QCD calculations in Heavy Flavour Physics and Soft-Collinear Effective Theory

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Outline

The role of B-decays in flavour physics

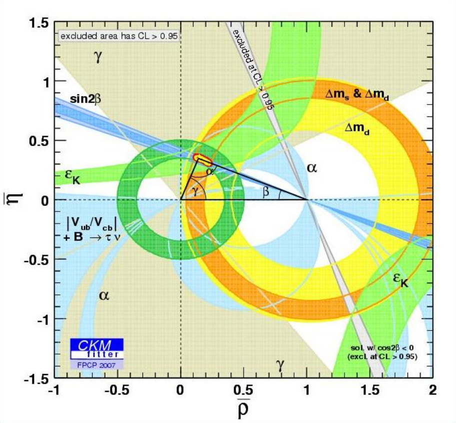
- Determination of CKM elements $|V_{cb}|$ ($\rightarrow$ HQET) and $|V_{ub}|$ from semi-leptonic decays.
- Determination of CKM elements $|V_{td}|$ and $|V_{ts}|$ from penguin decays and B-$\bar{B}$ mixing (virtual top quarks).
- Determination of angles α, β, γ in CKM unitarity triangle from CP asymmetries in various non-leptonic B decays.

  $\Rightarrow$ Test of CKM mechanism ($\rightarrow$ minimal flavour violation)

- Test of Wilson coefficients in effective electroweak Hamiltonian

  $\Rightarrow$ Constraints on new-physics models (e.g. charged Higgs, SUSY, ...)

Consistency Check of CKM Mechanism in Quark Transitions:



Global CKM Fit ("Unitarity Triangle")

$$\sqrt{\bar\rho^2 + \bar\eta^2} = \left| \frac{V_{ud} V_{ub}^*}{V_{cd} V_{cb}^*} \right|$$

$$\sqrt{(1-\bar\rho)^2 + \bar\eta^2} = \left| \frac{V_{td} V_{tb}^*}{V_{cd} V_{cb}^*} \right|$$

- $K^0 - \bar{K}^0$ mixing
- CP asymmetry in $B \to J/\psi K_s$ *
- $b \to c\ell\nu$ and $b \to u\ell\nu$ decays
- $B_s^0 - \bar{B}_s^0$ and $B_d^0 - \bar{B}_d^0$ mixing
- $B \to DK$ decays
- $B \to \pi\pi$ etc.

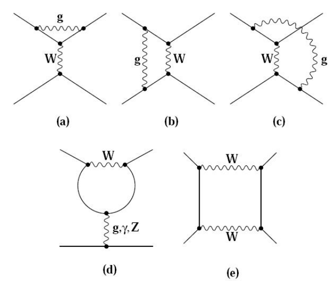
• Essential (perturbative and non-perturbative) QCD input required. (except *)

The weak effective Hamiltonian

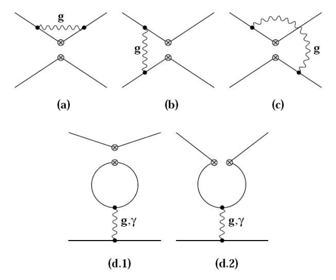
- Integrate out W, Z bosons, top quark, Higgs and possible new heavy particles.

$\rightarrow$ current-current operators, strong/electroweak penguin operators.

$$H_{\text{eff}} = \lambda_{\text{CKM}} \sum_i C_i(\mu)\, \mathcal{O}_i$$



$\longrightarrow$

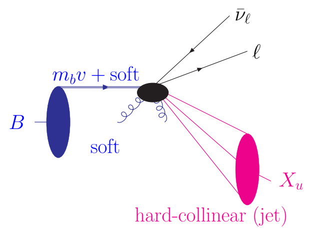


- QED/QCD matching calculation at $\mu = m_W$.
- Renormalization-group evolution to $\mu \sim m_b$ (operator mixing).

Momentum regions and Factorization

$$\boxed{\lambda_{\rm CKM} \sum_i C_i(m_b)\, \langle h_1 h_2 \ldots | \mathcal{O}_i | B \rangle_{\mu=m_b}}$$

Momentum regions in $B \to$ hadronic jet $(b \to u\ell\nu,\ b \to s\gamma,\ b \to s\ell^+\ell^-)$



B meson at rest:

Energetic jet:

$E_X = \mathcal{O}(m_b)\,, \quad p_X^2 = \mathcal{O}(\Lambda m_b)$

(kinematics set by experimental cuts)

long-distance modes

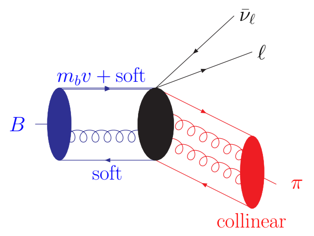
- HQET fields: $\Delta p \sim \Lambda\ [= \mathcal{O}(\Lambda_{QCD})]$
- soft quarks and gluons: $p_s^\mu \sim \Lambda$

$\longrightarrow$ *B*-meson shape functions

short-distance modes

- hard modes: $p_h^2 \sim m_b^2$
- hard-collinear jet modes: $p_{\rm hc}^2 \sim \Lambda m_b$ (duality)

Momentum regions in semi-leptonic $B \to \pi$ decays etc.



