f(x/z,\mu^2)}{f(x,\mu^2)} \right]$$

- Emitted partons should have *resolvable* energy (or p_T) with: $p_T > q_{t\,\rm cut} \sim 1 {\rm Ge}V$

$$z_{\rm dyn} = 1 - \frac{q_{t\ \rm cut}}{\mu'}$$

- With $z_{\rm dyn} \ll 1$ soft gluons with $p_T < 1 {\rm Ge}V$ are neglected.

- What is the role of these soft gluons ?

Parton densities including photons

DGLAP evolution equation – including photons

- Start from QCD evolution

$$\frac{dq_i(x,\mu^2)}{d\log\mu^2} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{d\xi}{\xi}\left[q_i(\xi,\mu^2)P_{qq}\left(\frac{x}{\xi}\right) + g(\xi,\mu^2)P_{qg}\left(\frac{x}{\xi}\right)\right]$$

$$\frac{dg(x,\mu^2)}{d\log\mu^2} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{d\xi}{\xi}\left[\sum_i q_i(\xi,\mu^2)P_{gq}\left(\frac{x}{\xi}\right) + g(\xi,\mu^2)P_{gg}\left(\frac{x}{\xi}\right)\right]$$

- extend to include γ

$$\frac{dq_i(x,\mu^2)}{d\log\mu^2} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{d\xi}{\xi}\left[q_i(\xi,\mu^2)P_{qq}\left(\frac{x}{\xi}\right) + g(\xi,\mu^2)P_{qg}\left(\frac{x}{\xi}\right) + \gamma(\xi,\mu^2)P_{q\gamma}\left(\frac{x}{\xi}\right)\right]$$

$$\frac{dg(x,\mu^2)}{d\log\mu^2} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{d\xi}{\xi}\left[\sum_i q_i(\xi,\mu^2)P_{gq}\left(\frac{x}{\xi}\right) + g(\xi,\mu^2)P_{gg}\left(\frac{x}{\xi}\right)\right]$$

$$\frac{d\gamma(x,\mu^2)}{d\log\mu^2} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{d\xi}{\xi}\left[\sum_i q_i(\xi,\mu^2)P_{\gamma q}\left(\frac{x}{\xi}\right)\right] \quad + \gamma_0(x,\mu^2)$$

The Collinear Photon Density and DIS

- The photon density and its relation to DIS

$\sum x q(x,Q^2) = F_2$

$$\gamma(z,Q^2) = \sum_f \frac{\alpha_{em} e_f^2}{2\pi} \int_{Q_0^2}^{Q^2} \frac{d\mu^2}{\mu^2} \int_z^1 \frac{dx}{x} P_{\gamma q}\left(\frac{z}{x}\right) \left[q_f(x,\mu^2) + \bar{q}(x,\mu^2)\right) + \gamma(z,Q_0^2)$$

$$= \frac{\alpha_{em} e_f^2}{2\pi} \int_{Q_0^2}^{Q^2} \frac{d\mu^2}{\mu^2} \int_z^1 \frac{dx}{x} P_{\gamma q}\left(\frac{z}{x}\right) \frac{F_2(x,\mu^2)}{x} + \gamma(z,Q_0^2)$$

- offers possibility to determine photon density directly from DIS measurement

$\frac{d\sigma}{dy\,dQ^2} = \dots \frac{1}{y}\frac{1}{Q^2}\left(1+(1-y)^2\right) \cdot F_2(x,Q^2)$

$P_{\gamma q}$

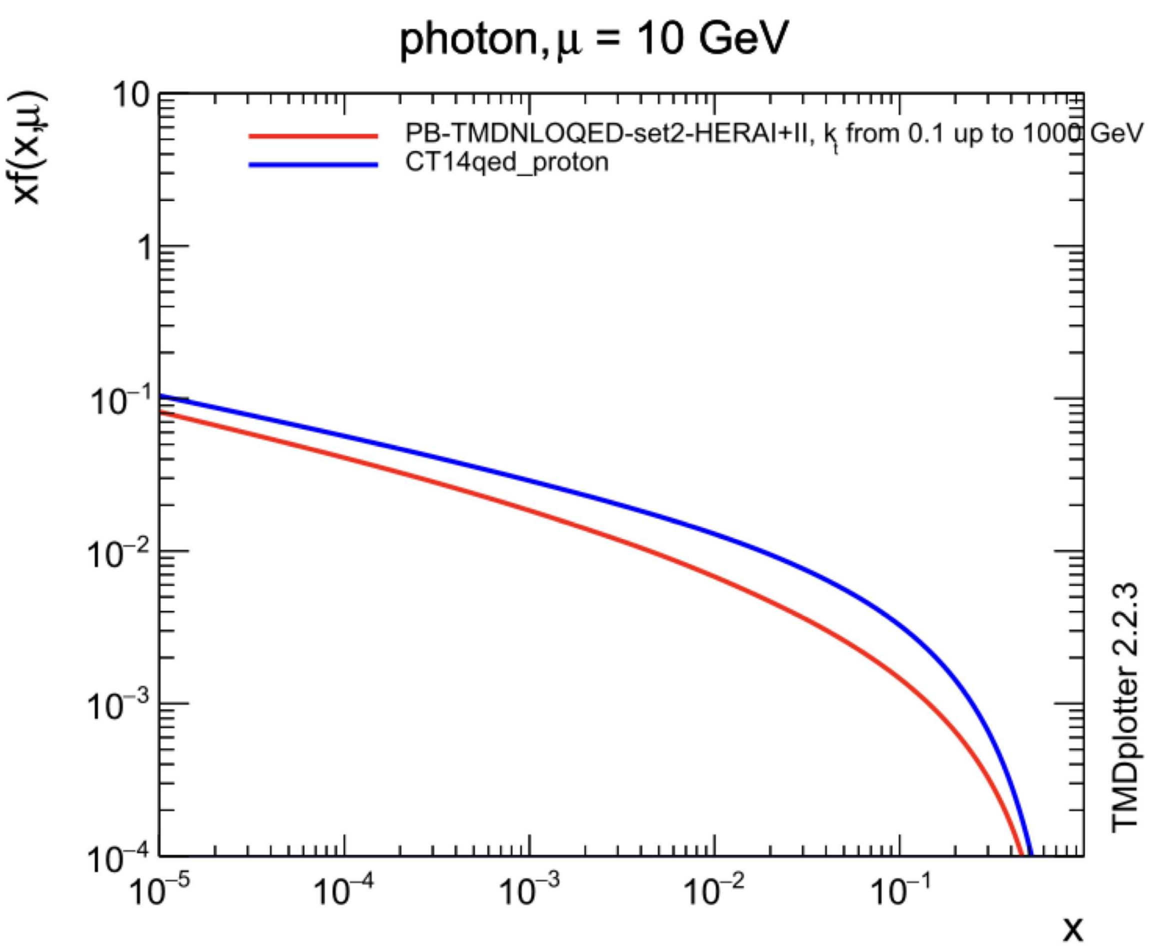
$\gamma(z,Q^2) \sim \frac{d\sigma^{DIS}}{dy\,dQ^2}$

