Structure of hadrons from lattice QCD

Philipp Hägler

TECHNISCHE UNIVERSITÄT MÜNCHEN

Overview

- physics case
- introduction to GPDs
- introduction to lattice calculations
- lattice results – highlights
- summary

Physics case – why GPDs?

- all-encompassing framework
- fundamental sum rules
- distribution of quarks and gluon in coordinate space
- spin structure of hadrons
- relevance for phenomenology and experiment

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→
- Jaffe/Ji/Manohar nucleon spin sum rule
- momentum sum rule
- vanishing of the total anomalous gravitomagnetic moment

Physics case – why GPDs?

- all-encompassing framework
- fundamental sum rules
- distribution of quarks and gluon in coordinate space →
  - FFs: Breit frame + NR approximation
  - densities in transverse impact parameter space (Burkardt PRD 2000)
- spin structure of hadrons
- relevance for phenomenology and experiment

Physics case – why GPDs?

- all-encompassing framework
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- longitudinal spin
- transversity/transverse spin
  - correlations of spin (OAM) and coordinate DOFs

Physics case – why GPDs?

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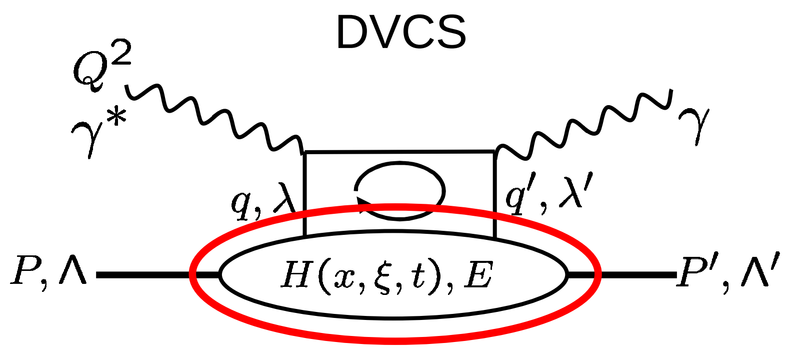

- QCD factorization
- DVCS & exclusive meson production
- wide angle Compton scattering
- azimuthal asymmetries in SIDIS and DY

Introduction to GPDs

- definition

$$\int \frac{d\eta}{4\pi} e^{i\eta x} \langle P' | \overline{q}(-\frac{\eta n}{2}) \gamma^{\mu} \mathcal{U} q(\frac{\eta n}{2}) | P \rangle = \overline{U}(P') \left(\gamma^{\mu} H(x,\xi,\Delta^2) + i \frac{\sigma^{\mu\nu} \Delta_{\nu}}{2M} E(x,\xi,\Delta^2) \right) U(P)$$

$$t^2 = \Delta^2 = (P'-P)^2$$

- basic properties
  - relation to PDFs
  - relation to FFs
  - higher moments
  - in impact parameter space
- spin sumrule
- what is known quantitatively

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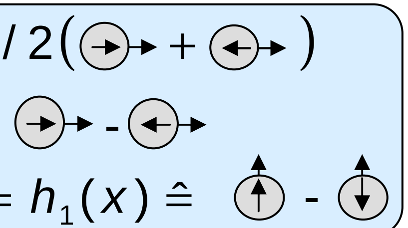
$$H(x,0,0) = q(x) \hat{=} 1/2(\quad + \quad)$$

$$\widetilde{H}(x,0,0) = \Delta q(x) \hat{=} \quad - \quad$$

$$H_T(x,0,0) = \delta q(x) = h_1(x) \hat{=} \quad - \quad$$



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- basic properties
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  - relation to FFs →
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$$\int dx\, H(x,\xi,t) = F_1(t),$$
$$\int dx\, \tilde{H}(x,\xi,t) = g_A(t),$$
$$\int dx\, H_T(x,\xi,t) = g_T(t) \text{ etc.}$$

↓

$$\langle 1 \rangle_q = g_V = A_{10}(t=0) = F_1(t=0)$$
$$\langle 1 \rangle_{\Delta q} = g_A = \tilde{A}_{10}(t=0) = g_A(t=0)$$
$$\langle 1 \rangle_{\delta q} = g_T = A_{T10}(t=0) = g_T(t=0)$$

Introduction to GPDs

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$$\int dx\, x^{n-1} H(x,\xi,t) = A_{n0}(t) + \ldots$$

$$\int dx\, x^{n-1} E(x,\xi,t) = B_{n0}(t) + \ldots$$

$$\ldots$$

$$\int dx\, xH(x,\xi,t) = A_{20}(t) + (-2\xi)^2 C_{20}(t)$$

$$\int dx\, xE(x,\xi,t) = B_{20}(t) - (-2\xi)^2 C_{20}(t)$$

$$\ldots$$

$$\langle P',\Lambda' | T_{\mu\nu} | P,\Lambda \rangle = \bar{U} \left\{ \gamma_{\{\mu} \bar{P}_{\nu\}} A_{20}(t) - \frac{i\Delta^{\rho}\sigma_{\rho\{\mu}\bar{P}_{\nu\}}}{2m} B_{20}(t) + \frac{\Delta_{\mu}\Delta_{\nu}}{m} C_{20}(t) \right\} U$$

Introduction to GPDs

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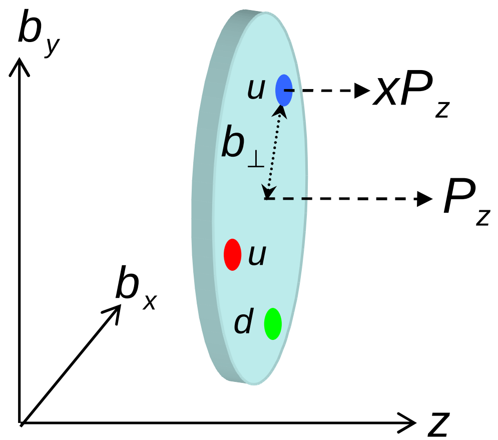
- what is known quantitatively

$$q(x, b_{\perp}^2) = \int d^2 \Delta_{\perp} e^{-i\Delta_{\perp} \cdot b_{\perp}} H(x, \xi = 0, \Delta^2)$$

M. Burkardt, PRD 2000



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- what is known quantitatively

$$\frac{1}{2} = \frac{1}{2}\left(A_{20}(t=0) + B_{20}(t=0)\right) = \frac{1}{2}\left(\langle x \rangle_q + \langle x \rangle_g + B_{20}^{q+g}(t=0)\right) = J_q + J_g$$

$$L_q \equiv J_q - \Delta\Sigma_q / 2, \qquad L_g \equiv J_g - \Delta G$$

Jaffe&Manohar 1989, Ji 2001

Introduction to GPDs

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- basic properties
  - relation to PDFs
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  - higher moments
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- spin sumrule
- what is known quantitatively
  - a lot
    - FFs ($\int dx$)
    - PDFs ($t{=}\Delta^2{=}0$)
    - data, phenomenology&models (HERMES PRL 2001, CLAS PRL 2001, Diehl/Feldmann/Jakob/Kroll EPJC 2004,…)
  - not much
    - correlations in $x,\xi,t{=}\Delta^2$
    - helicity flip/chiral odd/tensor sector
    - …

Lattice QCD simulations

- calculations from first principles with controllable systematic uncertainties
- numerical evaluation of PI using MC methods in discretized Euclidean space time
- large number of different discretizations/actions being used
- dynamical (unquenched) calculations are standard
- pion masses as low as $m_{\pi,\mathrm{lat}} \approx 2m_{\pi,\mathrm{phys}}$

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$$\int \mathcal{D}[q]\mathcal{D}[\bar{q}]\mathcal{D}[A] e^{iS[q,\bar{q},A]} qqq \ldots \overline{q}\overline{q}\overline{q}$$

$$\int \prod_n \underbrace{dU_n \, \mathrm{det}\, M(U) e^{-S_g[U]}}_{\text{MC sampling}} M^{-1}(U) \cdots M^{-1}(U)$$

Lattice QCD simulations

- calculations from first principles with controllable systematic uncertainties
- numerical evaluation of PI using MC methods in discretized Euclidean space time
- large number of different discretizations/actions being used →
  - Wilson
  - staggered
  - Ginsparg-Wilson →
    - overlap
    - domain wall
  - twisted mass
- dynamical (unquenched) calculations are standard
- pion masses as low as $m_{\pi,lat} \approx 2m_{\pi,phys}$

Lattice QCD simulations

- calculations from first principles with controllable systematic uncertainties
- numerical evaluation of PI using MC methods in discretized Euclidean space time
- large number of different discretizations/actions being used
- dynamical (unquenched) calculations are standard →
  - sea quark loops included
  - ~~det(M)=1~~
- pion masses as low as $m_{\pi,lat} \approx 2m_{\pi,phys}$

