Flavors from time-dependent D7 embeddings

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Outline

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Viscous Hydrodynamics

- Choose proper time/rapidity coordinates (τ, y, x^2, x^3)

$$x^0 = \tau \cosh y \qquad\qquad x^1 = \tau \sinh y$$

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- Assume
  - boost invariance ($\not{y}$)
  - infinitely large nuclei ($\not{x}^2$, $\not{x}^3$)
  - energy momentum conservation $\partial_\mu T^{\mu\nu} = 0$
  - tracelessness $T_\mu{}^\mu = 0$

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$$T_{\tau\tau} = \varepsilon(\tau) \quad T_{yy} = -\tau^2 \left(\varepsilon(\tau) + \tau\varepsilon'(\tau)\right) \quad T_{xx} = \varepsilon(\tau) + \frac{1}{2}\tau\varepsilon'(\tau)$$

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$$T_{\mu\nu} = (\varepsilon + p)u_\mu u_\nu + p\eta_{\mu\nu} + \eta(\nabla_\mu u_\nu + \nabla_\nu u_\mu) \qquad u^2 = -1$$

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$$\implies \varepsilon(\tau) \sim \frac{\varepsilon_0}{\tau^{4/3}}(1 - \frac{2\eta_0}{\varepsilon_0^{1/4}\tau^{2/3}} + \dots) \qquad \eta = \eta_0 T^3$$

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$$\varepsilon \sim T^4$$

Dual geometry

Dual geometry of $\mathcal{N} = 4$ SYM viscous fluid

Janik/Peschanski, Nakamura/Sin, Janik/Heller

$$ds^2 = \frac{r^2}{L^2}\left(-e^{\mathcal{A}(v,\tau)}\, d\tau^2 + e^{\mathcal{B}(v,\tau)}\tau^2\, dy^2 + e^{\mathcal{C}(v,\tau)}\, dx_\perp^2\right) + \frac{L^2\, dr^2}{r^2}$$

$$v = \frac{\varepsilon_0^{1/4}}{r\,\tau^{1/3}}$$

- $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$ ...
- How to obtain them
  - Expand the coefficients

$$\mathcal{A} = a_0(v) + a_1(v)\tau^{-2/3} + \ldots$$

  - Solve Einstein's equation order by order
- From horizon: $T^4 \sim \varepsilon$
- Regularity $\implies$ Bjorken dynamics and $\eta/s = 1/4\pi$

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Ansatz and Boundary Conditions

- Full metric

$$ds^2 = ds_4^2 + \frac{1}{r^2}\left(d\rho^2 + \rho^2 d\Omega_3^2 + (dX^8)^2 + (dX^9)^2\right)$$
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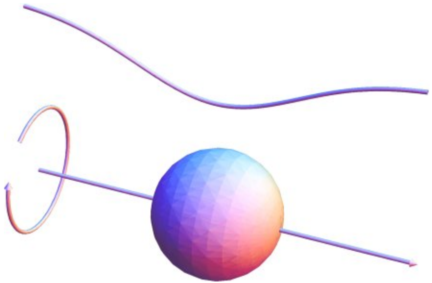

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- DBI action $(X^9 = \Phi(\rho, \tau),\ X^8 = 0)$

$$S \sim \int d\tau\, d\rho\, \tau \rho^3\, \# \sqrt{1 + \#(\partial_\rho \Phi)^2 - \# \frac{(\partial_\tau \Phi)^2}{r^4}}$$

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$$\Phi \xrightarrow[\rho\to\infty]{} m + \frac{c}{\rho^2} + \dots$$

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- Consequence for the asymptotics

$$\Phi \xrightarrow[\rho\to\infty]{} (m + \sum m_k \tau^{-\frac{k}{3}}) + \frac{1}{\rho^2}(\sum c_k \tau^{-\frac{k}{3}}) + \dots$$

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- Moreover: *regularity* in the interior

