High energy limit of R-current scattering in N=4 SYM

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Quantum chromodynamics: string theory meets collider physics, DESY

Based on:
J. Bartels, A. M. Mischler, M. Salvadore, to appear.

Outline

1. Introduction and Motivations
   - Why high energy, $N = 4$ SYM and R-currents?
   - Review of $\gamma^*\gamma^*$ scattering in QCD
2. Four point function of R-currents
   - $N = 4$ SYM, R-currents and Ward identities
   - Perturbative computation at weak coupling

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Why high energy?

- QCD is very complicated;
- QCD in the high energy (Regge) limit,

$$s \gg -t \gg \Lambda_{\rm QCD}\,,$$

  is much simpler;
- The longitudinal d.o.f. can be integrated, leading to resummation of large terms $\log(s/\mu^2)$, $\mu^2 \sim -t$;
- The dynamics reduced to the $2-$dim transverse plane;
- Such $2-$dim theory enjoys remarkable properties (in LLA: Moebius invariance, holomorphic separability, integrability).

Why $N = 4$ SYM?

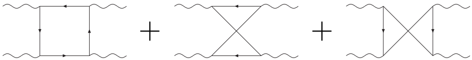
- To some extend $N = 4$ SYM is very similar to QCD. In LLA they both lead to the same BFKL equation;
- It enjoys much more symmetry than QCD and it is therefore more tractable;
- It is likely dual to type IIB superstring theory (weak-strong coupling duality), therefore it is possible to address the strong ('t Hooft) coupling regime.

Why $R-$currents?

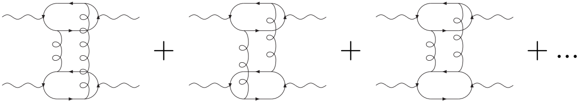
- In QCD the cleanest environment to study the Regge limit is virtual photon scattering (highly virtual photon couple perturbatively to the non-Abelian gauge system in a gauge invariant way);
- The $R-$currents of $N = 4$ SYM look quite similar to the EM currents which couple the photons of QED and scalar-QED to QCD;
- The $R-$currents correspond to global symmetries, while the EM currents correspond to gauge symmetries. This fact introduces important subtleties.

$\gamma^*\gamma^*$ scattering in QCD.

- The lowest order diagrams (1-loop) are boxes:



- They behave in the Regge limit as $\log^2 s$;
- Starting from three loops there are diagrams where only gluons are exchanged:



- They behave in the Regge limit as $\alpha_s^2 s$; They dominate at high energy!

Impact factors.

- These diagrams factorize as transverse space convolution of Impact factors:

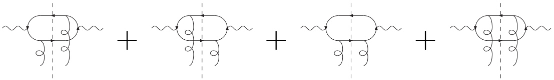
$$\mathcal{A}^{(0)}(s,t) = is \int \frac{d^{D-2}\boldsymbol{k}}{(2\pi)^{D-2}} \frac{\Phi(\boldsymbol{k},\boldsymbol{q}-\boldsymbol{k})\Phi(\boldsymbol{k},\boldsymbol{q}-\boldsymbol{k})}{\boldsymbol{k}^2(\boldsymbol{q}-\boldsymbol{k})^2}\,, \qquad t = -\boldsymbol{q}^2$$

- The leading term is purely imaginary:

$$\mathcal{A}(s,t) = i\,\mathrm{Im}\mathcal{A}(s,t)$$

- The Impact Factor is

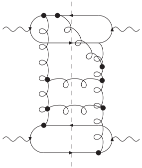
$$\Phi(\boldsymbol{k}_1,\boldsymbol{k}_2) =$$



LLA resummation.

- Higher order diagrams with gluons get factors $\alpha_s^n \log^n s \sim 1$. Summing up all these diagrams one gets:

$$\mathcal{A}_{\mathrm{QCD}}^{\mathrm{LLA}}(s,t) = is\, \Phi \otimes \mathcal{G}(s) \otimes \Phi = \sum$$



- $\mathcal{G}$ satisfies the BFKL equation (Balitski,Fadin,Kuraev,Lipatov),

$$\left(\frac{\partial}{\partial \log s} - \mathcal{H} \otimes\right)\mathcal{G}(s) = \delta(\log s)$$

- Properties of $\mathcal{H}$: Möbius invariance, holomorphic separability.

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$N = 4$ SYM

- All the field are in the adjoint representation of the gauge group $SU(N_c)$;
- There is a global symmetry $SU(4)_R$, R-symmetry, which reshuffle the different supecharges;
- The fields are:
  - 1 vector field A_μ, singlet of $SU(4)_R$;
  - 4 chiral spinors λ_I in the fundamental of $SU(4)_R$;
  - 6 real scalars X_M in the vector of $SU(4)_R$.