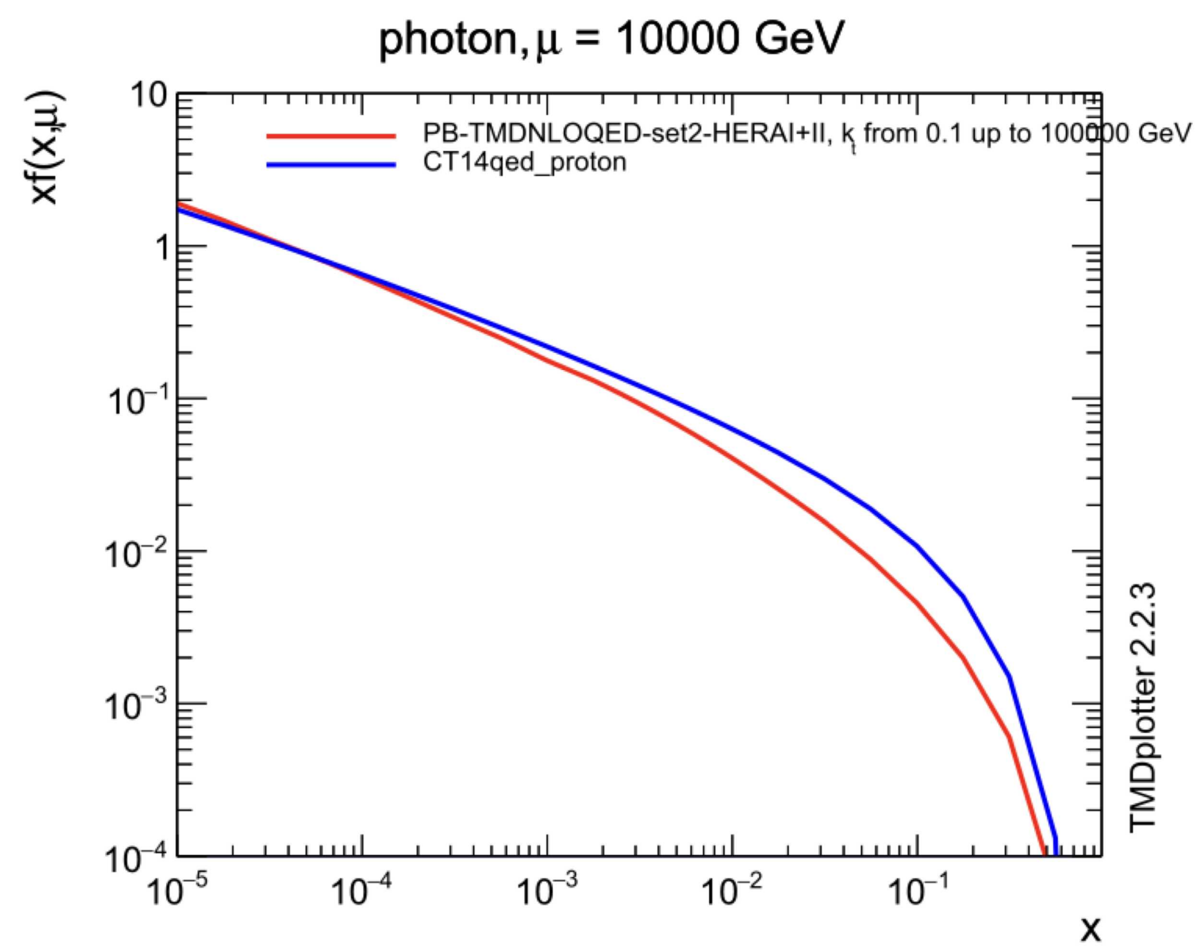
$z = y \cdot x = \frac{Q^2}{s}$

The Collinear Photon Density

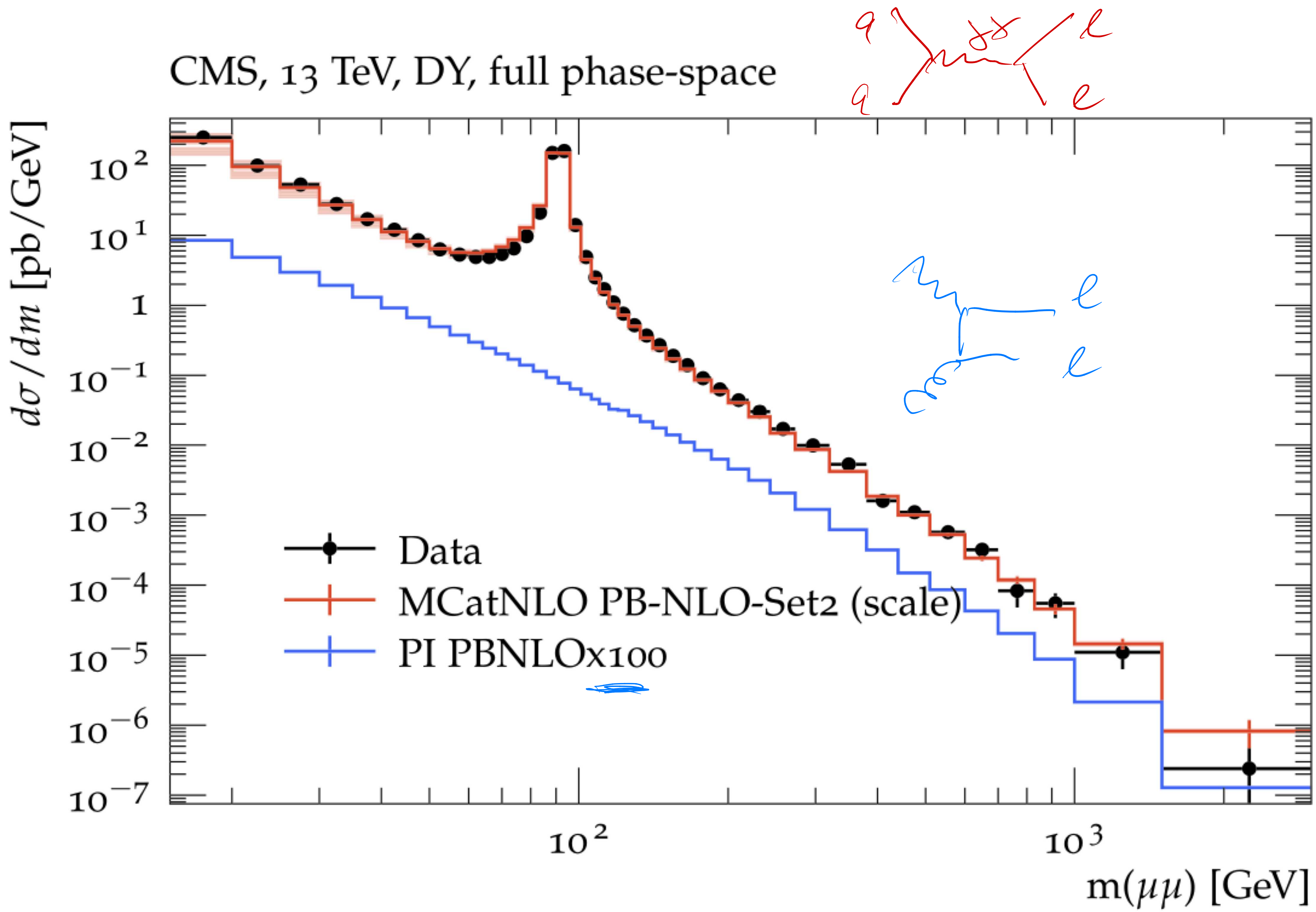
- The photon density determined from PB

https://arxiv.org/abs/2102.01494





Application of photon density to DY production

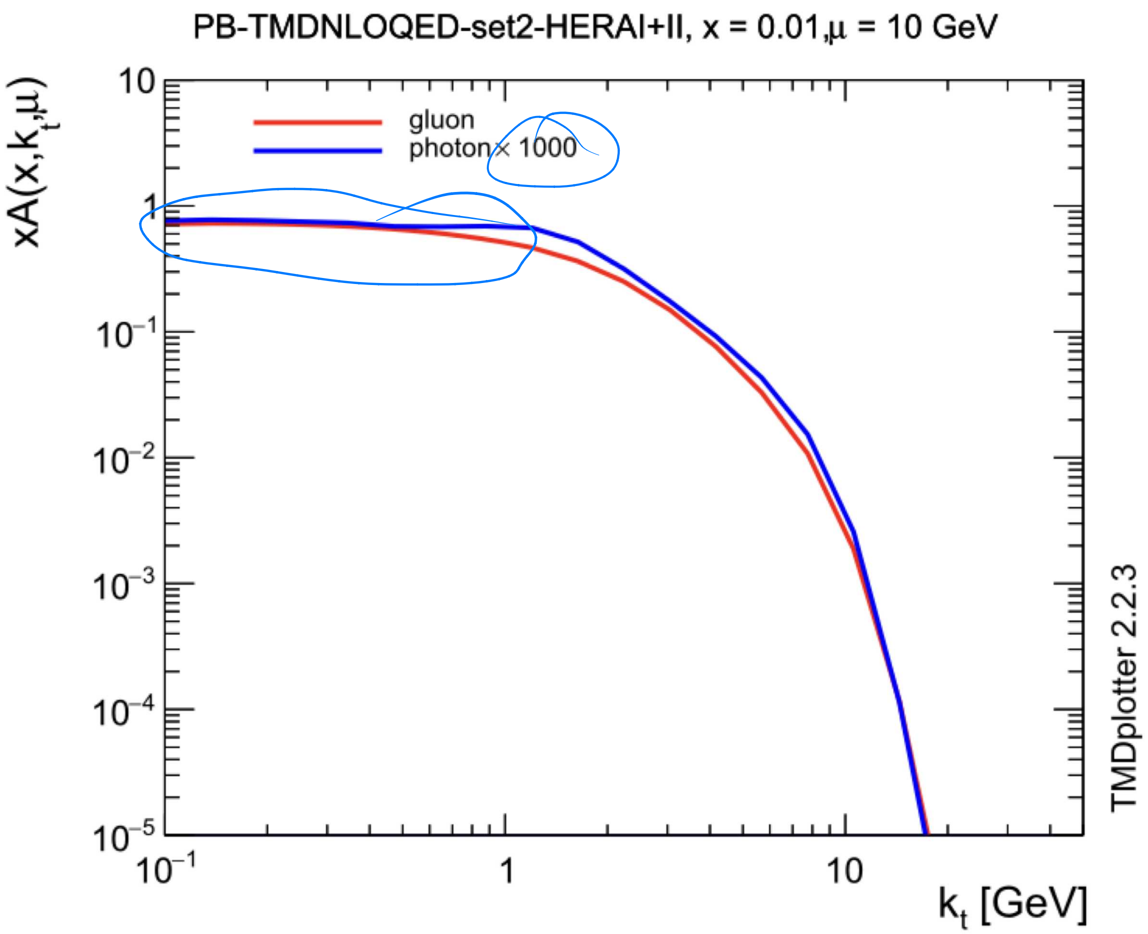


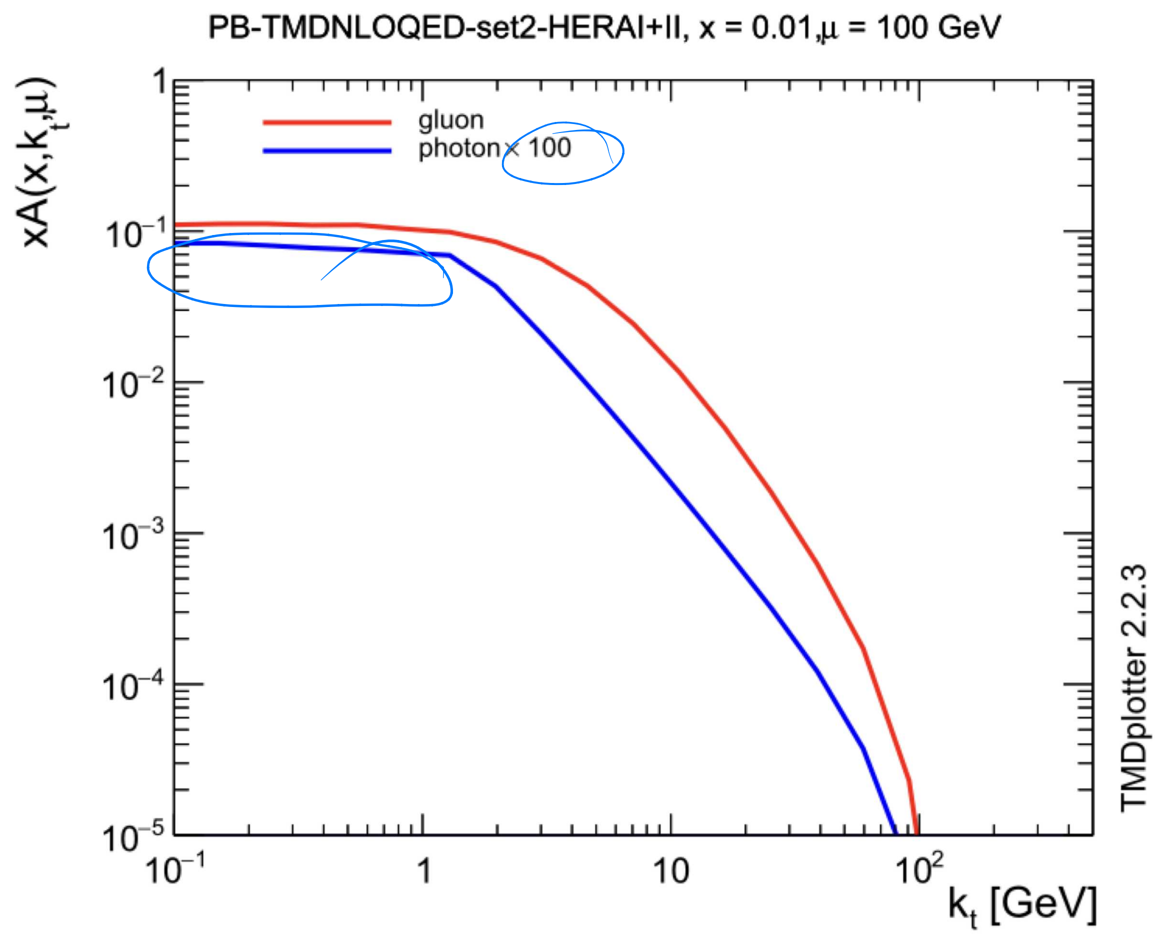
- calculate x-section in democratic order: same size of xsection or in formal order (order of expansion in coupling) (see https://arxiv.org/abs/1708.01256)