Lattice QCD simulations

- calculations from first principles with controllable systematic uncertainties
- numerical evaluation of PI using MC methods in discretized Euclidean space time
- large number of different discretizations/actions being used
- dynamical (unquenched) calculations are standard
- pion masses as low as $m_{\pi,lat} \approx 2m_{\pi,phys}$
  - QCDSF/UKQCD: Wilson $m_{\pi,lat} \approx 340$ MeV
  - LHPC: Asqtad+DW $m_{\pi,lat} \approx 360$ MeV
  - RBC-UKQCD: DW $m_{\pi,lat} \approx 300$ MeV
  - ETMC: $m_{\pi,lat} \approx 300$ MeV
  - JLQCD: overlap $m_{\pi,lat} \approx 288$ MeV

Ph. Hägler, DESY theory workshop 2007

Lattice QCD simulations

- from matrix elements to two- and three-point functions
- connected and disconnected diagrams
- renormalization of H(4) lattice operators
- extraction of generalized form factors
- systematic uncertainties
- chiral extrapolation

$$\langle P', \Lambda' | \bar{q}(0) \left[\Gamma_{Dirac} D^{\mu_1} D^{\mu_2} \ldots \right] q(0) | P, \Lambda \rangle$$

$$C_{3pt} = \langle J(P', t_{\mathrm{snk}}) \mathcal{O}_{\Gamma}^{\mu_1 \mu_2 \cdots}(\Delta, \tau) J^{\dagger}(P, t_{\mathrm{src}}) \rangle$$

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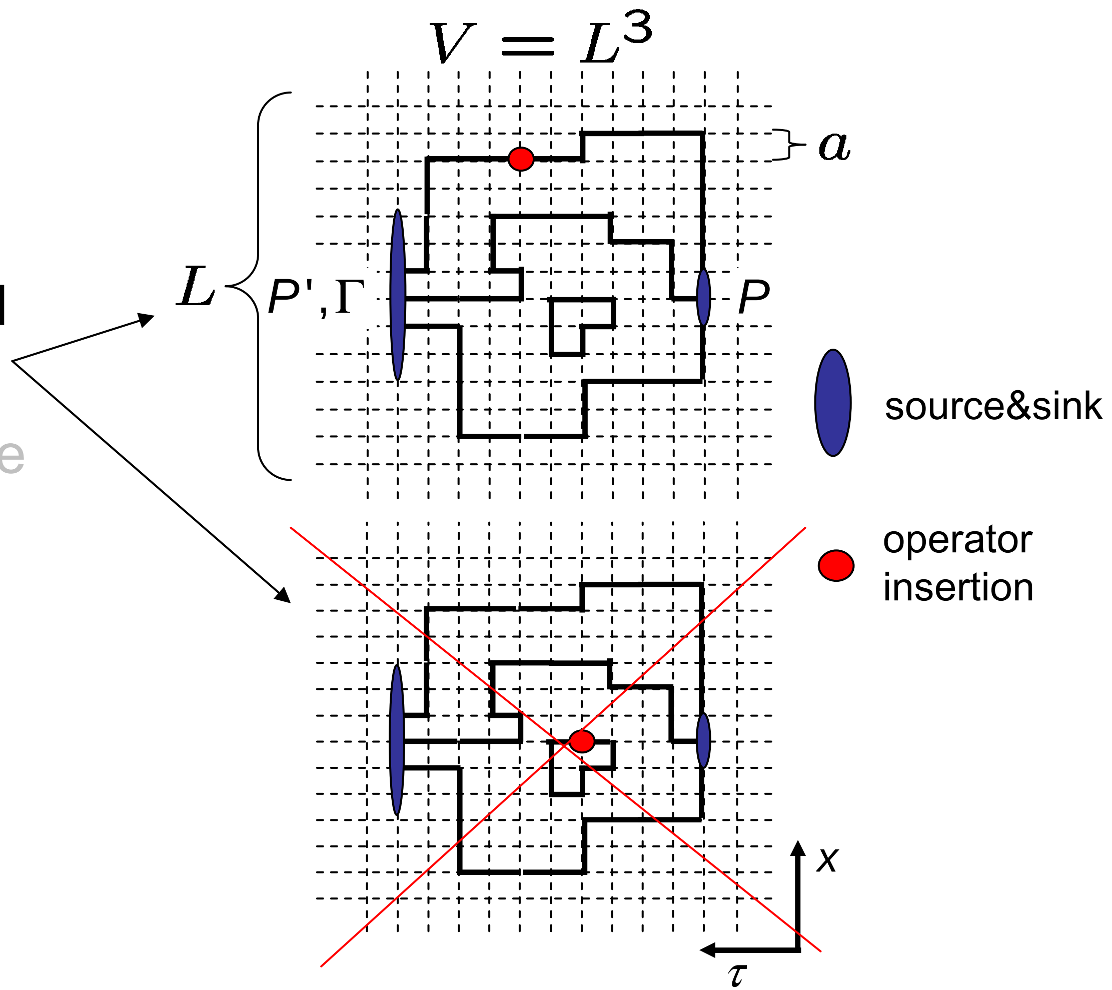


Lattice QCD simulations

- from matrix elements to two- and three-point functions
- connected and disconnected diagrams
- renormalization of H(4) lattice operators →
  - perturbative
  - non-perturbative (Rome-Southampton)
- extraction of generalized form factors
- systematic uncertainties
- chiral extrapolation

Lattice QCD simulations

- from matrix elements to two- and three-point functions
- connected and disconnected diagrams
- renormalization of H(4) lattice operators
- extraction of generalized form factors
- systematic uncertainties
- chiral extrapolation

$$\frac{C_{3pt}}{C_{2pt}} \to \Delta\Sigma^{\text{lat}}, \delta q^{\text{lat}}, \langle x \rangle^{\text{lat}}, F_1^{\text{lat}}(t), A_{20}^{\text{lat}}(t), B_{20}^{\text{lat}}(t), A_{T20}^{\text{lat}}(t), \ldots$$

Lattice QCD simulations

- from matrix elements to two- and three-point functions
- connected and disconnected diagrams
- renormalization of H(4) lattice operators
- extraction of generalized form factors
- systematic uncertainties →
  - discretization effects
  - finite size effects
  - large quark masses
- chiral extrapolation

Lattice QCD simulations

- from matrix elements to two- and three-point functions
- connected and disconnected diagrams
- renormalization of H(4) lattice operators
- extraction of generalized form factors
- systematic uncertainties
- chiral extrapolation →
  - chiral effective field theory (ChPT)
  - heavy baryon ChPT with and w/o the Delta
  - covariant baryon ChPT

Selected lattice results - overview

- nucleon
  - axial vector coupling constant
  - quark momentum fraction
  - spin sumrule & longitudinal spin structure
  - transverse spin structure
- pion
  - spin structure