$R-$currents

- The Nöther current of the R-symmetry is

$$J^{\mu A} = \mathrm{Tr}\Big\{ \frac{i}{2} X T^A (\overleftarrow{D^\mu} - \overrightarrow{D^\mu}) X - \lambda \sigma^\mu T^A \bar{\lambda} \Big\}$$

- $J^{\mu A}$ is akin to EM current in QED + scalar-QED

$$\gamma^*\gamma^* \text{scattering} \Rightarrow \text{“scattering of R-current”}$$

- Therefore we compute the four point function of R-current in momentum space

$$i\mathcal{A}(s,t) := \langle J^{\mu A}(p_A) J^{\nu B}(p_B) J^{\mu' A'}(-p_{A'}) J^{\nu' B'}(-p_{B'}) \rangle$$

Ward identities

- The EM current is a gauge current $\Rightarrow$ it satisfies at the quantum level the Ward identity

$$\partial_\mu \langle j^\mu(x)\mathcal{O}_1(x_1)\dots\mathcal{O}_n(x_n)\rangle = 0$$

- The R-currents are global $\Rightarrow$ their quantum Ward identities contain contact terms

$$\partial_\mu \langle J^{\mu A}(x)\mathcal{O}_1(x_1)\dots\mathcal{O}_n(x_n)\rangle = -i\sum_{i=1}^{n}\langle\dots\frac{\delta\mathcal{O}_i(x_i)}{\delta\epsilon_A(x)}\dots\rangle$$

- A consequence is that $\mathcal{A}(s,t)$ is logarithmically divergent $\Rightarrow$ it has to be renormalized.

Polarization vectors

- Since in general $p \cdot \mathcal{A} \neq 0$ one must introduce a four dimensional basis of polarization vectors $\epsilon^{\bar{L},L,\pm}$. The additional vector $\epsilon^{\bar{L}}$ is proportional to p itself;
- $\mathrm{Span}\{\epsilon^{L,\bar{L},\pm}\}$ is a complete basis of the four-dim. vector space where $J^{\mu A}$ belong to.
- The metric tensor is decomposed as

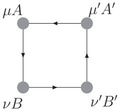
$$g_{\mu\nu} = - \sum_{i=L,\bar{L},\pm} \epsilon_\mu^{(i)}(p)^* \epsilon_\nu^{(i)}(p)$$

- The 16 d.o.f. of the four point function are encoded into the coefficients $\langle \lambda_A \lambda_B | \mathcal{A}_4 | \lambda_{A'} \lambda_{B'} \rangle$

$$\mathcal{A}_4 = \sum_{\lambda_i=L,\bar{L},\pm} \epsilon_\mu^{\lambda_A}(p_A)^* \epsilon_\nu^{\lambda_B}(p_B)^* \epsilon_{\mu'}^{\lambda'_A}(p'_A) \epsilon_{\nu'}^{\lambda'_B}(p'_B) \langle \lambda_A \lambda_B | \mathcal{A}_4 | \lambda_{A'} \lambda_{B'} \rangle$$

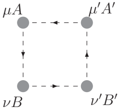
Box diagrams and renormalization

- The UV singularity of the fermionic box is



$$\rightarrow \frac{i\pi^2}{3\epsilon}\mathrm{Tr}(T^A T^{A'} T^{B'} T^B)(g_{\mu\mu'}g_{\nu\nu'}+g_{\mu\nu}g_{\mu'\nu'}-2g_{\mu\nu'}g_{\mu'\nu})$$

- In the sum of the three crossed diagram the singularity does not cancel due to the $SU(4)_R$ group structure;
- Similarly, the UV singularity of the scalar box is

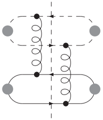


$$\rightarrow \frac{2i\pi^2}{3\epsilon}\mathrm{Tr}(T^A T^{A'} T^{B'} T^B)(g_{\mu\mu'}g_{\nu\nu'}+g_{\mu\nu}g_{\mu'\nu'}+g_{\mu\nu'}g_{\mu'\nu})$$

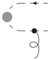
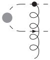
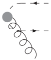
- This does not modify the s behaviour by powers $\Rightarrow$ Box diagrams are subleading at high energy $\sim \log^2 s$.

Two gluon exchange diagrams: Ward identities

- Let's first compute the imaginary part

$$2\mathrm{Im}\mathcal{A} = \sum$$ 

- The $SU(4)_R$ structure is trivial for all diagrams: $\delta^{AA'}\delta^{BB'}$;
- The fermionic diagrams satisfy the classical Ward identities $p_i \cdot \mathcal{A}_{\text{fermion}} = 0$ (same diagrams as in QCD!)
- The scalar diagrams also satisfy the classical Ward identities $p_i \cdot \mathcal{A}_{\text{scalar}} = 0$ thanks to the unitarity cut

$p \cdot \Big($  $+$  $+$  $\Big) = 0$

- At this order the projections on the funny polarization vector $\epsilon^{\bar{L}}$ vanish: $\epsilon^{\bar{L}} \cdot \mathcal{A} = 0$.

