Introduction to Heavy Ion Physics

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The purpose of this introduction is
to put two notions into wider context:

Shear Viscosity

Jet Quenching

‘Preface’

Starting point: Quantum Chromodynamics, QCD, the theory of strong interactions, is a mature theory with a precision frontier.
- background in search for new physics
- TH laboratory for non-abelian gauge theories

Open fundamental question: How do collective phenomena and macroscopic properties of matter emerge from the interactions of elementary particle physics?

Heavy Ion Physics: addresses this question in the regime of the highest temperatures and densities accessible in laboratories.

How? 1. Benchmark: establish baseline, in which collective phenomenon is absent.
2. Establish collectivity: by characterizing deviations from baseline
3. Seek dynamical explanation, ultimately in terms of QCD.

These lectures give examples of this ‘How?’

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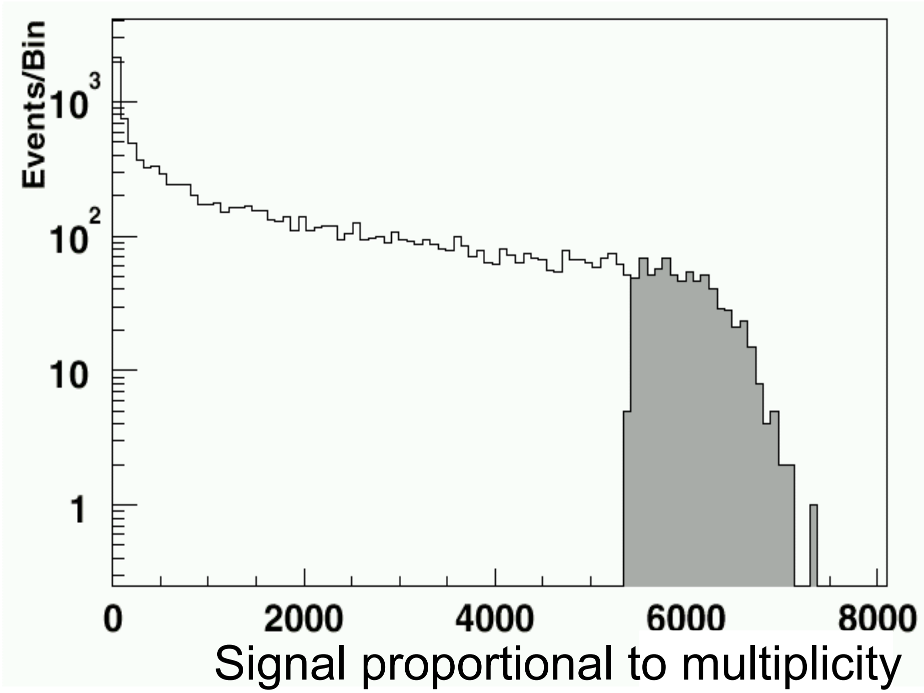
I.1. The very first measurement at an Heavy Ion Collider



What is the benchmark for multiplicity distributions?

Multiplicity in inelastic A+A collisions is incoherent superposition of inelastic p+p collisions.

(i.e. extrapolate p+p -> p+A -> A+A without collective effects)

⇒ **Glauber theory**

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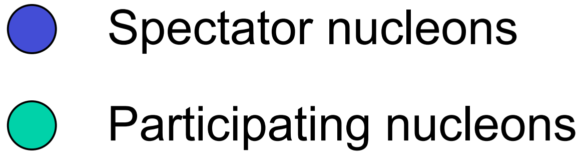
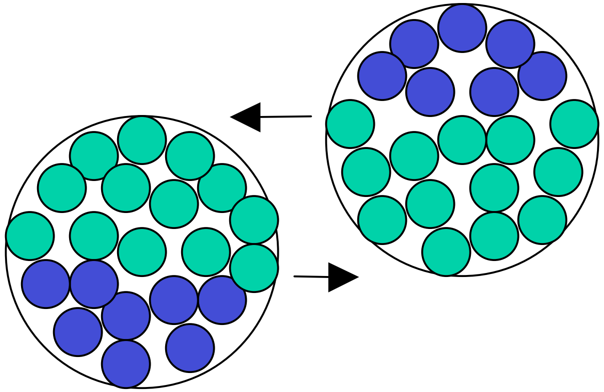
I.2. Glauber Theory

Assumption: inelastic collisions of two nuclei (A-B) can be described by incoherent superposition of the collision of “an equivalent number of nucleon-nucleon collisions”.
How many?
Establish counting based on

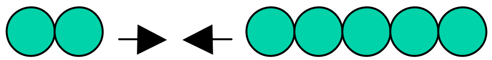




To calculate N_{part} or N_{coll}, take

σ = inelastic n-n cross section

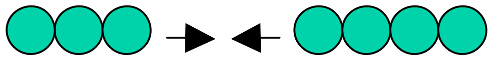
A priori, no reason for this choice other than that it gives a useful parameterization.



$N_{part} = 7$

$N_{coll.} = 10$

$N_{quarks+gluons} = ?$



$N_{inelastic} = 1$

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I.3. Glauber theory for n+A

We want to calculate:

N_{part} = number of participants = number of 'wounded nucleons',
which undergo at least one collision

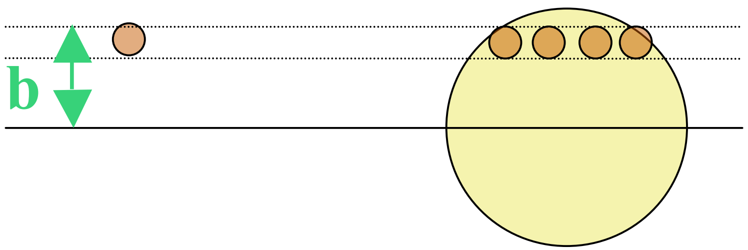
N_{coll} = number of n+n collisions,
taking place in an n+A or A+B collision

We know the single nucleon probability distribution within a nucleus A,
the so-called nuclear density

(1.1)

$$\rho(b,z)$$

$$\int dz\, db\, \rho(b,z) = 1$$



Normally, we are only interested in the transverse density,
the nuclear profile function

(1.2)

$$T_A(b) = \int_{-\infty}^{\infty} dz\ \rho(b,z)$$

I.4. Glauber theory for n+A

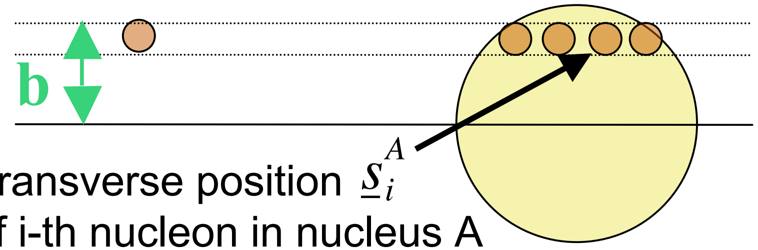
The probability that no interaction occurs at impact parameter b:

(1.3) $$P_0(\underline{b}) = \prod_{i=1}^{A}\left[1 - \int d\underline{s}_i^A T_A\left(\underline{s}_i^A\right)\sigma\left(\underline{b} - \underline{s}_i^A\right)\right] \qquad \int d\underline{s}\,\sigma(\underline{s}) = \sigma_{nn}^{inel}$$

If nucleon much smaller than nucleus

(1.4) $$\sigma(\underline{b} - \underline{s}) \approx \sigma_{nn}^{inel}\,\delta(\underline{b} - \underline{s})$$

(1.5) $$P_0(\underline{b}) = \left[1 - T_A\left(\underline{b}\right)\sigma_{nn}^{inel}\right]^A$$



The resulting nucleon-nucleon cross section is:

(1.6) $$\sigma_{nA}^{inel} = \int d\underline{b}\left(1 - P_0\left(\underline{b}\right)\right) = \int d\underline{b}\left[1 - \left[1 - T_A\left(\underline{b}\right)\sigma_{nn}^{inel}\right]^A\right]$$

$$\xrightarrow{A>>n} \int d\underline{b}\left[1 - \exp\left[-AT_A\left(\underline{b}\right)\sigma_{nn}^{inel}\right]\right]$$ Optical limit

(1.7) $$= \int d\underline{b}\left[AT_A\left(\underline{b}\right)\sigma_{nn}^{inel} - \frac{1}{2}\left(AT_A\left(\underline{b}\right)\sigma_{nn}^{inel}\right)^2 + \ldots\right]$$

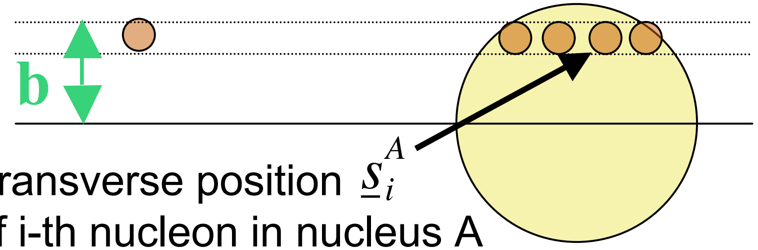
Double counting correction

I.5. Glauber theory for n+A

To calculate number of collisions: probability of interacting with i-th nucleon in A is

(1.8) $$p(\underline{b},\underline{s}_i^A) = \int d\underline{s}_i^A T_A\left(\underline{s}_i^A\right)\sigma\left(\underline{b}-\underline{s}_i^A\right) = T_A(\underline{b})\,\sigma_{nn}^{inel}$$

Probability that projectile nucleon undergoes n collisions
= prob that n nucleons collide and A-n do not



(1.9) $$P(\underline{b},n) = \binom{A}{n}(1-p)^{A-n}\,p^n$$

Average number of nucleon-nucleon collisions in n+A

(1.10) $$\overline{N}_{coll}^{nA}(\underline{b}) = \sum_{n=0}^{A} n\,P(\underline{b},n) = \sum_{n=0}^{A} n\binom{A}{n}(1-p)^{A-n}\,p^n = A\,p$$

$$= A\,T_A(\underline{b})\,\sigma_{nn}^{inel}$$

Average number of nucleon-nucleon collisions in n+A

(1.11) $$\overline{N}_{part}^{nA}(\underline{b}) = 1 + \overline{N}_{coll}^{nA}(\underline{b})$$

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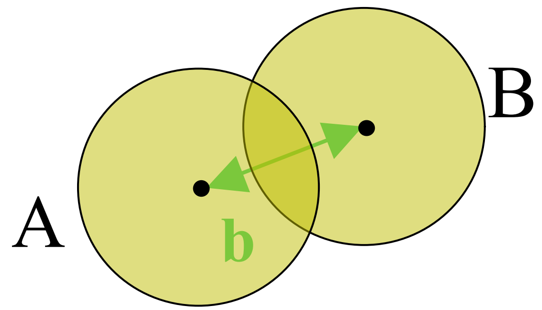
I.6. Glauber theory for A+B collisions

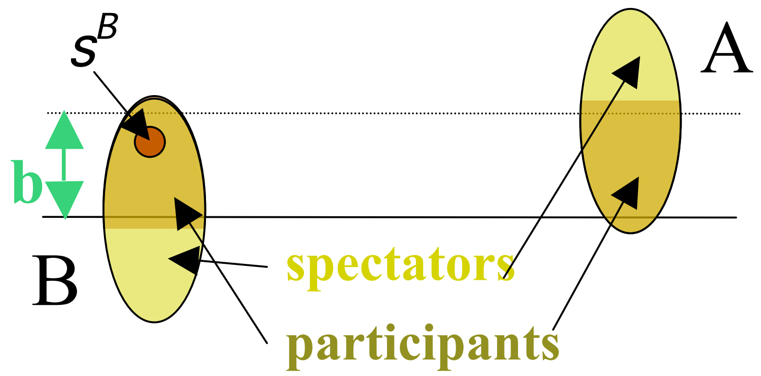
We define the nuclear overlap function

(1.12)
$$T_{AB}(\vec{b}) = \int_{-\infty}^{\infty} d\vec{s}\; T_A(\vec{s})\, T_B(\vec{b}-\vec{s})$$



The average number of collisions of nucleon at s^B with nucleons in A is

(1.13)
$$\overline{N}^{nA}_{coll}(\underline{b}-\underline{s}^B) = A\, T_A(\underline{b}-\underline{s}^B)\, \sigma^{inel}_{nn}$$



The number of nucleon-nucleon collisions in an A-B collision at impact parameter b is

(1.14)
$$\overline{N}^{AB}_{coll}(\underline{b}) = B \int d\underline{s}^B\, T_B(\underline{s}^B)\, \overline{N}^{nA}_{coll}(\underline{b}-\underline{s}^B)$$

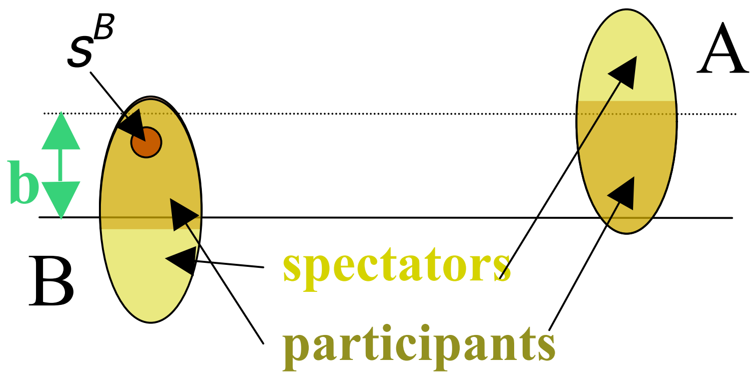
$$= AB \int d\underline{s}\, T_B(\underline{s})\, T_B(\underline{b}-\underline{s})\, \sigma^{inel}_{nn}$$

$$= A\, B\, T_{AB}(\underline{b})\, \sigma^{inel}_{nn}$$

determined in terms of nuclear overlap only

I.7. Glauber theory for A+B collisions

Probability that nucleon at s^B in B is wounded by A in configuration $\{s_i^A\}$



(1.15) $$p(\underline{s}^B,\{\underline{s}_i^A\}) = 1 - \prod_{i=1}^{A}\left[1 - \sigma(\underline{s}^B - \underline{s}_i^A)\right]$$

Probability of finding w_B wounded nucleons in nucleus B:

(1.16) $$P(w_b,\underline{b}) = \binom{B}{w_B}\left(\prod_{i=1}^{A}\prod_{j=1}^{B}\int d\underline{s}_i^A\, d\underline{s}_j^B\, T_A(\underline{s}_i^A)\, T_B(\underline{s}_j^B - \underline{b})\right) p(\underline{s}_1^B,\{\underline{s}_i^A\})\ldots$$

$$\ldots p(\underline{s}_{w_B}^B,\{\underline{s}_i^A\})\left[1 - p(\underline{s}_{w_B+1}^B,\{\underline{s}_i^A\})\right]\ldots\left[1 - p(\underline{s}_B^B,\{\underline{s}_i^A\})\right]$$

Nuclear overlap function defines inelastic A+B cross section.

(1.17) $$\sigma_{AB}^{inel} = \int d\underline{b}\,\sigma_{AB}(\underline{b}) = \int d\underline{b}\, P(w_B = 0,\underline{b})$$

$$= \int d\underline{b}\left[1 - \left(\prod_{i=1}^{A}\prod_{j=1}^{B}\int d\underline{s}_i^A\, d\underline{s}_j^B\, T_A(\underline{s}_i^A)\, T_B(\underline{s}_j^B - \underline{b})\right)\prod_{j=1}^{B}\left[1 - p(\underline{s}_j^B,\{\underline{s}_i^A\})\right]\right]$$

$$\approx \int d\underline{b}\left[1 - \left[1 - T_{AB}(b)\,\sigma_{NN}^{inel}\right]^{AB}\right]$$

I.8. Glauber theory for A+B collisions

It can be shown

Problem 1: derive the expressions (1.17), (1.19)
Use e.g. A. Bialas et al., Nucl. Phys. B111 (1976) 461

(1.18) Number of collisions: $\overline{N}_{coll}^{AB}(\underline{b}) = AB\,T_{AB}(\underline{b})\,\sigma_{NN}^{inel}$

(1.19) Number of participants: $\overline{N}_{part}^{AB}(\underline{b}) = \dfrac{A\sigma_B^{inel}(\underline{b})}{\sigma_{AB}^{inel}(\underline{b})} + \dfrac{B\sigma_A^{inel}(\underline{b})}{\sigma_{AB}^{inel}(\underline{b})} \neq \overline{N}_{coll}^{AB}(\underline{b}) + 1$

1. There is a difference between 'analytical' and 'Monte Carlo' Glauber theory: For 'MC Glauber, a random probability distribution is picked from T_A.

2. The nuclear density is commonly taken to follow a Wood-Saxon parametrization (e.g. for A > 16)

(1.20)
$$\rho(\vec{r}) = \rho_0 \Big/ \left(1 + \exp\left[-(r-R)/c\right]\right); \qquad R \equiv 1.07\,A^{1/3}\,fm, c = 0.545\,fm.$$

C.W. de Jager, H.DeVries, C.DeVries, Atom. Nucl. Data Table 14 (1974) 479

3. The inelastic Cross section is energy dependent, typically

(1.21)
$$\sigma_{nn}^{inel} \approx 40\,mb \qquad at \qquad \sqrt{s_{nn}} = 100\,GeV.$$

But σ_{nn}^{inel} is sometimes used as fit parameter.

I.9 Event Multiplicity in wounded nucleon model

Model assumption: If $\bar{n}_{nn}$ is the average multiplicity in an n-n collision, then

(1.22)
$$\bar{n}_{AB}(b) = \left(\frac{1-x}{2} \bar{N}^{AB}_{part}(b) + x\, \bar{N}^{AB}_{coll}(b) \right) \bar{n}_{NN}$$

is average multiplicity in A+B collision
(x=0 defines the wounded nucleon model).

The probability of having w_b wounded nucleons fluctuates around the mean,,
so does the multiplicity n per event (the dispersion d is a fit parameter, say d~1)

(1.23)
$$P(n,\underline{b}) = \frac{1}{\sqrt{2\pi\, d\, \bar{n}_{AB}(\underline{b})}} \exp\left(- \frac{\left[n - \bar{n}_{AB}(\underline{b}) \right]^2}{2d\, \bar{n}_{AB}(\underline{b})} \right)$$

How many events dN_{events} have event multiplicity dn?

