ADS/CFT

DESY 09/07

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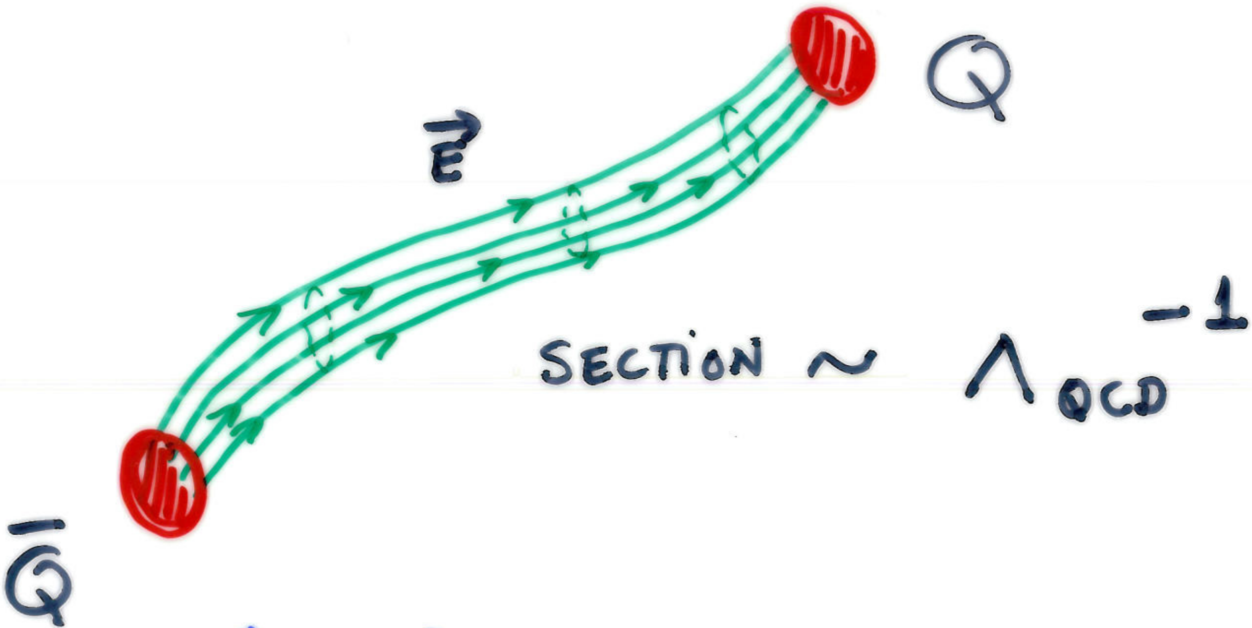
IFT MADRID

SUBTITLE:

AdS/CFT AND THE SEARCH FOR A THIN QCD STRING

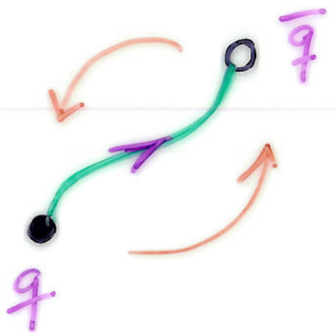
QCD STRING IS OFTEN A HEURISTIC PICTURE OF CONFINEMENT BY ELECTRIC FLUX TUBES



A "FAT", COMPLICATED STRING...

UNLIKE ABRIKOSOV VORTICES, A FULLY NON-PERTURBATIVE EFFECT IN QCD VARIABLES

STRING PICTURE OF DUAL MODELS OF HADRONS



REGGE TRAJECTORIES

$$\text{Spin} \sim \underbrace{\alpha'}_{\text{Regge slope}} M^2 + \text{constant}$$

$$\boxed{\text{Regge slope}} \sim (1\ \text{GeV})^{-2}$$

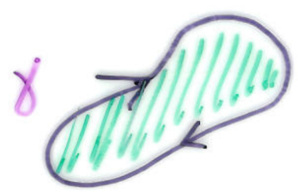
↳ CONFINING POTENTIAL (LINEAR)

$$V_{q\bar{q}}(r) \sim (\alpha')^{-2} \cdot r$$

↳ SMOKING GUN FOR STRINGS (= GLUE FLUX TUBES)

AREA LAW OF WILSON LOOP

$$\left\langle \mathcal{P} \exp i \oint_\gamma A_\mu dx^\mu \right\rangle \longrightarrow e^{-\sigma \,\text{Area}(\gamma)}$$



σ = STRING TENSION $\sim 1/\alpha'$

$$\sigma \sim \Lambda_{QCD}^2$$

* CAN WE "DERIVE" THE QCD STRING FROM FUNDAMENTAL GLUON DYNAMICS ?

XI.

STRONG COUPLING EXPANSION ON THE LATTICE

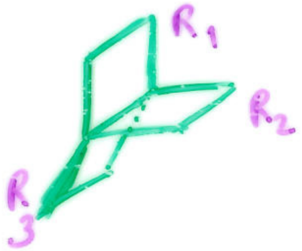
$$\int \mathcal{D}A\, e^{-\frac{1}{4g^2}\int \mathrm{Tr}\, F^2} \leftarrow \int \prod_{\to} dU_{\to}\, e^{-\frac{1}{g_0^2}\sum_{\square} \mathrm{Tr}\, U_\square}$$

$$\boxed{e^{-\frac{1}{g_0^2}\mathrm{Tr}\, U_\square} = \sum_R f_R\, \chi_R(U_\square)}$$

EXPAND IN POWERS OF $1/g_0^2$

↳ USE $\int dU\, \chi_R(U)\, \chi_{R'}(U) \sim \delta_{R,R'}$

↳ GET INCIDENCE RULES

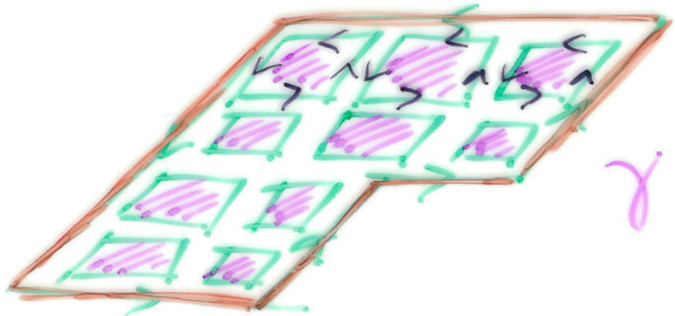


if $R_1 \otimes R_2 = \cdots \oplus R_3 \oplus \cdots$

↳ INTERSECTION RULES FOR $\boxed{\text{RANDOM SURFACES}}$

* WILSON LOOP SATURATED BY PLAQUETTE-TESELATED SURF.

$$\left\langle \mathrm{Tr} \prod_{\ell\in\gamma} U_\ell \right\rangle \sim$$

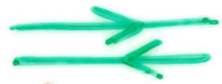


$1/N$ EXPANSION

$\frac{1}{\mathrm{rank}\,(G)}$ IS THE ONLY UNIFORM EXPANSION PARAMETER IN YM THEORY

$SU(N\to\infty)$ WITH $g^2N \equiv \lambda$ = FIXED

TOPOLOGICAL CLASSIFICATION OF DIAGRAMS

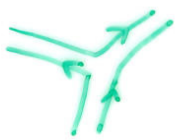
 $\sim$ Adjoint $= N \otimes \bar{N}$ $\sim$ 

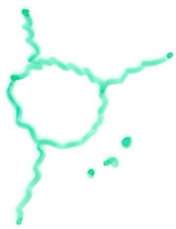
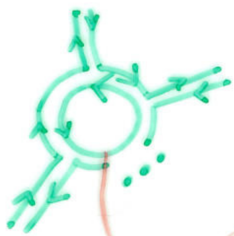
A RIBBON SENSITIVE TO TWIST 

$\boxed{\text{DIAGRAMS TESSELATE SURFACES}}$

NORMALIZE $\quad \mathcal{L} = \frac{N}{g^2N}\,\mathrm{Tr}\,F^2 = \frac{N}{\lambda}\,\mathrm{Tr}\,F^2$

PROPAGATOR  $\sim \frac{1}{N}$

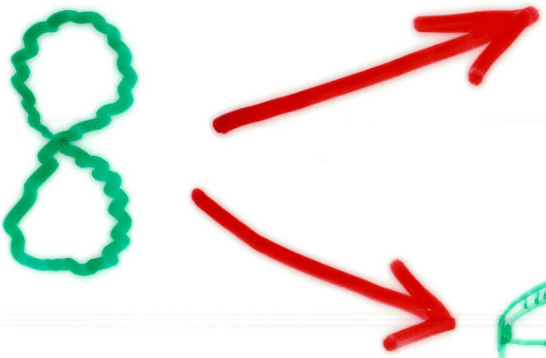
VERTEX  $\sim N$

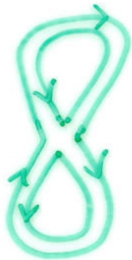
INDEX LOOP  $\to$  N

OVERALL POWER OF N

$$N^{V-I+L} \overset{\text{EULER}}{=\!=} N^{2-2h}$$

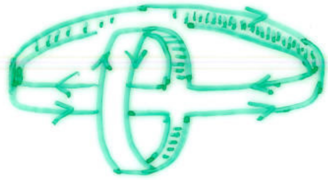
h = # OF HANDLES OF THE TESELATED SURFACE





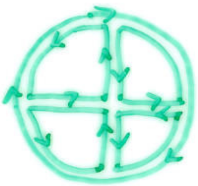
$$\sim g^2 N^3 = \lambda N^2 = O(N^2)$$

(PLANAR)



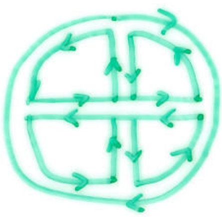
$$\sim g^2 N = \lambda = O(N^0)$$

(TOROIDAL)



$$\sim g^2 \cdot g^2 \cdot g^2 \cdot N^5 = \lambda^3 N^2 = O(N^2)$$

(PLANAR)



$$\sim g^4 \cdot N^2 = \lambda^2 = O(N^0)$$

(TOROIDAL)

$$\boxed{T_{eff} = \sum_{h \geq 0} N^{2-2h} \, T_h(\lambda)}$$

* CORRELATION FUNCTIONS

$$G = \operatorname{Tr} F^n$$

ADD SOURCES TO LAGRANGIAN

$$\mathcal{L} \to \mathcal{L} + J_G \cdot N \operatorname{Tr} F^n$$

$$\left\langle G_1 G_2 \cdots G_n \right\rangle_{\text{connected}} = \frac{1}{N^n} \frac{\partial^n}{\partial J_{G_1} \cdots \partial J_{G_n}} \underbrace{\log Z[J]}_{O(N^2)}$$

$$= \boxed{N^{2-n} \left(1 + O(1/N^2) \right)}$$

$$\langle G G \rangle \sim O(1)$$

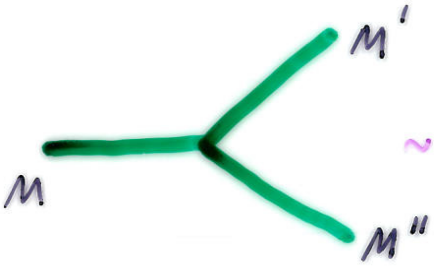
$$\langle G G G \rangle \sim O\left(\frac{1}{N}\right)$$

HOW SMALL is $1/3$??