The TMD photon

- The TMD photon density determined from PB

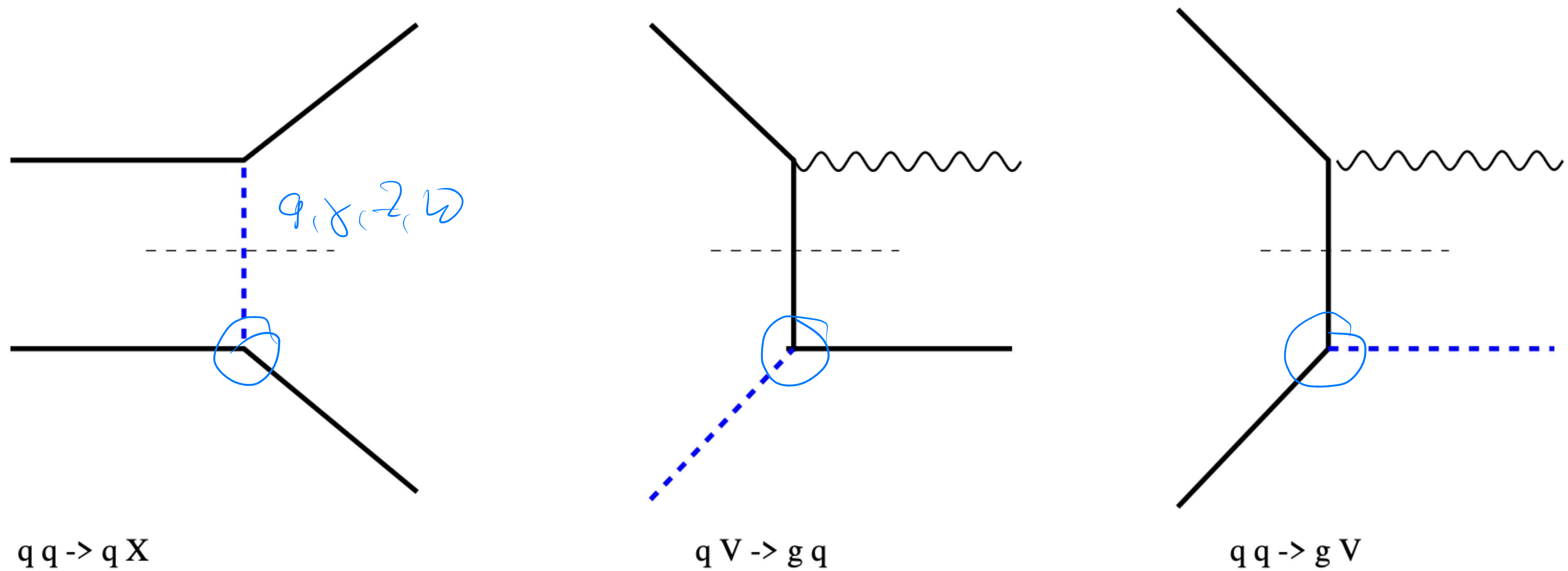
https://arxiv.org/abs/2102.01494





Including W/Z bosons

General Splitting functions



- First attempts to calcualte equivalendt W approximation in 1980's
  - Kane, G. L., Repko, W. W., and Rolnick, W. B.. The Effective W+-, Z0 Approximation for High-Energy Collisions, Phys. Lett. B, 148(1984), 367
  - Dawson, S. The Effective W Approximation, Nucl. Phys. B, 249(1985), 42
- for discussion of SSC !