B meson at rest:

Pion energetic:

$E_\pi = \mathcal{O}(m_b)$

long-distance modes

- HQET fields: $\Delta p \sim \Lambda$
- soft quarks and gluons: $p_s^\mu \sim \Lambda$
- collinear quarks and gluons:
  $E_c \sim m_b\,, \quad p_c^2 \sim \Lambda^2$

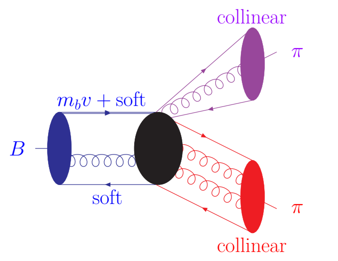
$\longrightarrow$ Light-cone distribution amplitudes

short-distance modes

- hard modes:
  (heavy + collinear)$^2 \sim m_b^2$
- hard-collinear modes:
  (soft + collinear)$^2 \sim m_b \Lambda$

+ non-factorizable "soft" form factors

Momentum regions in non-leptonic $B \to \pi\pi$ decays etc.



B meson at rest:

Pions energetic:

$2E_\pi = 2\bar{E}_\pi \simeq m_b\,, \quad p_\pi \cdot \bar{p}_\pi \simeq m_b^2/2$

long-distance modes

- HQET fields: $\Delta p \sim \Lambda$
- soft quarks and gluons: $p_s^\mu \sim \Lambda$
- collinear quarks and gluons: $E_c,\ \bar{E}_c \sim m_b\,, \quad p_c^2,\ \bar{p}_c^2 \sim \Lambda^2$

$\longrightarrow$ Light-cone distribution amplitudes

short-distance modes

- hard modes: (heavy + collinear)$^2 \sim m_b^2$
- hard-collinear modes: (soft + collinear)$^2 \sim m_b\Lambda$

+ non-factorizable corrections

Factorization of short- and long-distance QCD effects: What for?

- Express cross-sections/amplitudes in terms of a few simple universal (process-independent) hadronic quantities.
  (from experimental data or non-perturbative methods)

- Separate different short-distance scales in renormalization-group improved perturbation theory:

electroweak scale:	$\mu \sim m_W$	$\alpha_s(m_W) \approx 0.1$	$\surd$
hard scale:	$\mu \sim m_b$	$\alpha_s(m_b) \approx 0.2$	
hard-collinear (jet) scale:	$\mu_{\rm hc} \sim \sqrt{\Lambda m_b}$	$\alpha_s(\mu_{\rm hc}) \approx 0.3 - 0.4$	

Resum large logarithms $\ln \frac{\sqrt{\Lambda m_b}}{m_b}$ into short-distance coefficients

Effective theory construction

[Bauer/Fleming/Pirjol/Stewart, Beneke/TF et al., Chay/Kim, Neubert et al., ..., 2001 – today]

Separate short- and long-distance modes:

$$p_c^2,\ p_s^2 \ll p_{\rm hc}^2 \ll p_{\rm h}^2$$

$\to$ Use dimensional regularization: $d^4k \to \mu^{2\epsilon} d^{4-2\epsilon}k$

- modes with $p^2 > \mu^2$ in short-distance coefficients
- modes with $p^2 < \mu^2$ in matrix elements

Two-step matching procedure:

- integrate out hard modes at $\mu_{\rm I}^2 \sim m_b^2$
  $\longrightarrow$ SCET$_{\rm I}$
- renormalization group in SCET$_{\rm I}$:
  $\to$ evolution to $\mu_{\rm II}^2 \sim m_b\Lambda$
- integrate out hard-collinear modes
  $\longrightarrow$ SCET$_{\rm II}$ / HQET
- RG evolution in SCET$_{\rm II}$ / HQET:
  $\to$ hadronic scales Λ

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  $\rightarrow$ hadronic scales Λ

QCD factorization theorem (leading power)

[Neubert 93; Bigi/Shifman/Uraltsev/Vainshtein 93; …Bauer/Rothstein/Stewart et al. 00+; Neubert et al. 04+]

For instance P_+ spectrum in $B \to X_u \ell \nu$:

$$\propto \int_0^1 \underbrace{dy\, y^{-2a}\, H_u(y;\mu_h)\, U(\mu_h,\mu_i)}_{\text{hard function}} \int_0^{P_+} \underbrace{d\hat\omega\, y m_b\, J\left(y m_b(P_+ - \hat\omega);\mu_i\right)}_{\text{jet function}} \underbrace{\widehat{S}(\hat\omega;\mu_i)}_{\text{shape f.}}$$