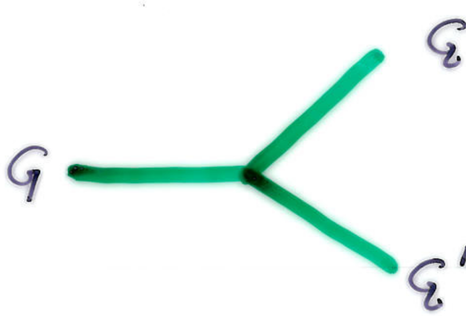
* $1/N$ POWER COUNTING GIVES A GOOD CARICATURE OF THE HADRONIC WORLD

(ZWEIG RULE etc)

$$M \text{ ——— } M' \sim O(1) \sim G \text{ ——— } G'$$



$\sim O\left(\frac{1}{\sqrt{N}}\right)$



$\sim O\left(\frac{1}{N}\right)$

* FACTORIZATION OF GAUGE INVARIANT CORRELATORS

$$\langle \mathrm{Tr}\, \mathcal{O} \; \mathrm{Tr}\, \mathcal{O}' \rangle \rightarrow \langle \mathrm{Tr}\, \mathcal{O} \rangle \langle \mathrm{Tr}\, \mathcal{O}' \rangle + O(1/N^2)$$

↳ A CLASSICAL LIMIT AT $N = \infty$?

↳ PATH INTEGRAL SATURATED BY A MASTER FIELD ?

$S \sim O(N^2)$ # VARIABLES $\sim O(N^2)$

WE MUST FIRST PROJECT ON THE GAUGE INVARIANT DEGREES OF FREEDOM

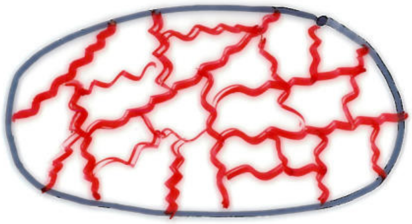
(THE $O(N)$ EINGENVALUES)

$$\mathrm{Tr}\, F, \; \mathrm{Tr}\, F^2, \; \ldots, \; \mathrm{Tr}\, F^N$$

'T HOOFT'S PROPHECY:

NONPERTURBATIVE (CONFINEMENT?) EFFECTS FILL THE HOLES AND WE GET WORLDSHEETS

$$\left\langle \frac{1}{N} \mathrm{Tr}\, \mathcal{P}\, e^{i\oint A} \right\rangle \equiv \sum_{\text{diagr.}}$$



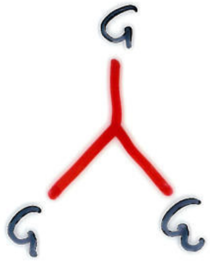
$$\stackrel{?}{=} \sum_{\text{worldsheets}} e^{-\sigma\, \text{Area}} \sim$$

$+$

$+ \ldots$

$$\boxed{g_s \sim}$$

$\sim$

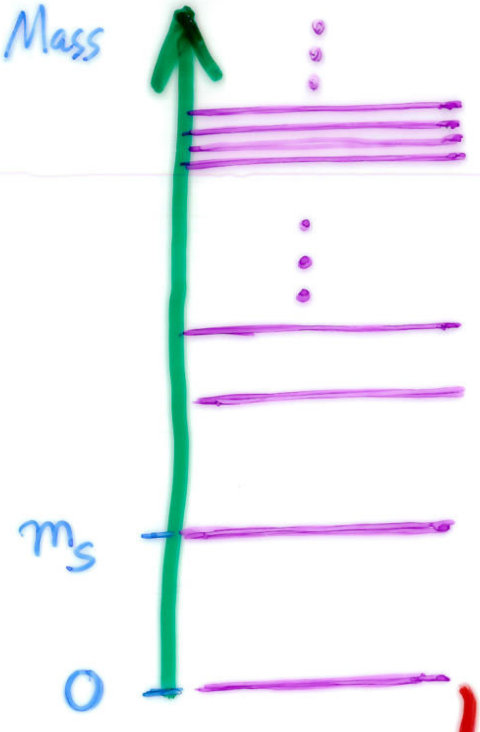


$$\sim \frac{1}{N}$$

* WHAT ABOUT THE "THICKNESS" QUESTION?

* CHECK OUT THIN (CRITICAL) STRINGS... (Polyakov's paradigm...)

UNIVERSALITY OF STRING SPECTRUM



DENSE "HAGEDORN" SPECTRUM

$$\rho(M) \sim M^a \exp\left(\frac{cM}{m_s}\right)$$

MASS GAP $\sim m_s \sim \frac{1}{\ell_s} \sim \frac{1}{\sqrt{\alpha'}}$

MASSLESS SPECTRUM (SHORT STRINGS)

Open: a thin ribbon

$\lim_{E m_s \to 0}$  $= \langle A_\mu(p) A_\nu(-p) \rangle$

Yang Mills fields.

closed: a thin tube

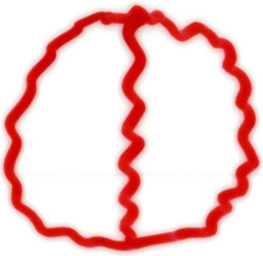
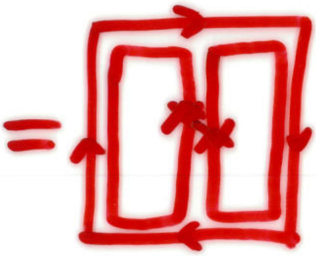
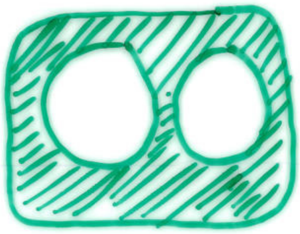
$\lim_{E m_s \to 0}$  $= \langle g_{\mu\nu}(p)\, g_{\rho\sigma}(-p) \rangle$

Gravity

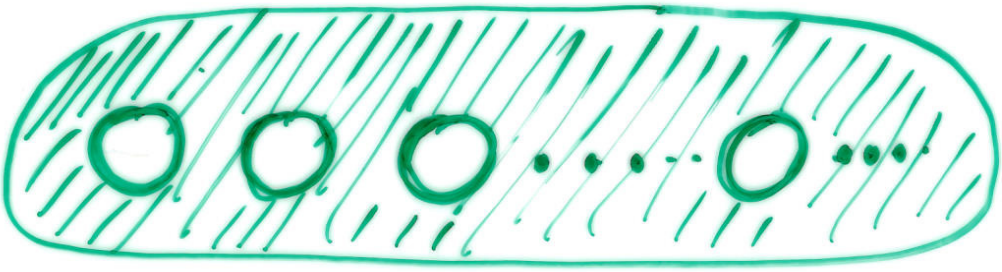
NOT GOOD...

STILL . . .

ONE CAN USE CRITICAL $\boxed{\text{OPEN}}$ STRINGS AS A REGULARIZATION OF YANG-MILLS THEORY.

 =  $= \lim\limits_{E\, m_s \to 0}$ 

ANY PLANAR GLUON DIAGRAM FOLLOWS FROM A LOW ENERGY LIMIT OF



PERHAPS SUMMING HOLES IN OPEN STRING WORLDSHEETS IS EASIER !!

CAN WE USE FUNDAMENTAL STRINGS TO SUM UP PLANAR DIAGRAMS ?

OPEN STRING WITH QUARK / ANTIQUARK QUANTUM NUMBERS ON ENDPOINTS

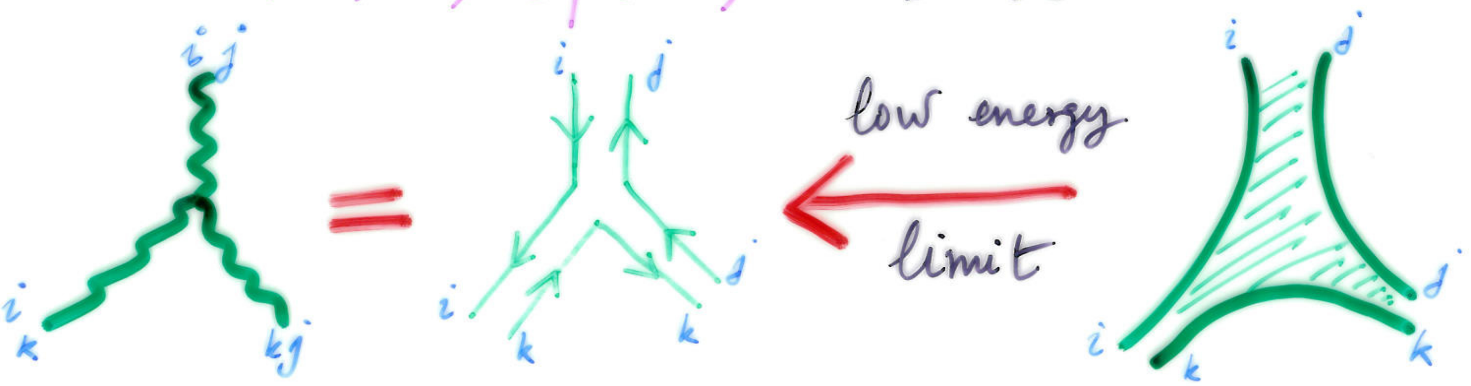
CHAN - PATON FACTORS

$$\boxed{\left| \, {}_{i}\!\longrightarrow{}^{j} \right\rangle \rightarrow U_{ii'} \, U^{*}_{jj'} \left| \, {}_{i'}\!\longrightarrow{}^{j'} \right\rangle}$$

Gauge transformation

CARRY THE $N \otimes \bar{N}$ OF $U(N)$

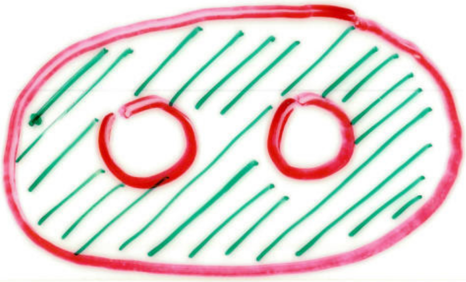
GIVING UP ORIENTABILITY, CAN CONSTRUCT $SO(N)$, $Sp(N)$ AS WELL



WHY A YANG-MILLS / GRAVITY DUALITY AT LARGE N ?