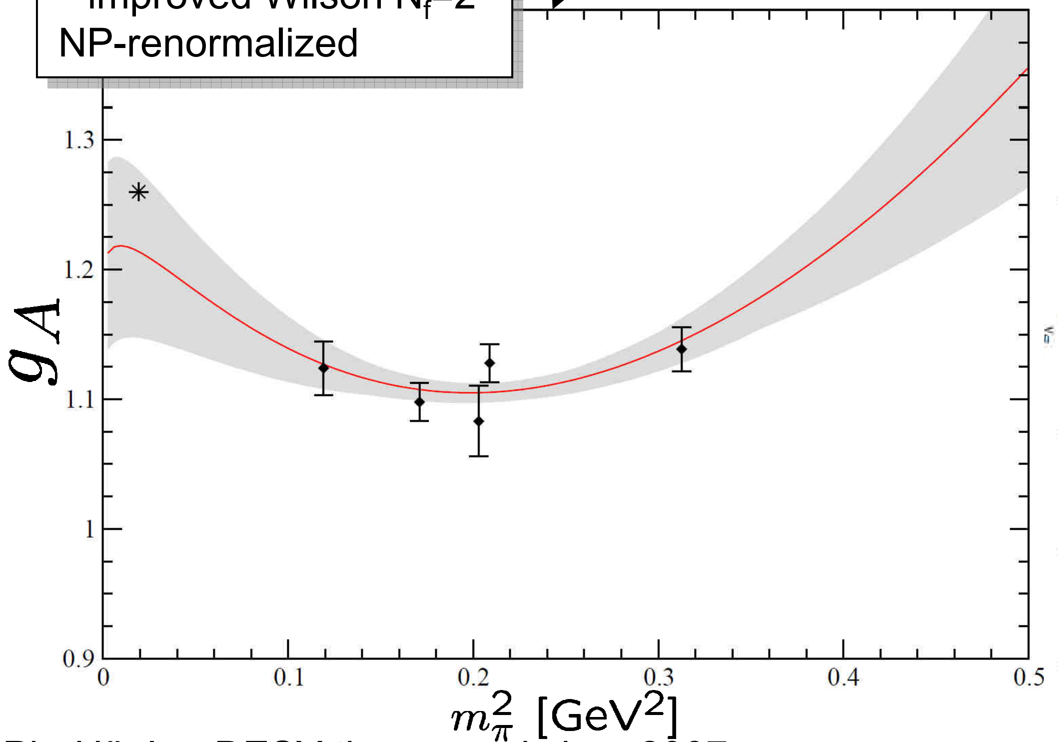
Axial vector coupling constant g_A

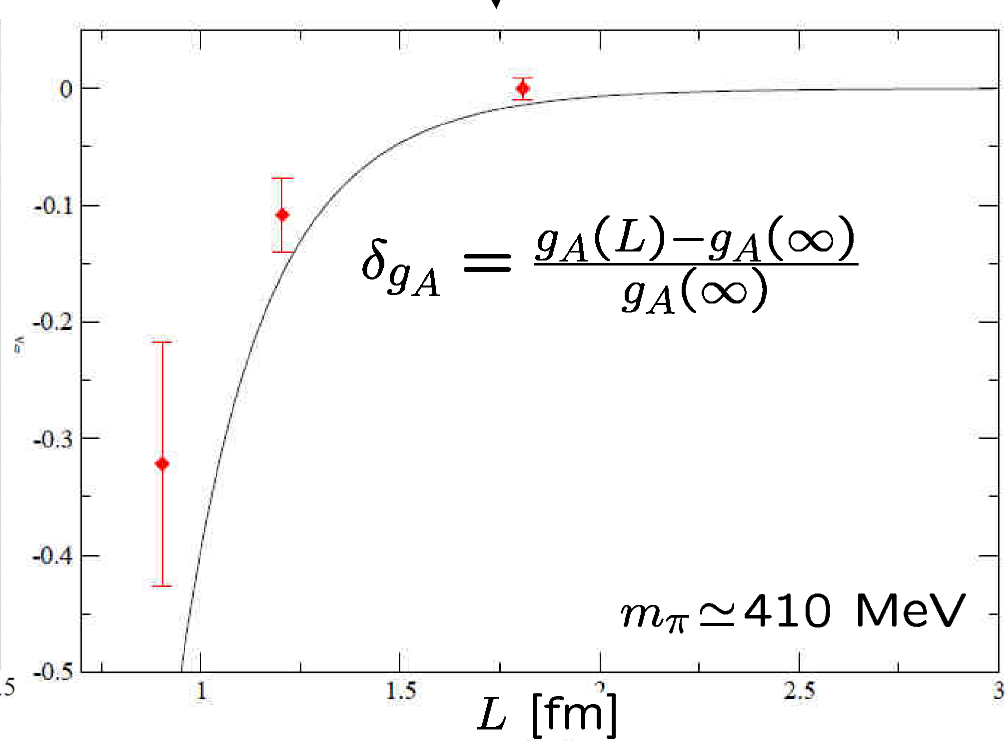
QCDSF/UKQCD preliminary

- precisely known from β-decay
- u-d quark spin contribution
- fundamental coupling constant
- benchmark observable g_A
- no discon. contr. (isovector)

$$\langle P|\bar{u}\gamma_\mu\gamma_5 u - \bar{d}\gamma_\mu\gamma_5 d|P\rangle = g_A \bar{U}(P)\gamma_\mu\gamma_5 U(P)$$

- volume dependence
- HBChPT including the Δ-resonance in a finite volume (Hemmert et al, Savage&Beane)
- improved Wilson N_f=2 NP-renormalized





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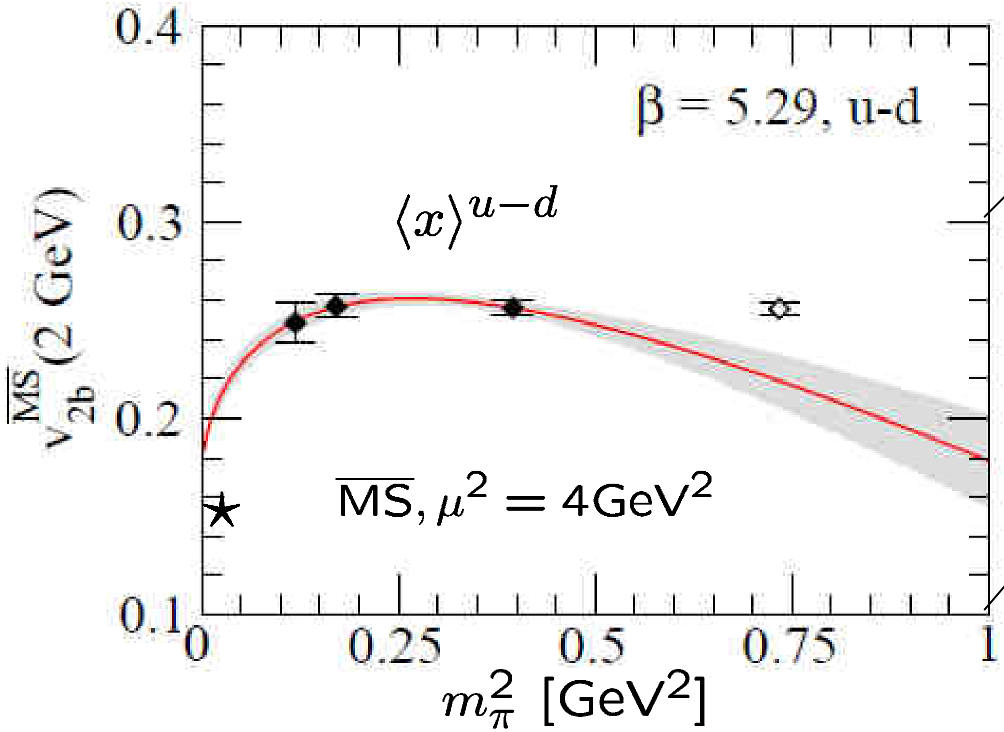
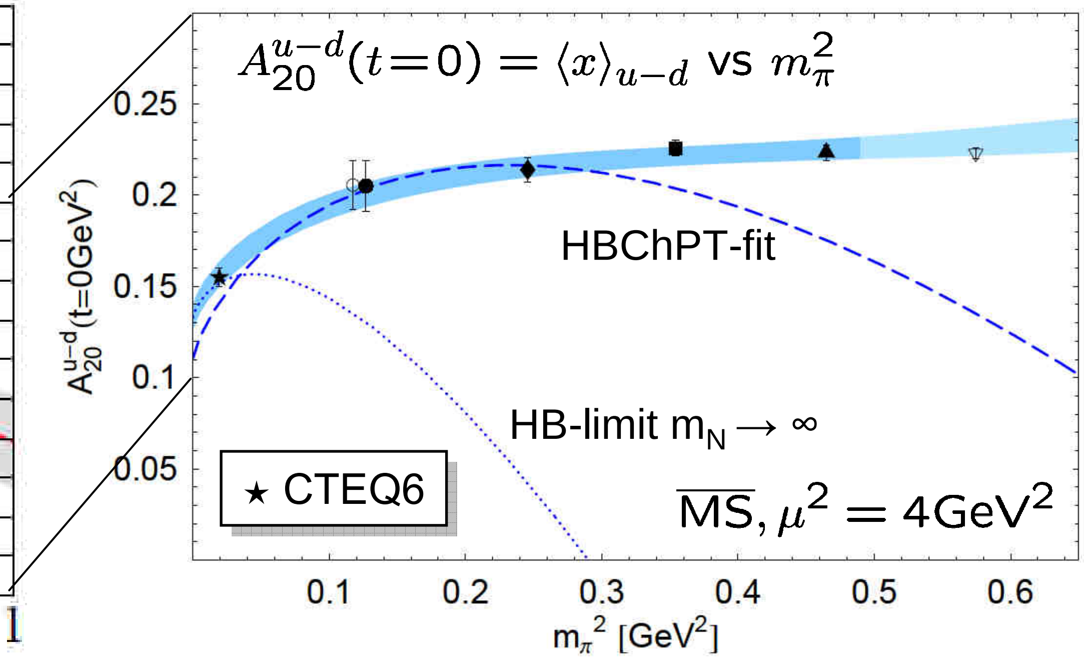
Moments of nucleon PDFs

$$\langle P|\bar{q}\,\Gamma D^{\mu_1} D^{\mu_2}\cdots q|P\rangle \propto \langle x\rangle_q, \langle x\rangle_{\Delta q}, \langle x\rangle_{\delta q}, \langle x^2\rangle_q \ldots$$