- $y = (E_X + |\vec{P}_X|)/m_B$?!
- Spectral variable: $P_+ = E_X - |\vec{P}_X| \sim \mathcal{O}(\Lambda)$
- Cut $P_+ \leq \Delta < M_D^2/M_B \simeq 0.66$ GeV suppresses charm background
- RG-evolution functions $U(\mu_h,\mu_i)$ and

$$a = a_\Gamma(\mu_h,\mu_i) = -\int_{\alpha_s(\mu_h)}^{\alpha_s(\mu_i)} d\alpha\, \frac{\Gamma_{\rm cusp}(\alpha)}{\beta(\alpha)} = \frac{\Gamma_0}{2\beta_0} \ln \frac{\alpha_s(\mu_i)}{\alpha_s(\mu_h)} + \ldots \qquad \text{(known to 3-loops)}$$

$$d\Gamma = H\, J \otimes S$$

Status of the calculations

- Hard matching coefficients from QCD loop calculations:
  - $b \to u\ell\nu$ (NLO)
    [e.g. Bauer/Manohar hep-ph/0312109, Bosch/Lange/Neubert/Paz hep-ph/0402094]
  - $b \to s\gamma$ (NNLO)
    [e.g. Melnikov/Mitov hep-ph/0505097, Asatrian et al. hep-ph/0607316, Becher/Neubert hep-ph/0610067]

$$d\Gamma = H\, J \otimes S$$

Status of the calculations

- Jet function (massless quarks) known at NNLO,
  [Neubert, hep-ph/0506245; Becher/Neubert, hep-ph/0603140]
- Jet function for massive charm quarks in $\mathrm{SCET_I}$ known at NLO
  [Boos/TF/Mannel/Pecjak, hep-ph/0512157]
  [see also Chay/Kim/Leibovich, hep-ph/0505030, for an expansion in m^2]

$$d\Gamma = H\,J \otimes S$$

Status of the calculations

- Two-loop evolution kernel for leading-power shape function [Becher/Neubert, hep-ph/0512208]
- Sub-leading shape functions (massless case) classified in [Lee/Stewart, hep-ph/0409045; Bosch/Neubert/Paz, hep-ph/0409115; Beneke/Campanario/Mannel/Pecjak; hep-ph/0411395; Tackmann, hep-ph/0503095]
- Sub-leading shape functions (massive case) classified in [Boos/TF/Mannel/Pecjak; hep-ph/0504005]

$$d\Gamma = H\, J \otimes S$$

The BLNP scheme

Bosch/Lange/Neubert/Paz hep-ph/0504071

Application to $B \to X_u \ell \nu$:

- smooth transition between SF and OPE phase-space regions,
- leading shape function extracted from $B \to X_s \gamma$,
- sub-leading shape functions modelled according to moments constraints,
- additional uncertainties from annihilation topologies.

[see e.g. discussion in Paz hep-ph/0612077]

Phenomenological implications for $|V_{ub}|$

(see e.g. discussion in Lacker arXiv:0708.2731 [hep-ph])

- HFAG: $|V_{ub}|_{\rm incl.} = (4.52 \pm 0.19 \pm 0.27) \times 10^{-3}$

  (Shape-Function scheme)

- CKMfit: $|V_{ub}|_{\rm incl.} = (4.52 \pm 0.23 \pm 0.44) \times 10^{-3}$

  (more conservative error estimate)

Remark:
Depending on the treatment of theoretical uncertainties, one obtains marginal consistency or slight inconsistency with $|V_{ub}|_{\rm excl.}$ from $B \to \pi \ell \nu$ and $\sin 2\beta$ from $B \to J/\psi K_s$.

Phenomenological implications for $B \to X_s\gamma$

- Experimental issue: Cut on photon energy, $E_\gamma > E_0 = 1.8 - 2.0\ \mathrm{GeV}$
  Recent world average (extrapolated to $E_0 = 1.6\ \mathrm{GeV}$):

$$\mathrm{Br}(B \to X_s\gamma)_{\mathrm{exp.}} = (3.55 \pm 0.24^{+0.09}_{-0.10} \pm 0.03) \cdot 10^{-4}$$

[HFAG hep-ph/0603003]

Phenomenological implications for $B \to X_s \gamma$

$$\mathrm{Br}(B \to X_s\gamma)_{\mathrm{exp.}} = (3.55 \pm 0.24^{+0.09}_{-0.10} \pm 0.03) \cdot 10^{-4} \qquad (E_0 = 1.6\ \mathrm{GeV})$$

- Fixed-order 2-loop calculation: [Misiak et al. hep-ph/0609232]

$$\mathrm{Br}(B \to X_s\gamma)_{\mathrm{NNLO}} = (3.15 \pm 0.23) \cdot 10^{-4}$$

[non-perturbative (4%), parametric (3%), higher-order (3%), m_c-interpolation (3%)]