$\xrightarrow{E \ll m_s}$ $A^{\mu}{}_{ij}$

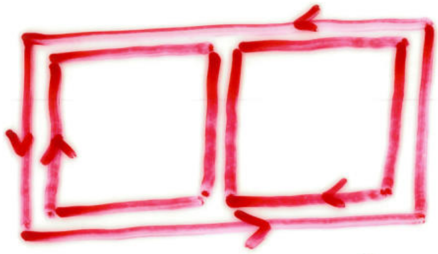
$\xrightarrow{E \ll m_s}$ $g_{\mu\nu}$



$\frac{1}{g_s^2} \times (g_s N)^3$

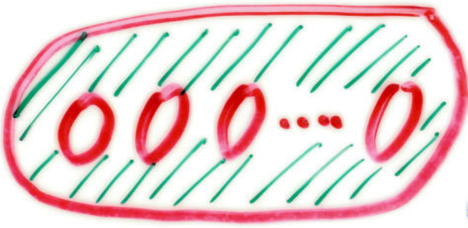
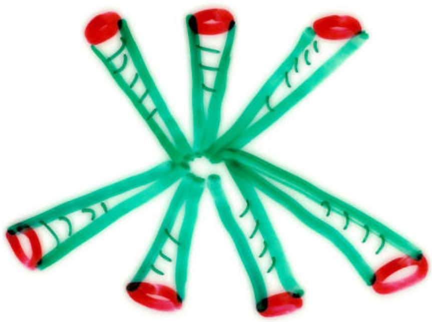
$\xrightarrow{\langle E_{open} \rangle \ll m_s}$

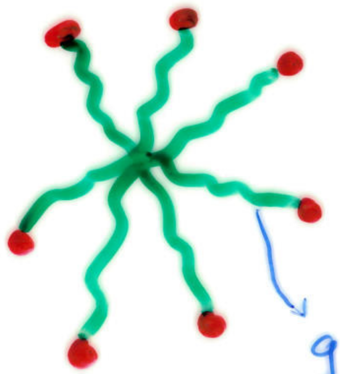
$g_s \sim g_{YM}^2$



$g_{YM}^2 \cdot N^3 = N^2 \cdot \lambda$

CAN WE SUM UP PLANAR DIAGRAMS ?

$\sum_{holes}$  $\xrightarrow{\langle E_{closed} \rangle \ll m_s}$ $\sum$ 

$\rightarrow \sum_{legs}$ 

$g_{\mu\nu}$

$\rightarrow$ $S_{eff} = \int \sqrt{-g}\, R + \dots$

* NORMALLY, RESULTING GRAVITY PICTURE HAS NOTHING TO DO WITH THE GAUGE DYNAMICS i.e. THE TWO LOW ENERGY LIMITS

$$\boxed{\begin{array}{l} \langle E_{open} \rangle \ell_s \ll 1 \\ \langle E_{closed} \rangle \ell_s \ll 1 \end{array}}$$

DO NOT OVERLAP

* THE GOOD NEWS:

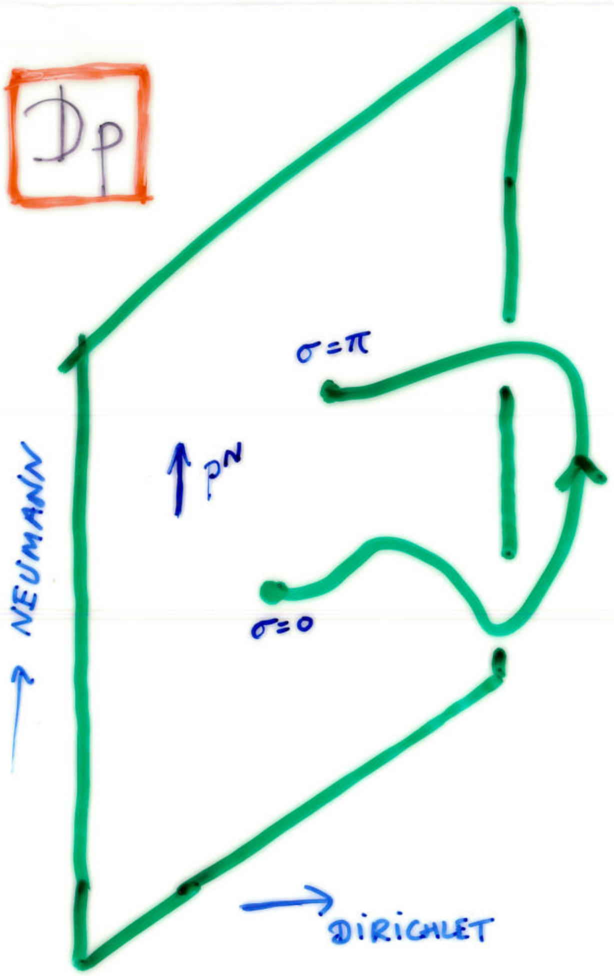
OPEN STRINGS ON D-BRANES BYPASS THIS PROBLEM.

ONE CAN DEFINE A DYNAMICAL REGIME WHERE BOTH LOW ENERGY LIMITS ARE COMPATIBLE

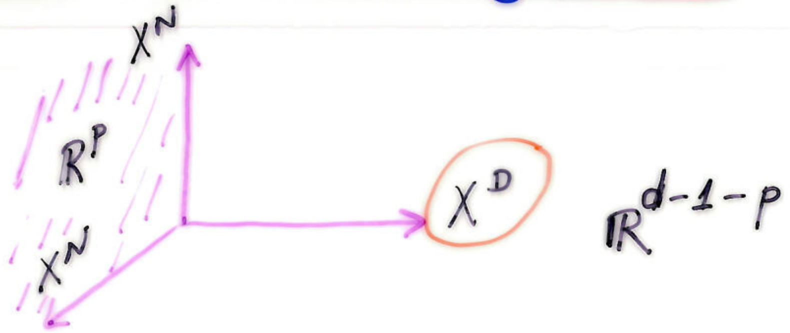
D-BRANES

* HYPERPLANES OF NEUMANN BOUNDARY CONDITIONS WHERE OPEN STRINGS END

$\boxed{Dp}$



$X^D =$ constant



OPEN STRINGS ARE CONFINED TO D-BRANE WITHIN $O(l_s)$ DISTANCE

$\sim l_s \sim \sqrt{\alpha'}$

CLOSED STRINGS FLY AROUND

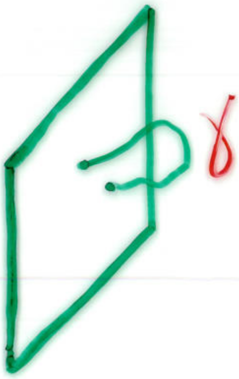
* LORENTZ GROUP

$$\boxed{SO(1,d-1) \to SO(1,p) \times SO(d-1-p)}$$

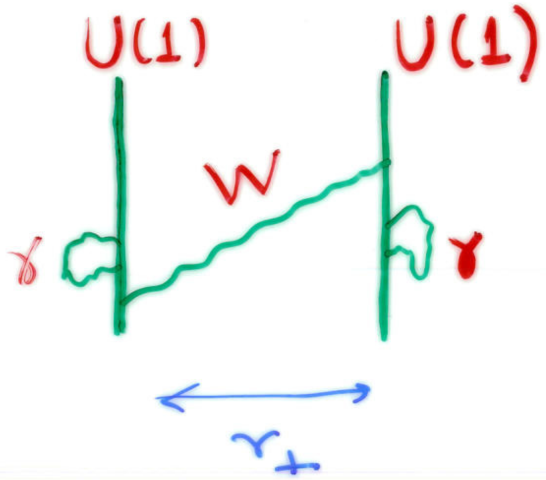
AMBIENT LORENTZ

Dp-BRANE LORENTZ

TRANSVERSE ROTATIONS "INTERNAL SYM"



U(1) photon on world-volume



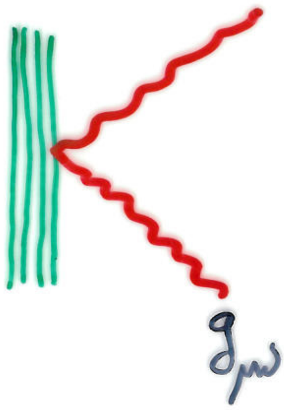
$$M_W \sim r_\perp / \alpha'$$

DESCRIBES THE HIGGSING $U(2) \to U(1) \times U(1)$ BY ADJOINT HIGGSES $\langle \Phi \rangle \sim r_\perp / \alpha'$

* N D-BRANE STACK $\rightarrow$ $U(N)_{YM}$

$$\boxed{g_s = g_{YM}^2}$$

* GRAVITY SOURCES OF TENSION $T \sim N / g_s$



$$\boxed{R^4_{grav} \sim N g_s \ell_s^4}$$

ADS / CFT

AT LOW ENERGIES ON DP-BRANE

$$\mathcal{L}_{eff} = \frac{1}{4g^2} \mathrm{Tr}\left[F^2 + 2|D\Phi|^2 + \bar{\Psi} \not{D} \Psi + \bar{\Psi} \Gamma \cdot \Phi \Psi + \sum_{I,J} [\Phi^I, \Phi^J]^2 \right]$$

$\hookrightarrow$ $\mathcal{N}=4$ SYM_{p+1} $\quad g^2 \sim g_s \quad + O(\alpha'^2 F^4)$

$\boxed{\text{FOR } p=3 \text{ WE GET A VERY SYMMETRIC THEORY}}$

$$\underbrace{SO(1,3) \times (16 \text{ SUSY'S}) \times SO(6)_R}$$

$$\Downarrow$$

$$\underbrace{SO(2,4)_{\text{conformal}} \times (32 \text{ SUSY'S}) \times SO(6)_R}_{\text{UNIQUE SUPERCONFORMAL GROUP}}$$

VANISHING BETA FUNCTION

$$\frac{d\, g_{ym}(\mu)}{d \log \mu} = 0$$

S-DUALITY $\quad \tau = \frac{4\pi i}{g^2} + \frac{\theta}{2\pi} \to -\frac{1}{\tau}$

- VERY CONSTRAINED THEORY!