(1.24)
$$\frac{dN_{events}}{dn} = \int db\, P(n,b) \underbrace{\left[1 - \left(1 - \sigma_{NN} T_{AB}(b)\right)^{AB} \right]}_{1-P_0(b)}$$

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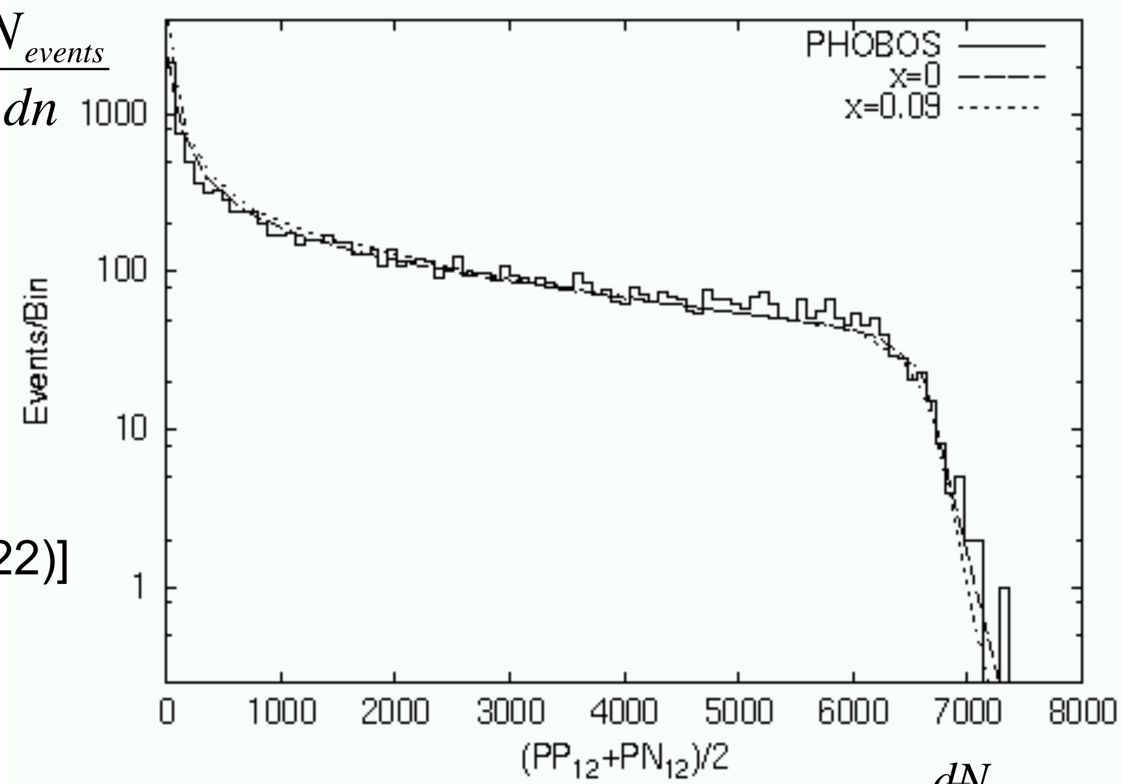
I.10 Wounded nucleon model vs. multiplicity

Compare data to multiplicity distribution (1.24): $\frac{dN_{events}}{dn} = \int d\underline{b}\, P(n,\underline{b})\left[1 - P_0(\underline{b})\right]$

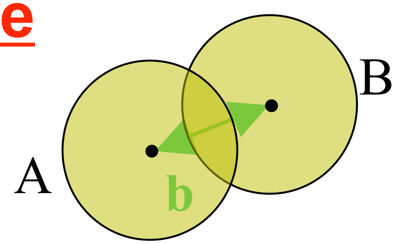


- determined by geometry only
- insensitive to details of particle production [there is almost no dependence on parameter x in (1.22)]
- insensitive to collective effects

Sensitivity to geometry but insensitivity to model-dependent dynamics makes $\frac{dN_{events}}{dn}$

→ **A well-suited centrality measure**

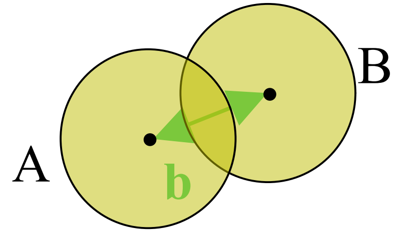
(i.e. a measure of the impact parameter b)



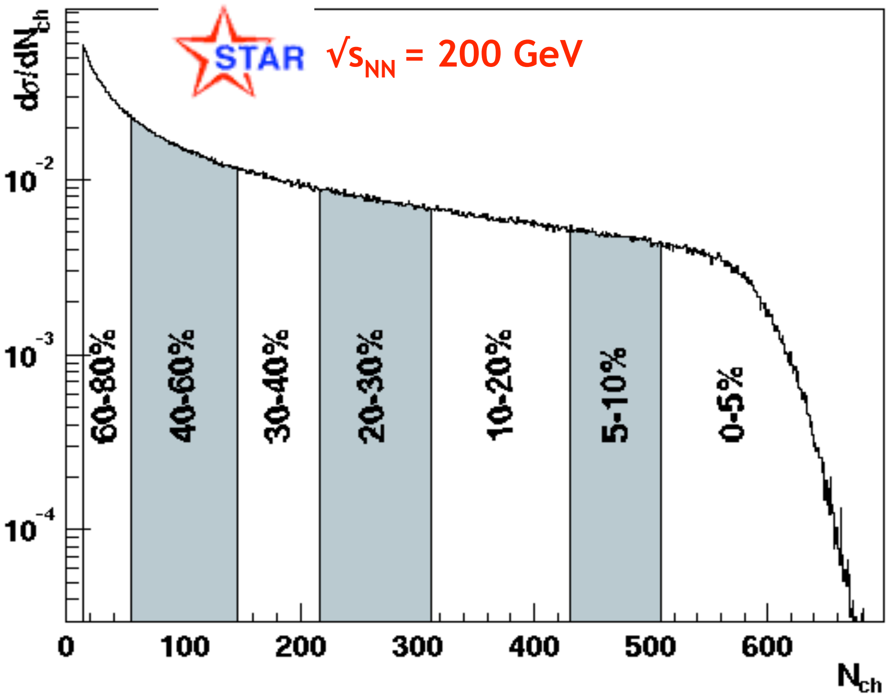
I.11. Multiplicity as a Centrality Measure

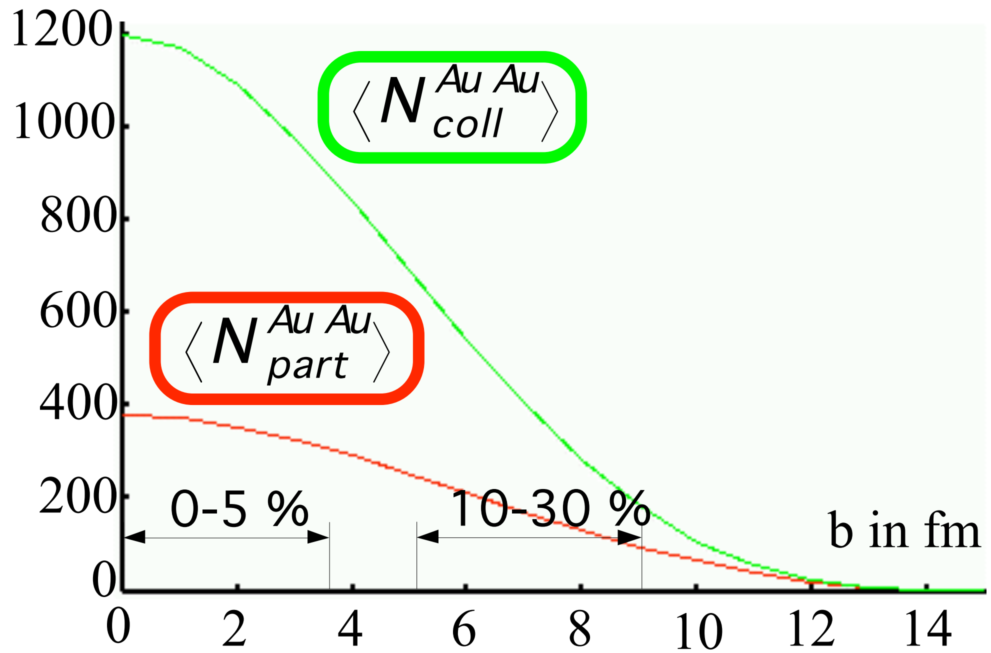
The connection between centrality and event multiplicity can be expressed in terms of

(1.25) $$\left\langle N_{part}^{Au+Au} \right\rangle_{n>n_0} = \frac{\int_{n_0} dn \int db\, P(n,\underline{b})\left[1-P_0(\underline{b})\right] N_{part}(\underline{b})}{\int_{n_0} dn \int d\underline{b}\, P(n,\underline{b})\left[1-P_0(\underline{b})\right]}$$



- Centrality class = percentage of the minimum bias cross section
- Centrality class specifies range of impact parameters





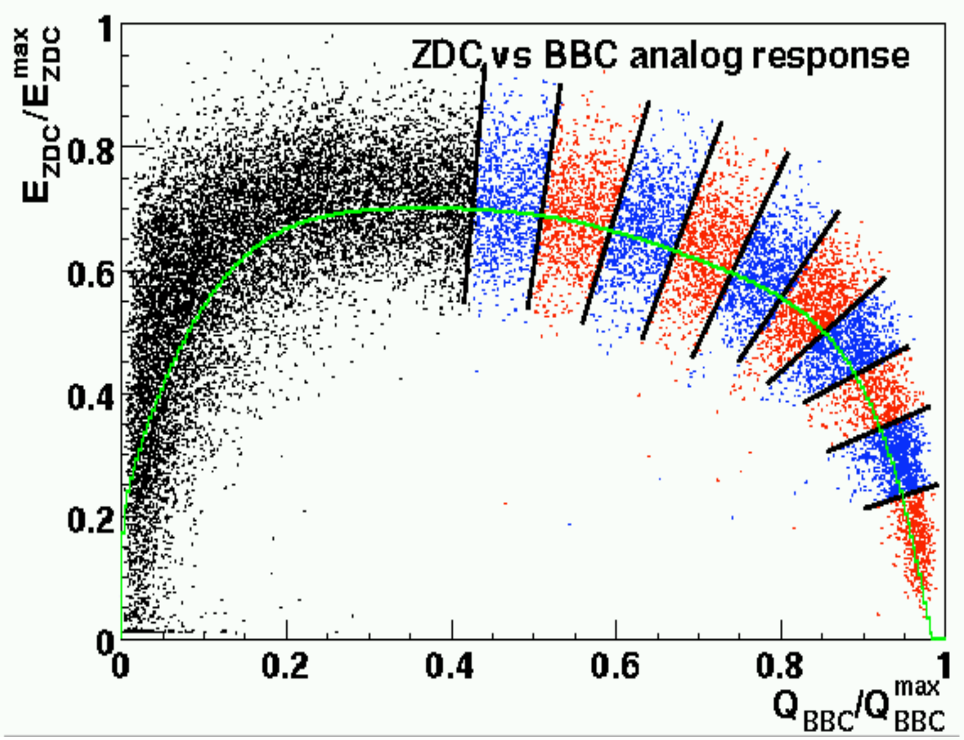
I.12. Cross-Checking Centrality Measurements

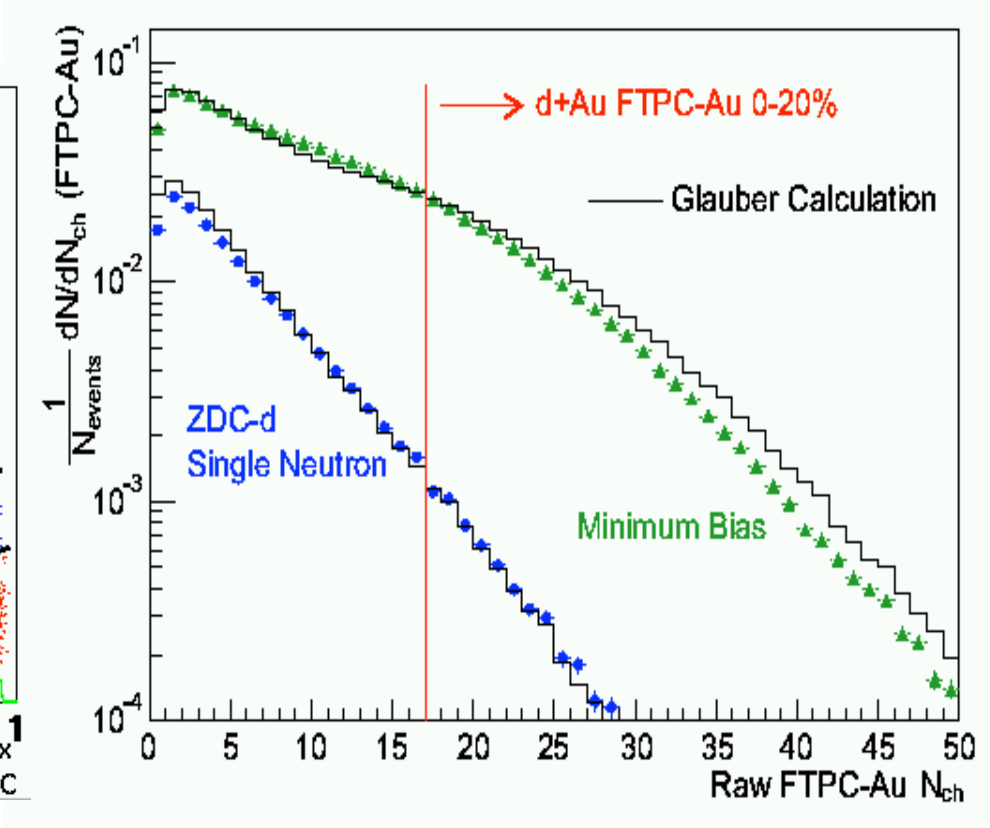
The interpretation of min. bias multiplicity distributions in terms of centrality Measurements can be checked in multiple ways, e.g.

1. Energy E_F of spectators is deposited in Zero Degree Calorimeter (ZDC)

2. Testing Glauber in d+Au and in p+Au(+ n forward)

$$E_F = \left(A - N_{part}(b)/2\right)\sqrt{s}/2$$





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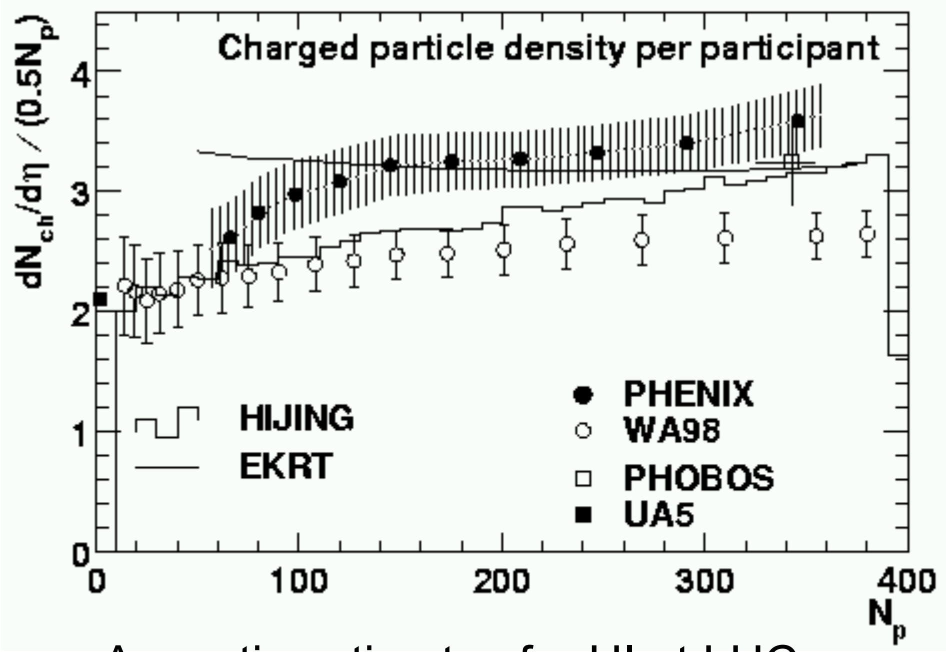
I.13. Final remarks on event multiplicity in A+B

There is no 1st principle QCD calculation of event multiplicity, neither in p+p nor in A+B

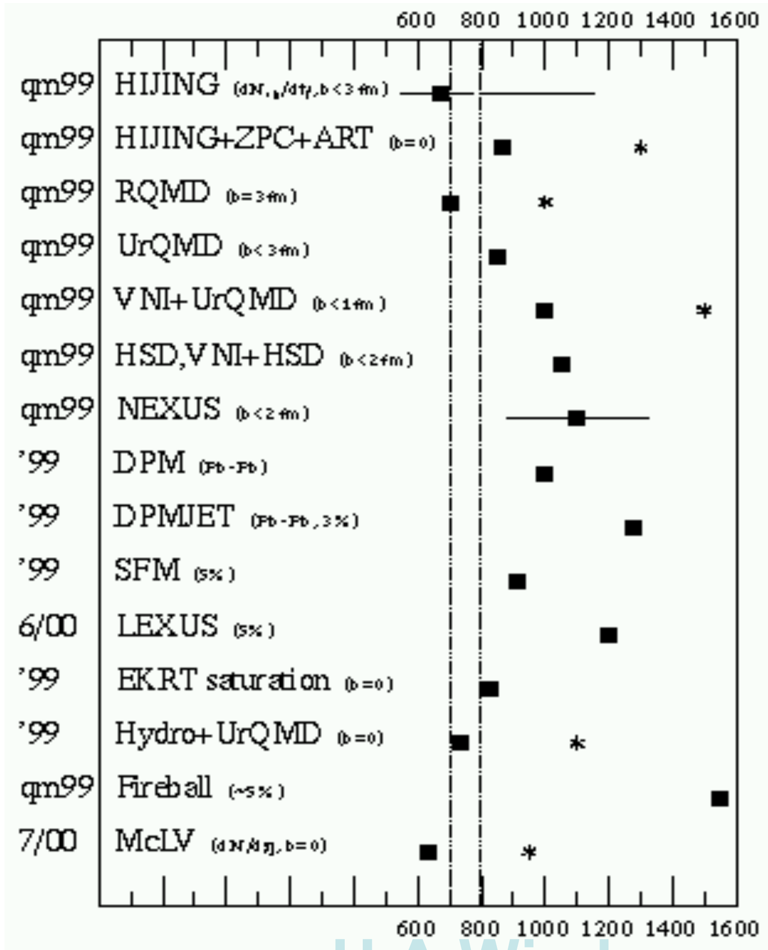
- Clear deviations from multiplicity of wounded nucleon model



<u>Agnostic estimates</u> for HI at LHC:

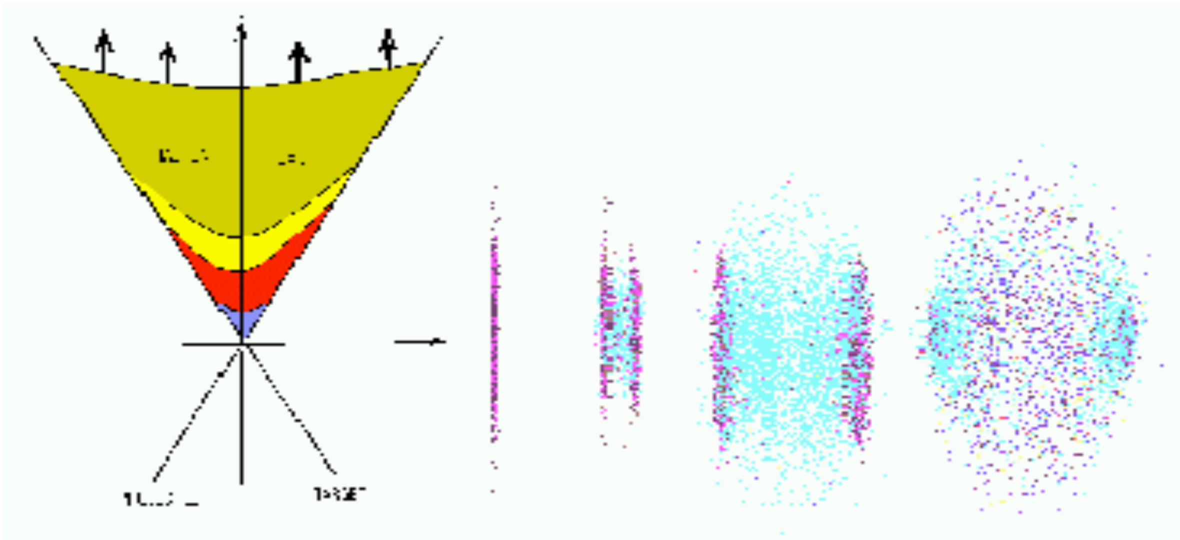
$$\left.\frac{dN_{ch}}{d\eta}\right|_{\eta=0}^{LHC} = 1000 - 8000$$

- Total charged event multiplicity: models failed at RHIC



I.14. Final remarks on event multiplicity

Multiplicity distribution is not only used as centrality measure but:



Multiplicity (or transverse energy) thought to determine properties of produced matter

Bjorken estimate

$$\varepsilon(\tau_0) = \frac{1}{\pi R^2} \frac{1}{\tau_0} \frac{dE_T}{dy}$$

$$\frac{dE_T}{dy} \approx \frac{dN}{dy} \langle E_T \rangle$$

This estimate is based on geometry, thermalization is not assumed, numerically:

$$\varepsilon^{SPS}(\tau_0 \cong 1\, fm/c) = 3 - 4\, GeV/fm^3$$

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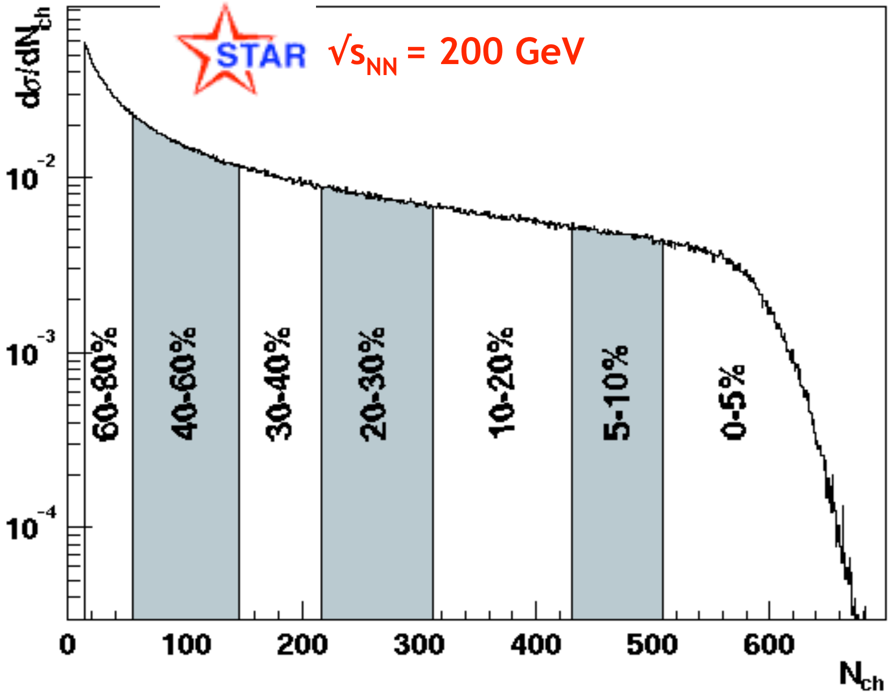
II.1. Azimuthal Anisotropies of Particle Production

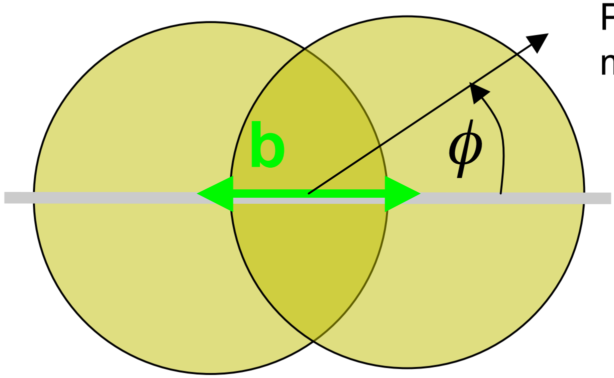
We know how to associate an impact parameter range $b \in [b_{\min}, b_{\max}]$ to an event class in A+A, namely by selecting a multiplicity class.



What can we learn by characterizing not only the modulus b , but also the orientation $\underline{b}$?

II.2. Particle production w.r.t. reaction plane

Consider single inclusive particle momentum spectrum



(2.1) $$f(\vec{p}) \equiv dN/d\vec{p}$$

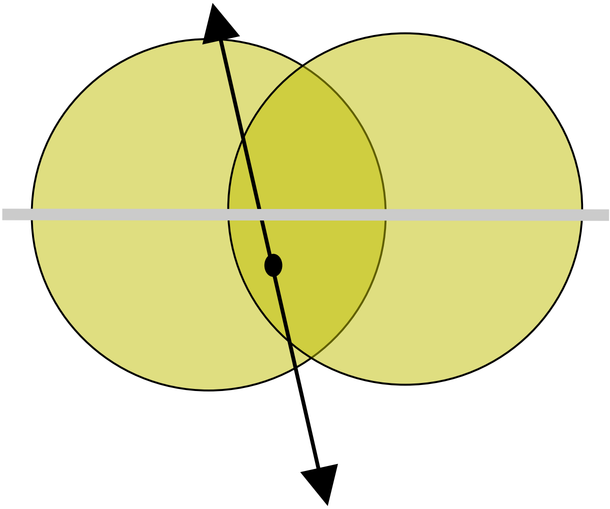
(2.2) $$\vec{p} = \begin{pmatrix} p_x = p_T \cos\phi \\ p_y = p_T \sin\phi \\ p_z = \sqrt{p_T^2 + m^2}\,\sinh Y \end{pmatrix}$$

To characterize azimuthal asymmetry, measure n-th harmonic moment of (2.1) in some detector acceptance *D* [phase space window in (p_T,Y)-plane].

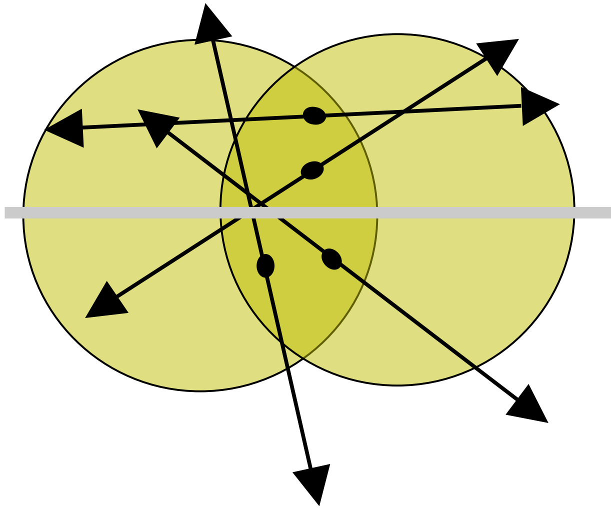
(2.3) $$v_n(D) \equiv \left\langle e^{in\phi} \right\rangle_D = \frac{\int_D d\vec{p}\, e^{in\phi} f(\vec{p})}{\int_D d\vec{p}\, f(\vec{p})}$$ **n-th order flow**

Problem: Eq. (2.3) cannot be used for data analysis, since the orientation of the reaction plane is not known a priori.

II.3. Why is the study of v_n interesting?



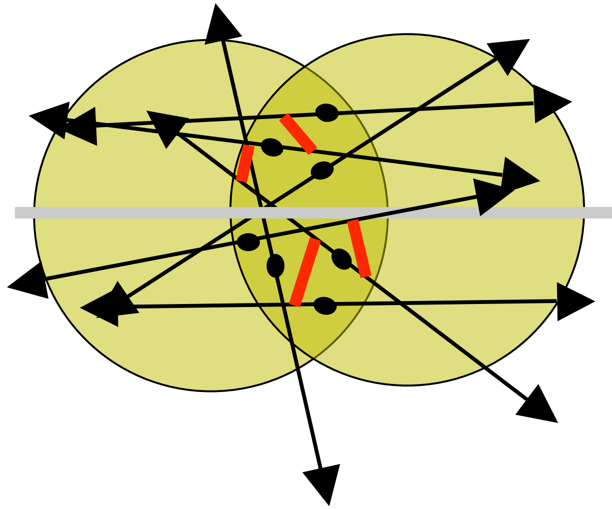
- Single 2->2 process
- Maximal asymmetry
- NOT correlated to the reaction plane



- Many 2->2 or 2-> n processes
- Reduced asymmetry

$$\sim 1/\sqrt{N}$$

- NOT correlated to the reaction plane



- final state interactions
- asymmetry caused not only by multiplicity fluctuations
- collective component is correlated to the reaction plane

The azimuthal asymmetry of particle production has a collective and a random component. Disentangling the two requires a statistical analysis of finite multiplicity fluctuations.

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II.4. Cumulant Method

If reaction plane is unknown, consider particle correlations

(2.4) $$\left\langle e^{in(\phi_1-\phi_2)} \right\rangle_{D_1 \wedge D_2} = \frac{\int_{D_1 \wedge D_2} d\vec{p}_1 d\vec{p}_2 \, e^{in(\phi_1-\phi_2)} f(\vec{p}_1,\vec{p}_2)}{\int_{D_1 \wedge D_2} d\vec{p}_1 d\vec{p}_2 \, f(\vec{p}_1,\vec{p}_2)}$$

A two-particle distribution has an uncorrelated and a correlated part

(2.5) $$f(\vec{p}_1,\vec{p}_2) = f(\vec{p}_1) f(\vec{p}_2) + f_c(\vec{p}_1,\vec{p}_2)$$

(2.6) Short hand $$(1,2) = (1)(2) + (1,2)_c$$

Correlated part

Assumption: Event multiplicity N>>1

→ correlated part is O(1/N)-correction to $f(\vec{p}_1) f(\vec{p}_2)$

(2.7) $$\left\langle e^{in(\phi_1-\phi_2)} \right\rangle_{D_1 \wedge D_2} = v_n(D_1)\, v_n(D_2) + \underbrace{\left\langle e^{in(\phi_1-\phi_2)} \right\rangle^{corr}_{D_1 \wedge D_2}}_{O(1/N)}$$

"Non-flow effects"

(2.8) If $v_n(D) >> \frac{1}{\sqrt{N}}$,then non-flow corrections are negligible.

What, if this is not the case?

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II.5. 4-th order Cumulants

2nd order cumulants allow to characterize v_n, if $v_n \gg 1/\sqrt{N}$.
Consider now 4-th order cumulants:

(2.9)
$$(1,2,3,4) = (1)(2)(3)(4) + (1,2)_c(3)(4) + \ldots$$
$$+ (1,2)_c(3,4)_c + (1,3)_c(2,4)_c + (1,4)_c(2,3)_c$$
$$+ (1,2,3)_c(4) + \ldots$$
$$+ (1,2,3,4)_c$$

If the system is isotropic, i.e. $v_n(D)=0$, then k-particle correlations are unchanged by rotation $\phi_i \rightarrow \phi_i + \phi$ for all i, and only labeled terms survive. This defines

(2.9)
$$\left\langle\left\langle e^{i\,n(\phi_1+\phi_2-\phi_3-\phi_4)}\right\rangle\right\rangle$$
$$\equiv \left\langle e^{i\,n(\phi_1+\phi_2-\phi_3-\phi_4)}\right\rangle - \left\langle e^{i\,n(\phi_1-\phi_3)}\right\rangle\left\langle e^{i\,n(\phi_2-\phi_4)}\right\rangle - \left\langle e^{i\,n(\phi_1-\phi_4)}\right\rangle\left\langle e^{i\,n(\phi_2-\phi_3)}\right\rangle$$

For small, non-vanishing v_n, one finds

Borghini, Dinh, Ollitrault, PRC (2001)

(2.10)
$$\left\langle\left\langle e^{i\,n(\phi_1+\phi_2-\phi_3-\phi_4)}\right\rangle\right\rangle = -v_n^4 + O\left(\frac{1}{N^3}, \frac{v_{2n}^2}{N^2}\right)$$

Improvement: signal can be separated from fluctuating background, if
$$v_N \gg \frac{1}{N^{3/4}}$$

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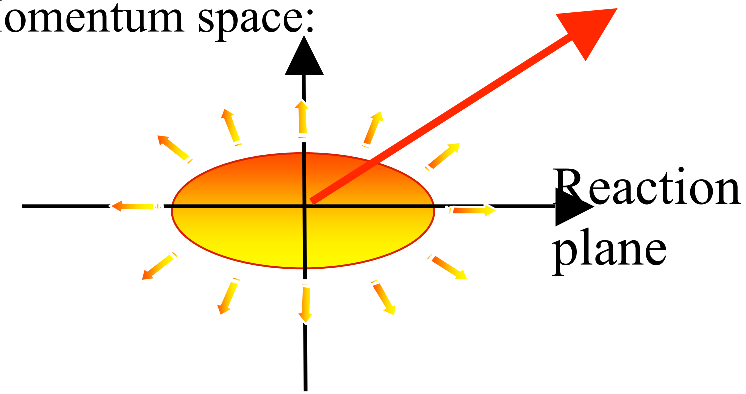
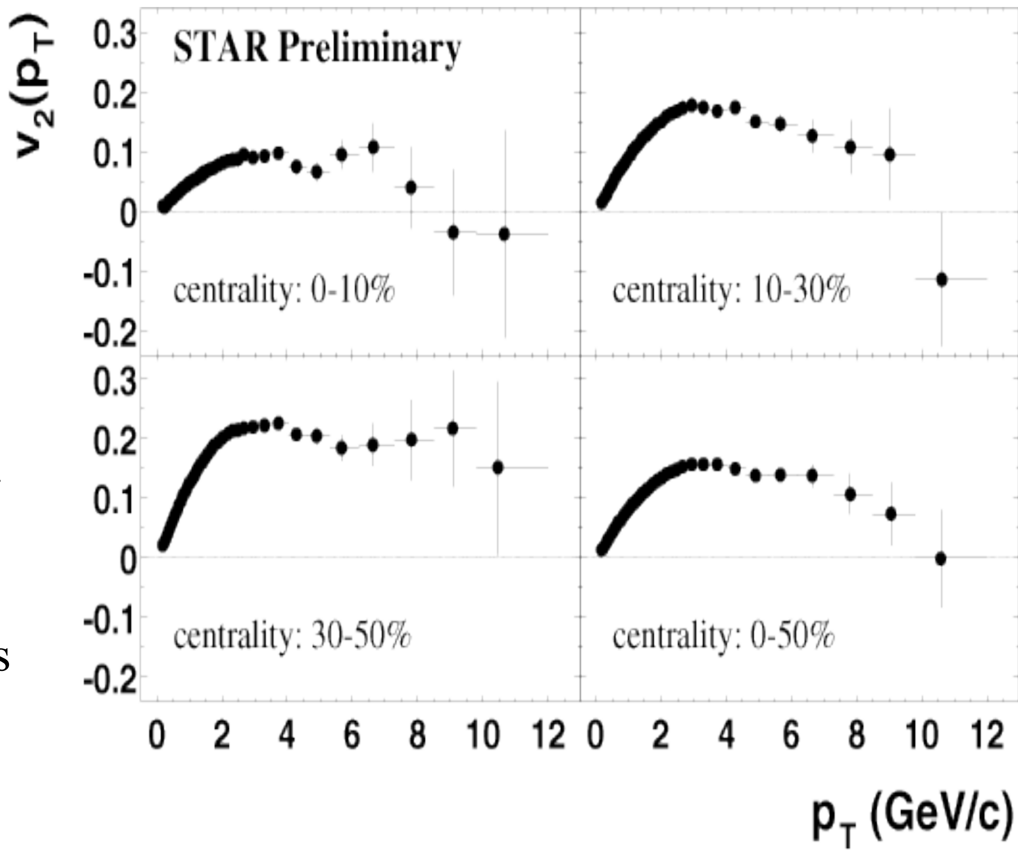
II.6. RHIC Data on Elliptic Flow: v_2

(2.11) $$E\frac{dN}{d^3p} = \frac{1}{2\pi}\frac{dN}{p_T dp_T d\eta}\left[1 + 2v_2(p_T)\cos\left(2(\phi - \psi_{reaction\ plane})\right)\right]$$

- Momentum space:





- Signal $v_2 \approx 0.2$ implies 2-1 asymmetry of particles production w.r.t. reaction plane.