Embedding solution and chiral condensate

- Equation of motion

$$\frac{1}{\rho^3}\partial_\rho(\rho^3\partial_\rho\phi_k(\rho)) = \#(\rho)\cdot\begin{cases}1 & \text{if } k=8\\ -4\eta_0\varepsilon_0^{-\frac{1}{4}} & \text{if } k=11\\ 0 & \text{else; provided } k<14\end{cases}$$

- Homogeneous Solution

$$\frac{1}{\rho^3}\partial_\rho(\rho^3\partial_\rho\phi_k(\rho)) = 0$$
$$\implies \phi_k = m_k + \frac{c_k}{\rho^2}$$

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- Final Solution

$$\Phi(\rho,\tau) = m + c\,\frac{\rho^4+3\rho^2m^2+3m^4}{(m^2+\rho^2)^3},$$

$$c = -\frac{\varepsilon_0^2L^{16}}{54m^5}\tau^{-\frac{8}{3}}\left(1-4\eta_0\varepsilon_0^{-\frac{1}{4}}\tau^{-\frac{2}{3}}+\dots\right)$$

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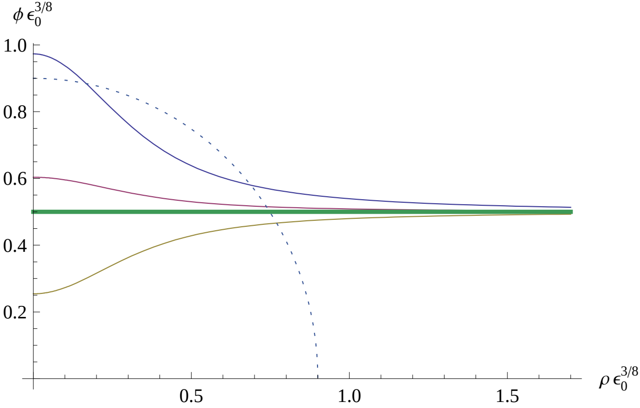
$$\langle\mathcal{O}\rangle = \frac{1}{216\pi^4}\frac{N_f\lambda^3}{N_c^3}\frac{\varepsilon^2}{m_q^5}\sim N_cN_fT^8.$$

Time-dependent embedding

$$m = \frac{1}{2}\varepsilon_0^{-3/8}$$



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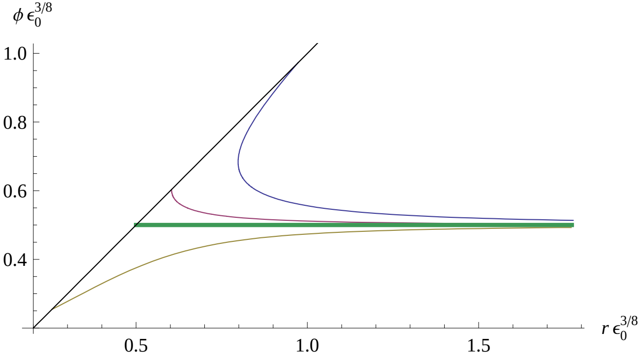