Phenomenological implications for $B \to X_s\gamma$

$$\mathrm{Br}(B \to X_s\gamma)_{\mathrm{exp.}} = (3.55 \pm 0.24^{+0.09}_{-0.10} \pm 0.03) \cdot 10^{-4} \qquad (E_0 = 1.6\ \mathrm{GeV})$$

$$\mathrm{Br}(B \to X_s\gamma)_{\mathrm{NNLO}} = (3.15 \pm 0.23) \cdot 10^{-4}$$

- $E_0 = 1.6$ GeV implies presence of soft scale: $\Delta = m_b - 2E_0 \simeq 1.4$ GeV
- Factorization in SCET allows resummation of logs $\ln \Delta/m_b$.
- Δ may be large enough for perturbative estimate of soft functions.

$$\mathrm{Br}(B \to X_s\gamma)_{\mathrm{resum}} = (2.98 \pm 0.26) \cdot 10^{-4}$$

[Becher/Neubert hep-ph/0610067]

Phenomenological implications for $B \to X_s\gamma$

$$\mathrm{Br}(B \to X_s\gamma)_{\mathrm{exp.}} = (3.55 \pm 0.24^{+0.09}_{-0.10} \pm 0.03) \cdot 10^{-4} \qquad (E_0 = 1.6\ \mathrm{GeV})$$

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$$\mathrm{Br}(B \to X_s\gamma)_{\mathrm{resum}} = (2.98 \pm 0.26) \cdot 10^{-4}$$

$\Rightarrow$ e.g. loser constraints on type-II two-Higgs-doublet models

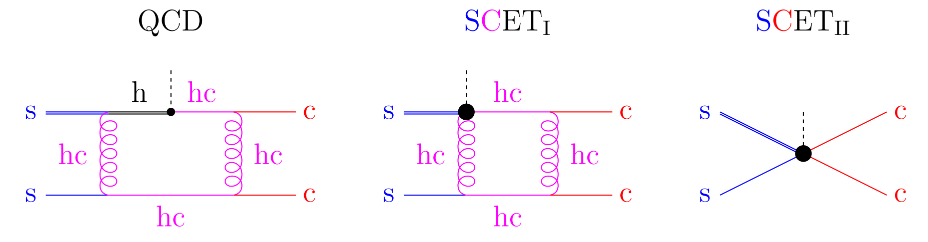
Factorization Theorem for Decay Amplitudes (leading power)

$$
\begin{aligned}
\mathcal{A}_i(B \to \gamma + \text{lept.}) &= && + T_i^{\rm II}(\mu) \otimes \phi_B(\mu) \\
\mathcal{A}_i(B \to M + \text{lept.}) &= \xi_M(\mu) \cdot T_i^{\rm I}(\mu) && + T_i^{\rm II}(\mu) \otimes \phi_B(\mu) \otimes \phi_M(\mu) \\
\mathcal{A}_i(B \to MM') &= \xi_M(\mu) \cdot T_i^{\rm I}(\mu) \otimes \phi_{M'}(\mu) && + T_i^{\rm II}(\mu) \otimes \phi_B(\mu) \otimes \phi_M(\mu) \otimes \phi_{M'}(\mu)
\end{aligned}
$$

- universal transition form factors ξ_M (“non-factorizable”)
- two-particle LCDAs for B-meson and light hadrons
- perturbative coefficient functions, $T_i^{\rm I}$ and $T_i^{\rm II} = H_i \otimes J$
- power-corrections induce more factorizable and non-factorizable terms

[Beneke/Buchalla/Neubert/Sachrajda 99; Beneke/TF 00; Bauer/Pirjol/Stewart 02; Lunghi/Pirjol/Wyler 02; Bosch/Hill/Lange/Neubert 03; Beneke/TF 03; Becher/Hill/Neubert 05 . . .]

Example: a particular momentum region in $B \to \pi$



- part of tree-level matching for QCD $\longrightarrow$ $\mathrm{SCET_I}$
- part of one-loop matching for $\mathrm{SCET_I}$ $\longrightarrow$ $\mathrm{SCET_{II}}$

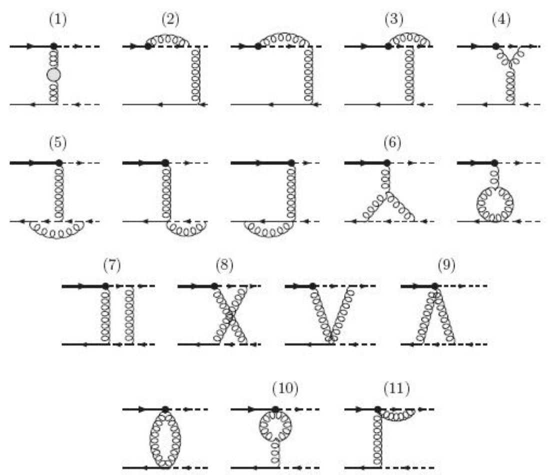
- repeat for all possible diagrams, momentum regions, and electroweak operators ...
- classify all $\mathrm{SCET_I}$ and $\mathrm{SCET_{II}}$ operators that contribute to leading power
- investigate endpoint behaviour of convolution integrals in $\mathrm{SCET_{II}}$:
  factorizable $\leftrightarrow$ non-factorizable