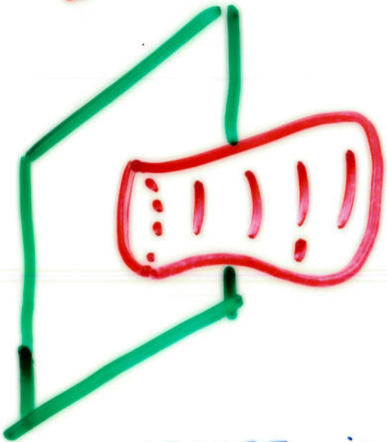
WHAT ABOUT STRING EXCITED STATES ?

IF INTEGRATED OUT AT ENERGIES $\ll m_s$

THEY CONTRIBUTE HIGH DIMENSION OPERATORS

SUPPRESSED BY POWERS OF $\alpha' = 1/m_s^2$

LEADING EFFECTIVE ACTION IS OBTAINED FROM LOWEST ORDER OPEN-STRING DIAGRAM : (DISK)



$$\sim \; g_s^{\text{Euler character}} = g_s^{-1}$$

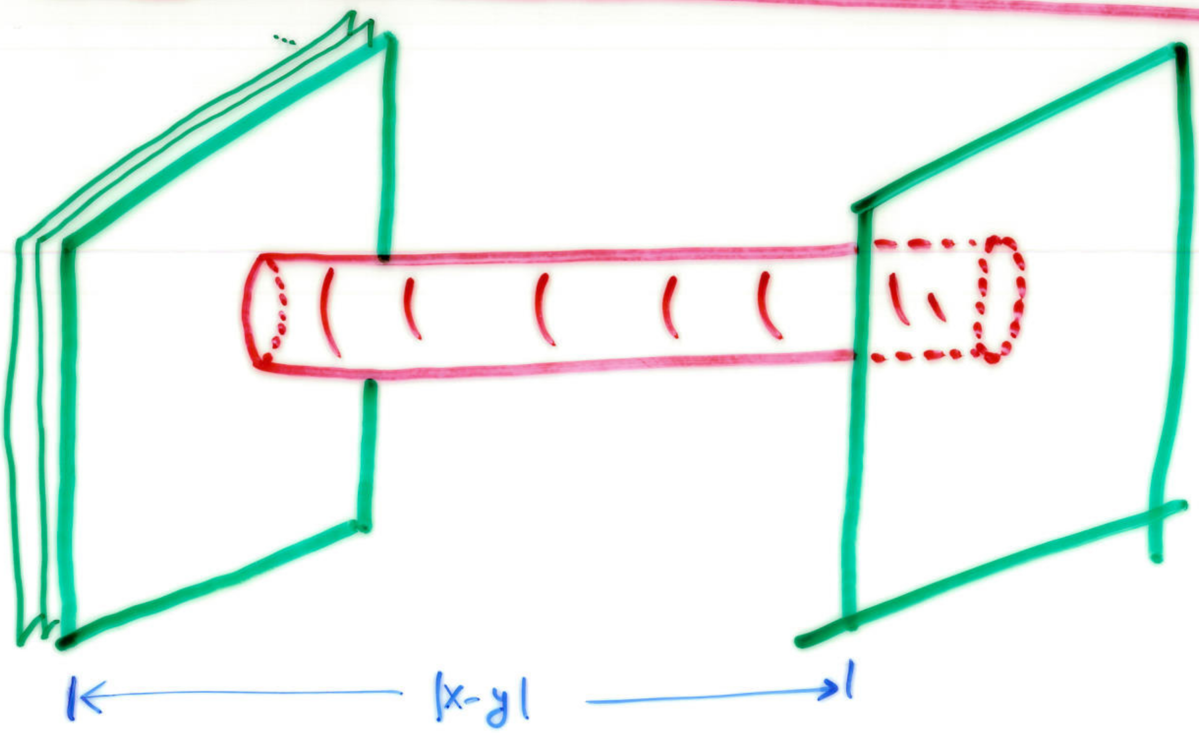
THERE IS A RESUMMATION FOR SLOWLY-VARYING FIELDS : (Dirac-Born-Infeld action)

$$\mathcal{L}_{DBI} \sim \frac{1}{g_s}\sqrt{\det\left(1 + 2\pi\alpha' F\right)} \sim$$

$$\sim 1/g_s + \frac{1}{g_{YM}^2} F^2 + O(\alpha'^4 F^4)$$

$$\boxed{g_{YM}^2 = g_s \cdot (\alpha')^{\frac{p+1}{2}}}$$

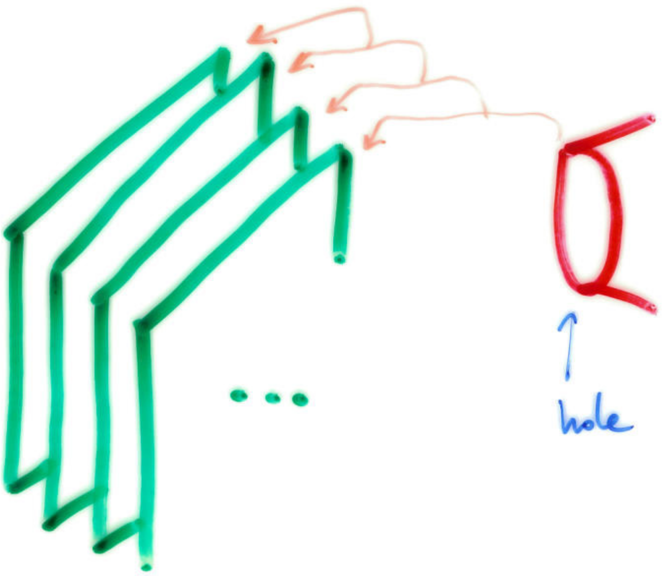
BASIC OPEN/CLOSED STRING DUALITY



$$\langle x | \frac{1}{\Delta_{closed}} | y \rangle = \frac{1}{2} \operatorname{Tr} \log \Delta_{open}$$

IF $\boxed{g_s N = g_{YM}^2 N = \lambda < 1}$

USE THE OPEN STRING FOR CALCULATIONS

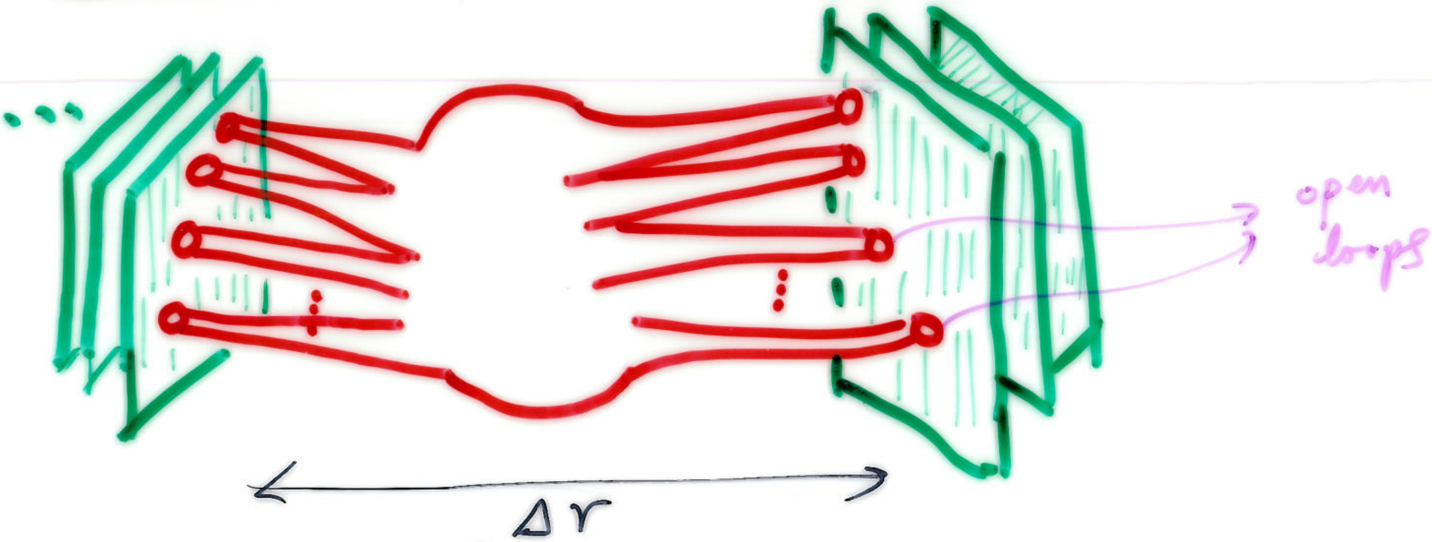


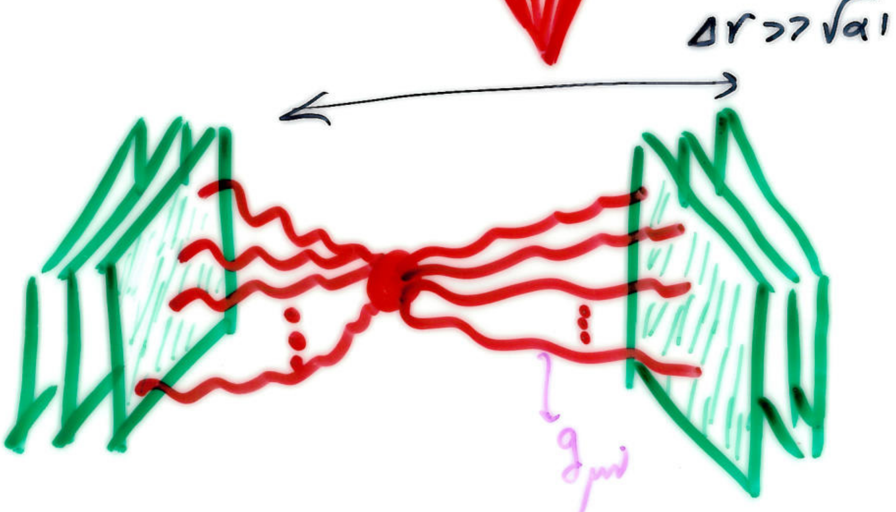
EFFECTIVE EXPANSION PARAMETER

$$\sum_1^N g_s = g_s N = \lambda$$

$\boxed{\text{WHAT IF } \lambda > 1 \text{ ?}}$

If $\Delta r \gg \sqrt{\alpha'} \sim l_s$ graviton exchange is good approximation. We sum-up holes in diagrams (open loops) by gravity nonlinear corrections





CAN WE CONTINUE THE GRAVITY DESCRIPTION BACK INTO THE GAUGE THEORY REGIME ($\Delta r \ll \sqrt{\alpha'}$) WITHOUT LOOSING CONTROL ?