The effective coupling of V to quarks

$$\sigma(q\bar{q} \to \gamma^*) = \frac{1}{3}\frac{\pi}{2}\left[4\pi\alpha_{em}^2\right] = \frac{1}{3}\pi\;\left[4\pi\alpha_{em}\right]$$

$$\sigma(q\bar{q} \to \mathrm{Z}) = \frac{1}{3}\frac{\pi}{m_{\mathrm{Z}}^2}\left[\frac{2G_F}{\sqrt{2}}m_{\mathrm{Z}}^4(V_f^2 + A_f^2)\right] = \frac{1}{3}\pi\;\left[\sqrt{2}G_F m_{\mathrm{Z}}^2(V_f^2 + A_f^2)\right]$$

$$\sigma(q_i\bar{q}_j \to \mathrm{W}) = \frac{1}{3}\frac{\pi}{m_{\mathrm{W}}^2}\left[\frac{2|V_{qq}|^2 G_F m_{\mathrm{W}}^4}{\sqrt{2}}\right] = \frac{1}{3}\pi\;\left[\sqrt{2}G_F m_{\mathrm{W}}^2|V_{qq}|^2\right]$$

to $\quad \mathrm{Z}\!:\; \alpha_{eff} = \frac{\alpha_{em}}{4\sin^2\theta_W\cos^2\theta_W}(V_f^2 + A_f^2)\mathrm{W} : \alpha_{eff} = \frac{\alpha_{em}|V_{qq}|^2}{4\sin^2\theta_W}$

Effective Coupling and Splitting functions

Splitting function	$V = \mathrm{g}$	$V = \gamma$	$V = \mathrm{Z}$	$V = \mathrm{W}$
$P_{qq}(z) = \frac{\alpha_{eff}}{2\pi}\frac{1+z^2}{1-z}$	$\frac{4}{3}\alpha_\mathrm{s}$	α_{em}	$\frac{\alpha_{em}(V_f^2+A_f^2)}{4\sin^2\theta_W\cos^2\theta_W}$	$\frac{\alpha_{em}\lvert V_{qq}\rvert^2}{4\sin^2\theta_W}$
$P_{qV}(z) = \frac{\alpha_{eff}}{2\pi}\frac{1}{2}\left(z^2+(1-z)^2\right)$	α_s	α_{em}	$\frac{\alpha_{em}(V_f^2+A_f^2)}{4\sin^2\theta_W\cos^2\theta_W}$	$\frac{\alpha_{em}\lvert V_{qq}\rvert^2}{4\sin^2\theta_W}$
$P_{Vq}(z) = \frac{\alpha_{eff}}{2\pi}\frac{1+(1-z)^2}{z}$	$\frac{4}{3}\alpha_\mathrm{s}$	α_{em}	$\frac{\alpha_{em}(V_f^2+A_f^2)}{4\sin^2\theta_W\cos^2\theta_W}$	$\frac{\alpha_{em}\lvert V_{qq}\rvert^2}{4\sin^2\theta_W}$
$P_{VV'}(z) = \frac{\alpha_{eff}}{2\pi}2\left(\frac{1-z}{z}+\frac{z}{1-z}+z(1-z)\right)$	$3\alpha_\mathrm{s}$	0	$4\pi\alpha_{em}\cot\theta_W$	

DGLAP evolution equation – including V-bosons

- extend DGLAP to include γ, W, Z bosons

$$\frac{dq_i(x,\mu^2)}{d\log\mu^2} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{d\xi}{\xi}\left[q_i(\xi,\mu^2)P_{qq}\left(\frac{x}{\xi}\right) + g(\xi,\mu^2)P_{qg}\left(\frac{x}{\xi}\right) + \gamma(\xi,\mu^2)P_{q\gamma}\left(\frac{x}{\xi}\right) + \mathrm{W}(\xi,\mu^2)P_{q\mathrm{W}}\left(\frac{x}{\xi}\right) + \mathrm{Z}(\xi,\mu^2)P_{q\mathrm{Z}}\left(\frac{x}{\xi}\right)\right]$$

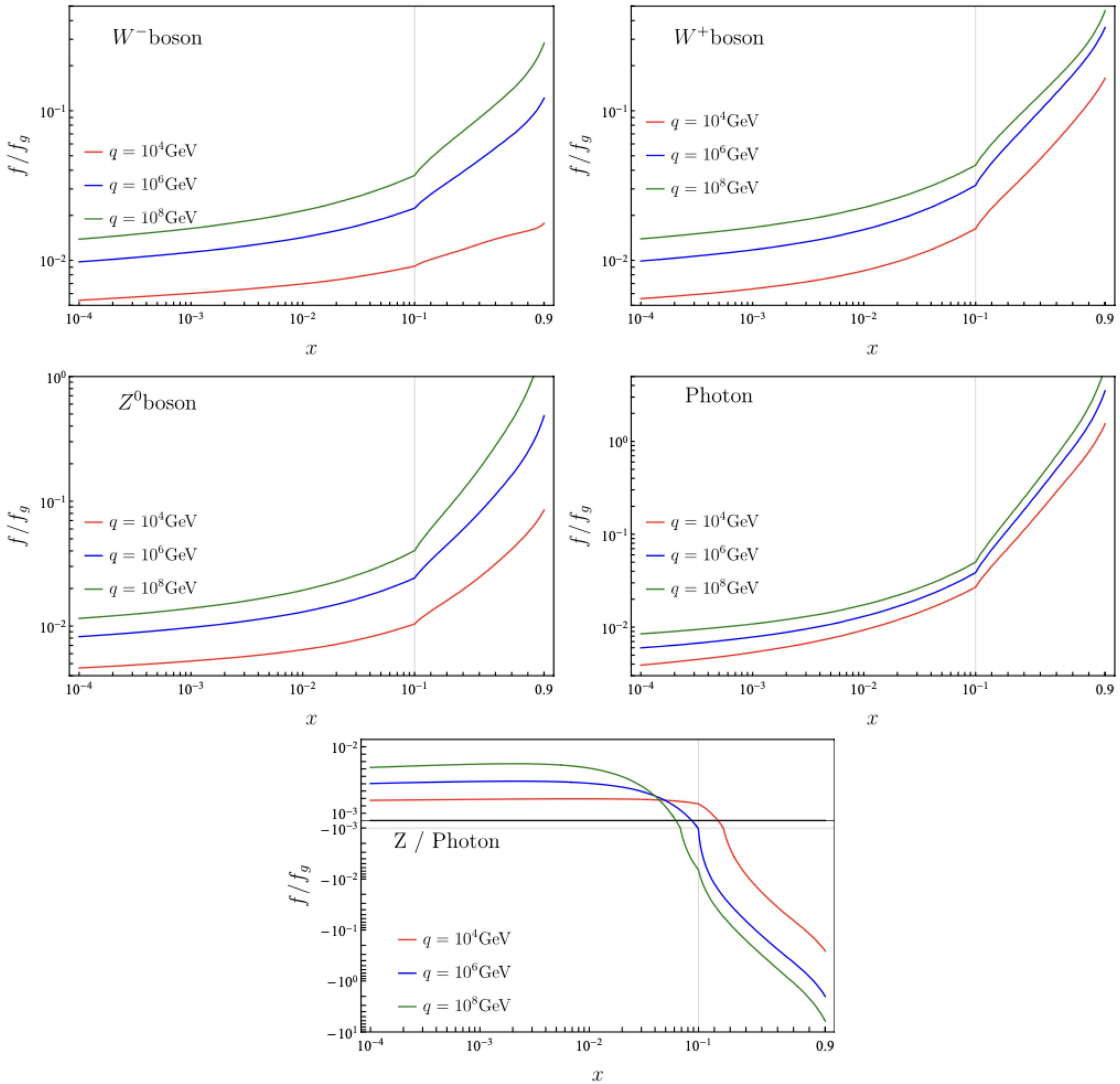
$$\frac{d\mathrm{W}(x,\mu^2)}{d\log\mu^2} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{d\xi}{\xi}\left[\sum_i q_i(\xi,\mu^2)P_{\mathrm{W}q}\left(\frac{x}{\xi}\right)\right]$$