- QCDSF/UKQCD preliminary; improved N_f=2 Wilson 24^3,32^3
- LHPC preliminary hybrid N_f=2+1 DW valence + Asqtad staggered sea on $20(8)^3$ and L_S=16
- NP-renormalization → RI'-MOM→ $\overline{\text{MS}}$
- NP-improved PT renormalization





LHPC, arXiv:0705.4295

- covariant baryon ChPT (Dorati et al. 2007)

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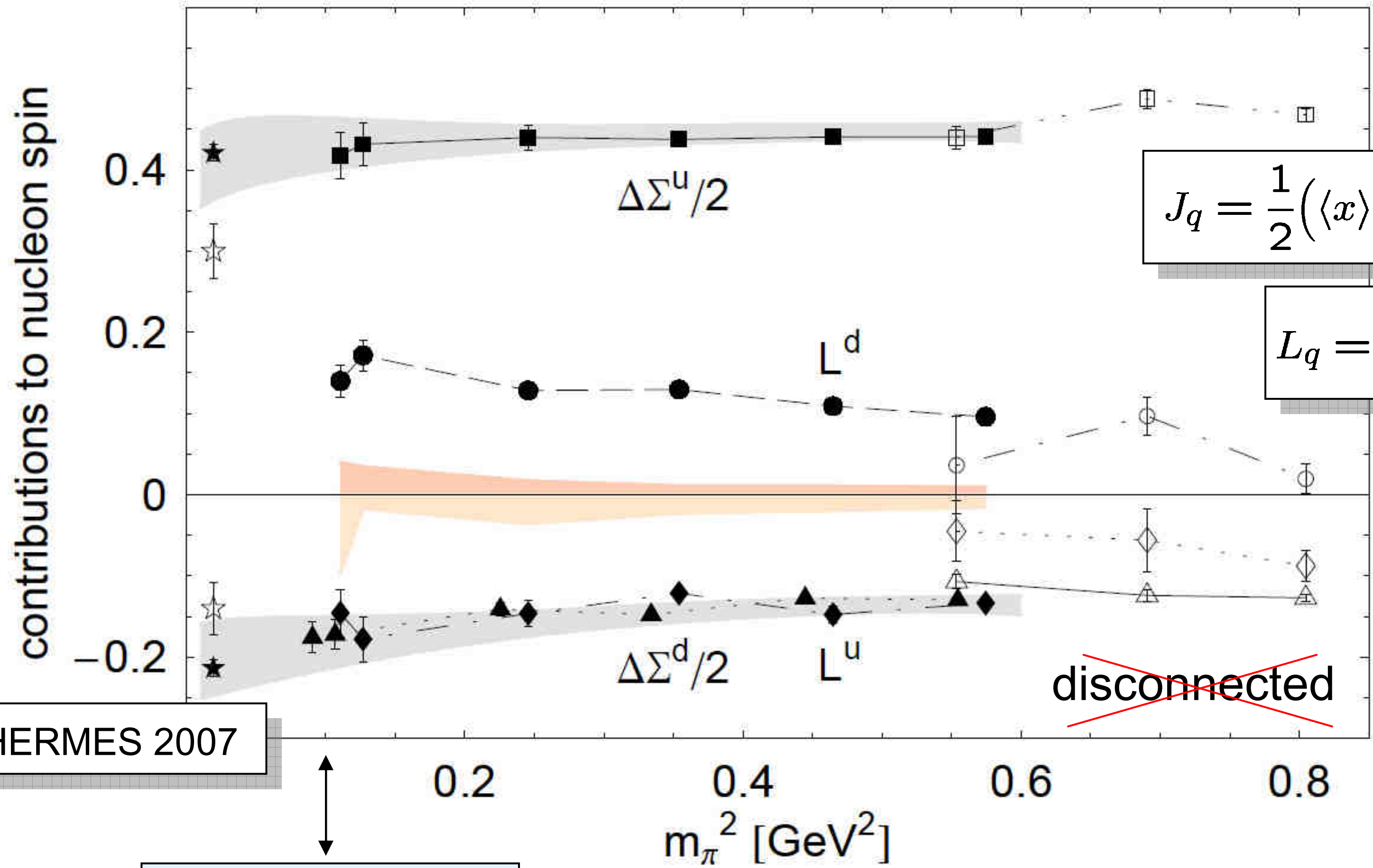
Quark spin and OAM contributions to the nucleon spin

LHPC, arXiv:0705.4295; hybrid Asqtad sea + DW valence



$$J_q = \frac{1}{2}\left(\langle x\rangle_q + B^q_{20}(t=0)\right)$$

$$L_q = J_q - \frac{1}{2}\Delta\Sigma$$

$m_\pi \simeq 350$ MeV

$L^d \approx -L^u \approx 30\%$ of $\frac{1}{2}$

$L^u + L^d \approx 0$

$J^u = 0.26(1)$

$J^d = -0.02(1)$

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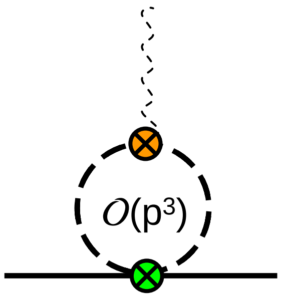
Isosinglet $B_{20}(t)$ form factor LHPC, arXiv:0705.4295

including quark anomalous gravitomagnetic moment AGM

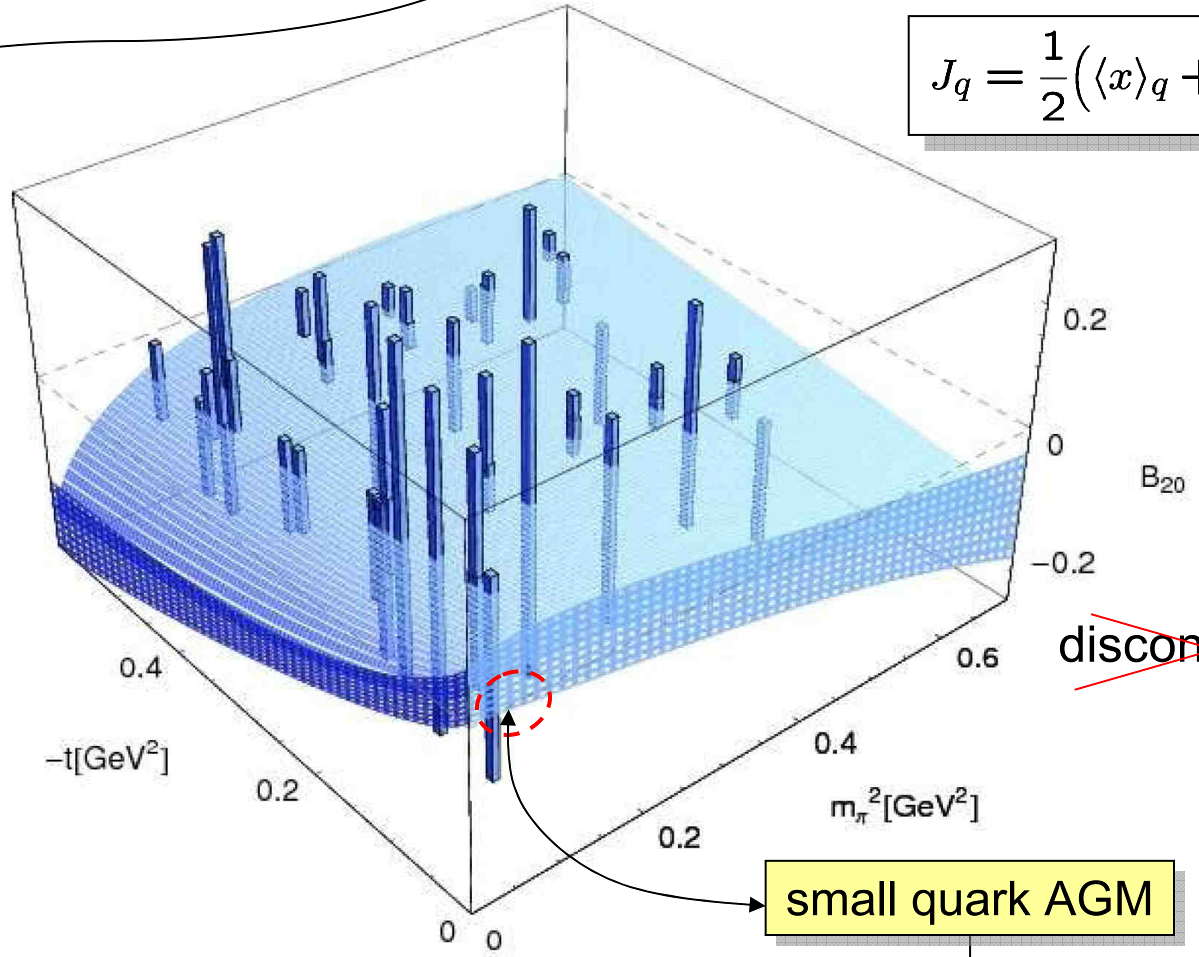
- based on HBChPT by Diehl, Manashov, Schäfer EJPA 2006, Ando, Chen, Kao PRD 2006

$$J_q = \frac{1}{2}\left(\langle x\rangle_q + B^q_{20}(t=0)\right)$$

including



non-linear correlation in t and m_π



~~disconnected~~

small quark AGM

$$B^{u+d}_{20}(t=0, m_{\pi,\text{phys}}) = 0.05(5)$$

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Angular momentum of quarks: Lattice vs phenomenology+experiment