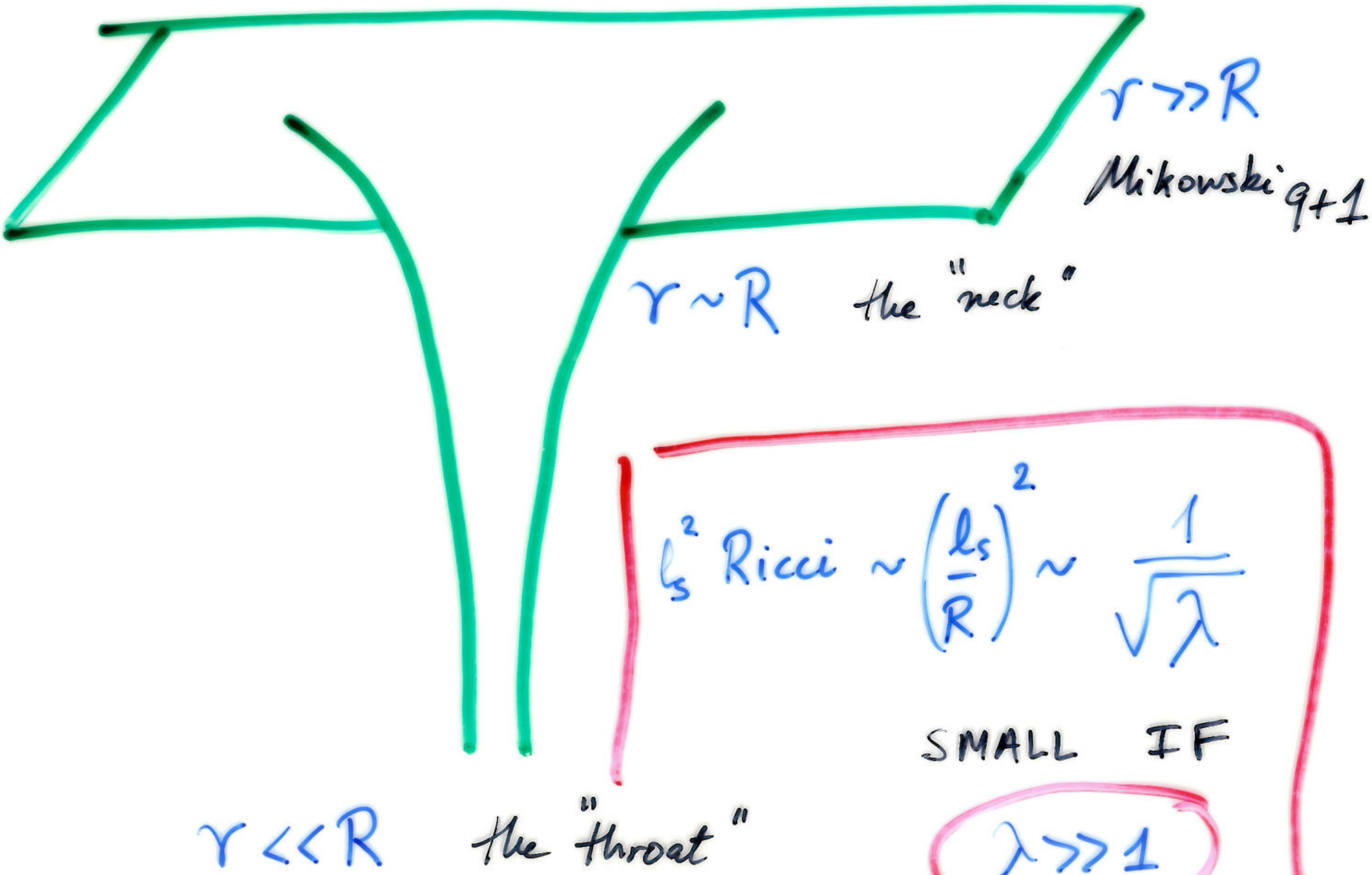
YES! IF WE KEEP $\boxed{\lambda N \gg 1}$

SUPERGRAVITY PICTURE

FOR N COINCIDENT D3-BRANES:

$$ds^2 = \frac{1}{\sqrt{H}} dx_{\parallel}^2 + \sqrt{H}\, dx_{\perp}^2$$

$$H = 1 + (R/r)^4 \qquad R = l_s (g_s N)^{1/4} = l_s \lambda^{1/4}$$



$$l_s^2 \,\mathrm{Ricci} \sim \left(\frac{l_s}{R}\right)^2 \sim \frac{1}{\sqrt{\lambda}}$$

SMALL IF

$\lambda \gg 1$

MALDACENA CONJECTURE

$\mathcal{N}=4$ SYM$_{3+1}$ AT $E \ll m_s$ $\lambda \gg 1$

IS DESCRIBED BY IIB SUGRA IN THE THROAT

$$ds^2_{\text{throat}} = \underbrace{\frac{r^2}{R^2}dx_{\parallel}^2 + \frac{R^2}{r^2}dr^2}_{(AdS_5)_R} + \underbrace{R^2 d\Omega_5^2}_{(S^5)_R}$$

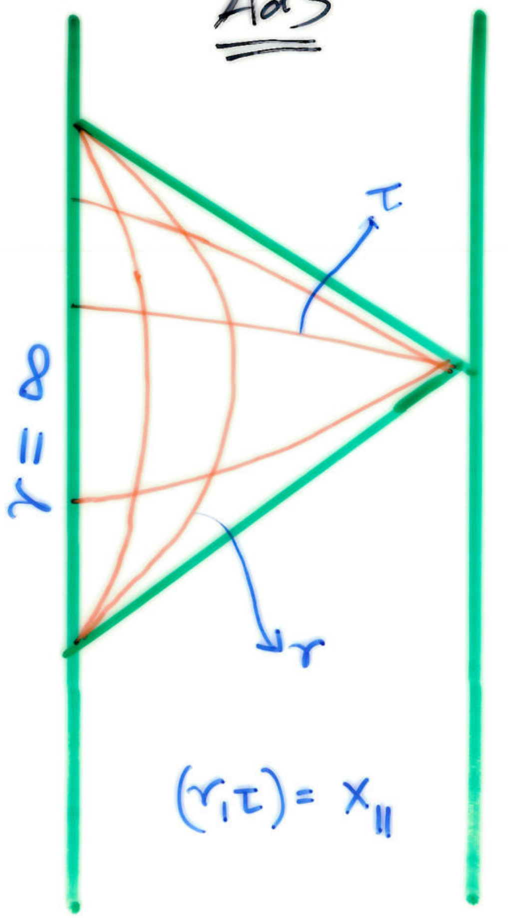
$$(AdS_5)_R \times (S^5)_R$$

Isometries: $SO(4,2) \times SO(6)$

$SO(4,2) \to$ (SUPER) CONFORMAL GROUP

$SO(6) \to$ R-SYMMETRY GROUP

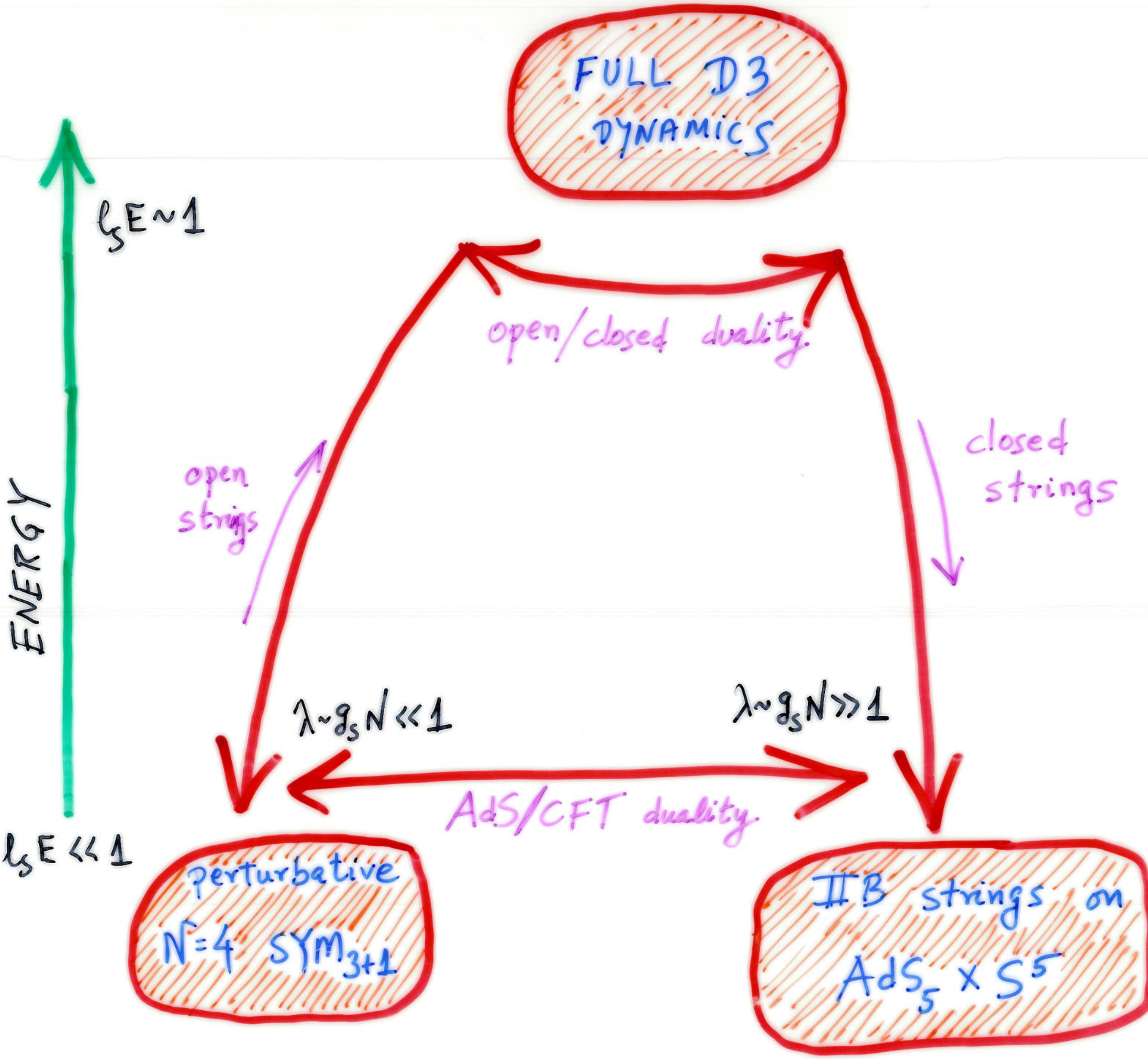
AdS



NEED BOUNDARY CONDITIONS AT $r=\infty$

$(AdS_5)_{r=\infty}$ is CONFORMAL TO M_4

THE DUALITY



FULL D3 DYNAMICS
$l_s E \sim 1$
open/closed duality.
closed strings
open strings
ENERGY
$\lambda \sim g_s N \ll 1$
$\lambda \sim g_s N \gg 1$
AdS/CFT duality.
$l_s E \ll 1$
perturbative
$\mathcal{N}=4$ SYM_{3+1}
IIB strings on
$AdS_5 \times S^5$