$$\frac{d\mathrm{Z}(x,\mu^2)}{d\log\mu^2} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{d\xi}{\xi}\left[\sum_i q_i(\xi,\mu^2)P_{\mathrm{Z}q}\left(\frac{x}{\xi}\right) + \mathrm{W}(\xi,\mu^2)P_{\mathrm{ZW}}\left(\frac{x}{\xi}\right)\right]$$

$$\frac{d\gamma(x,\mu^2)}{d\log\mu^2} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{d\xi}{\xi}\left[\sum_i q_i(\xi,\mu^2)P_{\gamma q}\left(\frac{x}{\xi}\right)\right]$$

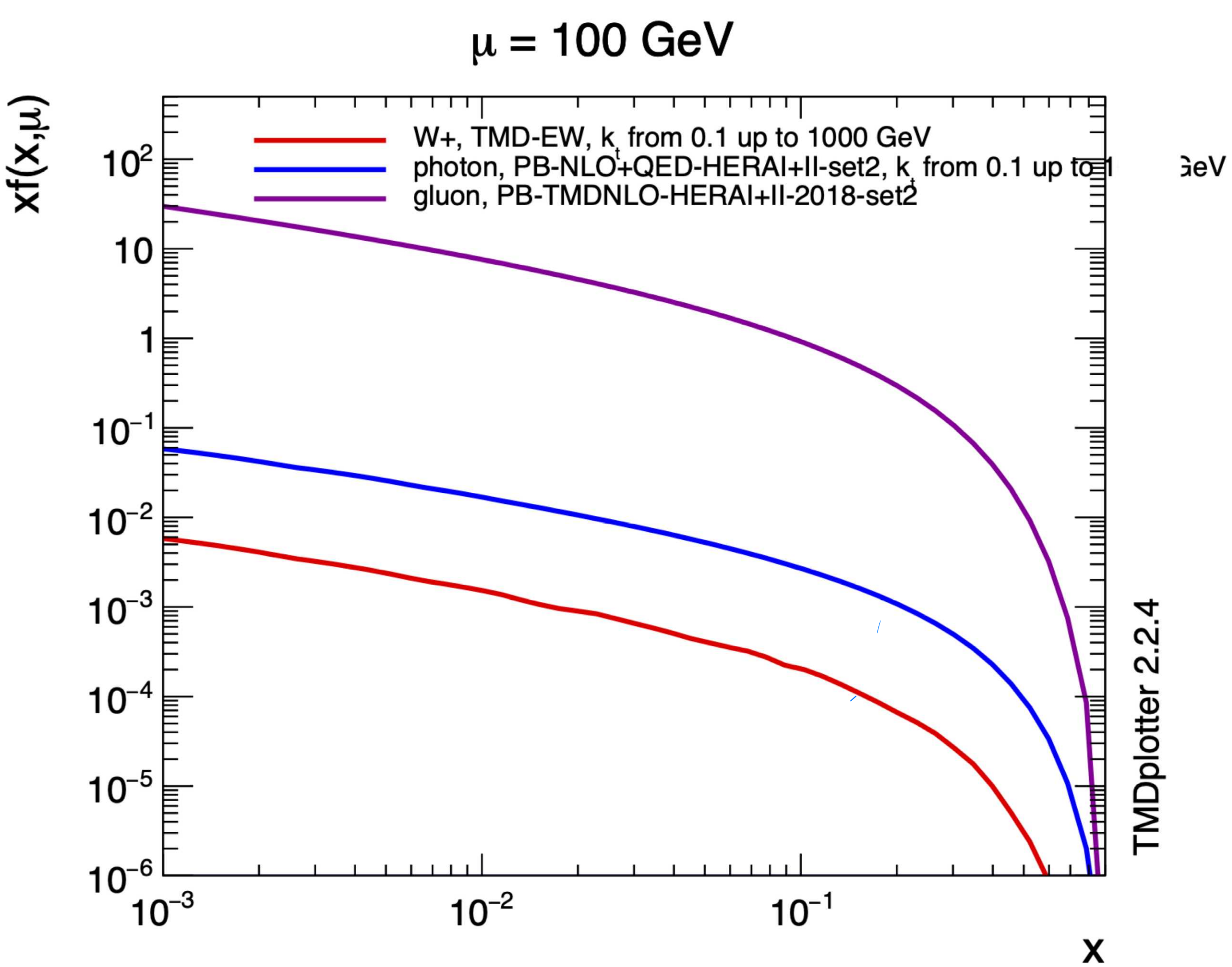
Collinear densities for V bosons

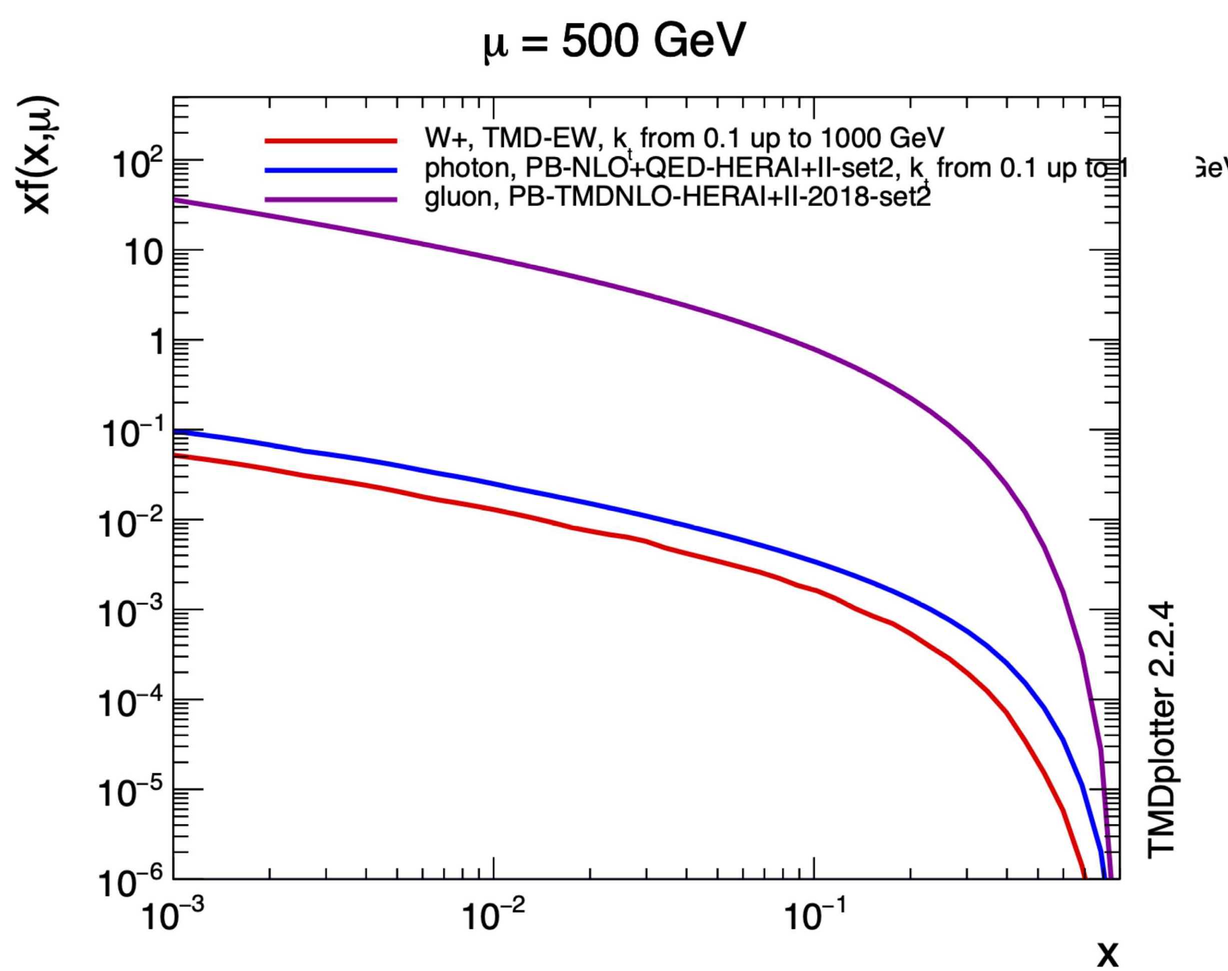
https://arxiv.org/abs/1703.08562