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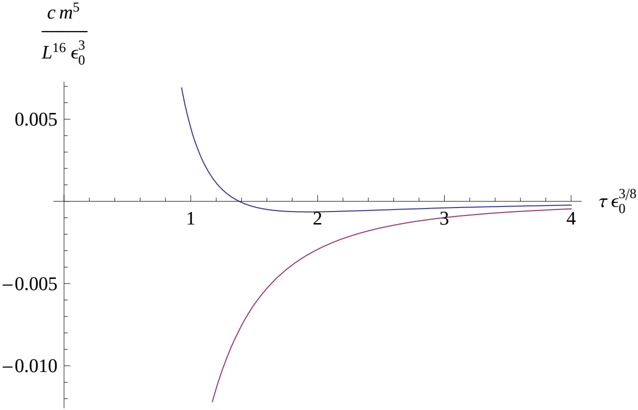
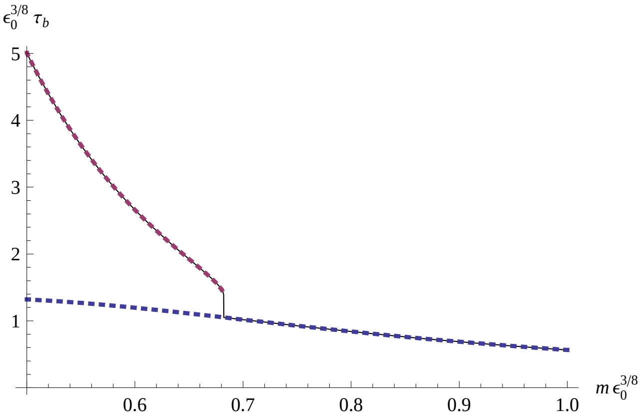


Break-down of perturbative solution

- Time to approach the horizon or bending backwards



Outline

Boost invariant mesons

- usual 5d ansatz: mesons from $\Phi = \Phi_{embedding} + \delta\Phi$

$$\delta\Phi(\rho) = \delta\phi(\rho)\,\exp(ikx)\,\mathcal{Y}^{\ell}(S^3)$$

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$$\Box\phi = \left[-\frac{1}{\tau}\partial_\tau \tau \partial_\tau + \tau^{-2}\partial_y^2 + \partial_x^2\right]\phi = M^2\phi$$

$$\Longrightarrow\ \phi = J_0(\omega\tau)\,e^{\pm i k_\perp x_\perp}$$

$$M^2 = \omega^2 + k_\perp^2 \qquad k_\perp^2 = k_3^2 + k_4^2$$

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Holographic ansatz

- 5d ansatz

$$\delta\Phi = \delta\phi(\rho, \tau)\, J_0\big(\omega(\tau)\, \tau\big)\, \mathcal{Y}^\ell(S^3)$$

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- equation of motion $\mathbf{y} := -\rho^2/m^2$

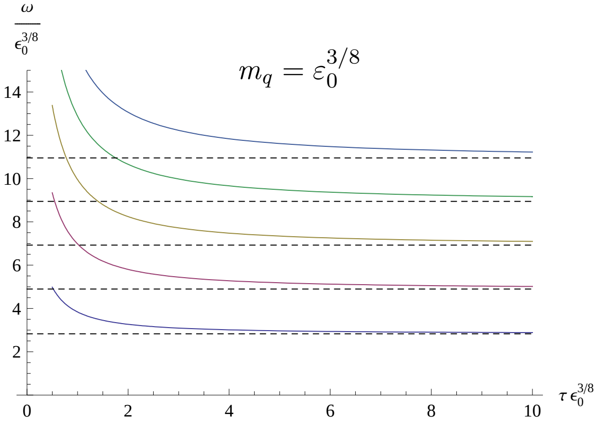
$$\left[\mathbf{y}(1-\mathbf{y})\partial_{\mathbf{y}}^2 + (\mathbf{c} - (\mathbf{a}+\mathbf{b}-1)\mathbf{y})\partial_{\mathbf{y}} - \mathbf{a}\,\mathbf{b}\right]\phi_k(\mathbf{y}) = \mathcal{I}_k$$