QCD factorization in $B \to \pi(\rho)\ell\nu$ form factors

- Form factor ratios obey approximate symmetry relations [Charles et al. '98]
- Short-distance corrections can be estimated from SCET/QCDF:
  - vertex corrections at the hard scale:
    2-body operators [Beneke/TF 00, Bauer/Fleming/Pirjol/Stewart 01],
    3-body operators [Beneke/Kiyo/Yang 04, Becher/Hill 04]
  - RG evolution between hard and hard-collinear scale:
    2-body operators [Bauer et al. 01],
    3-body operators [Hill/Becher/Lee/Neubert 04, Beneke/Yang 05]
  - spectator scattering at the hard-collinear scale,
    involving LCDAs of light and heavy meson:
    LO: [Beneke/TF 00],
    NLO: [Becher/Hill et al 04, Beneke/Yang 05]

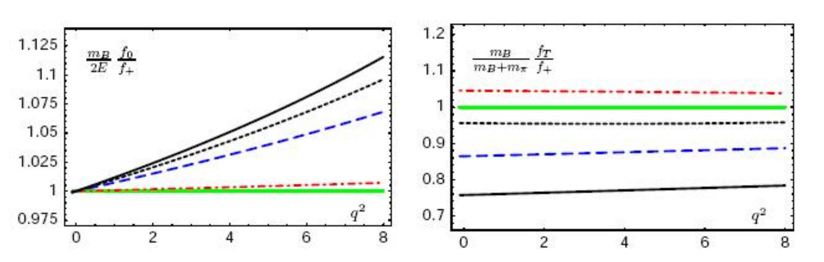
QCD factorization in $B \to \pi(\rho)\ell\nu$ form factors

NLO spectator-scattering diagrams in SCET_I:



Example: Form factor ratios in $B \to \pi \ell \nu$

[Beneke/Yang hep-ph/0508250]



— full NLO spectator terms and LL resummation (default values for hadronic parameters)

– – only LO spectator scattering

- – - without any spectator scattering

- - - QCD sum rule estimate [Ball/Zwicky 04]

— symmetry limit [Charles et al. 98]

QCD factorization in $B \to K^* \ell^+ \ell^-$

[Beneke/TF/Seidel 01,03; Ali/Kramer/Zhu 06]

- "naive" factorization: Wilson coefficients $\times$ $B \to K^*$ form factors.

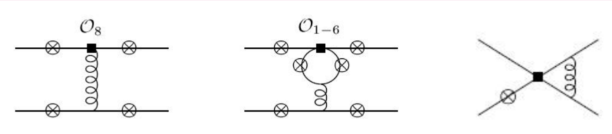
  form factor symmetry limit $\Rightarrow$ prediction for FB asymmetry zero $m_{\ell\ell}^2 = q_0^2$:

$$C_9 + \mathrm{Re}(Y(q_0^2)) + \frac{2 m_b m_B}{q_0^2} \, C_7^{\mathrm{eff}} = 0$$

  ($Y(q^2)$: short-distance function involving 4-quark operators)

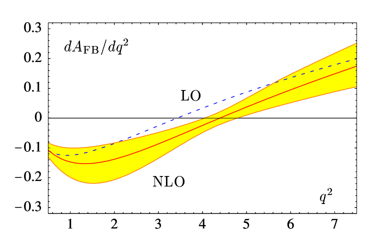
- corrections to form factor symmetry relations (s.a.)
- corrections from $B \to K^* \gamma^*$,

  non-trivial insertions of 4-quark and chromomagnetic operators, e.g.

$\langle B |$  $| K^* \rangle$

$\otimes$ = virtual photon insertion

FB asymmetry in $B \to K^* \ell^+ \ell^-$ [from Beneke/TF/Seidel 01]



$q^2 = m_{\ell\ell}^2$

Theoretical uncertainties incl.:

- hadronic input parameters
- electro-weak SM parameters
- variation of factorization scale

Asymmetry zero in SM: $q_0^2 = 4.2\ \mathrm{GeV}^2 \ \pm\ $ (10-15)%

- perturbative error can be reduced using RG evolution in SCET [Ali et al. 06]
- systematic theoretical uncertainty limited by $1/m_b$ power corrections $\gtrsim$ 10%

- $b \to s\ell^+\ell^-$ occurs only at 1-loop level in the SM, $d\Gamma \propto \frac{|V_{tb}V_{ts}^*|^2}{16\pi^2}$
- Generic new physics effects compete with SM contribution!

QCD-F in non-leptonic decays $B \to \pi\pi,\ \pi K$ etc.