- 'Non-flow' effect for 2nd order cumulants

(2.12) $$N \sim 100 \Rightarrow 1/\sqrt{N} \sim O(v_2)$$

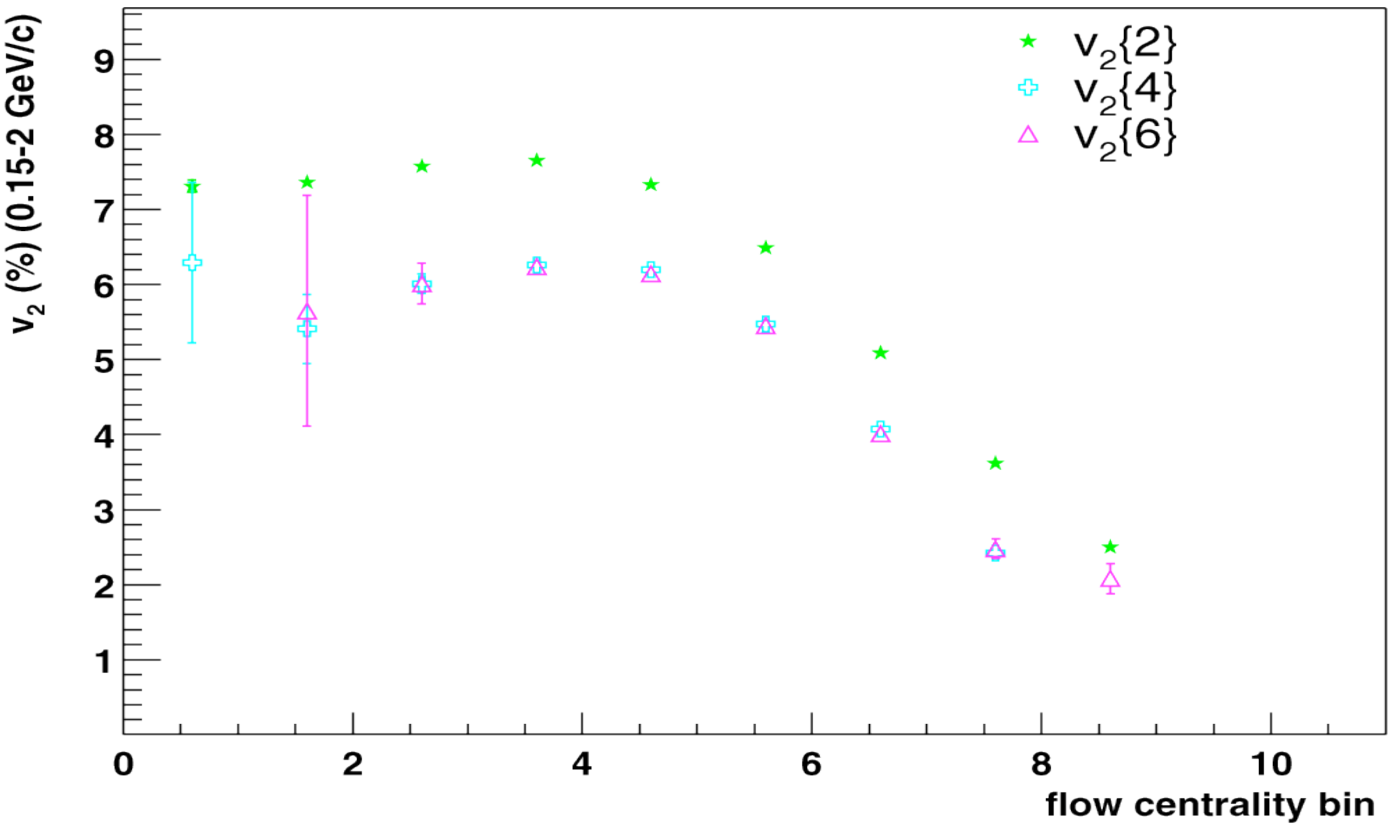
2nd order cumulants do not characterize solely collectivity.

(2.13) $$1/N^{3/4} \sim 0.03 \ll v_2$$

Non-flow effects should disappear if we go from 2nd to 4th order cumulants.

II.7. Pt-integrated elliptic flow: v_2

STAR Coll, Phys. Rev. C66 (2002) 034904



Elliptic flow signal is stable if reconstructed from higher order cumulants.

We have established a **strong collective effect**, which cannot be mimicked by multiplicity fluctuations in the reaction plane.

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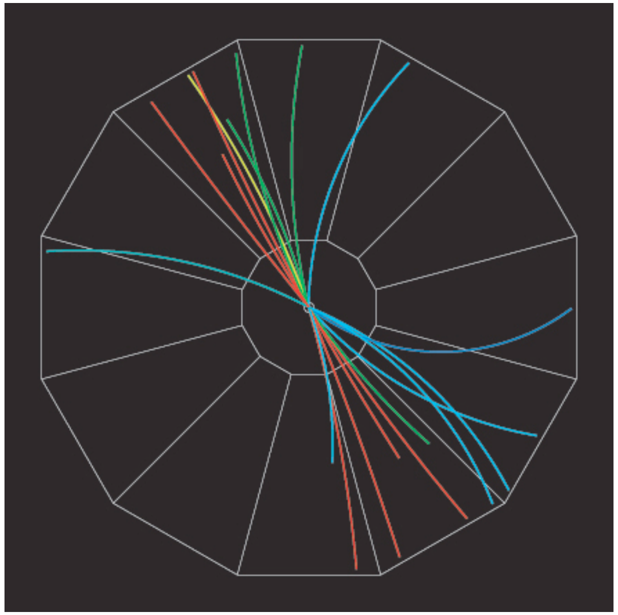
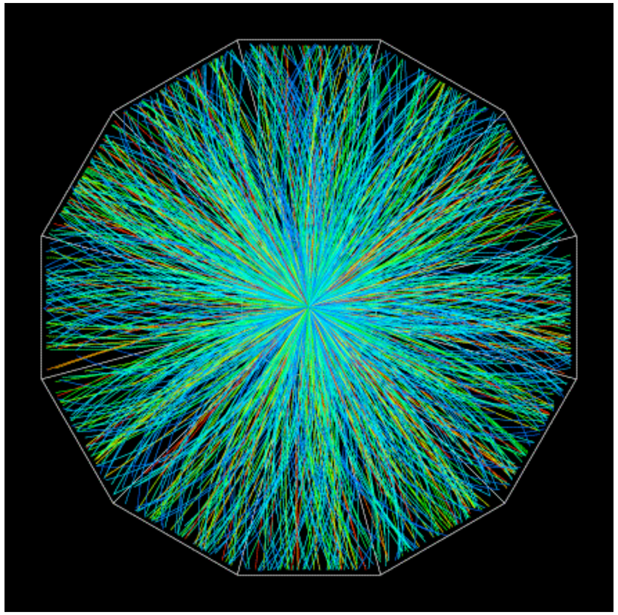
II.8. First Conclusion about elliptic flow

p+p @ RHIC



Au+Au @ RHIC



- compared to the reaction plane, this is rotationally symmetric
- azimuthal asymmetry comes from non-flow effects (here:momentum conservation)

- compared to the reaction plane, this is rotationally asymmetric for semi-central collisions
- azimuthal asymmetry is much larger than non-flow effects allow

To understand the size of v2, let us study a theoretical baseline: the zero mean free path limit of final state interactions:

→ **Hydrodynamics**

III.1. Hydrodynamics - the basics

Consider matter in local equilibrium, characterized locally by its energy momentum tensor, the density of n charges, and a flow field:

- energy momentum tensor $T^{\mu\nu}$ .... 10 indep. components
- conserved charges N_i^{μ} .... 4n indep. components

Tensor decomposition w.r.t. flow field $u_\mu(x)$ projector $\Delta_{\mu\nu} = g_{\mu\nu} - u_\mu u_\nu$

(3.1) $$N_i^{\mu} = n_i u^{\mu} + \overline{n}_i$$

(3.2) $$T^{\mu\nu} = \varepsilon u^{\mu} u^{\nu} - p\Delta^{\mu\nu} + q^{\mu} u^{\nu} + q^{\nu} u^{\mu} + \Pi^{\mu\nu}$$

(3.3) (1 comp.) $\varepsilon \equiv u_\mu T^{\mu\nu} u_\nu$ <u>energy density</u>

(3.4) (1 comp.) $p \equiv -T^{\mu\nu}\Delta_{\mu\nu}/3$ <u>isotropic pressure</u>

(3.5) (3 comp.) $q^{\mu} \equiv \Delta^{\mu\alpha} T_{\alpha\beta} u^{\beta}$ <u>heat flow</u>

(3.6) (5 comp.) $\Pi^{\mu\nu} \equiv \left[\left(\Delta^{\mu}_{\alpha}\Delta^{\nu}_{\beta} + \Delta^{\mu}_{\beta}\Delta^{\nu}_{\alpha}\right)/2 - \Delta^{\mu\nu}\Delta_{\alpha\beta}/3\right] T^{\alpha\beta}$ <u>shear viscosity</u>

In Local Rest Frame (LRF) $u_\mu = (1,0,0,0)$

Convenient choice of frame: Landau frame: $u = u_L \Rightarrow q^{\mu} = 0$

Eckard frame: …

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III.2. Equations of motion for a perfect fluid

A fluid is perfect if it is locally isotropic at all space-time points. This implies

(3.7) $$N_i^\mu = n_i\, u^\mu + \bar{n}_i$$ (n comp.)

(3.8) $$T^{\mu\nu} = \varepsilon\, u^\mu u^\nu - p\Delta^{\mu\nu} + q^\mu u^\nu + q^\nu u^\mu + \Pi^{\mu\nu}$$ (5 comp.)

The equations of motion are then determined by conservation laws

(3.9) $$\partial_\mu N_i^\mu \equiv 0$$ (n constraints)

(3.10) $$\partial_\mu T^{\mu\nu} \equiv 0$$ (4 constraints)

and the equation of state

(3.11) $$p = p(\varepsilon, n)$$ (1 constraint)

Here, information from ab initio calculations (lattice) or models enters.

Hydrodynamic simulations are numerical solutions of (3.7),(3.8).
'Systematic' model uncertainties arise from

- specifying initial conditions
- specifying the decoupling of particles ('freeze-out')
- assuming that non-perfect terms in (3.7),(3.8) can be dropped
- specifying (3.11)

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III.3. Two-dimensional Bjorken Hydrodynamics

Main assumption: initial conditions for thermodynamic fields do not depend on space-time rapidity

(3.12) $$\eta = \frac{1}{2}\ln\left[\frac{t+z}{t-z}\right]$$

Longitudinal flow has 'Hubble form':

(3.13) $$v_z = z/t$$

Bjorken scaling means that hydrodynamic equations preserve Hubble form

(3.14) $$u^\mu = \cosh y_T \left(\cosh\eta, v_x, v_y, \sinh\eta\right)$$ **Longitudinally boost-invariant flow profile**

(3.15) at mid-rapidity $$v_r(\tau, r, \eta = 0) \equiv \tanh y_T(\tau, r)$$

(3.16) at forward rapidity $$v_r(\tau, r, \eta) \equiv \frac{v_r(\tau, r, \eta = 0)}{\cosh\eta}$$

Problem: show that e.o.m. (3.10) preserve longitudinal boost-invariance of initial conditions.
solution see e.g. Kolb+Heinz, PRC62 (2000) 054909

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III.4. 2-dim ‘perfect’ Hydro Simulations: Input

Initialization: thermo-dynamic fields $\varepsilon(\tau, r, \eta = 0)$ have to be initialized, e.g. by

(3.17) $$\varepsilon_{init}(\underline{r}) = \varepsilon(\tau_0, \underline{r}, \eta = 0) \propto \left(\frac{1-x}{2} \overline{N}^{AB}_{part}(\underline{b}, \underline{r}) + x \overline{N}^{AB}_{coll}(\underline{b}, \underline{r}) \right)$$

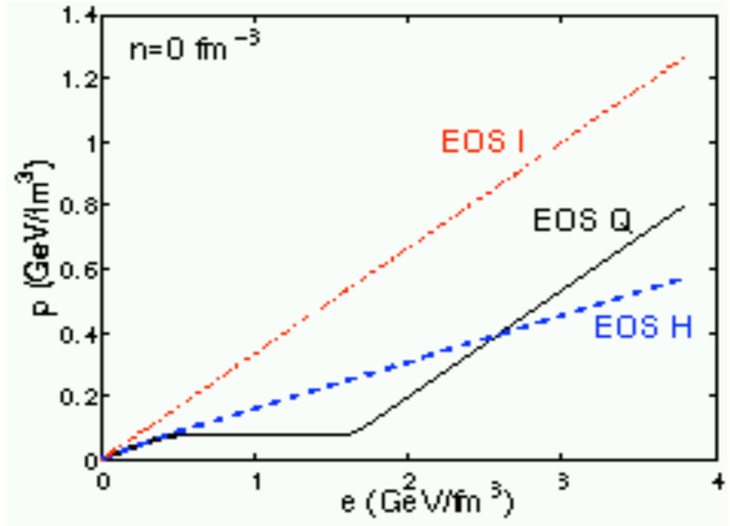
Equation of state: $p(\varepsilon, n)$

(3.18) Velocity of sound: $$c_s^2 = \frac{\partial p}{\partial \varepsilon}$$

(3.19) Expectations: $c_s^2 \approx 0.15$ **Soft EOS**

$c_s^2 = 1/3$ **Hard EOS**

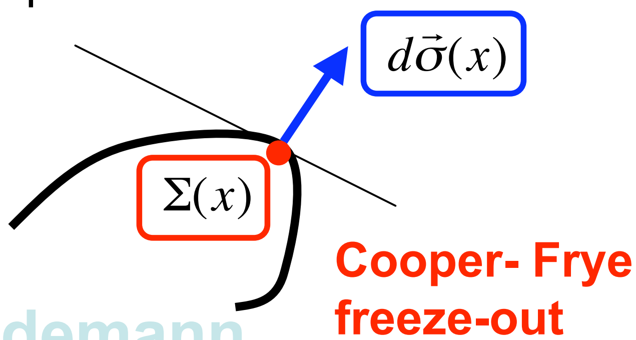
Input from (many) models and from lattice QCD.



Freeze-out: local temperature $T(x) = T_{fo}$ defines space-time hypersurface $\Sigma(x)$, from which particles decouple with spectrum

(3.20) $$E \frac{dN_i}{d\vec{p}} = \frac{g_i}{(2\pi)^3} \int_\Sigma \vec{p}.d\vec{\sigma}(x)\, f_i(p.u(x), x)$$

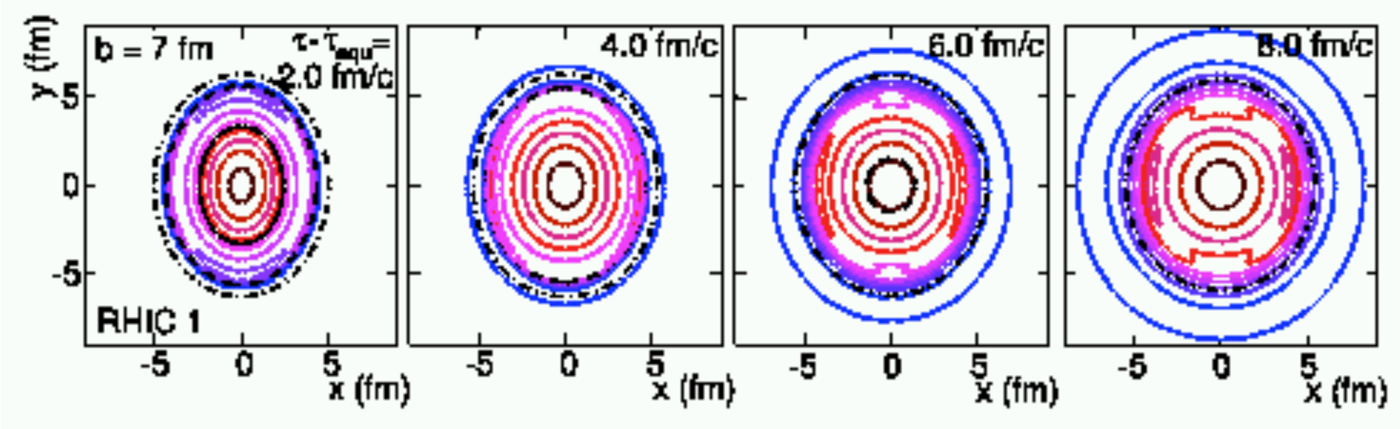
(3.21) $$f_i(E, x) = \frac{1}{\exp[(E - \mu_i(x))/T(x)] \pm 1}$$



III.5. Elliptic flow vs. hydrodynamic simulations

Results of simulations: time evolution in transverse plane

Kolb, Heinz nucl-th/0305084

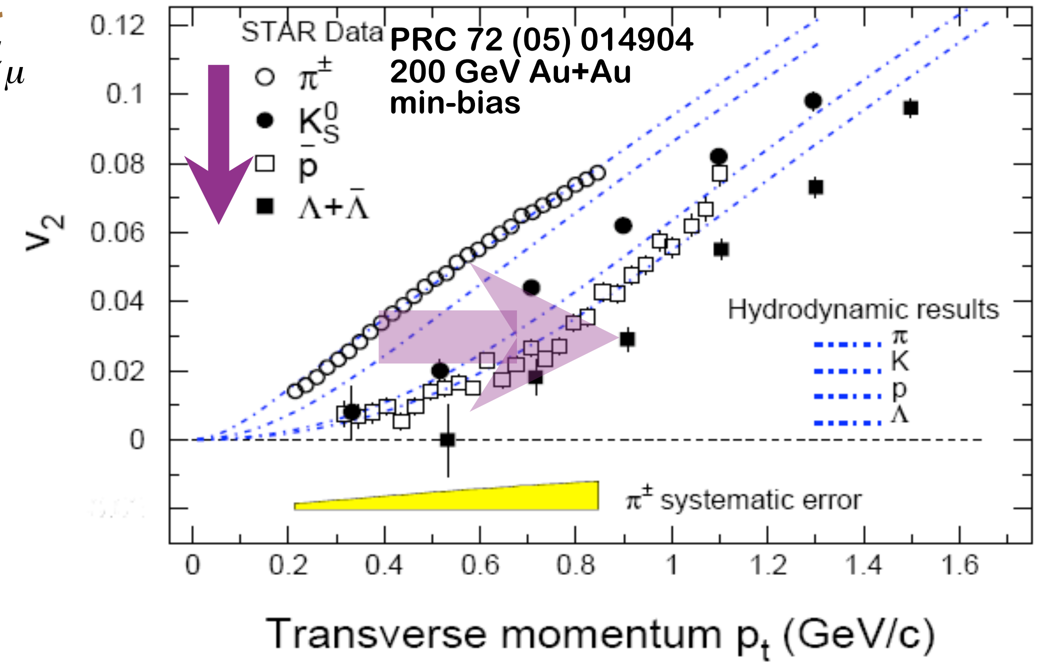


Conclusions from such studies:

- initial **transverse pressure gradient**
  - ⇒ ϕ - dependence of flow field u_μ
  - ⇒ elliptic flow $v_2(p_T)$

- size and pt-dependence of v_2 data accounted for by hydro ('maximal')

- characteristic **mass dependence**, since all particle species emerge from common flow field u_μ



Strong claims at RHIC …

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April 18, 2005

Scientists Serve Up 'Perfect' Liquid

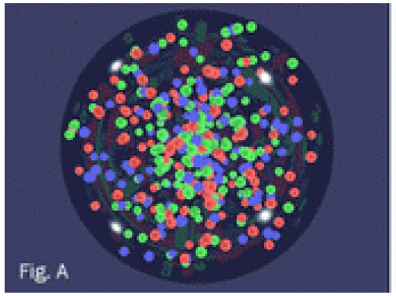
Fig. A

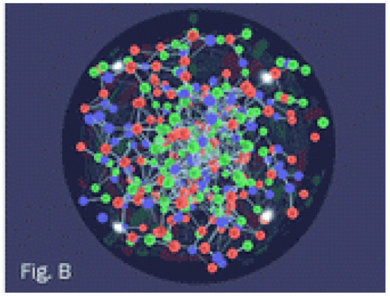
Fig. B

New state of matter more remarkable than predicted -- raising many new questions

The four detector groups conducting research at the Relativistic Heavy Ion Collider (RHIC) -- a giant atom "smasher" located at the U.S. Department of Energy's Brookhaven National Laboratory -- say they've created a new state of hot, dense matter out of the quarks and gluons that are the basic particles of atomic nuclei, but it is a state quite different and even more remarkable than had been predicted. In peer-reviewed papers summarizing the first three years of RHIC findings, the scientists say that instead of behaving like a gas of free quarks and gluons, as was expected, the matter created in RHIC's heavy ion collisions appears to be more like a

III.6. Dissipative corrections to a perfect fluid

Small deviations from a locally isotropic fluid can be accounted for by restoring

(3.7) $$N_i^\mu = n_i u^\mu + \bar{n}_i$$ (4n comp.)