$$\mathbf{a} = -n-1, \qquad \mathbf{b} = -n, \qquad \mathbf{c} = 2.$$

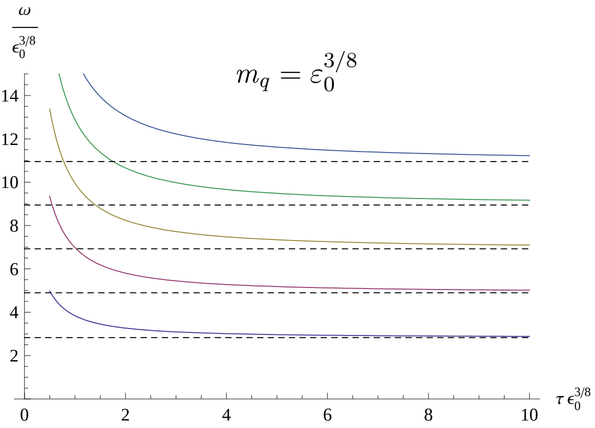
Kruczenski/Mateos/Myers/Winters

Mass spectrum

$$M_\phi(n=0) = \frac{4\pi}{\sqrt{\lambda}} \cdot \left[m_q + \frac{9\lambda^2 \varepsilon_0}{80\pi^4 \tau^{4/3} m_q^3} \cdot \left(1 - \frac{2\eta_0}{3\tau^{\frac{2}{3}} \varepsilon_0^{\frac{1}{4}}} \right) \right]$$



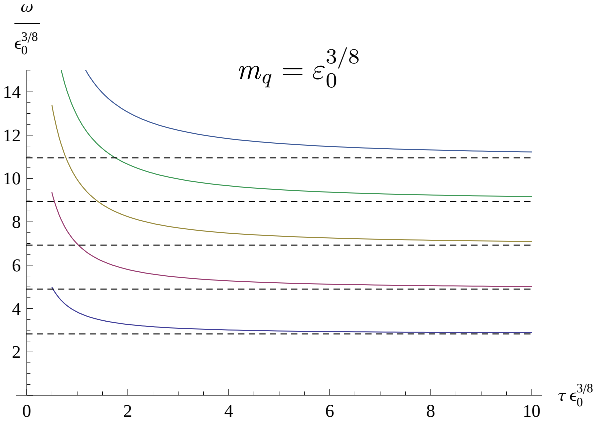
Mass spectrum



- scalars and pseudoscalars are degenerate
- vectors deviate about 3%

But there is something wrong...

Mass spectrum



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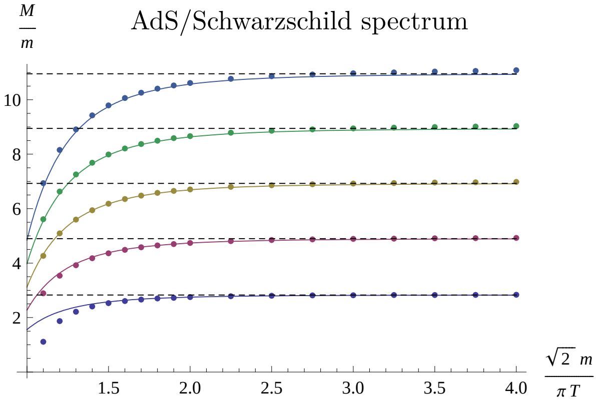
Does not agree with AdS/Schwarzschild?



- horizontal lines: SUSY spectrum (LO)
- dots: numerical solution
- solid curve: NLO low temperature expansion (static)

DOES agree with AdS/Schwarzschild!

We can replace $T \sim t^{-4/3}$ in

- static meson mass

$$M = \frac{4\pi}{\sqrt{\lambda}} \left[m_q - \frac{9\lambda^2}{320} \frac{T^4}{m_q^3} \right]$$

- metric

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☞ agrees with perfect fluid

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☞ Why?

DOES agree with AdS/Schwarzschild!

We can replace $T \sim t^{-4/3}$ in

- static meson mass

$$\partial_t \text{ is } \mathcal{O}(1) \text{ in } T \text{ expansion}$$

- metric

$$\partial_t \text{ is } \mathcal{O}(1/t) \text{ in } 1/t \text{ expansion}$$

Conclusions

Done

- time-dependent D7 embedding
- chiral condensate
- meson spectra

Surprise

- meson spectra not adiabatic

To do

- “black hole” solutions
- thermodynamics

The end

Thank you