Collinear densities for W bosons

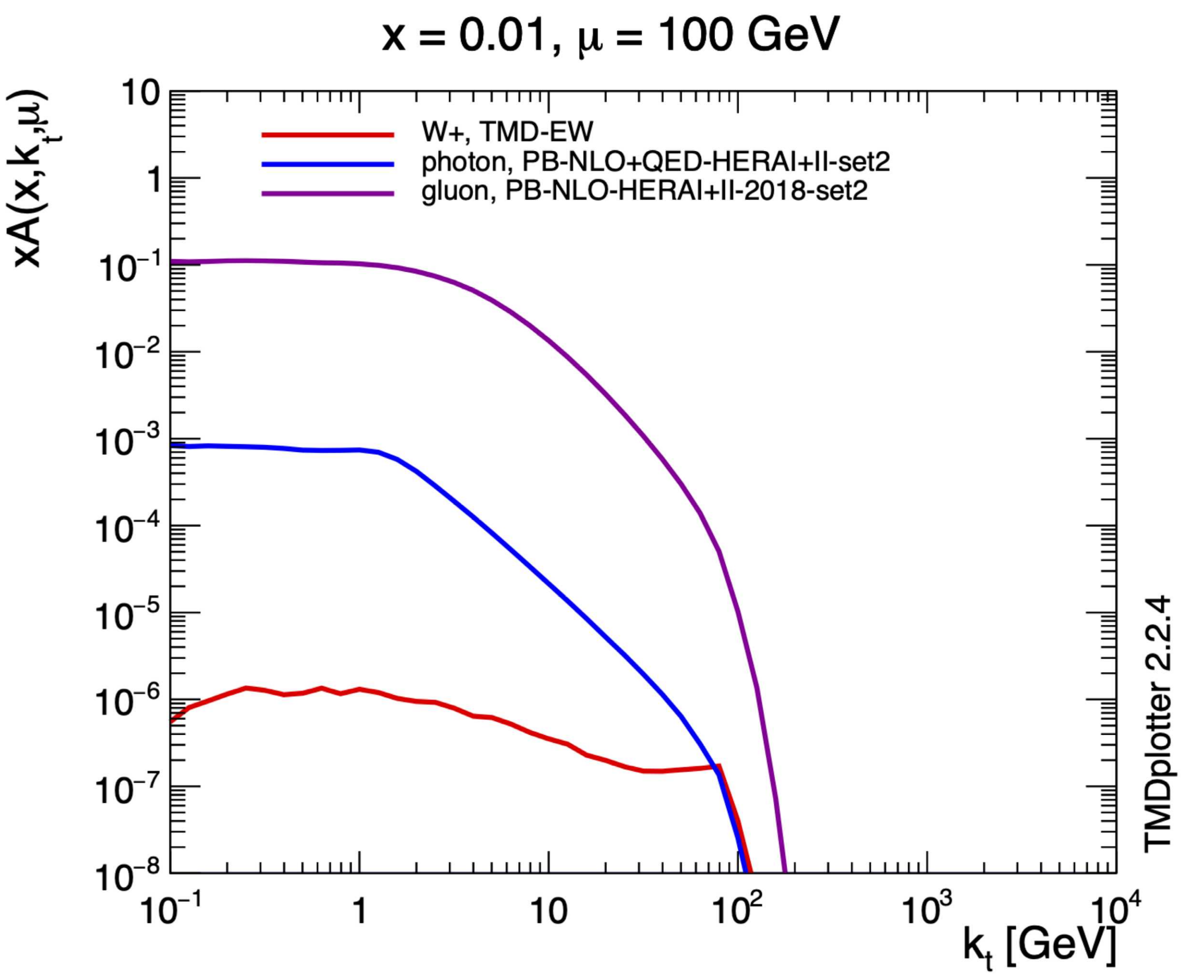
- determination with PB method

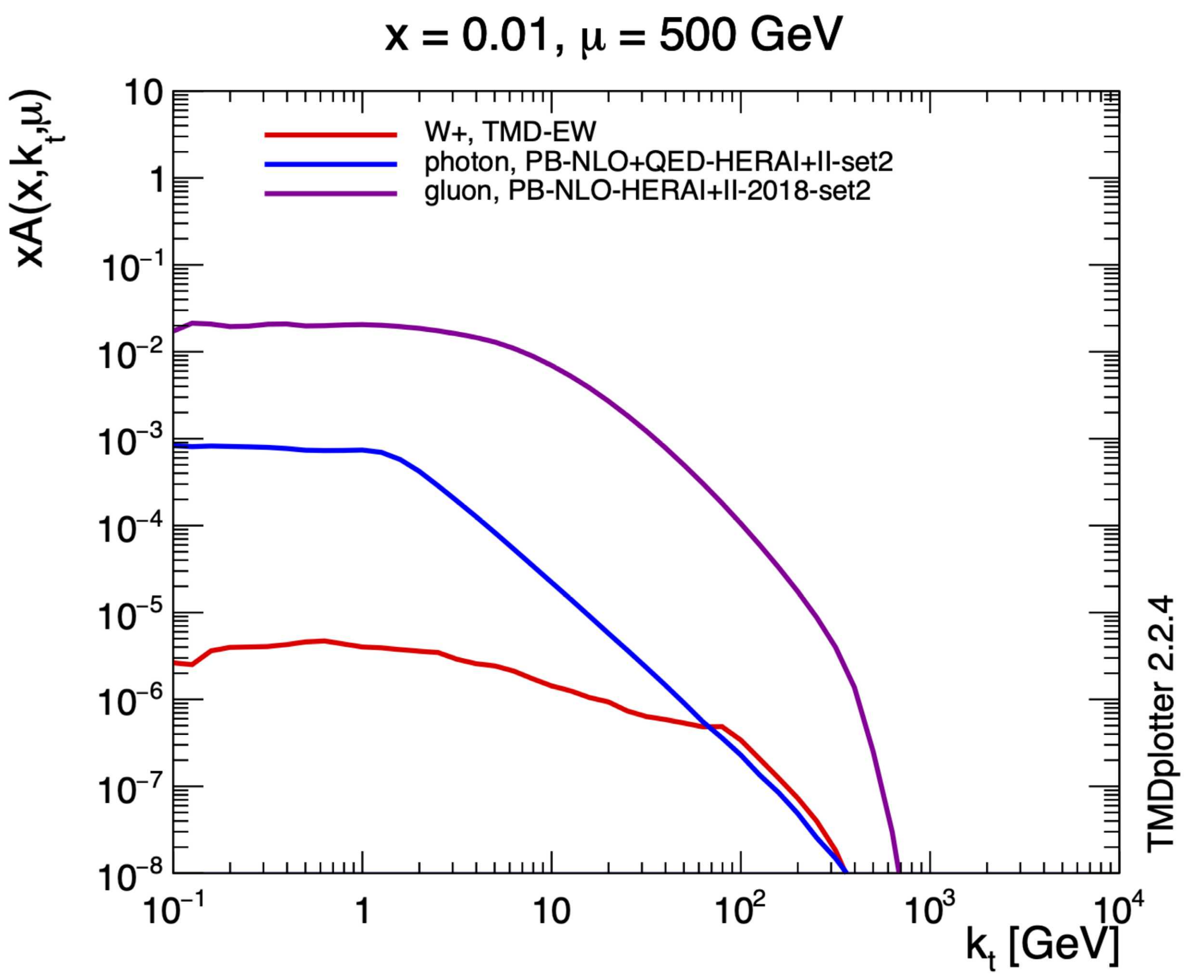




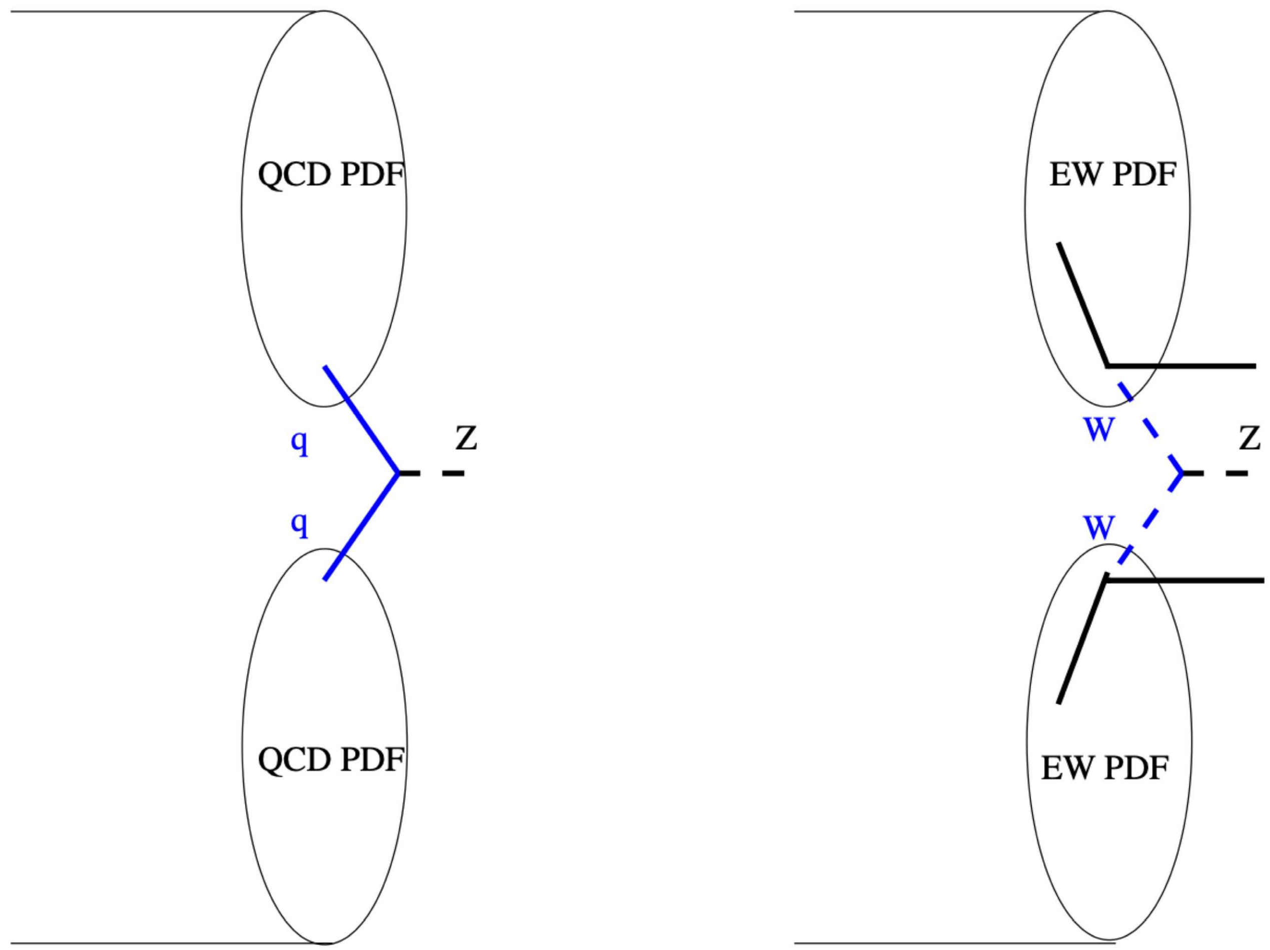
TMD densities for W bosons

- determination with PB method



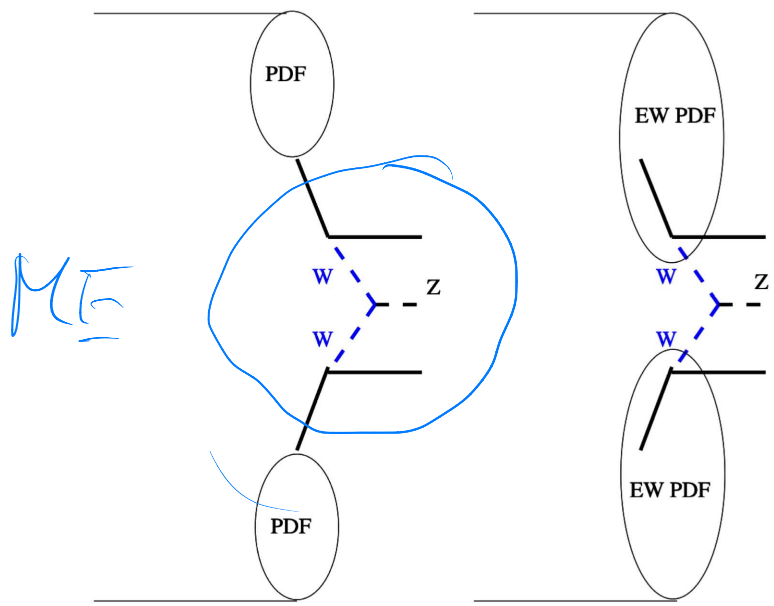


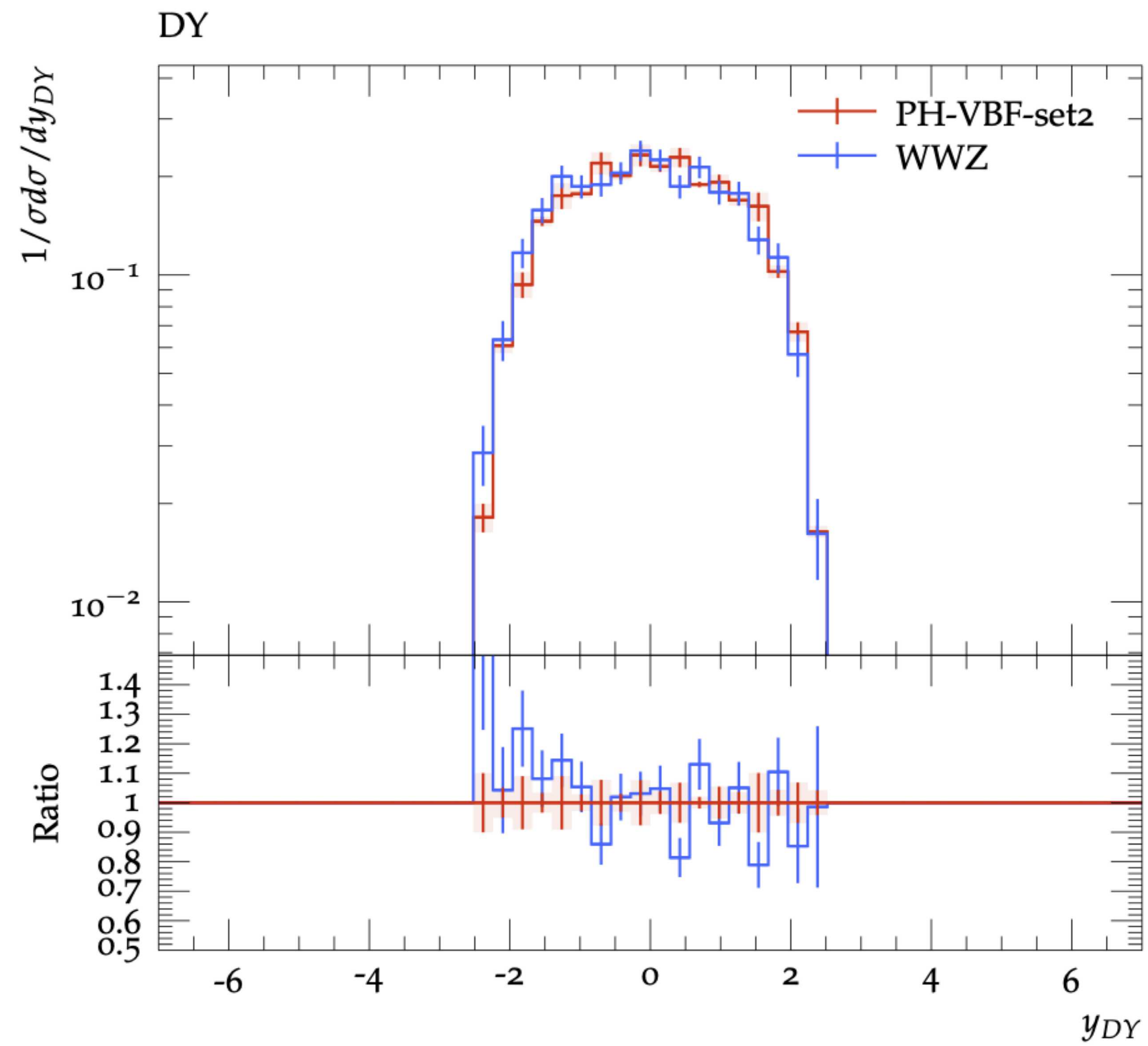
W-densities and Z production



- Similarity of Z production off quarks with Z production off W's !
  - generalization of parton densities
  - natural extension of TMDs

W-densities and VBF process



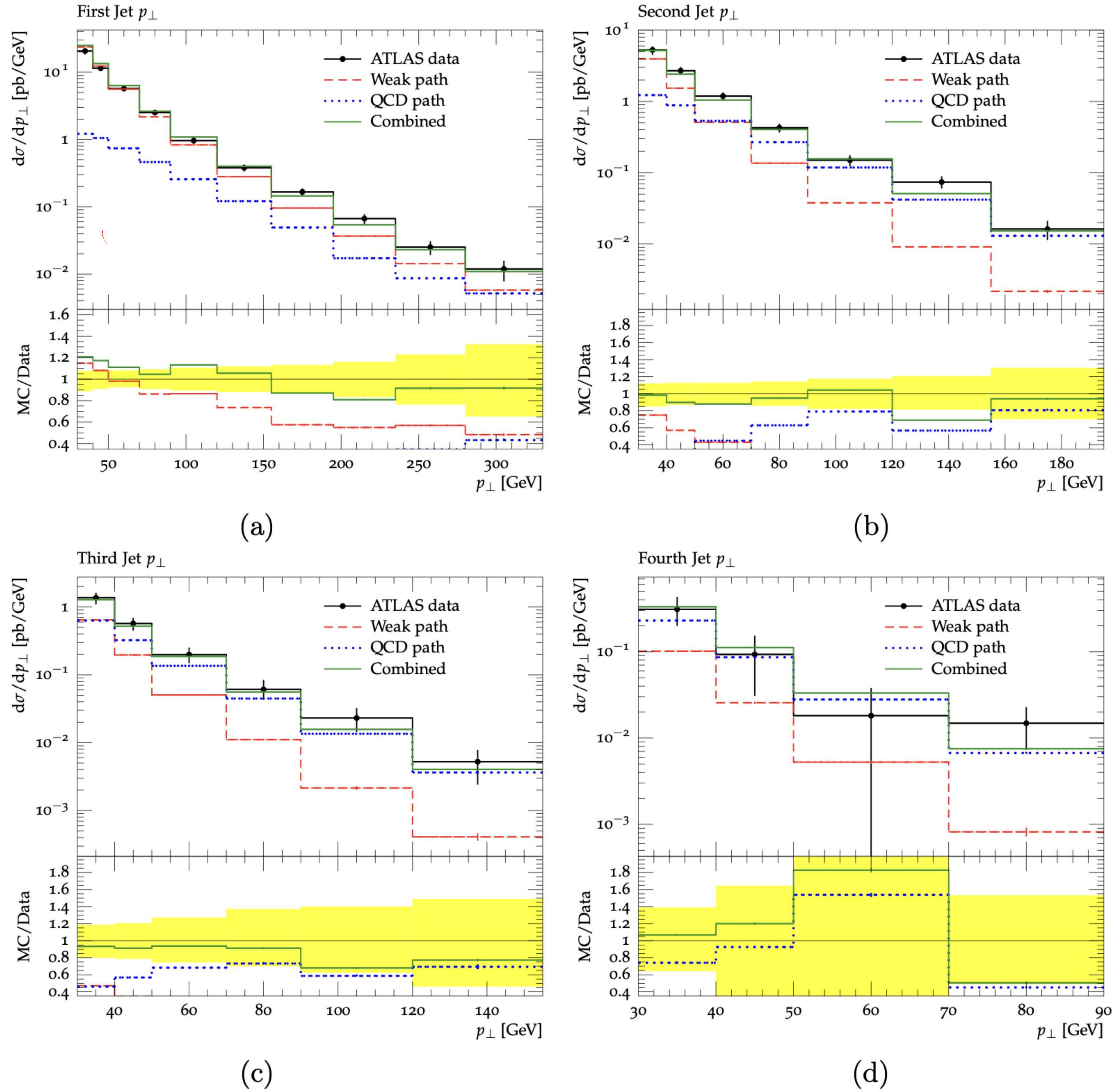