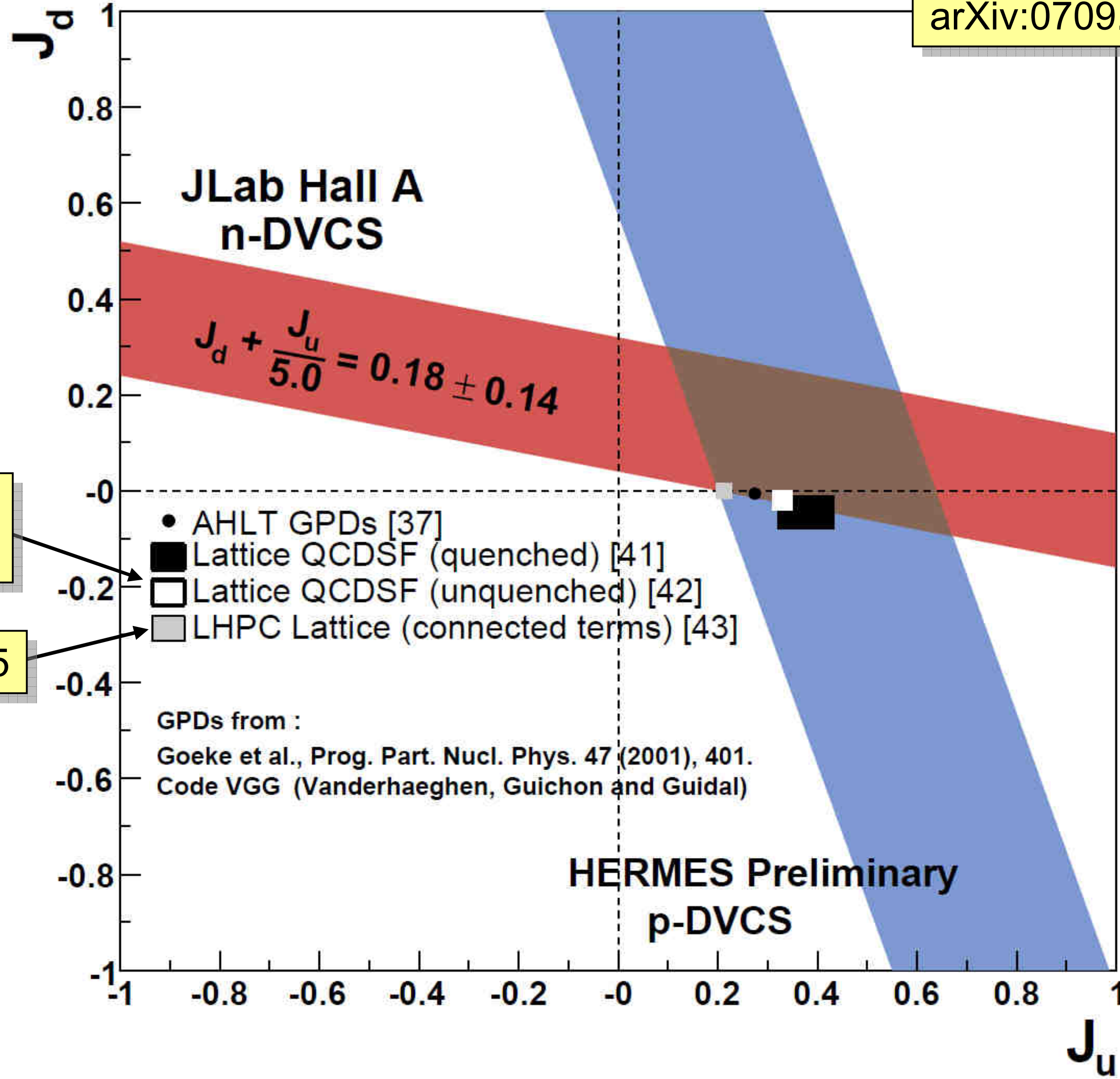
$$J_q = \frac{1}{2}\left(\langle x \rangle_q + B^q_{20}(t=0)\right)$$

extrapolated to $m_{\pi,phys}$ using covariant baryon ChPT

unpublished preliminary

arXiv:0705.4295

arXiv:0709.0450



disclaimer:
JLab band from integral over VGG model, constrained at a single $x=\xi$-point

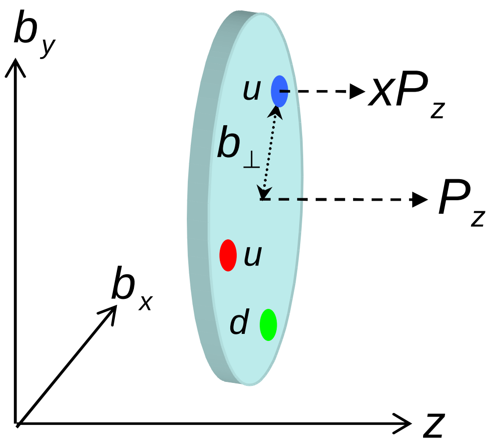
Tranverse nucleon structure

$$q(x, b_\perp^2) = \int d^2\Delta_\perp e^{-i\Delta_\perp \cdot b_\perp} H(x, \xi = 0, \Delta^2)$$

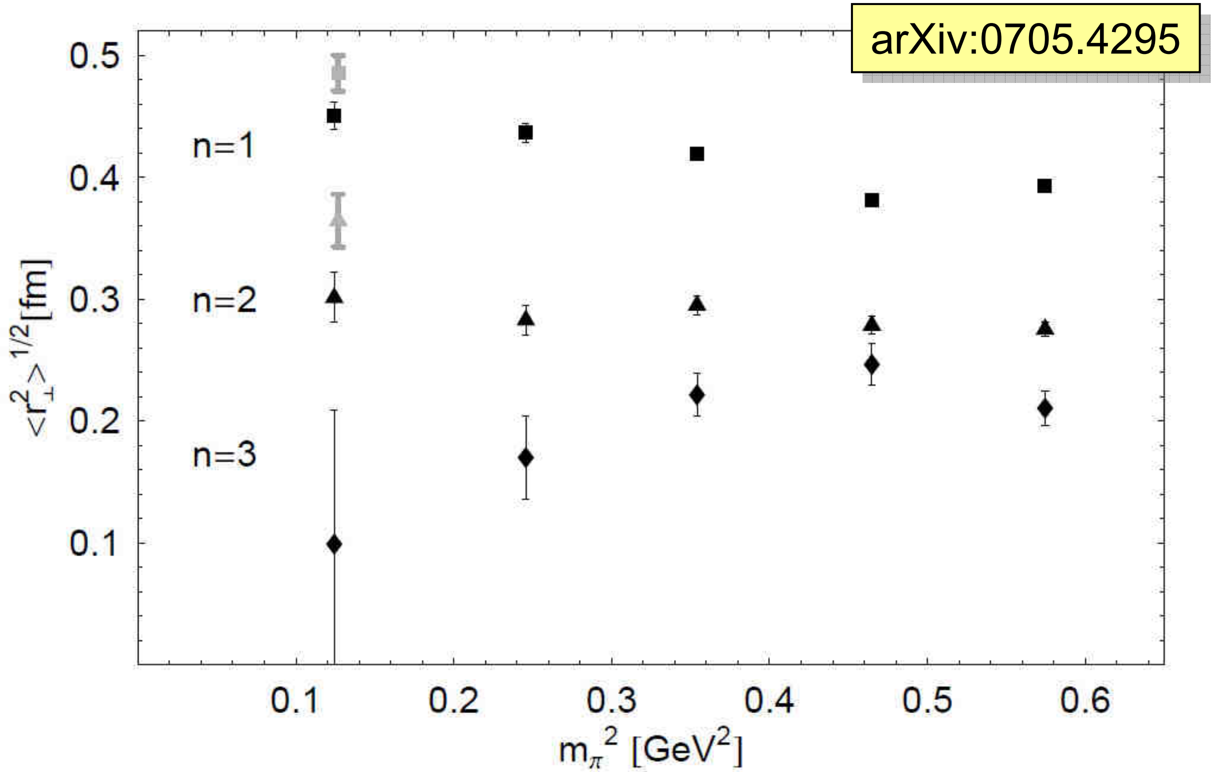
M. Burkardt, PRD 2000 (ξ=0)