Two gluon exchange diagrams: Fermion loop

Very similar to QCD (Dirac $\rightarrow$ Weyl)

$\Phi_F =$

$$= N_c \alpha_s \delta^{AA'} \delta^{aa'} \int_0^1 d\alpha \int \frac{d^2 \boldsymbol{l}}{(2\pi)^2} \varphi_F$$

$$\varphi_F^{LL} = Q_A Q_{A'} \alpha^2 (1-\alpha)^2 \left(\frac{1}{D_1} - \frac{1}{D_2} \right) \left(\frac{1}{D_1'} - \frac{1}{D_2'} \right)$$

$$\varphi_F^{hh'} = \delta^{hh'} \left(\frac{\boldsymbol{N}_1}{D_1} - \frac{\boldsymbol{N}_2}{D_2} \right) \cdot \left(\frac{\boldsymbol{N}_1'}{D_1'} - \frac{\boldsymbol{N}_2'}{D_2'} \right) + \ldots$$

$$\varphi_F^{hL} \neq 0$$

where $\boldsymbol{N}$ and D are functions of $\boldsymbol{l}, \boldsymbol{k}, \boldsymbol{q}$ and α.

Two gluon exchange diagrams: Scalar loop

- In principle there are new diagrams,

which seem to break Regge factorization!

- Explicit computation shows that their contribution is suppressed by powers of s.
- Eventually we are left again with

$\Phi_F =$ 

Two gluon exchange diagrams: Fermion + Scalar

- The φ_S^{ij} are similar to the φ_F^{ij}. For the specific number of fermions and scalar in $N = 4$ SYM one gets

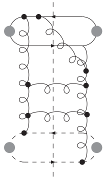
$$
\begin{aligned}
\varphi^{LL} &= Q_A Q_{A'} \alpha(1-\alpha)\left(\frac{1}{D_1} - \frac{1}{D_2}\right)\left(\frac{1}{D_1'} - \frac{1}{D_2'}\right) \\
\varphi^{hh'} &= \delta^{hh'}\left(\frac{\mathbf{N}_1}{D_1} - \frac{\mathbf{N}_2}{D_2}\right)\cdot\left(\frac{\mathbf{N}_1'}{D_1'} - \frac{\mathbf{N}_2'}{D_2'}\right) \\
\varphi^{hL} &= 0
\end{aligned}
$$

- All the integrals are finite in $D = 4$;
- Only diagonal terms are non-vanishing;
- ϕ^{ij} are gauge invariant. They satisfy the Ward indenties $\phi^{ij}(\boldsymbol{k} = 0) = \phi^{ij}(\boldsymbol{k} = \boldsymbol{q}) = 0$.

LLA resummation

- Summing up the leading-logs one gets the same expression as in QCD with the impact factor just shown

$$i\ \mathrm{Im}\mathcal{A}_{N=4}^{\mathrm{LLA}}(s,t) = is\ \Phi \otimes \mathcal{G}(s) \otimes \Phi = \sum$$



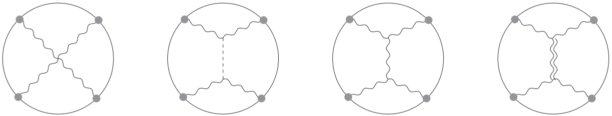
- $\mathcal{G}$ satisfies the same BFKL equation as in QCD;
- Proof of Regge factorization for the four point functions of R-currents in $N = 4$ SYM in Leading-Log Approximation.

Summary

- Studying $N = 4$ SYM might give some clues about the dynamics of QCD at arbitrary value of the 't Hooft coupling;
- A safe gauge invariant environment similar to $\gamma^*\gamma^*$ scattering is provided by correlation functions of R-currents;
- We have computed the high energy limit of the four point function $\mathcal{A}(s,t)$;
- We have proved that $\mathcal{A}(s,t)$ satisfies Regge factorization in the Leading-Log Approximation;

Outlook

- $\mathcal{A}(s,t)$ can be computed at strong coupling via AdS/CFT;
- Planar + large 't Hooft coupling $\Rightarrow$ Supergravity;
- The R-currents of the boundary theory are sources for the vector fields in the bulk of AdS_5;
- The relevant Witten diagrams to be computed are



- There exist a simple consistent truncation of the full supergravity action which contains only one (abelian) gauge field (one R-charge) and the graviton,

$$e^{-1}\mathcal{L}_5 = -\frac{1}{2k_5^2}\left(R + 3g^2 - \frac{1}{4}F^2 + \frac{1}{12\sqrt{3}}\epsilon^{\mu\nu\rho\sigma\tau}F_{\mu\nu}F_{\rho\sigma}A_\tau\right)$$