* AT WEAK COUPLING $\lambda \sim g_s N \ll 1$

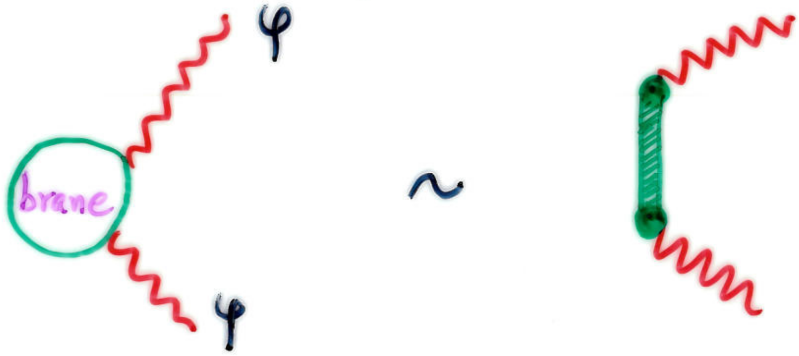
GRAVITATIONAL FIELDS ARE SOURCES FOR CFT OPERATORS

$$\mathcal{L}_{CFT} \sim \frac{1}{g^2} \mathrm{Tr}\, F^2 \sim \frac{1}{g_s} \mathrm{Tr}\, F^2 \sim e^{-\varphi}\, \mathrm{Tr}\, F^2$$

$$\hookrightarrow \frac{\delta}{\delta \varphi} \langle \cdots \rangle_{CFT} \propto \left\langle \frac{1}{g^2} \mathrm{Tr}\, F^2 \cdots \right\rangle_{CFT}$$

i.e. THERE IS A DILATON / "GLUEBALL" COUPLING LOCALIZED ON BRANE

DILATON SCATTERING OFF BRANE

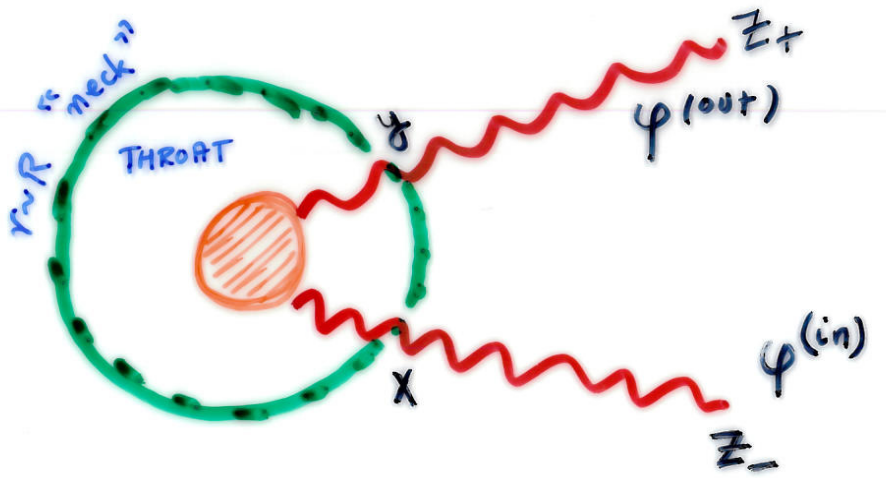


AFTER LSZ EXTERNAL PROPAGATOR AMPUTATION :

$$\text{Amplitude} \sim \text{Fourier}\left\{ \langle \mathrm{Tr}\, F^2(x)\, \mathrm{Tr}\, F^2(y) \rangle_{CFT} \right\}$$

THIS GENERALIZES ...

AT $\lambda = g_s N \gg 1$ BOUNDARY FIELD VALUES "AT THE NECK" ARE SOURCES FOR INNER THROAT SCATTERING



$$\sim \int_x \int_y \langle z_- | \frac{1}{D^2_{flat}} | x \rangle \langle x | \frac{1}{D^2_{throat}} | y \rangle \langle y | \frac{1}{D^2_{flat}} | z_+ \rangle$$

AMPUTATING FLAT-SPACE PROPAGATORS, THE SCATTERING OFF THE THROAT IS

$$\text{Amplitude} \underset{LSZ}{\sim} \text{Fourier}\left(\langle x | \frac{1}{D^2_{throat}} | y \rangle \right)$$

AT NECK

* IN DECOUPLING LIMIT THE NECK RECEDES TO INFINITY (BOUNDARY OF AdS) AND WE GET THE FOLLOWING IDENTIFICATION (**GKPW** PRESCRIPTION)

$$\boxed{\langle \mathrm{Tr}\, F^2(x)\, \mathrm{Tr}\, F^2(y) \rangle_{CFT} \sim \langle x | \frac{1}{D^2_{throat}} | y \rangle}$$

ON ∂AdS

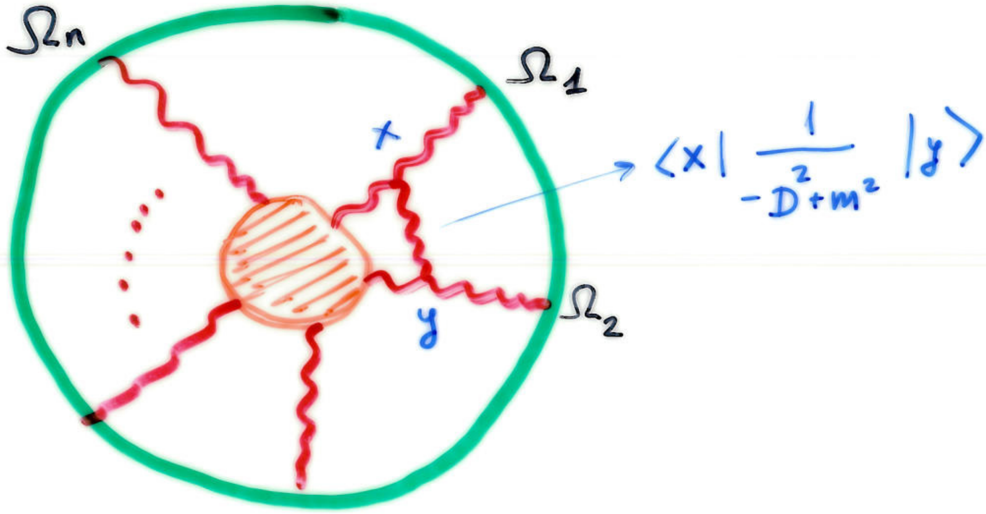
GKP/W OR AdS/CFT PROPER

SUGRA FIELDS ARE SOURCES FOR CFT OPERATORS

$$\left\langle e^{-\int_{\partial(AdS)} \varphi_{\mathcal{O}} \cdot \mathcal{O}} \right\rangle_{CFT} = e^{-I^{SUGRA}_{eff}\left[\varphi|_{\partial AdS} \to \varphi_{\mathcal{O}}\right]}$$

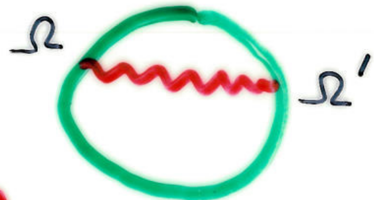
$$= \left\langle \mathcal{O}(\Omega_1)\, \mathcal{O}(\Omega_2) \cdots \mathcal{O}(\Omega_n) \right\rangle_{CFT} =$$

$$=$$



CONFORMAL WEIGHT :

$$\left\langle \mathcal{O}_\Delta(\Omega)\, \mathcal{O}_\Delta(\Omega') \right\rangle = \frac{1}{|\Omega - \Omega'|^{2\Delta}} =$$



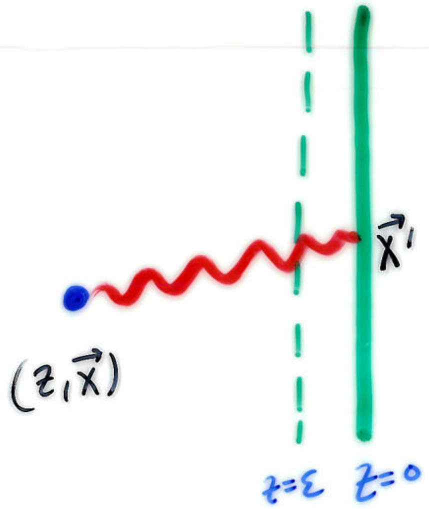
$$\Rightarrow \boxed{\Delta = \frac{d}{2} + \sqrt{\frac{d^2}{4} + R^2 m^2}}$$

BPS CHECKS : ALL OK !

FOR SCALAR FIELDS is AdS_{d+1}

$$z = R^2/r \qquad \left.\frac{ds^2}{R^2}\right|_{AdS} = \frac{1}{z^2}\left(dz^2 + d\vec{x}^{\,2}\right)$$

$\mathbb{R}^d = \partial AdS_{d+1}$



WHAT is BOUNDARY TO BULK PROPAGATOR FOR

$$S = \frac{1}{2}\int_{AdS_{d+1}} \sqrt{g}\left(|d\phi|^2 + m^2\phi^2\right) \; ?$$

THAT is A SOLUTION OF THE WAVE EQUATION

$$\left(-D^2 + m^2\right) K(z,\vec{x};\vec{x}') = 0$$

THAT VANISHES AT BOUNDARY EXCEPT FOR A DELTA FUNCTION AT $\vec{x}'$.