- Test of x -dependence of W parton density
    - for p_T a full simulation of the initial state EW parton shower is needed

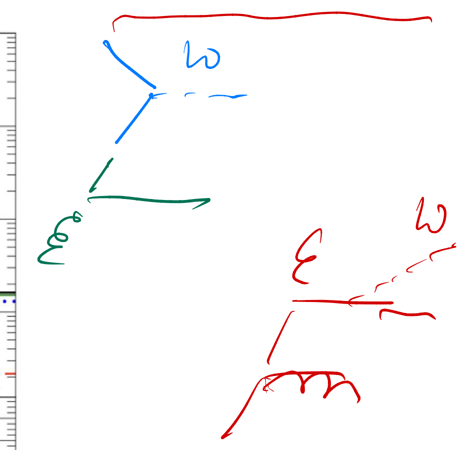
EW bosons in final state

W/Z bosons in Final State Parton Shower

https://arxiv.org/abs/1401.5238



(a) (b) (c) (d)



Great prospects for highest energies and highest luminosities:
democratic zoo of particles, including bosons

Eventually determine PDF and TMD of Higgs !

.... what you should have learned

- Lecture:
  - QCD is still a interesting and challenging field
  - basic QCD calculations
  - important role of gluons, which comes from gluon self-coupling
  - understanding of parton evolution equations in terms of parton radiation
  - importance of “soft” gluon resummation
- Exercises:
  - basic Monte Carlo techniques
  - MC integration and generation of variables according to distributions
  - solution of parton evolution equation with MC technique
  - advantage of MC technique to solve complex problems like multiparton radiation (example pt spectrum of Drell Yan)

There are many open questions in QCD at highest energies and at highest scales ….

Your ideas, imagination and help is very much needed !

The End