(3.8) $$T^{\mu\nu} = \varepsilon u^\mu u^\nu - p\Delta^{\mu\nu} + q^\mu u^\nu + q^\nu u^\mu + \Pi^{\mu\nu}$$ (10 comp.)

When does perfect fluid assumption fail? Consider conserved current:

(3.22) $$\partial_\mu j^\mu = \partial_\mu\left(\rho\, u^\mu\right) = \rho \underbrace{\partial_\mu u^\mu}_{\text{expansion scalar}} + \underbrace{u^\mu \partial_\mu}_{\text{comoving } t-\text{derivative}} \rho = 0$$

Spatio-temporal variations of macroscopic fluid should be small if compared to microscopic reaction rates

(3.23) $$\Gamma \cong n\sigma >> \theta = \partial_\mu u^\mu$$

Dissipative corrections characterized by gradient expansion!

Now, the conservation laws and equation of state

$$\partial_\mu N_i^\mu \equiv 0$$ (n constraints)

$$\partial_\mu T^{\mu\nu} \equiv 0$$ (4 constraints)

$$p = p(\varepsilon, n)$$ (1 constraint)

are not sufficient to constrain all independent thermo-dynamic fields in (3.7),(3.8).

How do we obtain additional constraints?

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III.7. 1st order dissipative hydrodynamics

Since conservation laws + eos do not close equations of motion, one seeks additional constraints from expanding 2nd law of thdyn to 1st order

(3.24) $S^\mu = s\,u^\mu + \beta\, q^\mu$ Entropy to first order

Use $\varepsilon + p = \mu n + Ts$ and $u_\nu \partial_\mu T^{\mu\nu} \equiv 0$ to write:

(3.25) $$T\partial_\mu S^\mu = \left(T\beta - 1\right)\partial.q + q.\left(\dot{u} + T\partial.\beta\right) + \Pi^{\mu\nu}\partial_\nu u_\mu + \Pi\theta \geq 0$$

To warrant that entropy increases, require:

$$\beta \equiv 1/T$$

Navier-Stokes
1st order hydro

(3.26) bulk viscosity $$\Pi \equiv \varsigma\,\theta$$

(3.27) heat conductivity $$q^\mu \equiv \kappa T \Delta^{\mu\nu}\left(\partial_\nu \ln T - \dot{u}_\nu\right)$$

(3.28) shear viscosity $$\Pi^{\mu\nu} \equiv 2\eta\left[\left(\Delta^\mu_\alpha \Delta^\nu_\beta + \Delta^\mu_\beta \Delta^\nu_\alpha\right)/2 - \Delta^{\mu\nu}\Delta_{\alpha\beta}/3\right]\partial^\alpha u^\beta$$

Determines $\Pi, q^\mu, \Pi^{\mu\nu}$ in terms of flow, energy density and dissipative coeff.

(3.29) $$\partial_\mu S^\mu = \frac{\Pi^2}{\varsigma T} - \frac{q.q}{\kappa T^2} + \frac{\Pi^{\mu\nu}\Pi_{\mu\nu}}{2\eta T} \geq 0$$

Problem: instantaneous acausal propagation.

U.A.Wiedemann

III.8. A model illustrating viscosity

Model: fluid with Bjorken scaling and no transverse gradients

1. Zeroth order ideal fluid dynamics

(3.30) $$\partial_\tau \varepsilon = -\frac{\varepsilon + p}{\tau}$$

This e.o.m. implies that entropy s is conserved

(3.31) $$\frac{d(\tau s)}{d\tau} = \frac{\frac{4}{3}\eta}{\tau T}$$

2. First order Navier-Stokes dissipative hydrodynamics

(3.32) $$\partial_\tau \varepsilon = -\frac{\varepsilon + p}{\tau} + \frac{4\eta}{3\tau^2} \qquad \frac{d(\tau s)}{d\tau} = \frac{\frac{4}{3}\eta}{\tau T}$$

A 'perfect liquid' description is applicable, if the change of entropy is small compared to its absolute size

(3.33) $$\frac{\eta}{\tau T}\frac{1}{s} << 1$$

Put in numbers $\tau \sim 1\, fm/c, \quad T \sim 200\, MeV$ ⇒ $\frac{\eta}{s} << 1$

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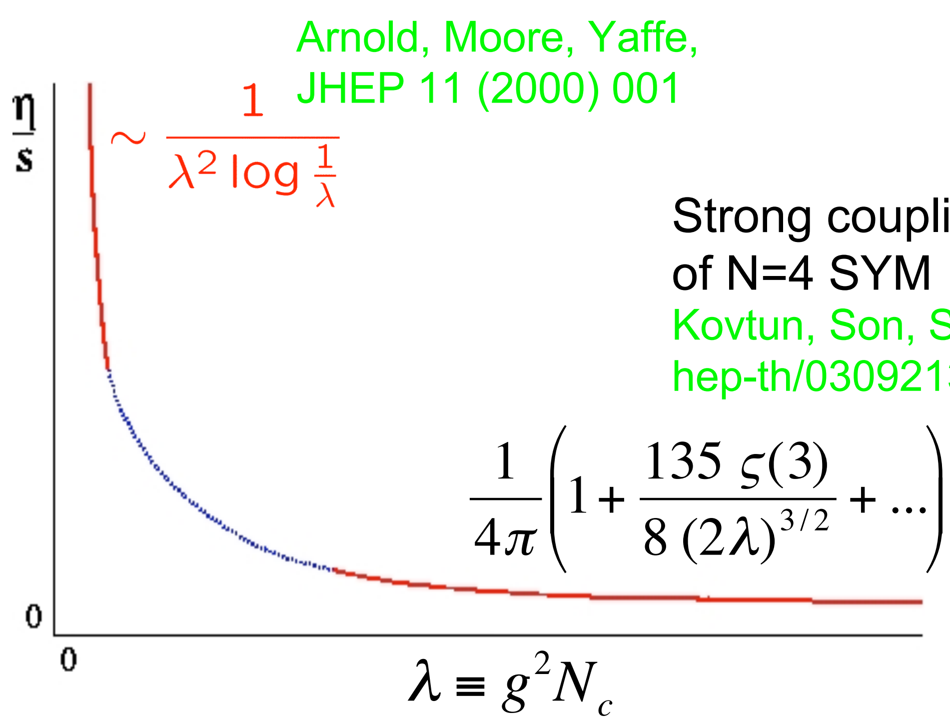
III.9. Viscosity: Bounds from theory

Final remark:

There are calculations of the viscosity over entropy ratio, which indicate that the constraint $\eta/s \ll 1$ may be realized by QCD in the strong coupling regime



$$\frac{\eta}{s} > \frac{1}{4\pi}$$

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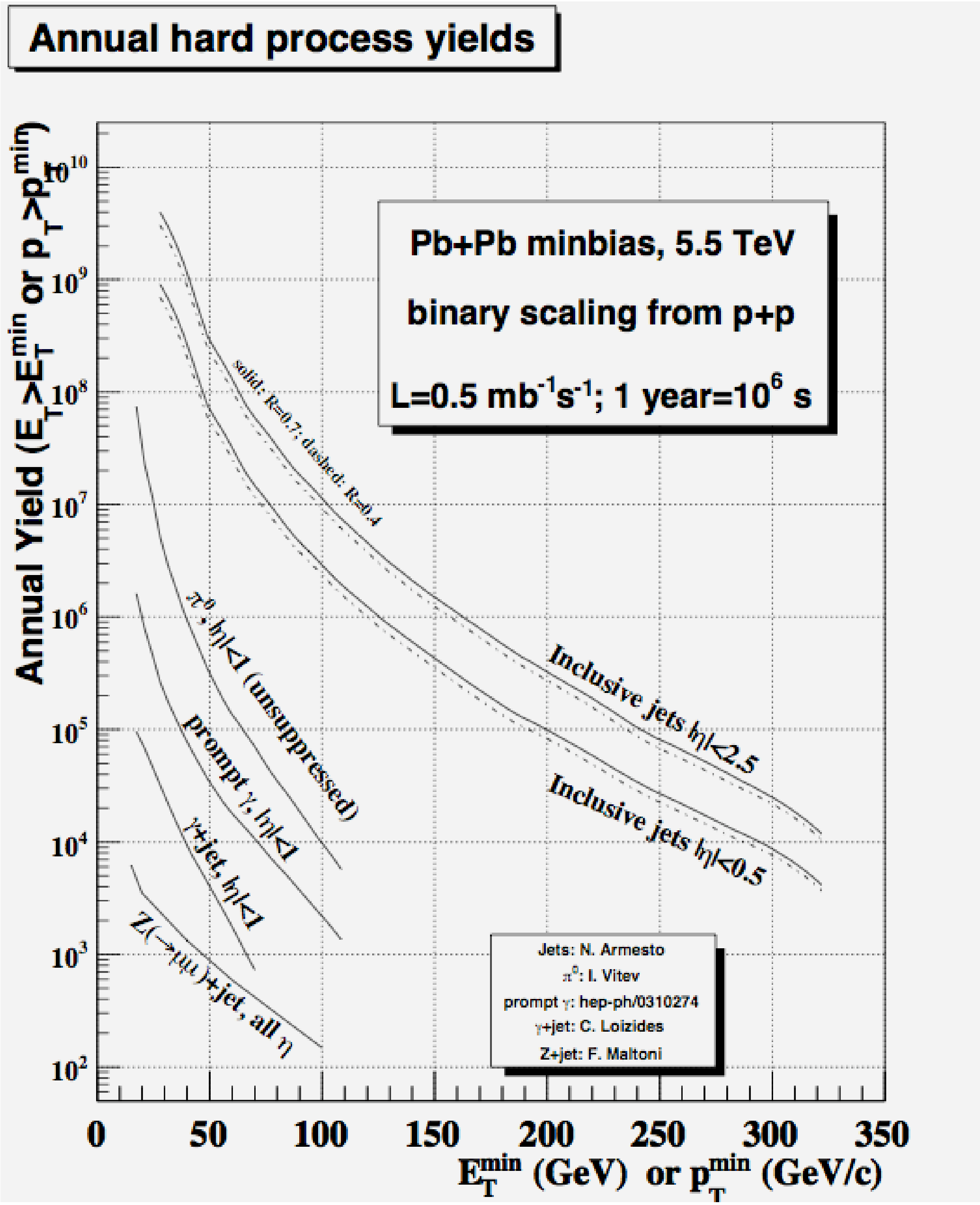
So now turn to ‘Hard Probes’:



Hard probes
= hard processes
embedded in dense nuclear matter (and sensitive to its ‘properties’)

These are produced abundantly at the LHC.

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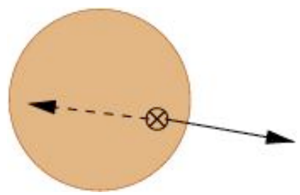
Bjorken's original estimate and its correction

Bjorken 1982: consider jet in p+p collision, hard parton interacts with underlying event ⇒ collisional energy loss

$$dE_{coll}/dL \approx 10\,GeV/fm$$

(error in estimate!)

Bjorken conjectured monojet phenomenon in proton-proton



But: radiative energy loss expected to dominate

$$\Delta E_{rad} \approx \alpha_s \hat{q} L^2$$

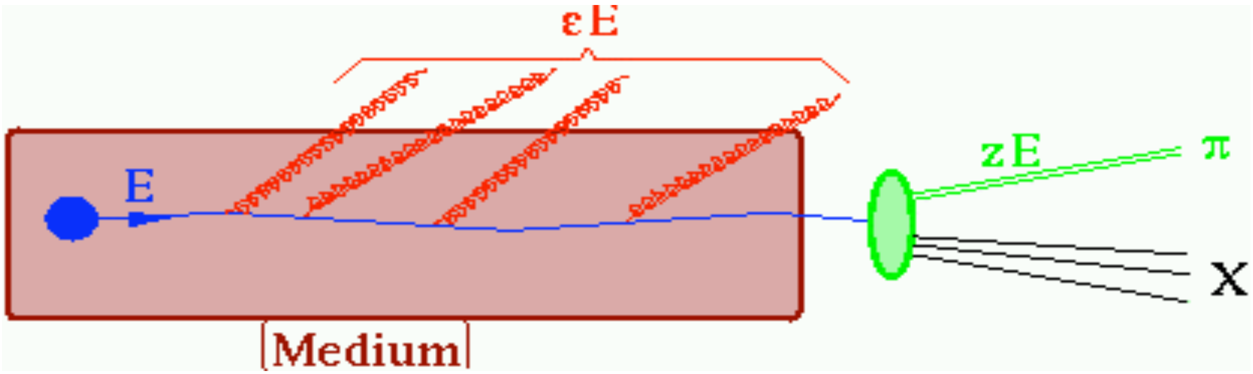
Baier Dokshitzer Mueller Peigne Schiff 1995

- p+p: $L \approx 0.5\ fm, \quad \Delta E_{rad} \approx 100\ MeV$ Negligible !
- A+A: $L \approx 5\ fm, \quad \Delta E_{rad} \approx 10\, GeV$ Monojet phenomenon! Observed at RHIC

Explain how these estimates arise
and how energetic partons lose energy in dense matter.

IV.1 Jet Quenching

So far, ‘jet quenching’ is mainly tested by suppressed leading hadron production:



Nuclear modification factor characterizes medium-effects:

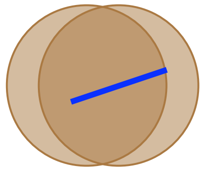
$$R_{AA}(p_T) = \frac{dN^{AA}/dp_T}{n_{coll}\ dN^{NN}/dp_T}$$

$R_{AA}(p_T) = 1.0$ no suppression

$R_{AA}(p_T) = 0.2$ factor 5 suppression

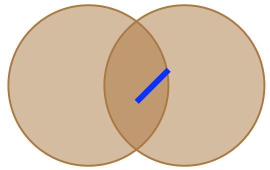
IV.2. Suppression persists to highest p_T

Centrality dependence:



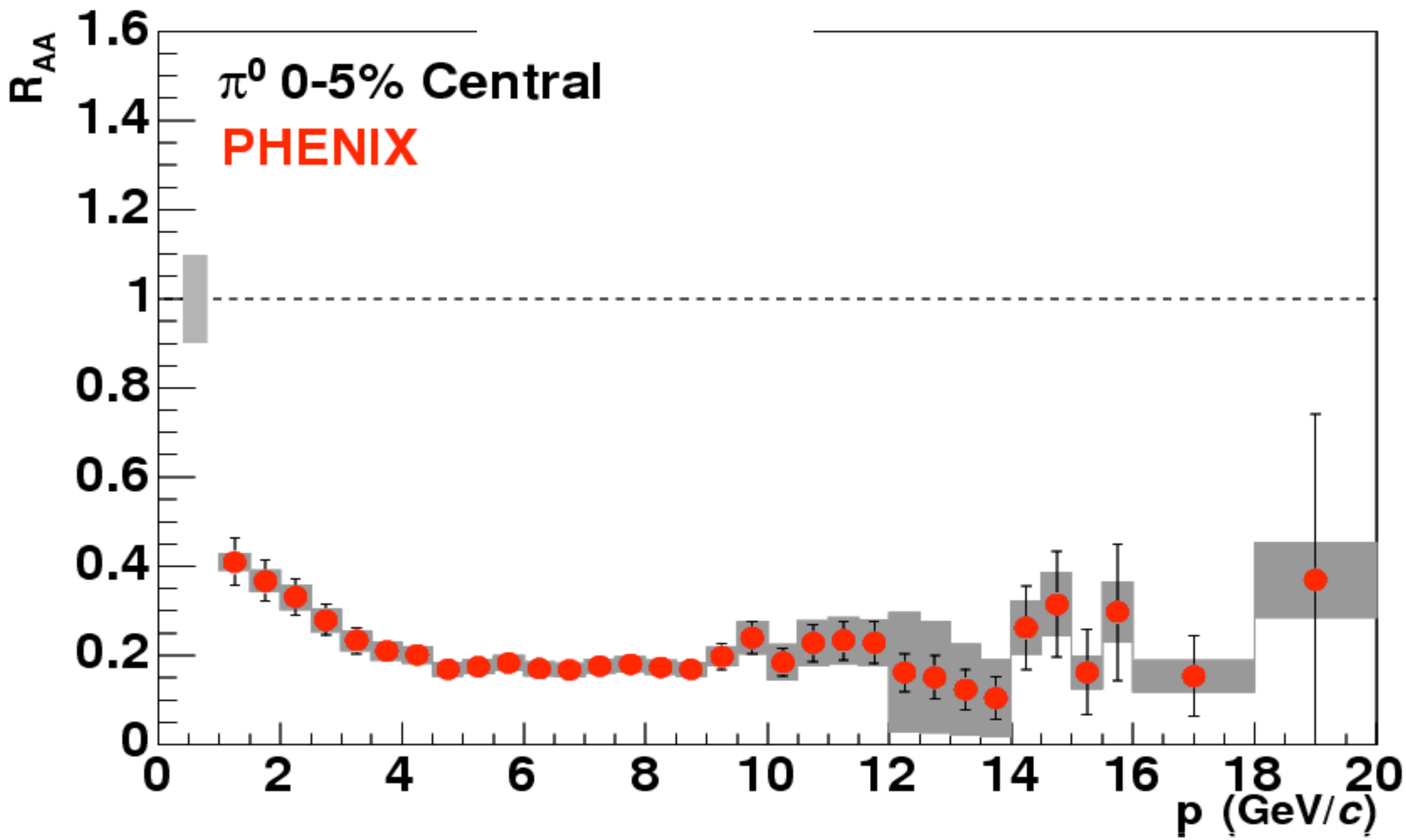
0-5%

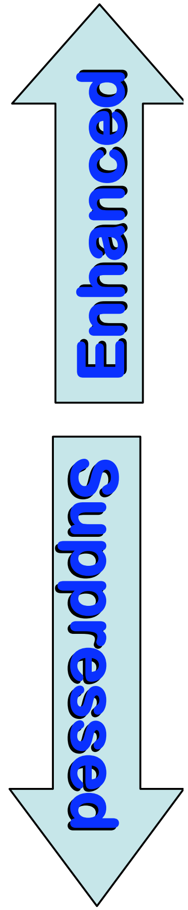
L large



70-90%

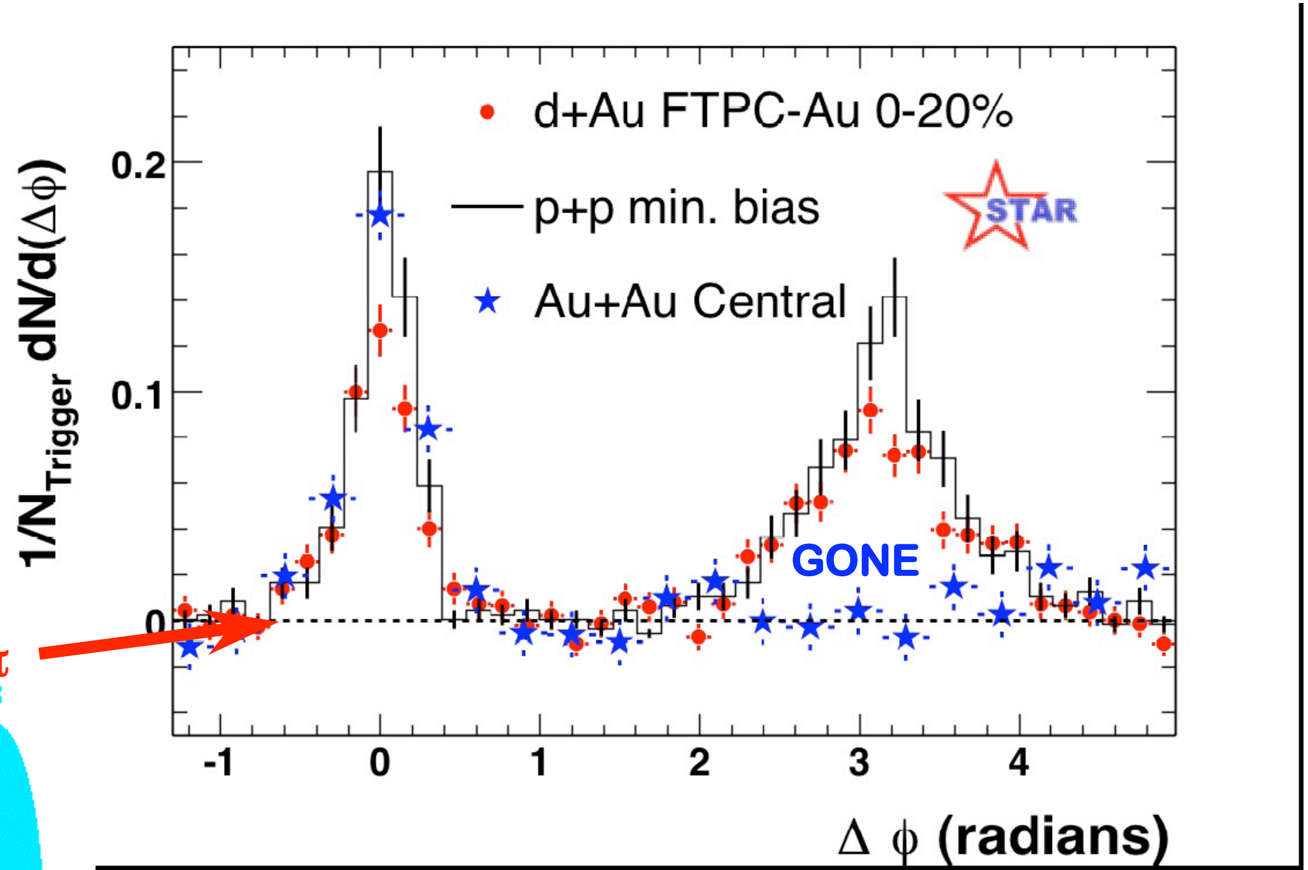
L small

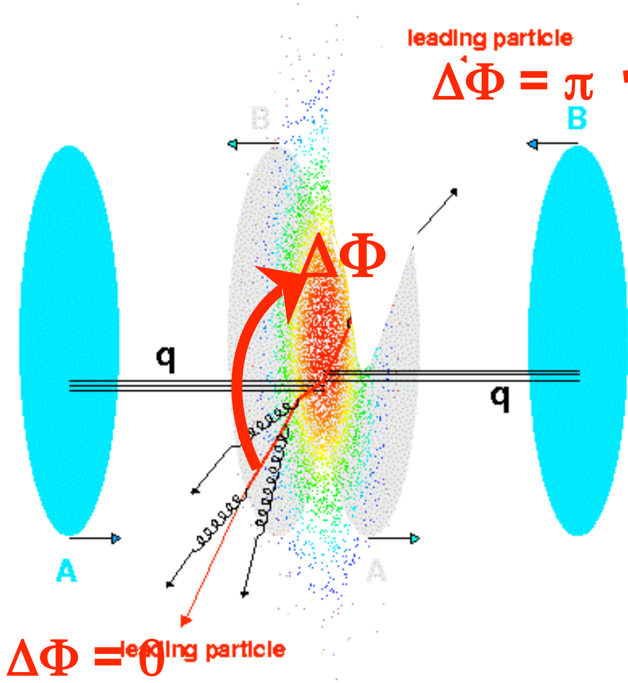




IV.3. The Matter is Opaque

- STAR azimuthal correlation function shows ~ complete absence of “away-side” jet





✦ Partner in hard scatter is *completely absorbed* in the dense medium

IV.4. Issues

Problem: How does a parton propagate and fragment in spatially extended dense QCD matter? By studying its hadronic remnants, what can we learn about *properties of this matter*?

Physics: *Propagation/fragmentation* of highly energetic parton in the vacuum *is modified* by the interaction of the parton with spatially extended color field of the medium.

Purpose of this lectures: sketch current state of the art of the 'theory of jet quenching' and its testable consequences.

Warning: This theory is far from complete! Our presentation is simplified.

IV.5. Eikonal formalism

Idea: at high energy, propagation time through target is short, partons propagate independently of each other, their transverse positions do not change during propagation.

Consider incoming hadronic projectile as superposition of partonic Fock states With color indices α_i and transverse coordinates x_i

(4.1) $$\Psi_{in} = \sum_{\{\alpha_i, x_i\}} \psi(\alpha_i, x_i) \left| \alpha_i, x_i \right\rangle$$

Scattering with a target at high energy implies that each partonic component acquires an **eikonal phase**

(4.2) $$W(x_i) = P \exp\left[i \int dz^- T^a A_a^+(x_i, z_-) \right]$$

namely

(4.3) $$\Psi_{out} = \hat{S}\Psi_{in} = \sum_{\{\alpha_i, x_i\}} \psi(\alpha_i, x_i) \left(\Pi W_{\alpha_i \beta_i}^{r_i}(x_i) \right) \left| \beta_i, x_i \right\rangle$$

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IV.6. Eikonal formalism

$$W(x_i) = P\exp\left[i\int dz^- T^a A_a^+(x_i, z_-)\right]$$

Here, A_a^+ is target gauge field, T^a is SU(3) generator in representation of the parton $|\alpha_i, x_i\rangle$, z_- is light cone coordinate.

Interpretation of

$$\Psi_{out} = \hat{S}\Psi_{in} = \sum_{\{\alpha_i, x_i\}} \psi(\alpha_i, x_i)\left(\Pi_i W_{\alpha_i\beta_i}^{r_i}(x_i)\right)|\beta_i, x_i\rangle$$

High-energy scattering ⟷ Rotation in color space

⟷ Components of Ψ_{in} decohere

⟷ Inelastic particle production

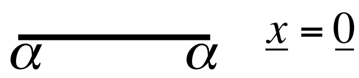
Measure of decoherence:

(4.4) $$|\delta\Psi\rangle = \left[1 - |\Psi_{in}\rangle\langle\Psi_{in}|\right]|\Psi_{out}\rangle$$

E.g. probability of inelastic scattering of projectile given by $\langle\delta\Psi\|\delta\Psi\rangle$

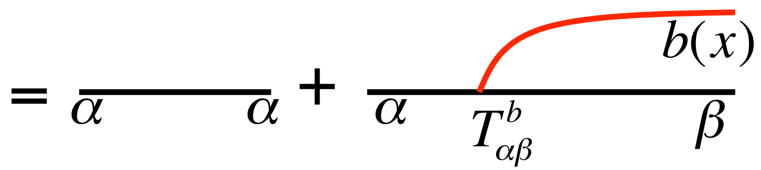
IV.7. Example: gluon production in q+A

Consider high energy quark centered at x=0 and projectile rapidity y=0.
The wave packet to zeroth order in coupling is

(4.5) $\left|\alpha(\underline{0},0)\right\rangle =$ 

But to 1st order in coupling, the quark is not any more a bare quark, it has a gluon in its wavefunction

(4.6) $$\left|\Psi_{in}^{q}\right\rangle = \left|\alpha(\underline{0},0)\right\rangle + \int d\underline{x}\, d\xi\, \vec{f}(\underline{x})\, T^{b}_{\alpha\beta} \left|\beta(\underline{0},0), b(\underline{x},\xi)\right\rangle + O(g^2)$$



This distribution of gluons is flat in rapidity. In transverse space, it follows a Coulomb-type Weizsäcker-Williams field

(4.7) $$\vec{f}(\underline{x}) \propto g \frac{\underline{x}}{\underline{x}^2}$$

IV.8. Example: gluon production in q+A

Note that $\left|\Psi_{in}^{q}\right\rangle$ results from unitary free time evolution of bare quark from the very past $t=-\infty$ to the present $t=0$

(4.8) $$\left|\Psi_{in}^{q}\right\rangle = U_{-}\left|\alpha\right\rangle = \exp\left[-\int d\underline{x}\left|\vec{f}(\underline{x})\right|^{2} + i\int dx\, d\xi\, \vec{f}(\underline{x})\left(a_{d}(x,\xi)+a_{d}^{+}(x,\xi)\right)T^{d}\right]\left|\alpha\right\rangle$$

i.e. U_{-} creates the cloud of gluons around the bare quark.

Now comes the scattering

(4.9) $$\left|\Psi_{out}^{q}\right\rangle = \hat{S}U_{-}\left|\alpha\right\rangle = W_{\alpha\gamma}^{F}(\underline{0})\left|\gamma\right\rangle + \int d\underline{x}\ \vec{f}\left(\underline{x}\right)T_{\alpha\beta}^{b}W_{\beta\gamma}^{F}(\underline{0})\,W_{bc}^{A}(\underline{x})\left|\gamma(\underline{0}),c(\underline{x})\right\rangle$$

Gluons are produced in those components of $\left|\Psi_{out}^{q}\right\rangle$, which lie in the subspace orthogonal to the incoming state with arbitrary color orientation γ

(4.10) $$\left|\delta\Psi_{\alpha}\right\rangle = U_{+}\left[\left|\Psi_{out}^{\alpha}\right\rangle - \sum_{\gamma}U_{-}\left|\gamma\right\rangle\left\langle\gamma U_{-}^{+}U_{+}^{+}\left|\Psi_{out}^{\alpha}\right\rangle\right.\right]$$

To calculate this, use $\left\langle\gamma U_{-}^{+}U_{+}^{+}\left|\Psi_{out}^{\alpha}\right\rangle\right. = \left\langle\gamma\left|W_{\alpha\delta}^{F}(\underline{0})\right|\delta\right\rangle + O(f^{2}) = W_{\alpha\gamma}^{F}(\underline{0})$

IV.9. Example: gluon production in q+A

(4.10) $$\left|\delta\Psi_\alpha\right\rangle = U_+\left[\left|\Psi_{out}^\alpha\right\rangle - \sum_\gamma U_-\left|\gamma\right\rangle\left\langle\gamma U_-^+ U_+^+\right|\Psi_{out}^\alpha\right\rangle\right]$$ Use **(4.6), (4.9)**

$$= U_+\left[\begin{array}{l} W_{\alpha\beta}^F(\underline{0})\left|\gamma\right\rangle + \int d\underline{x}\, \vec{f}(\underline{x}) T_{\alpha\beta}^b W_{\beta\gamma}^F(\underline{0}) W_{bc}^A(\underline{x})\left|\gamma, c(\underline{x})\right\rangle \\ -W_{\alpha\beta}^F(\underline{0})\left|\gamma\right\rangle - \int d\underline{x}\, \vec{f}(\underline{x}) W_{\alpha\gamma}^F(\underline{0}) T_{\gamma\beta}^b\left|\beta, b(\underline{x})\right\rangle \end{array}\right]$$

$$= \int d\underline{x}\, \vec{f}(\underline{x})\left[T_{\alpha\beta}^b W_{\beta\gamma}^F(\underline{0}) W_{bc}^A(\underline{x}) - T_{\beta\gamma}^c W_{\alpha\beta}^F(\underline{0})\right]\left|\gamma, c(\underline{x})\right\rangle$$

The number spectrum of produced gluons reads then

(4.11) $$N^{qA}(k_T) = \frac{1}{N_c}\sum_\alpha \left\langle\delta\Psi_\alpha\right| a_d^+(k_T)\, a_d(k_T)\left|\delta\Psi_\alpha\right\rangle$$

$$= \int d\underline{x}\, d\underline{y}\, e^{i\underline{k}.(\underline{x}-\underline{y})}\frac{1}{N_c}\sum_\alpha\left\langle\delta\Psi_\alpha\right| a_d^+(\underline{y})\, a_d(\underline{x})\left|\delta\Psi_\alpha\right\rangle$$

To calculate this expression, use

(4.12) $$a_d(\underline{x})\left|\delta\Psi_\alpha\right\rangle = \vec{f}(\underline{x})\left[\left(T^b W^F(\underline{0})\right)_{\alpha\gamma} W_{bd}^A(\underline{x}) - \left(W^F(\underline{0}) T^d\right)_{\alpha\gamma}\right]\left|\gamma\right\rangle$$

IV.10. Example: gluon production in q+A

Again:

(4.12)
$$a_d(\underline{x})|\delta\Psi_\alpha\rangle = \vec{f}(\underline{x})\left[\left(T^b W^F(\underline{0})\right)_{\alpha\gamma} W^A_{bd}(\underline{x}) - \left(W^F(\underline{0})T^d\right)_{\alpha\gamma}\right]|\gamma\rangle$$

$$\langle\delta\Psi_\alpha|a_d^+(\underline{y}) = \langle\gamma|\left[W^{A+}_{d\bar{b}}(\underline{y})\left(W^{F+}(\underline{0})T^{\bar{b}}\right)_{\gamma\alpha} - \left(T^d W^{F+}(\underline{0})\right)_{\gamma\alpha}\right]\vec{f}(\underline{y})$$

To calculate from this $\langle\delta\Psi_\alpha|a_d^+(\underline{y})a_d(\underline{x})|\delta\Psi_\alpha\rangle$, we use

$$Tr\left[T^a T^b\right] = \delta^{ab}/2 \qquad \vec{f}(\underline{x}).\vec{f}(\underline{y}) = \frac{\alpha_s}{2\pi}\frac{\underline{x}.\underline{y}}{x^2 y^2}$$

(4.13)
$$W^A_{ab}(\underline{x}) = 2Tr\left[T^a W^F(\underline{x})T^b W^{F+}(\underline{x})\right]$$

(4.14)
$$N^{qA}(k_T) = \frac{\alpha_s C_F}{2\pi}\int d\underline{x}\, d\underline{y}\, \frac{\underline{x}.\underline{y}}{x^2 y^2} e^{i\underline{k}.(\underline{x}-\underline{y})}\left[\begin{array}{l} 1 - \left\langle Tr\left[W^{A+}(\underline{0})W^A(\underline{x})\right]\right\rangle \\ -\left\langle Tr\left[W^{A+}(\underline{y})W^A(\underline{0})\right]\right\rangle + \left\langle Tr\left[W^{A+}(\underline{y})W^A(\underline{x})\right]\right\rangle \end{array}\right]$$

IV.11. Example: gluon production in q+A

(4.14)
$$N^{qA}(k_T) = \frac{\alpha_s C_F}{2\pi} \int d\underline{x}\, d\underline{y}\, \frac{\underline{x}.\underline{y}}{x^2 y^2} e^{i\underline{k}.(\underline{x}-\underline{y})} \begin{bmatrix} 1 - \left\langle Tr\left[W^{A+}(\underline{0}) W^{A}(\underline{x})\right]\right\rangle \\ - \left\langle Tr\left[W^{A+}(\underline{y}) W^{A}(\underline{0})\right]\right\rangle + \left\langle Tr\left[W^{A+}(\underline{y}) W^{A}(\underline{x})\right]\right\rangle \end{bmatrix}$$

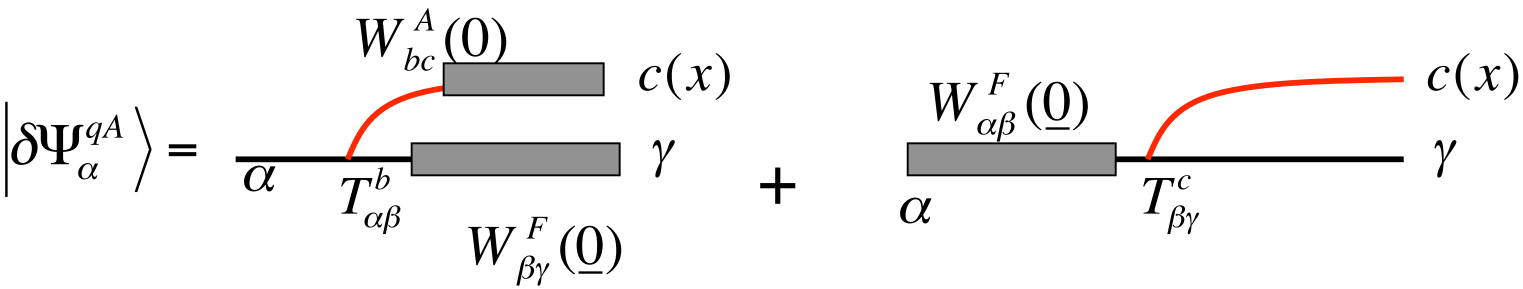
What does that mean?
If an ultra-relativistic quark scatters on a spatially extended target, its gluon radiation is characterized by a single non-perturbative quantity, the **target average**

$$\left\langle Tr\left[W^{A+}(\underline{y}) W^{A}(\underline{x})\right]\right\rangle$$

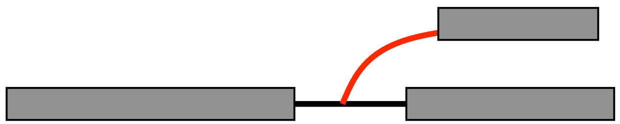
Let's see how this target average is parameterized in 'QCD-inspired models'.
We'll later calculate it from AdS/CFT.

V.1. Beyond the eikonal approximation

In eikonal approximation, gluons are produced before or after interaction with the target, but not within the target (since it is infinitely Lorentz contracted). Example:



But in heavy ion collision, gluons are produced within the target (no emission before target, but possibly after target). This would be a term like



Including this term amounts to

- Need info about spatial longitudinal resolution
- Keep leading $1/p^-$ energy corrections to eikonal amplitudes.

This will give access to interference terms (non-abelian LPM effect).