THE SOLUTION TO THIS BOUNDARY PROBLEM is:

$$\boxed{K(z,\vec{x};\vec{x}') = \frac{z^{\Delta}}{\left(z^2 + |\vec{x}-\vec{x}'|^2\right)^{\Delta}} \qquad \Delta = \frac{d}{2} + \sqrt{\frac{d^2}{4} + m^2R^2}}$$

CLASSICAL ON SHELL FIELD AS A FUNCTION OF BOUNDARY VALUE

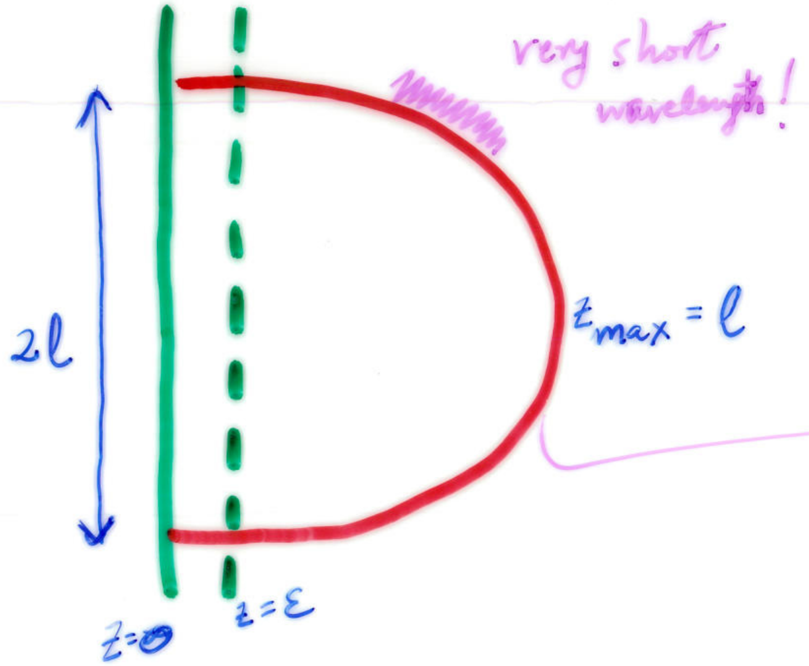
$$\varphi_{cl}(\vec{x},z) = C\int d\vec{x}'\, K(z,\vec{x};\vec{x}')\,\varphi_0(\vec{x}')$$

$$\boxed{S_{cl} = \frac{1}{2}\int \frac{\varphi_0(\vec{x})\,\varphi_0(\vec{x}')}{|\vec{x}-\vec{x}'|^{2\Delta}}} \longrightarrow \boxed{\langle \mathcal{O}_\varphi(\vec{x})\,\mathcal{O}_\varphi(\vec{x}')\rangle = \frac{1}{|\vec{x}-\vec{x}'|^{2\Delta}}}$$

EIKONAL APPROXIMATION FOR LARGE MASS

$m \gg 1/R$ approximate propagator by geodesic

semicircles!



$x = \ell \cos\vartheta$

$z = \ell \sin\vartheta$

$$S_\ell = m\int ds = mR\int_0^\pi d\vartheta \sqrt{\frac{1}{z^2}\frac{dz}{d\vartheta} + \frac{1}{z^2}\frac{dx}{d\vartheta}} = mR\int_0^\pi \frac{d\vartheta}{\sin\vartheta}$$

$$\simeq 2mR\int_{\varepsilon/\ell}^{\pi/2} \frac{d\vartheta}{\vartheta} \simeq -2mR \log \varepsilon/\ell$$

$$\boxed{\langle \mathcal{O}(-\ell)\, \mathcal{O}(\ell)\rangle \sim e^{-S_\ell} \sim \varepsilon^{2mR} \frac{1}{\ell^{2mR}}}$$

"cutoff" factor see next page!

OK with $\Delta = -d/2 + \sqrt{d^2/4 + m^2R^2} \underset{m \gg 1/R}{\sim} mR$

massive particles need a "push" or they do not reach boundary

IR/UV CONNECTION

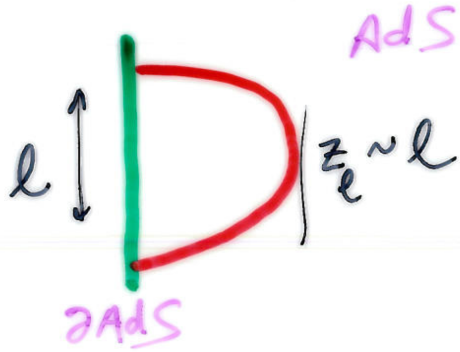
→ IMPORTANT LESSON FROM THIS EXAMPLE

A LENGTH SCALE ℓ IN THE GAUGE CFT

IS CHARACTERIZED SEMICLASSICALLY BY SOME AdS PROCESS THAT PROBES GEOMETRY UP TO A z-COORDINATE $z(\ell) \sim \ell$

* IN TERMS OF r COORDINATE :

$$\boxed{r_\ell \sim \frac{R^2}{z_\ell} \sim \frac{R^2}{\ell} \sim \frac{\alpha' \sqrt{\lambda}}{\ell}}$$



* IN TERMS OF THE U-COORDINATE, SET BY HIGGS EXPECTATION VALUES :

$$U_\ell = \frac{r_\ell}{\alpha'} \sim \sqrt{\lambda}/\ell$$

* TO MAKE TWO POINT FUNCTION FINITE, WE MUST RENORMALIZE OPERATORS

$$\mathcal{O}_{ren} = \varepsilon^{-\Delta}\, \mathcal{O}_{bare}$$

SO THAT THE ε-CUTOFF IN z IS A U.V. CUTOFF IN CFT !!

* INTEGRATING SLICES IN z IS EQUIVALENT TO RG!

* ALL OF THIS IS A RECURRENT IDEA IN AdS/CFT

OPERATOR / FIELD MAPPING

IN GENERAL : AN ART

SYMMETRIES HELP !

DILATON $\varphi \longrightarrow \mathrm{Tr}\, F^2$

RR AXION $a \longrightarrow \mathrm{Tr}\, F\wedge F$

GRAVITON $h_{\mu\nu} \longrightarrow T_{\mu\nu}$

SO(6) GAUGE FIELDS $g_{\mu I} \longrightarrow \mathrm{Tr}\left(\phi^I \partial_\mu \phi^J + \dots\right)$

SO(6) currents

ℓ'th SO(6) spherical HARMONIC $\longrightarrow \mathrm{Tr}\, \Phi^{(I_1} \dots \Phi^{I_\ell)}$

⋮

+ SCFT partners

$\boxed{\text{All BPS ops}}$

* KK states from massless 10 dim fields are BPS

String excited states are not BPS

EXPANSION PARAMETERS

BASIC RELATION $\qquad R^4 \sim g_s N \, l_s^4 = \lambda \, l_s^4$

WEAK COUPLING EXPANSION

$$\langle \cdots \rangle = {\sum_{h \geq 0}}' N^{2-2h} {\sum_n}' (g^2 N)^n {\sum_m}' e^{-8\pi^2/g^2 \cdot m} \langle \cdots \rangle_{h,n,m}$$

$\frac{1}{N}$ exp

perturb th

instanton exp

GRAVITY-SIDE EXPANSION

string loops

α' curvature exp.

D-instantons

$$\langle \cdots \rangle = \sum_h g_s^{2h-2} {\sum_k}' \left(\frac{\alpha'}{R^2}\right)^k \sum_m e^{-2\pi m/g_s} \langle \cdots \rangle_{h,k,m}$$

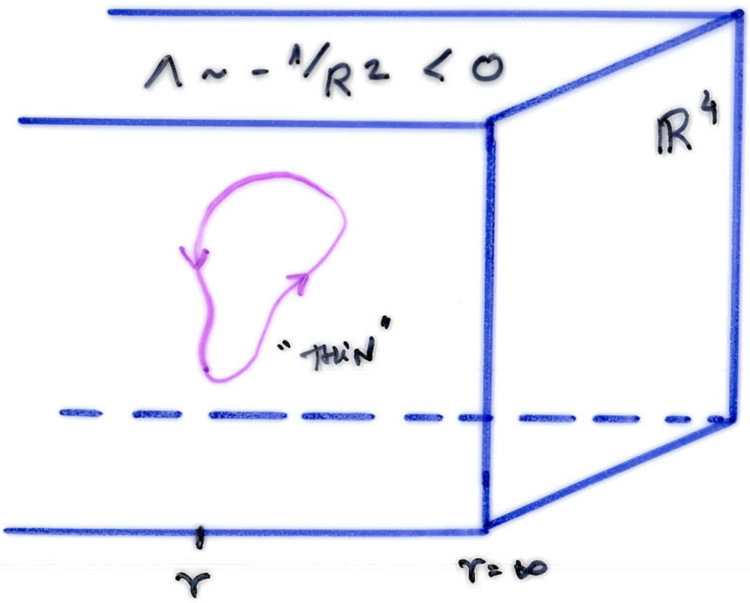
$$= \sum_h \underbrace{N^{2-2h} \lambda^{2h-2}} \sum_k \left(\frac{1}{\sqrt{\lambda}}\right)^k \sum_m e^{-8\pi^2 N m/\lambda}$$

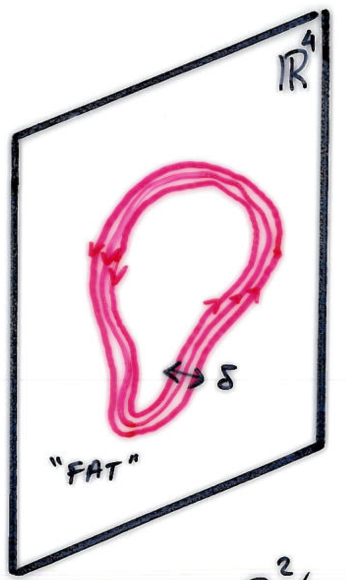
$1/N$ exp. with eff. expansion parameter $\frac{\lambda^2}{N^2} < 1$

strong coupling exp

instanton expansion $(N \gg \lambda)$

AdS   CFT





$\delta \sim R^2/r$

Comes in all sizes!