- $\mathcal{O}(\alpha_s)$ corrections to naive factorization [BBNS 99,00]
- $\mathcal{O}(\alpha_s^2)$ vertex corrections [Bell 07]
  - "tree" topology, imaginary part $\surd$
  - real part and "penguin" topology $(\surd)$
- $\mathcal{O}(\alpha_s^2)$ corrections from spectator scattering
  - "tree" amplitudes:
    [Beneke/Jäger 05, Kivel 06, Pilipp 07] $\surd$
  - "penguin" amplitudes:
    [Beneke/Jäger 06] $\surd$

Example: Imaginary part of "tree" amplitudes in $B \to \pi\pi$

[Bell 0705.3127 (hep-ph)]

$$\begin{aligned} \mathrm{Im}\alpha_1(\pi\pi) &= 0.012\,|_{V^{(1)}} + 0.031\,|_{V^{(2)}} - 0.012\,|_{S^{(2)}} \\ &= 0.031 \pm 0.015\,(\text{scale}) \pm 0.006\,(\text{param}) \pm 0.010\,(\text{power}) \end{aligned}$$

$$\begin{aligned} \mathrm{Im}\alpha_2(\pi\pi) &= -0.077\,|_{V^{(1)}} - 0.052\,|_{V^{(2)}} + 0.020\,|_{S^{(2)}} \\ &= -0.109 \pm 0.023\,(\text{scale}) \pm 0.010\,(\text{param}) \pm 0.045\,(\text{power}) \end{aligned}$$

- NNLO effects numerically important.
- Partial cancellation between vertex (V) and spectator (S) corrections.
- Absolute size of imaginary part (→ strong phases) still small.

- Consider correlation function in $\mathrm{SCET_I}$:
  exclusive final state (e.g. pion) is replaced by interpolating current.

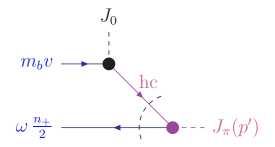
⇒ Factorization theorem for correlation function (soft ⊗ hard-collinear)

- Dispersion relation between
  - (unphysical) region of large (hc) space-like momenta
  - physical spectral function, containing the hadronic state

⇒ Sum rule for non-factorizable matrix elements in $\mathrm{SCET_I}$:
$\xi_\pi(q^2)$ in terms of light-cone distribution amplitudes of B meson

- correlation function at tree level:

$$\Pi_0(n_-p') = f_B m_B \int_0^\infty d\omega \, \frac{\phi_-^B(\omega)}{\omega - n_-p' - i\eta}$$



- radiative corrections from hc loops (excl. 3-particle DAs) √

Sum rule after continuum subtraction and Borel trafo

$$\omega_M \, \hat{B}\,[\Pi_0^{\rm res.}]\,(\omega_M, \omega_s) = \frac{1}{\pi} \int_0^{\omega_s} d\omega' \, e^{-\omega'/\omega_M} \, {\rm Im}\left[\Pi_0(\omega')\right] \equiv m_b \, f_\pi \, \xi_\pi$$

- Systematic uncertainties reflected by ω_s and ω_M:
  - $\Lambda > \omega_s > m^2_{[3\pi]}/m_b$: effective continuum threshold parameter
  - $\Lambda > \omega_M > \Lambda^2/m_b$: Borel parameter
- Numerical estimate: ($q^2 = 0$, incl. NLO radiative corrections, excl. 3-particle LCDA)

$$\frac{T_i^{\rm I}(\mu)}{T_i^{\rm I}(m_b)} \cdot \xi_\pi(\mu) = 0.27 \pm 0.02\Big|_\mu \pm 0.07\Big|_{f_B\phi_-} {}^{+0.05}_{-0.08}\Big|_{\rm s.r.} {}^{+0.00}_{-0.04}\Big|_{\rm sys.}$$

  - needs more information on LCDA $f_B \, \phi_-^B$
  - method to be tested in other observables
  - ...

Summary: Heavy Flavour Physics and SCET

Inclusive B Decays:

Presence of soft scale / sensitivity to B-meson shape functions.

- V_{ub} from $B \to X_u \ell \nu$
- New physics in $B \to X_s \gamma$
- $[M_X^{\rm cut}$ effects in $B \to X_s \ell^+ \ell^-]$

Exclusive B Decays:

Corrections to "naive" factorization:

- $\mathcal{O}(\alpha_s^2)$ corrections to form factor symmetry relations
- FB asymmetry zero in $B \to K^* \ell^+ \ell^-$
- $\mathcal{O}(\alpha_s^2)$ corrections to $B \to \pi\pi$ etc.

- Limiting factor: non-factorizable $1/m_b$ power corrections.

Other Applications: DIS at large x, Jet physics, ...