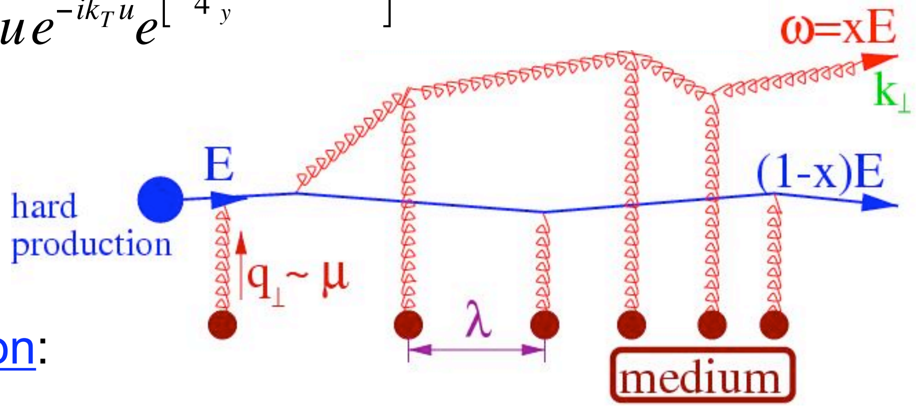
V.9. BDMPS gluon radiation spectrum

R. Baier et al. (BDMPS), NPB484:265, 1997
Wiedemann, NPB 588 (2000) 303

$$\frac{dI}{d\ln\omega\, dk_T} = \frac{\alpha_s C_R}{(2\pi)^2\omega^2} 2\mathrm{Re}\int_0^\infty dy \int_y^\infty d\bar{y} \int du\, e^{-ik_T u} e^{\left[-\frac{1}{4}\int_y^\infty d\xi\, \hat{q}(\xi) u^2\right]}$$

$$\times \frac{\partial}{\partial u} . \frac{\partial}{\partial s} K(s=0, y; u, y \mid \omega)$$

Radiation off produced parton



Target average includes Brownian motion:

$$K(s, y; u, \bar{y} \mid \omega) = \int_{s=r(y)}^{u=r(\bar{y})} Dr \exp\left[\int_y^{\bar{y}} d\xi \left\{\left(\frac{i\omega}{2}\dot{r}^2\right) - \frac{1}{4}\hat{q}(\xi) r^2\right\}\right]$$

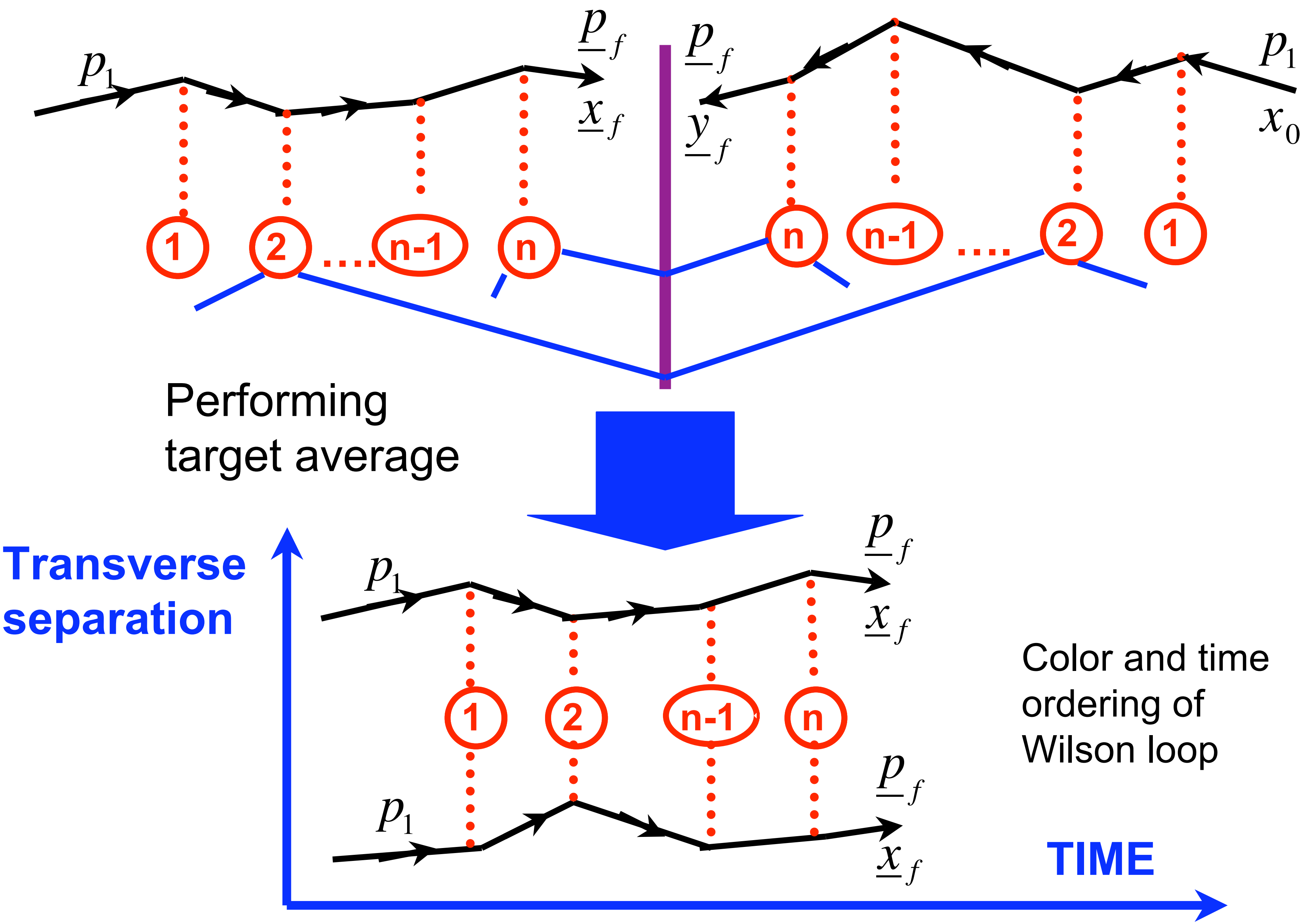
$$\xrightarrow{\omega\to\infty} \exp\left[-\frac{1}{4}\hat{q}\, L_{long}\, r^2\right]$$

BDMPS transport coefficient

$$\equiv \left\langle Tr\left[W^{A+}(0) W^{A}(r)\right]\right\rangle$$

Expectation value of light-like Wilson line

V.10. How Wilson loops arise in BDMPS



VI.1. Opacity Expansion - zeroth order

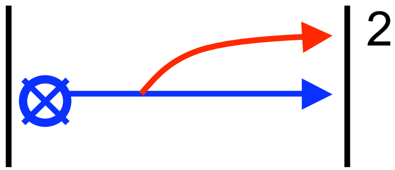
To understand in more detail the physics contained in

(6.1)
$$\frac{dI}{d\ln\omega\, dk_T} = \frac{\alpha_s C_R}{(2\pi)^2\omega^2} 2\mathrm{Re}\int_0^\infty dy \int_y^\infty d\bar{y} \int du\, e^{-ik_T u} e^{\left[-\int_y^\infty d\xi\, n(\xi) v(u)\right]}$$
$$\times \frac{\partial}{\partial u} \cdot \frac{\partial}{\partial s} K(s=0, y; u, y \mid \omega)$$

We expand this expression in **'opacity'** (=density of scattering centers times dipole cross section)

(6.2)
$$K(\underline{s}, y; \underline{u}, \bar{y}) = K_0(s; u) - \int d\underline{r}\, d\xi\, K_0(\underline{s}, y; \underline{r}, \xi)\, \boxed{n(\xi)\sigma(\underline{r})}\, K_0(\underline{r}, \xi; \underline{u}, \bar{y}) + \ldots.$$

To zeroth order, there is no medium (vaccum case), and one finds:

(6.3)
$$\omega \frac{dI^{(0)}}{d\omega\, dk_T} = \frac{\alpha_s C_F}{\pi^2} H(k_T) =$$

$$, \qquad H(k_T) = \frac{1}{k_T^2}$$

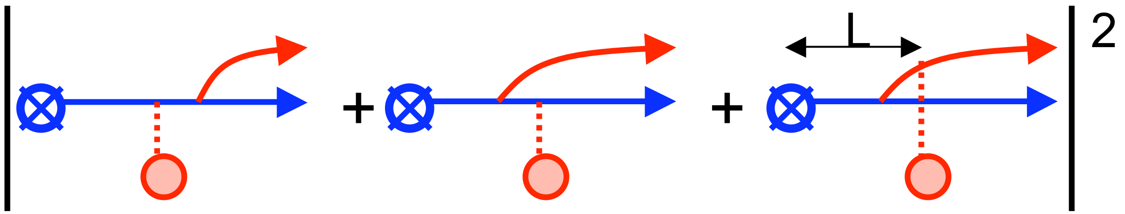
So, in the vacuum, the gluon energy distribution displays the dominant $1/\underline{k}^2$ piece of the DGLAP parton shower.

U.A.Wiedemann

VI.2. Opacity Expansion - up to 1st order

To first order in opacity, there is a generally complicate interference between vacuum radiation and medium-induced radiation.

(6.4) $$\omega \frac{dI^{(1)}}{d\omega\, dk_T} = \left| \quad + \quad + \quad \right|^2$$

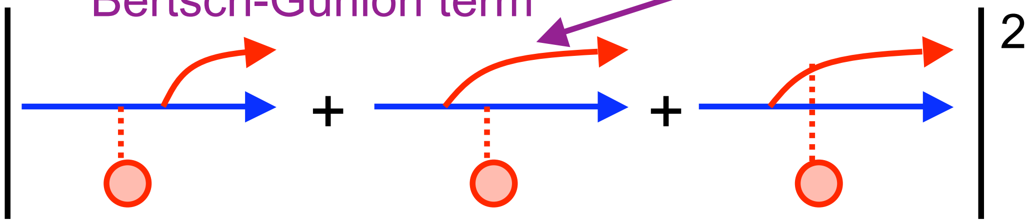


in the parton cascade limit $L \to \infty$, we identify three contributions:

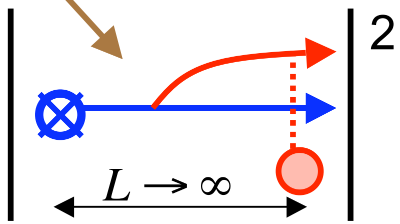
1. **Probability conservation** of medium-independent vacuum terms.
2. **Transverse phase space** redistribution of vacuum piece.
3. **Medium-induced gluon radiation** of quark coming from minus infinity

(6.5) $$\lim_{\substack{L\to\infty \\ nL=const}} \omega \frac{dI^{(1)}}{d\omega\, dk_T} = -w_1 H(k_T) + nL \int_{q_T} dq_T \left[R(q_T, k_T) + H(q_T + k_T) \right]$$

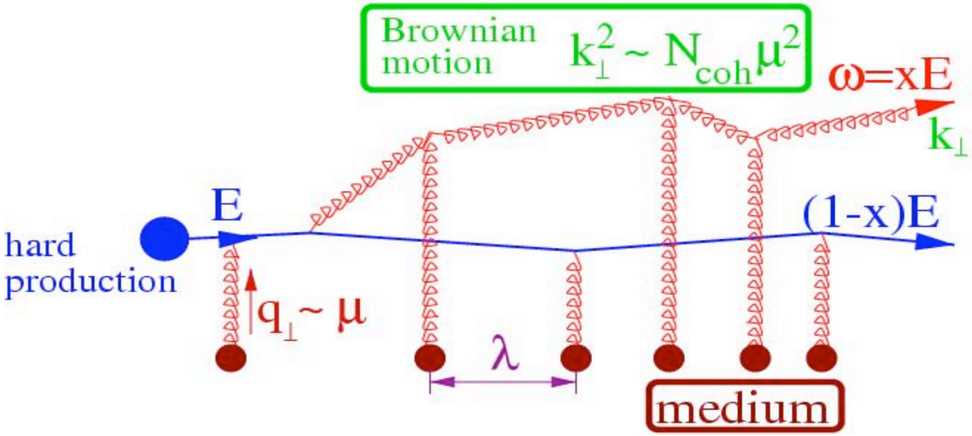
Bertsch-Gunion term



Rescattering of vacuum term



VI.3. Parton energy loss - what to expect?



Medium characterized by BDMPS transport coefficient:

$$\hat{q} \cong \frac{\mu^2}{\lambda}$$

- How much energy is lost?

(6.6) Phase accumulated in medium: $\left\langle k_T^2 \,\Delta z \big/ 2\omega \right\rangle \approx \frac{\hat{q}L^2}{2\omega} = \frac{\omega_c}{\omega}$ Characteristic gluon energy

(6.7) Number of coherent scatterings: $N_{coh} \approx \frac{t_{coh}}{\lambda}$, where $t_{coh} \approx \frac{2\omega}{k_T^2} \approx \sqrt{\omega/\hat{q}}$ ($k_T^2 \approx \hat{q}\, t_{coh}$)

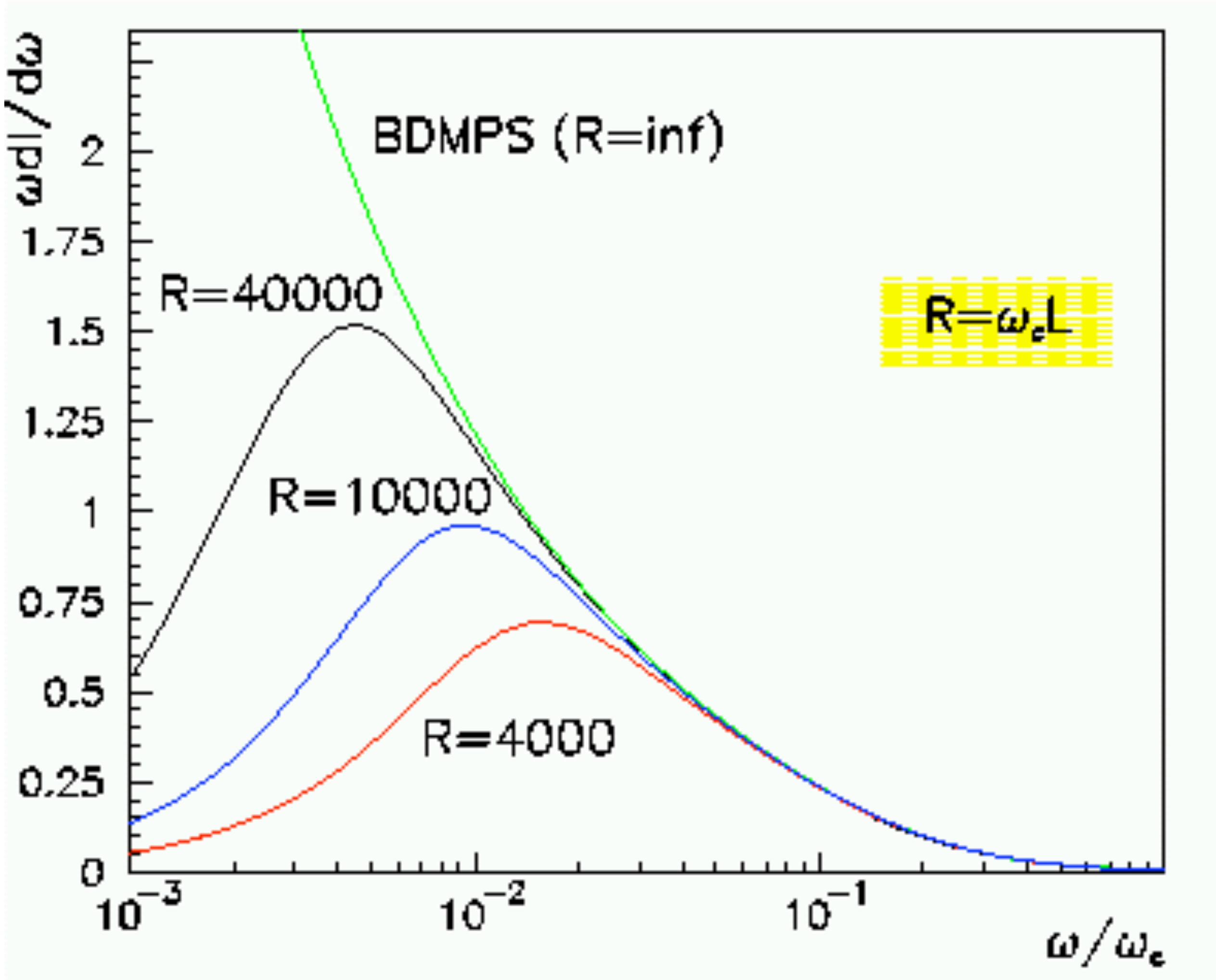
(6.8) Gluon energy distribution: $\omega \frac{dI_{med}}{d\omega\, dz} \approx \frac{1}{N_{coh}} \omega \frac{dI_1}{d\omega\, dz} \approx \alpha_s \sqrt{\frac{\hat{q}}{\omega}}$

(6.9) Average energy loss $\Delta E = \int_0^L dz \int_0^{\omega_c} d\omega\, \omega \frac{dI_{med}}{d\omega\, dz} \sim \alpha_s \omega_c \sim \alpha_s \hat{q} L^2$

Quadratic increase with L!