M. Diehl, EPJC 2002 ($\xi \neq 0$)

- interpretation as probability density



as $x \to 1$, the distribution peaks around $R_\perp$ and $\langle b_\perp^2 \rangle^{1/2} = \langle r_\perp^2 \rangle^{1/2} \to 0$



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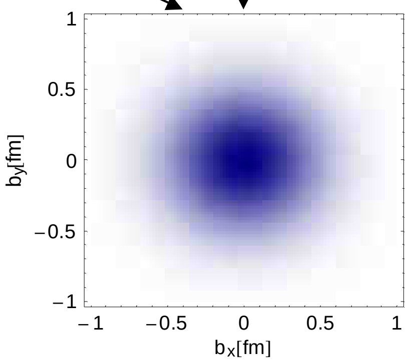
Transversely polarized quarks

transversely polarized quarks with spin vector $s_\perp$ in a transversely polarized nucleon with spin vector $S_\perp$

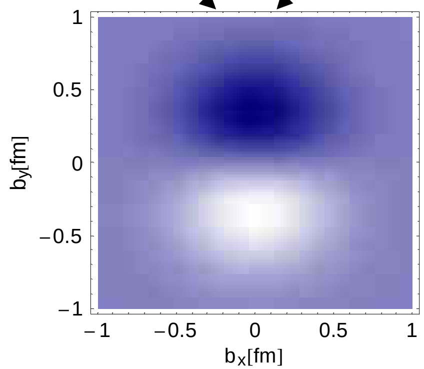
density in impact parameter ($b_\perp$)space

Diehl / PhH EPJC 2005

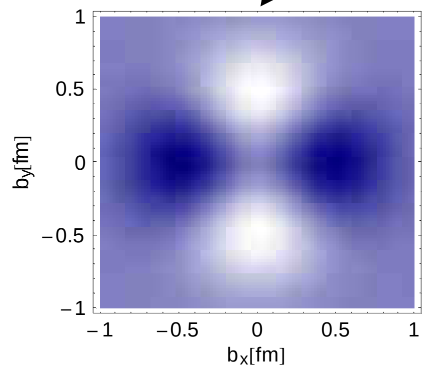
$$\langle P^+, 0_\perp, S_\perp | \widehat{\rho}_T(x, b_\perp; s_\perp) | P^+, 0_\perp, S_\perp \rangle$$
$$= \frac{1}{2}\Big\{ H + s^i_\perp S^i_\perp \Big(H_T - \frac{1}{4m^2}\Delta_b \widetilde{H}_T \Big) - \epsilon_{ij} S^i_\perp b^i_\perp \frac{1}{m} E' - \epsilon_{ij} s^i_\perp b^i_\perp \frac{1}{m} \overline{E}'_T + s^i_\perp (2 b^i_\perp b^j_\perp - b^2_\perp \delta^{ij}) S^j_\perp \frac{1}{m^2} \widetilde{H}''_T \Big\}$$



monopole



dipole



quadrupole

x-moments of density → on the lattice → calculate moments of the GPDs → parametrize t-dependence → Fourier-transform to $b_\perp$-space →

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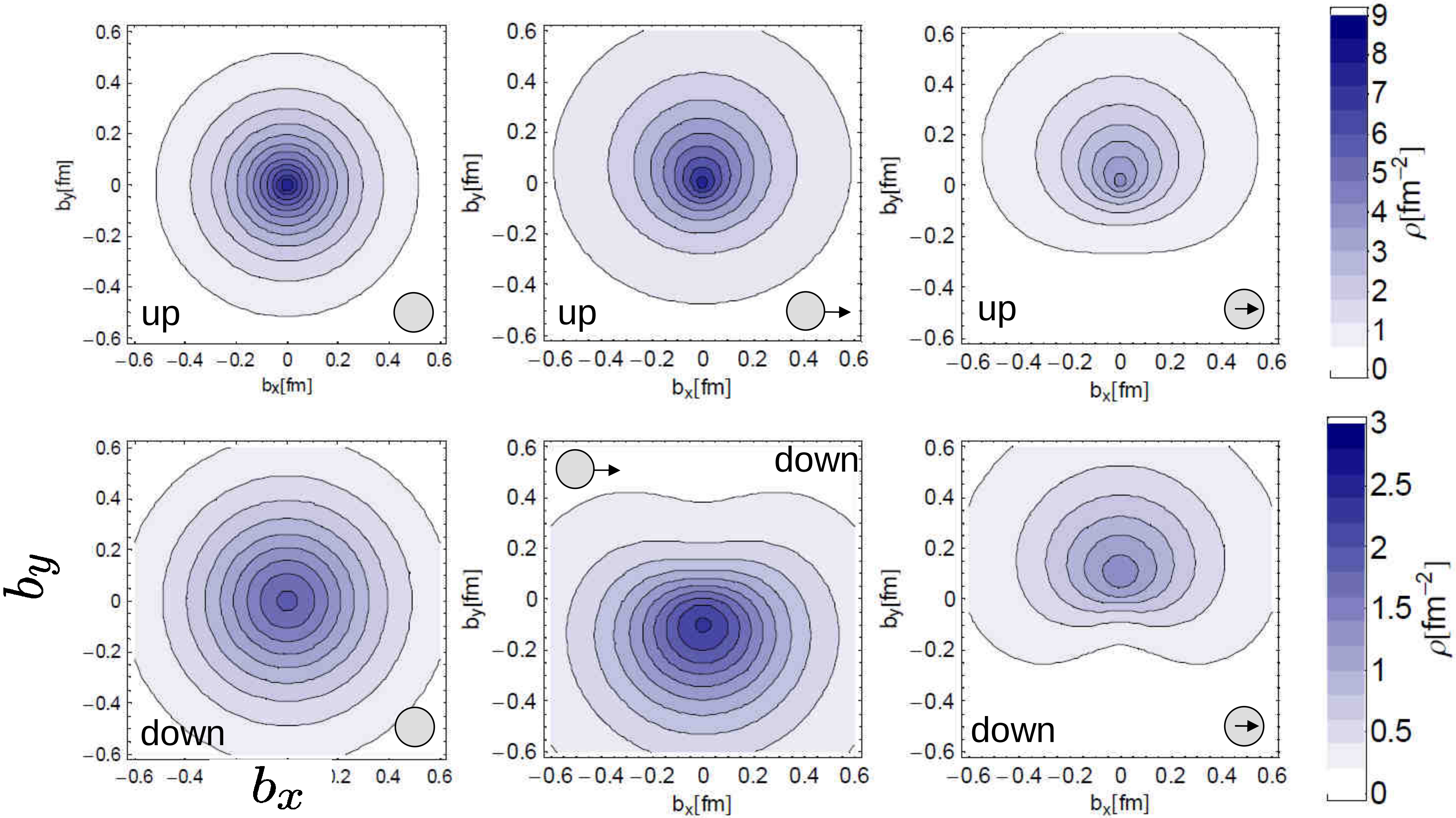
Lowest n=1 moments of up- and down-quark densities

QCDSF/UKQCD, PRL 2007 (hep-lat/0612032)



strong distortions for transverse polarizations

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Spin structure of the pion

QCDSF/UKQCD, arXiv:0708.2249 [hep-lat]

- longitudinal spin structure is trivial
- non-trivial transverse spin structure?
- finite volume effects
- discretization effects
- densities