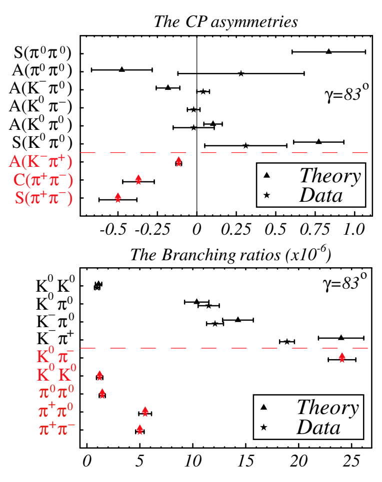
Different application of QCDF/SCET in $B \to \pi\pi$ etc.

- BBNS [Beneke/Buchalla/Neubert/Sachrajda 1999+]
- and BPRS ? [Bauer/Pirjol/Rothstein/Stewart 2004+]

	BBNS	BPRS
factorization formula:	reasonable values $\pm$ generous errors (form factor and LCDAs)	fit T_{I} and T_{II} to data (called ζ and ζ_J, real)
"charming penguins": [Ciuchini 97]	short-distance, (incl. in hard functions)	"charm-loop" left as phenomenological fit parameter (Δ^P)
non-factorizable power-corrections:	rough estimate of annihilation and sub-leading hard-scattering effects (X_A and X_H)	assumptions about systematic uncertainties

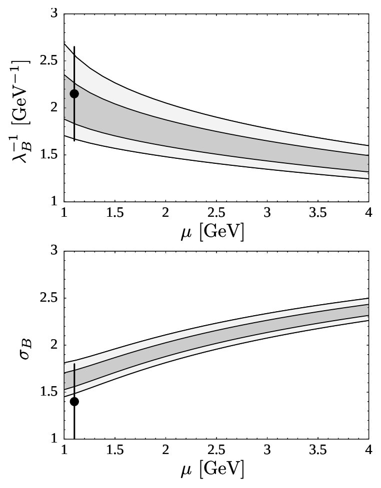
see also the controversial discussion in [Beneke et al., hep-ph/0411171] and [Bauer et al., hep-ph/0502094]

$B \to \pi\pi$ and $B \to \pi K$ in BPRS approach [from Bauer, hep-ph/0606018]



- red data points are used to fit free hadronic parameters
- results for iso-singlet final states in [Williamson/Zupan, hep-ph/0601214]

Moments of the *B*-meson distribution amplitude: Evolution



- $\lambda_B^{-1} = \int_0^\infty d\omega \, \frac{\phi_+^B(\omega,\mu)}{\omega}$
- $\sigma_B \, \lambda_B^{-1} = - \int_0^\infty d\omega \, \frac{\phi_+^B(\omega,\mu)}{\omega} \, \ln \frac{\omega}{\mu}$

- error band:
  from [Lee/Neubert, hep-ph/0509350]
- “data” points:
  sum-rule estimate from [Braun/Korchemsky/Ivanov, hep-ph/0309330]

Backup Slides

Shape-function independent relations between P_+ spectra in $B \to X_s\gamma$ and $B \to X_u\ell\nu$ (2-loop)

[Lange/Neubert/Paz, hep-ph/0508178], see also [Leibovich/Low/Rothstein 2000]

$$\Gamma_u(\Delta) = \int_0^\Delta dP_+ \underbrace{\frac{d\Gamma_u}{dP_+}}_{\text{exp. input}} = |V_{ub}|^2 \int_0^\Delta dP_+ \underbrace{W(\Delta, P_+)}_{\text{theory}} \underbrace{\frac{1}{\Gamma_s(E^*)} \frac{d\Gamma_s}{dP_+}}_{\text{exp. input}}$$

- Weight function $W(\Delta, P_+)$ can be calculated perturbatively, up to power corrections:
  - phase-space (can be treated exactly)
  - kinematical corrections $\propto \alpha_s \widehat{S}(\hat{\omega})$
  - hadronic corrections from sub-leading shape functions (model-dependent)

Backup Slides

Shape-function independent relations between $d\Gamma_u$ and $d\Gamma_s/\Gamma_s$
with more complicated experimental cuts
studied in [Lange, hep-ph/0511098]

SFIRs between $d\Gamma_u/dP_+$ and $d\Gamma_s/dP_\gamma$ (not normalized to Γ_s !),
and comparison of fixed-order and resummed results for $W(\Delta, P_+)$
investigated in [Hoang/Ligeti/Luke, hep-ph/0502134]

SFIRs between $d\Gamma(b \to u\ell\nu)$ and $d\Gamma(b \to c\ell\nu)$ at one loop,
for massive charm quarks in $\mathrm{SCET_I}$
studied in [Boos/TF/Mannel/Pecjak, hep-ph/0512157]

Factorization in rapidity space? [Manohar/Stewart 2006]

- **Problem:**
  Endpoint divergences in soft and collinear convolution integrals spoil (conventional) factorization (soft and collinear modes have same virtuality)
- **Idea:**
  Introduce additional factorization scales $\mu_\pm$ in *rapidity space*, identify $\mu^2 = \mu_+\mu_-$:
  1. Subtract endpoint configurations ("zero-bins") from integrand:
     Resulting integral has no endpoint divergence at $(n_\pm k) \to 0$, but develops new UV divergence at $(n_\pm k) \to \infty$
  2. Regulate UV divergence by factors $[(n_\pm k)/\mu_\mp]^\epsilon$ at soft-collinear operators:
     Endpoint divergences are turned into $1/\epsilon_{\rm UV}$ poles ("pull-up mechanism")
  3. Absorb terms of the form $\frac{1}{\epsilon}\,\Gamma[\epsilon]\,\mu^{2\epsilon}$ into special counter-term currents.