VI.4. Medium-induced gluon energy distribution

Consistent with estimate (3.6), spectrum is indeed determined by $\omega_c = \hat{q}L^2/2$

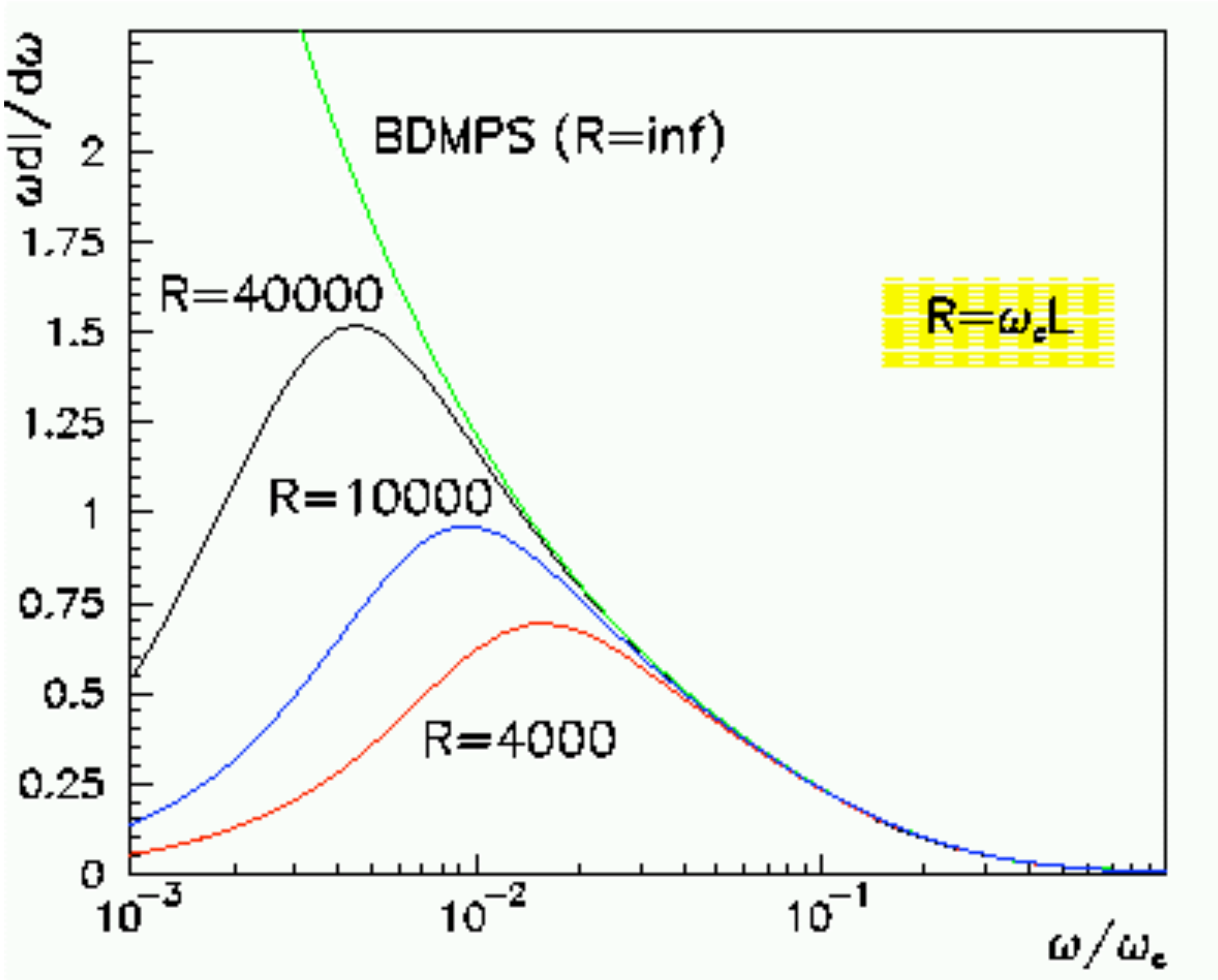


Salgado,Wiedemann PRD68:014008 (2003)

U.A.Wiedemann

VI.4. Medium-induced gluon energy distribution

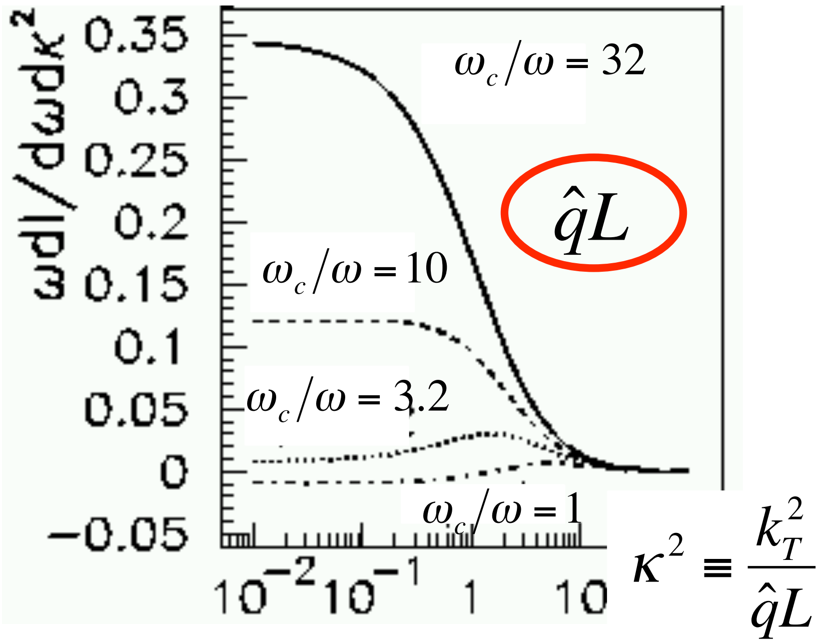
Consistent with estimate (3.6), spectrum is indeed determined by $\omega_c = \hat{q}L^2/2$



Salgado,Wiedemann PRD68:014008 (2003)

VI.5. Kt-distribution of medium-induced gluons

Follows transverse Brownian motion, consitent with (3.6).



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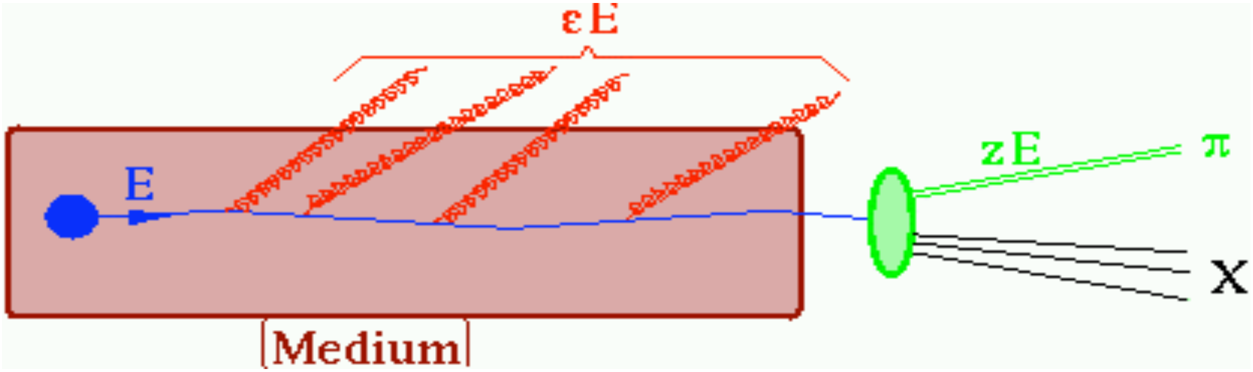
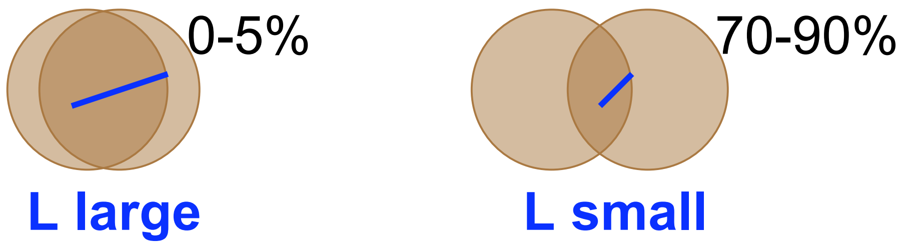
VI.11.Recall: High p_T Hadron Spectra



$$R_{AA}(p_T,\eta) = \frac{dN^{AA} / dp_T d\eta}{n_{coll}\, dN^{NN} / dp_T d\eta}$$

Centrality dependence
= dependence on in-medium path-length L



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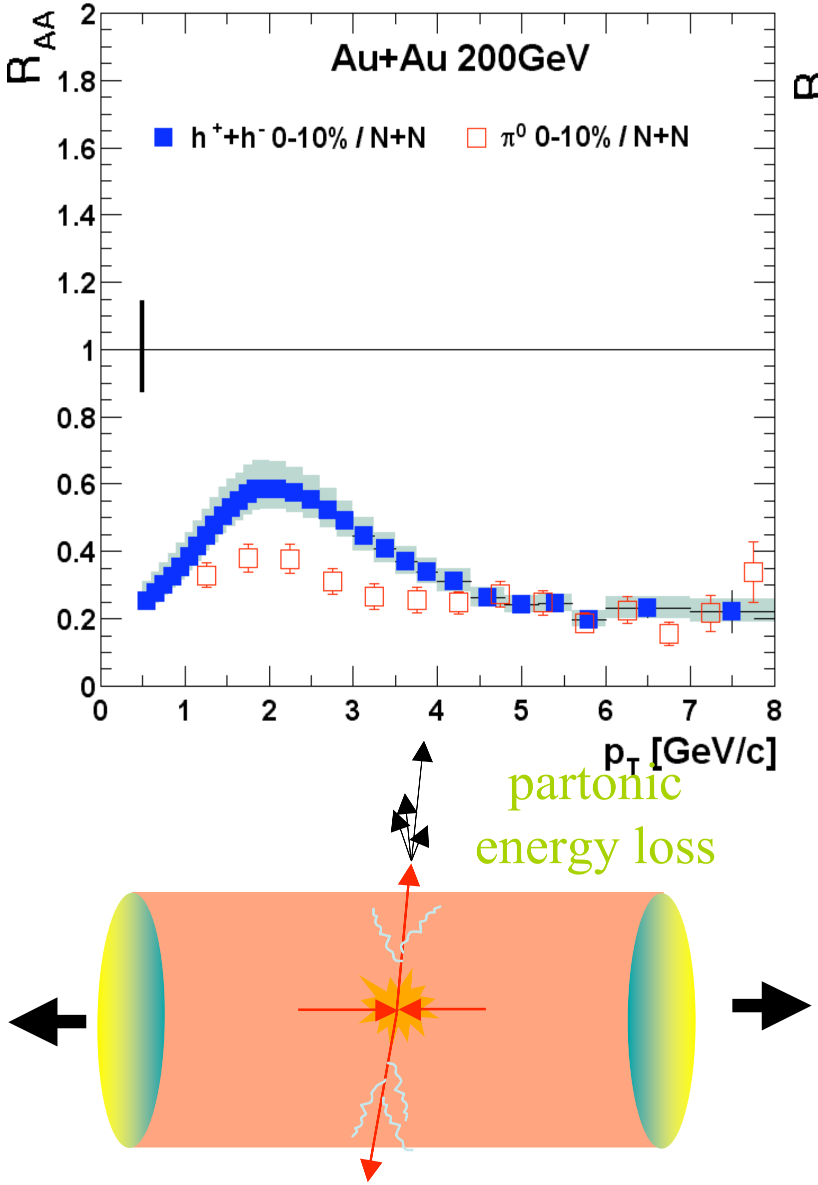
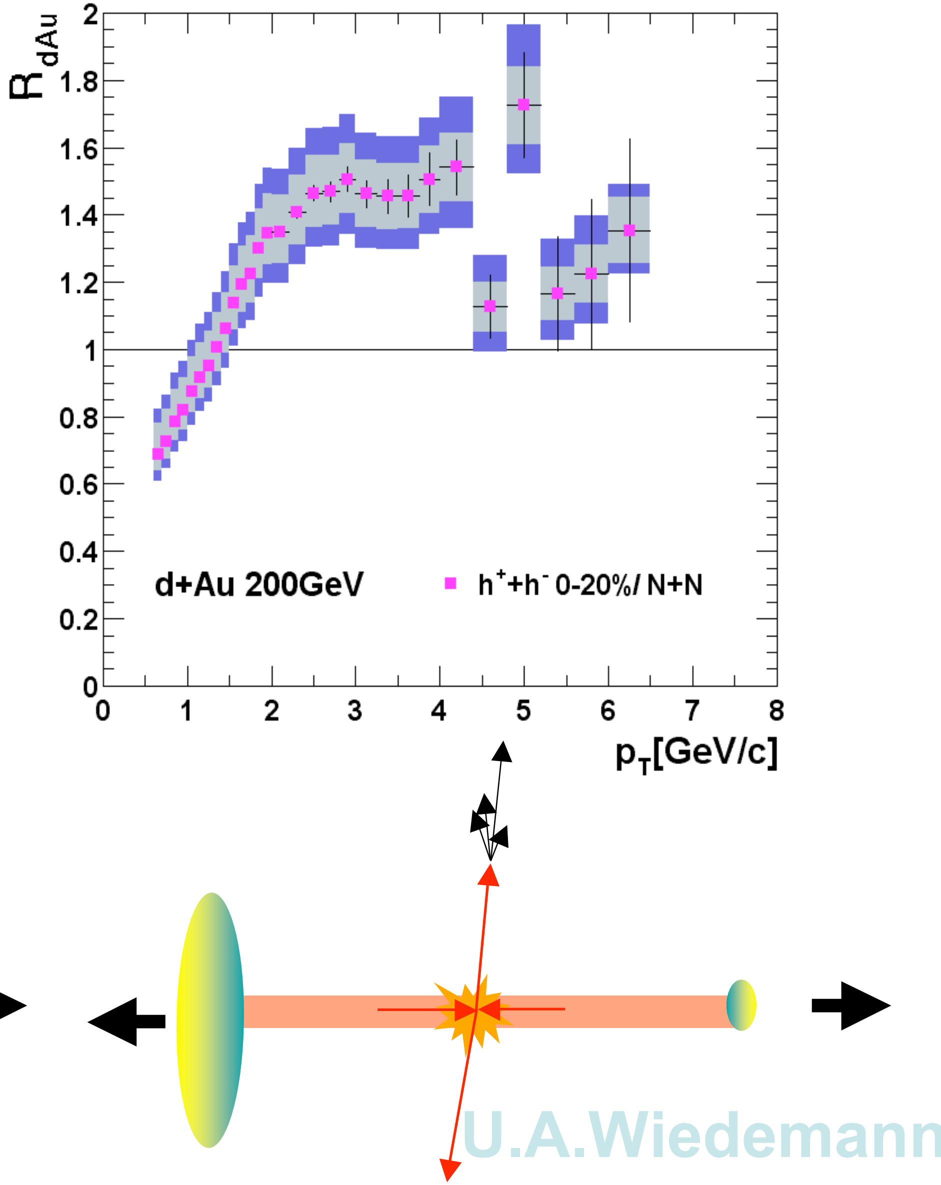
VI.12. Jet Quenching: Au+Au vs. d+Au

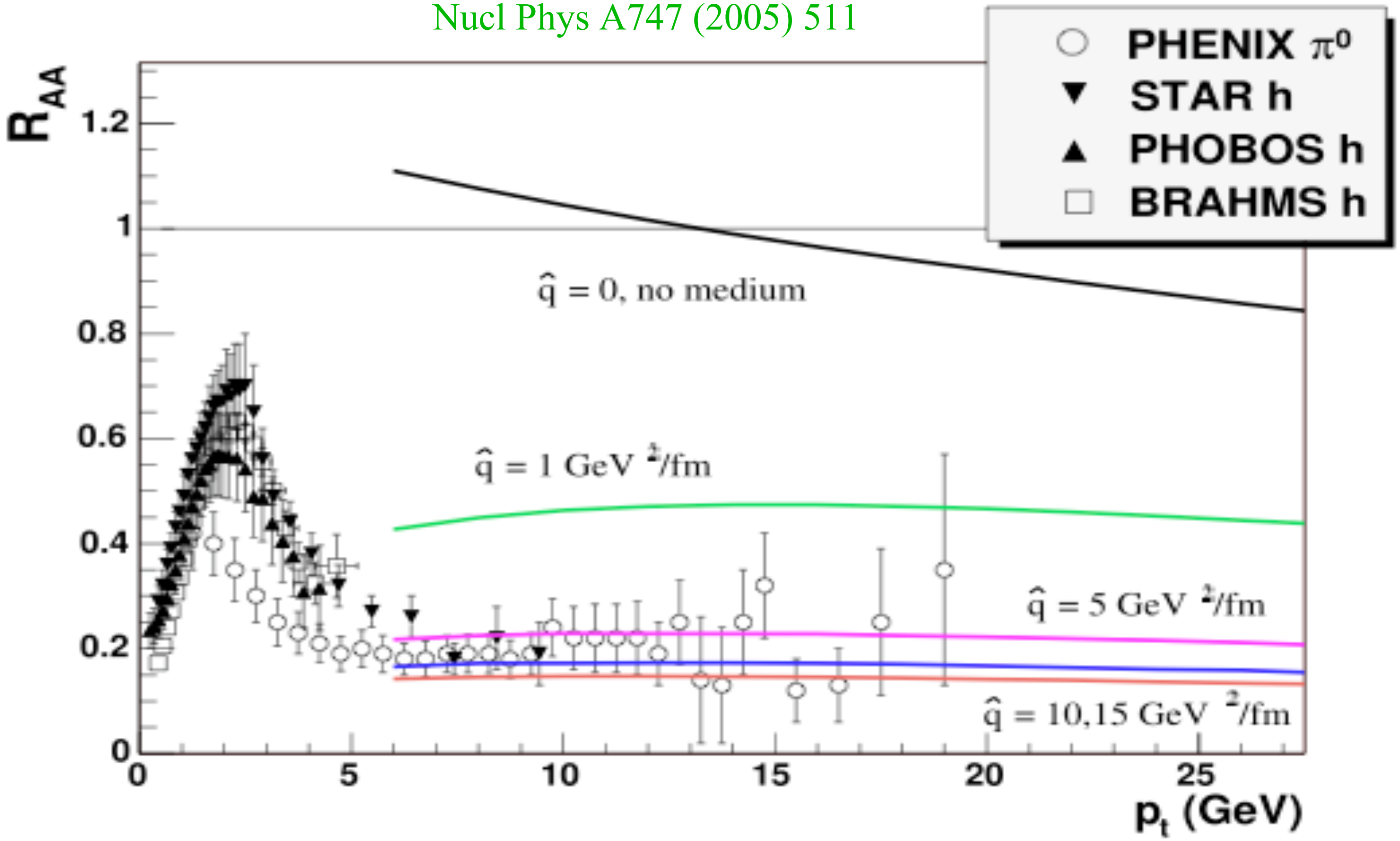
- Final state suppression
- Initial state enhancement





VI.14. Determining the quenching parameter

Eskola, Honkanen, Salgado, Wiedemann
Nucl Phys A747 (2005) 511



$$\bar{\hat{q}} = \frac{2}{L^2}\int_{\xi_0}^{L+\xi_0} (\xi - \xi_0)\,\hat{q}(\xi)\,d\xi = 5 - 15\,\frac{GeV^2}{fm}$$

U.A.Wiedemann

Non-perturbative calculation of qhat

- In QGP of QCD, parton energy loss described perturbatively up to non-perturbative quenching parameter.
- One can calculate quenching parameter in N=4 SYM (not necessarily a calculation of full energy loss of SYM), using AdS/CFT correspondence

$$\hat{q}_{SYM} = \frac{\pi^{3/2}\Gamma\left(\frac{3}{4}\right)}{\Gamma\left(\frac{5}{4}\right)}\sqrt{\lambda}T^3 \approx 26.68\sqrt{\alpha_{SYM}N_c}T^3$$

Liu, Rajagopal, UAW

- If we relate *N*=4 SYM to QCD by fixing $N_c = 3$ $\alpha_{SYM} = 1/2$

$\hat{q}_{SYM} = 4.4 \frac{GeV^2}{fm}$ for T = 300 MeV

$\hat{q}_{SYM} = 10.6 \frac{GeV^2}{fm}$ for T = 400 MeV

This is close to values from experimental fits.

Is this comparison meaningful?