Spin structure of the pion

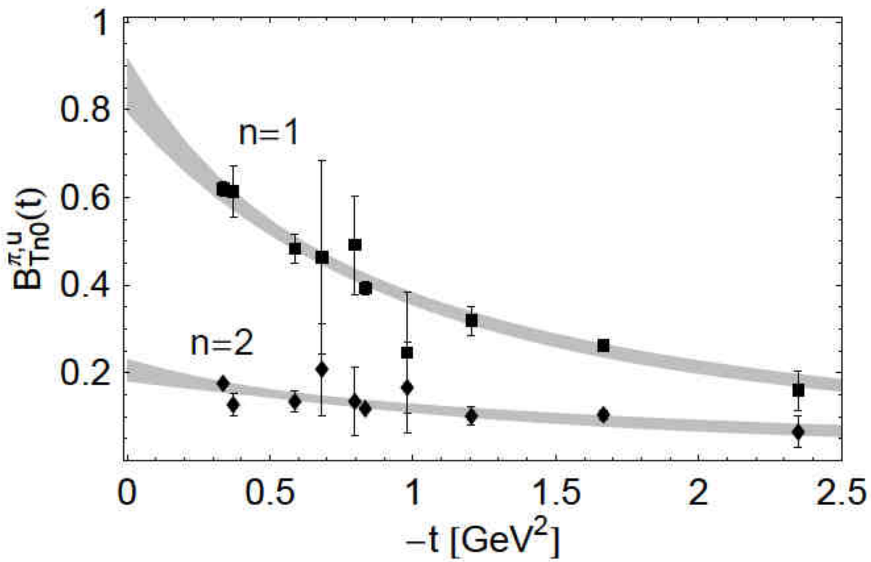
QCDSF/UKQCD, arXiv:0708.2249 [hep-lat]

- longitudinal spin structure is trivial
- non-trivial transverse spin structure? →

$$\rho_T^n(b_\perp; s_\perp) = \frac{1}{2}\Big\{A_{n0}^\pi(b_\perp^2) - \epsilon_{ij} s_\perp^i b_\perp^i \frac{1}{m_\pi} B_{Tn0}^{\pi\prime}\Big\}$$

- finite volume effects
- discretization effects
- densities

but is $B_{Tn0}^{\pi\prime}$ non-zero?



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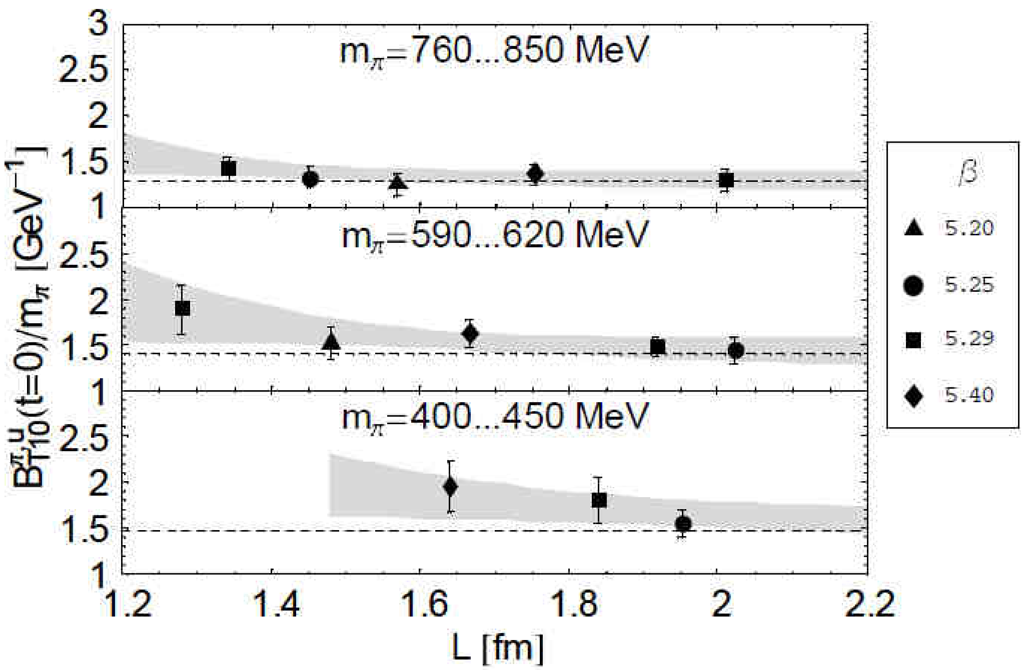

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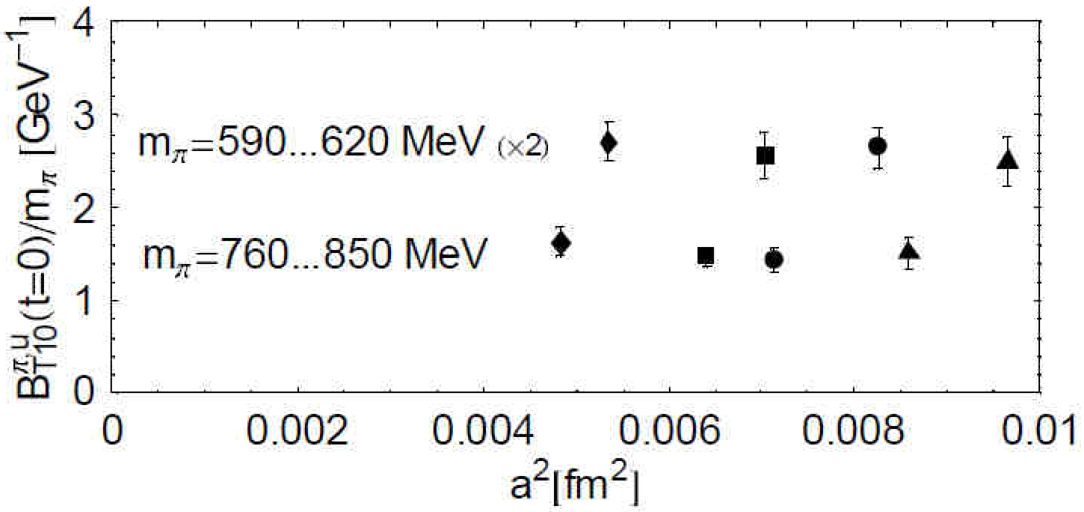

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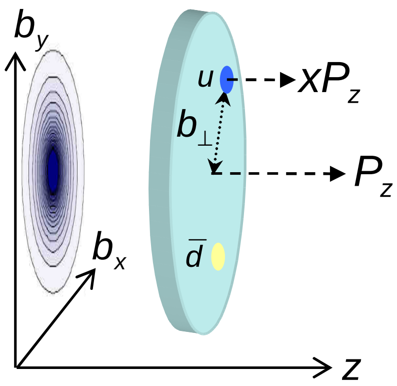

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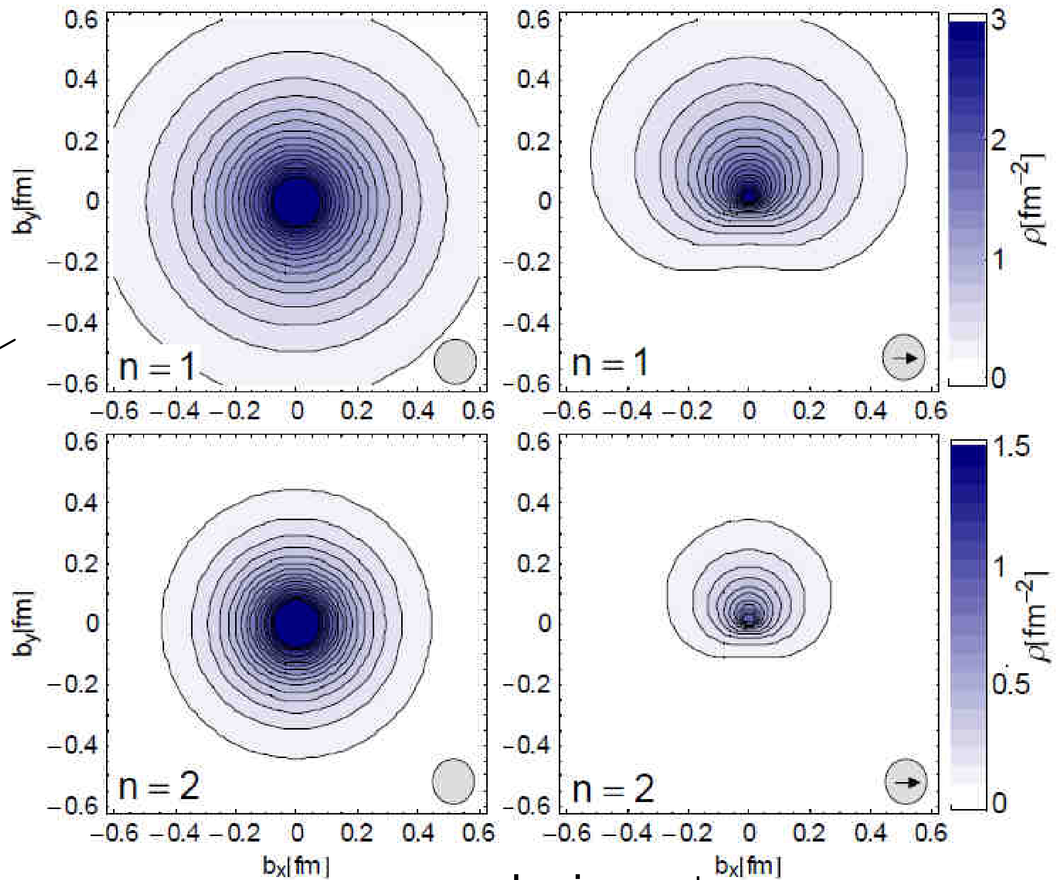
- discretization effects
- densities