Backup Slides

Example: Modification of would-be divergent integral

$$\langle \bar{x}^{-2} \rangle_\pi(\mu_-) \equiv \int_0^1 dx \, \frac{\phi_\pi(x,\mu) + (1-x)\,\phi'_\pi(1,\mu)}{(1-x)^2} - \phi'_\pi(1,\mu) \ln\left(\frac{m_b}{\mu_-}\right)$$

- **But:** Sensitive to higher Gegenbauer coefficients (quadratical growth with n)

$$\phi'_\pi(1) = -6 \left\{ 1 + 6a_2 + 15a_4 + \ldots + \begin{pmatrix} n+2 \\ 2 \end{pmatrix} a_n + \ldots \right\}$$

$\Rightarrow$ $\langle \bar{x}^{-2} \rangle_\pi(\mu_-)$ serves as independent (i.e. unknown) hadronic parameter!

- **Moreover:** Result equivalent to BBNS procedure, **if** $\mu_- \leftrightarrow \Lambda$:

$$\langle \bar{x}^{-2} \rangle_\pi(\Lambda) \equiv \int_0^{1-\Lambda/m_b} dx \, \frac{\phi_\pi(x,\mu)}{(1-x)^2} + \mathcal{O}(\Lambda/m_b)$$

Backup Slides

Application: Annihilation in $B \to \pi\pi$ [Arnesen/Ligeti/Rothstein/Stewart 06]

Leading-order annihilation coefficients take the form, [Eq. (24) in hep-ph/0607001]

$$a_i = \frac{C_F \pi \alpha_s(\mu_h)}{N_c^2} \times \langle F(x,y)\rangle_\pi(\mu_-,\mu_+) \times (\text{lin. comb. of Wilson coeff. } C_i(\mu_h))$$

Open questions:

- Scale-dependence does not drop out at the considered order !?!

$$\frac{\mu_\pm}{a_i}\,\frac{\partial a_i}{\partial \mu_\pm} \propto (\alpha_s)^0\,\phi'_\pi(1) = \mathcal{O}(1)$$

Something is still missing here!

- Compare with BBNS:
  - ! Logarithmic dependence on (ad-hoc) scale indicates non-factorizability, and induces irreducible model dependence in annihilation amplitudes !

$\Rightarrow$ Needs more theoretical studies!
(factorization proof? resummation of $\ln \mu_\pm$? cross-check with non-pert. methods? ...)

- Phenomenological implications remain unclear!
($\phi'_\pi(1)$, non-perturbative strong phases, ...)

Backup Slides

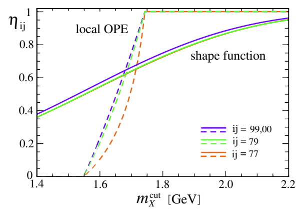
$M_X^{\rm cut}$ effects in $B \to X_s \ell^+\ell^-$ [Lee/Stewart 05, Lee/Ligeti/Stewart/Tackmann 05]

- Experimental studies on low-q^2 region $(1\ {\rm GeV}^2 \leq q^2 = m_{\ell\ell}^2 \leq 6\ {\rm GeV}^2)$ require additional cut on hadronic invariant mass (eliminate combinatorial background from $b \to c\ell\nu \to s\ell\ell\nu\nu$)

BaBar: $m_X \leq 1.8\ {\rm GeV}$, Belle: $m_X \leq 2.0\ {\rm GeV}$.

$\Rightarrow$ Large contribution from shape function region $\to$ Factorization in SCET [phenomenological studies so far used Fermi-motion model (Ali/Hiller)]

$m_X^{\rm cut}$ dependence of rate in low-q^2 region (theory)



$$\Gamma^{\rm cut}\,\tau_B = (1.20 \pm 0.15)\cdot 10^{-6} \quad \text{for } m_X^{\rm cut} = 1.8\ {\rm GeV}$$

$$\Gamma^{\rm cut}\,\tau_B = (1.48 \pm 0.14)\cdot 10^{-6} \quad \text{for } m_X^{\rm cut} = 2.0\ {\rm GeV}$$

- Uncertainty from sub-leading shape functions: 10% (5%)
- In the ratio $R = \Gamma^{\rm cut}(B \to X_s\ell^+\ell^-)/\Gamma^{\rm cut}(B \to X_u\ell\bar\nu)$ the $m_X^{\rm cut}$ effects cancel to a large extent.