up-quarks in a π^+

strong deformations for transversely polarized quarks

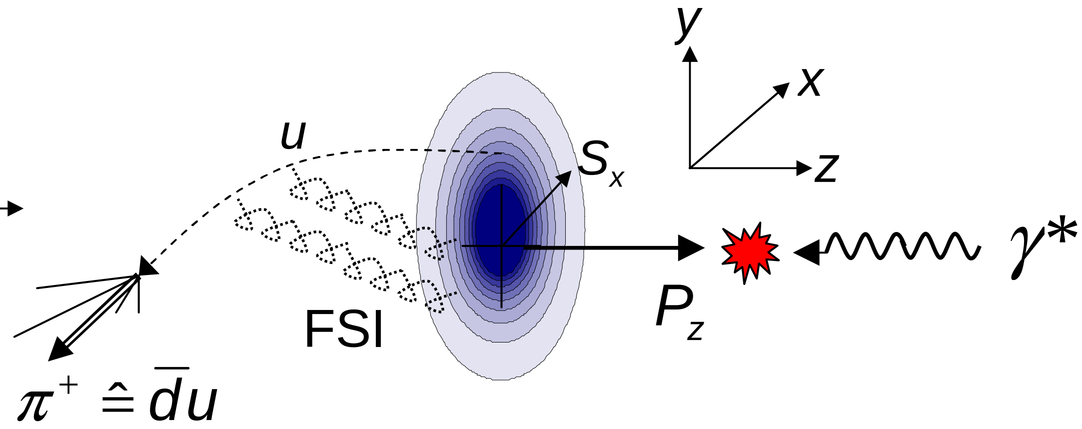
pion has a non-trivial (transverse) spin structure

transverse shifts for nucleon and pion are the same

Ph. Hägler, DESY theory workshop 2007

Implications for experiment

- asymmetric densities→asymmetries
- Sivers-asymmetry (unpolarized quarks in transversely polarized target)
- Boer-Mulders asymmetry (transversely polarized quarks in unpolarized target)



conjecture / hypothesis by Burkardt PRD 72 (2005); Meissner,Metz&Goecke 2007

Sivers	$f_{1T}^{\perp} \sim -E$
Boer-Mulders	$h_1^{\perp} \sim -\overline{E}_T$

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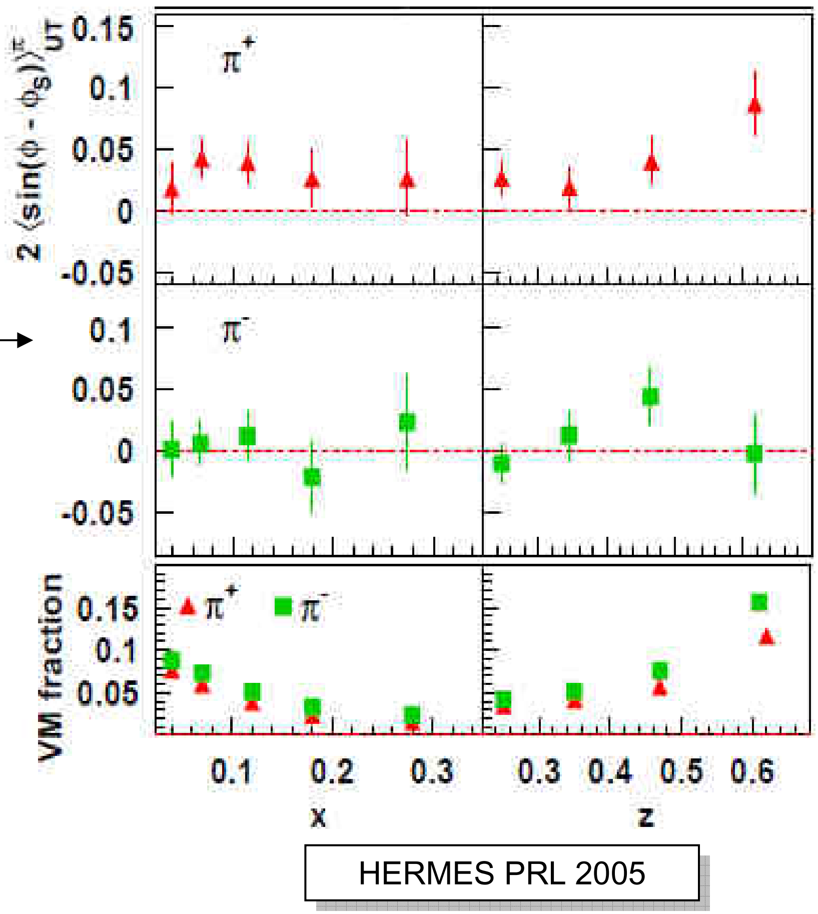

Ph. Hägler, DESY theory workshop 2007

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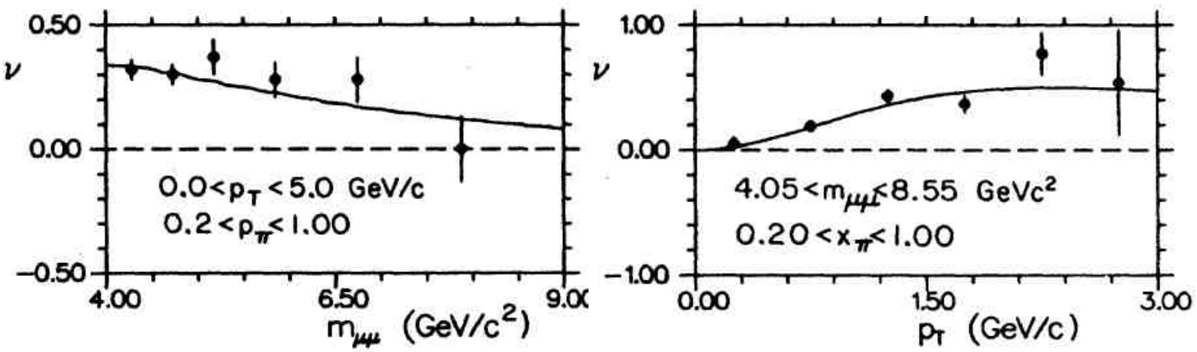

- $\cos(2\phi)$ in π^- on tungsten → DY production at Fermilab (E615); Conway et al. PRD 1989; Boer PRD 1999
- $\cos(2\phi)$ in unpolarized SIDIS at CLAS12/JLab
- (un-)polarized π^-p DY production at COMPASS/CERN

Summary&Outlook

- substantial progress in lattice QCD calculations of hadron structure observables
  - qualitative+quantitative insights
  - relevance for experiment
- addressing systematic uncertainties
- new&improved methods

Summary&Outlook

- substantial progress in lattice QCD calculations of hadron structure observables
- addressing systematic uncertainties
  - a priori: improved lattice fermions&actions
  - a posteriori: extrapolations; chiral effective field theory
- new&improved methods

Summary&Outlook

- substantial progress in lattice QCD calculations of hadron structure observables
- addressing systematic uncertainties
- new&improved methods
  - all-to-all propagators (disconnected diagrams etc.)
  - (partially) twisted boundary conditions (low Q^2)
  - improved HMC algorithms
  - new Fermion matrix inversion algorithms
  - multi-source-techniques (improved statistics)

in collaboration with

- A. Ali Khan, M. Göckeler, A. Schäfer (Regensburg U.), D. Brömmel, M. Diehl, Y. Nakamura, D. Pleiter, G. Schierholz (DESY), R. Horsley, J. Zanotti (Edinburgh U.), P. Rakow (Liverpool U.) H. Stüben (ZIB) [QCDSF/UKQCD]

- J. Bratt, J.W. Negele, A. Pochinsky (MIT), R.G. Edwards, D.G. Richards, K. Orginos (Jlab), M. Engelhardt (New Mexico), G. Fleming (Yale), B. Musch (TUM), D.B. Renner (Arizona), W. Schroers (DESY) [LHPC]