POLYAKOV (Liouville) (80's)

't HOOFT → SUSSKIND (Holography) 90's

D-branes
Sugra
⋮
→ MALDACENA (97)

Gubser, Klebanov, Polyakov 97

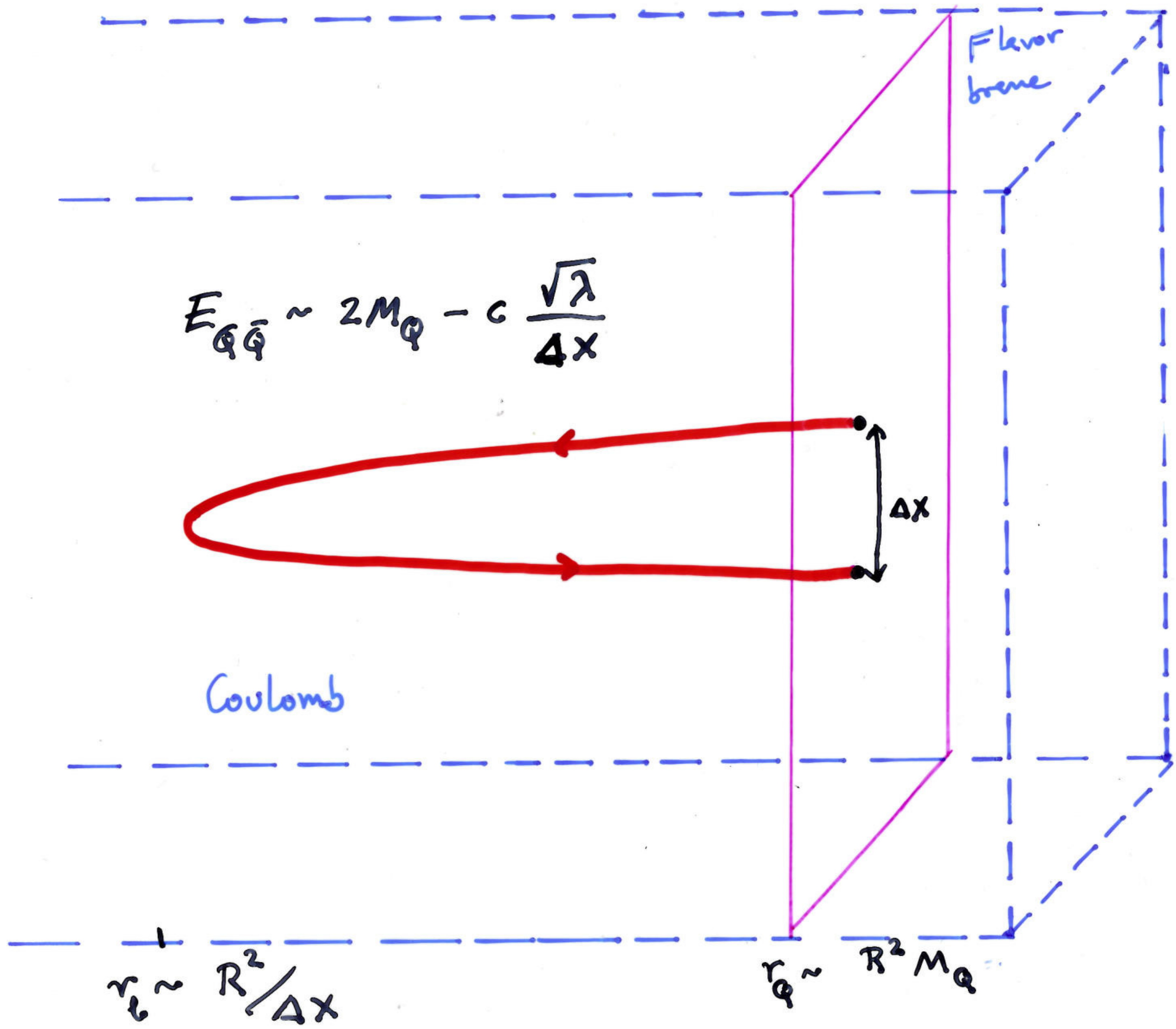
Witten 97

WHAT HAPPENED TO 'tHOOF'S PROPHECY ?

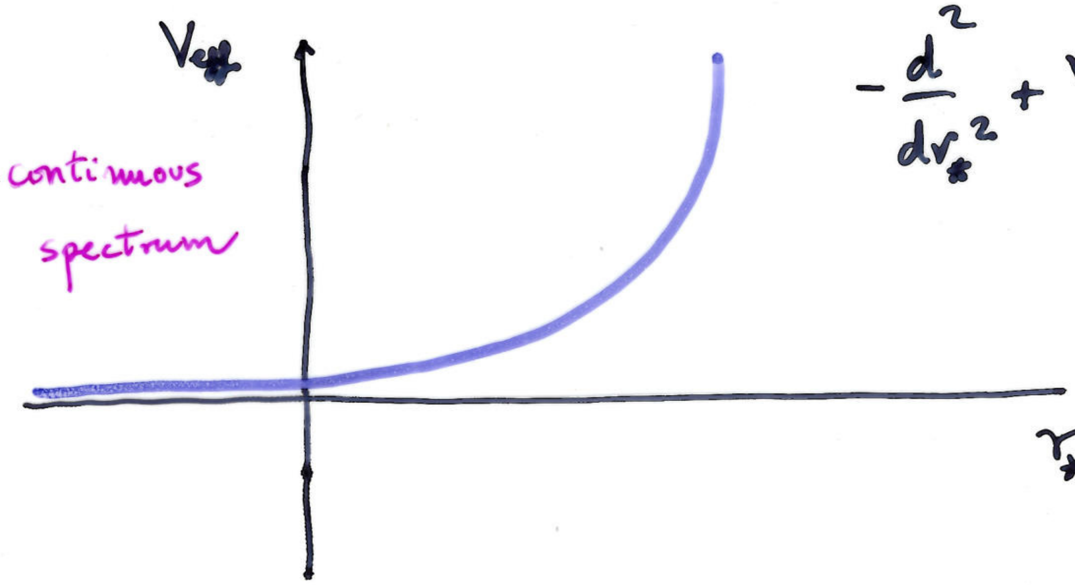
WE HAVE A "THIN" LARGE-N STRING , BUT IT IS NOT CONFINING !

* IT TURNS OUT THAT WE CAN "DEFORM" THE BASIC AdS/CFT EXAMPLE INTO QCD-LIKE MODELS , AT A QUALITATIVE LEVEL

CONFORMAL PHASE



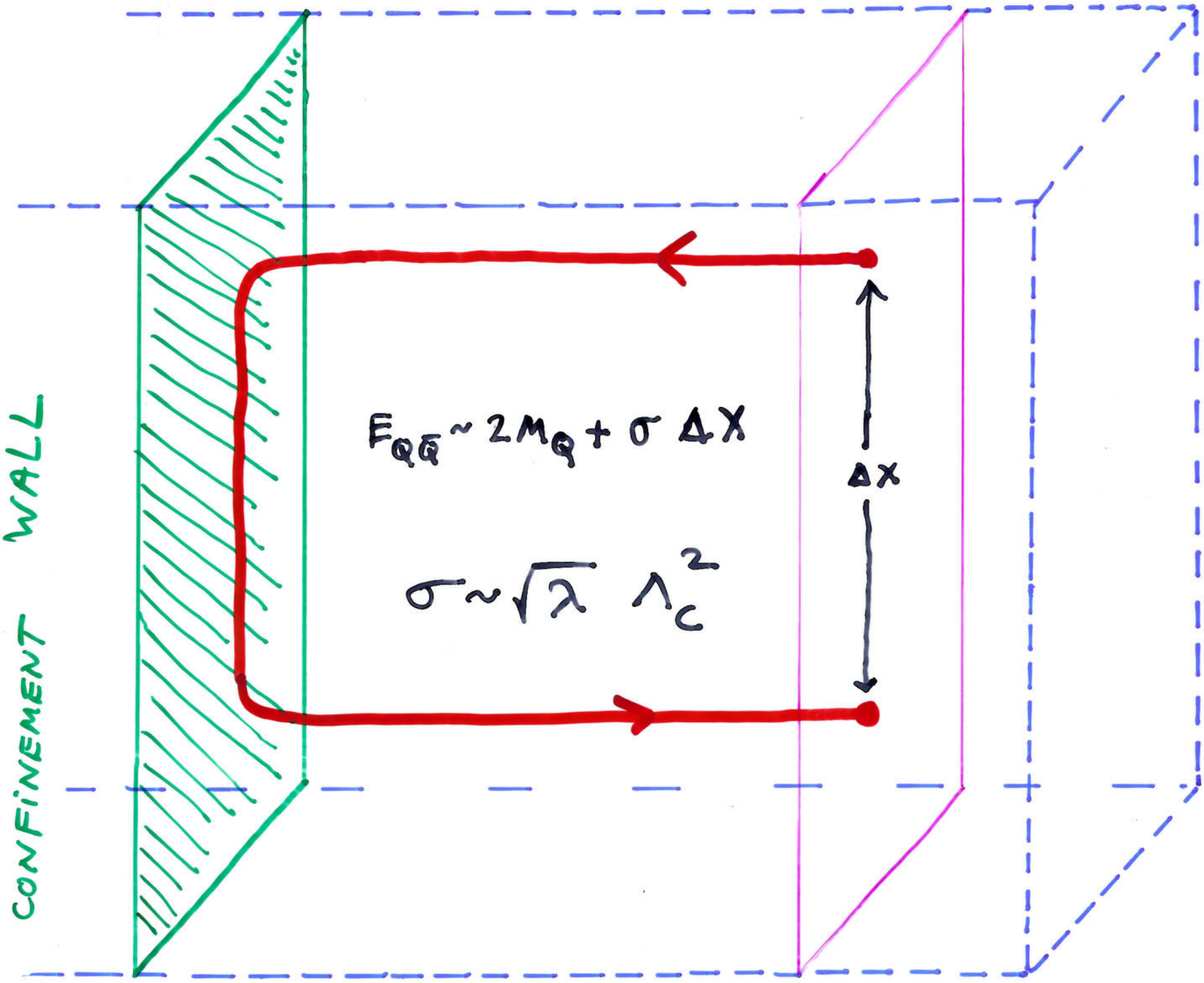
NO MASS GAP



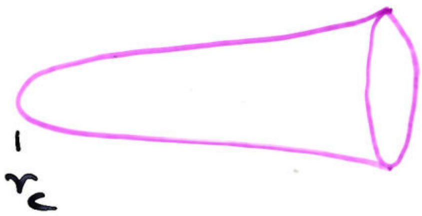
$$-\frac{d^2}{dr_*^2} + V_{eff}(r_*) = \omega^2$$

$$r_* = R\left(1 - R/r\right)$$

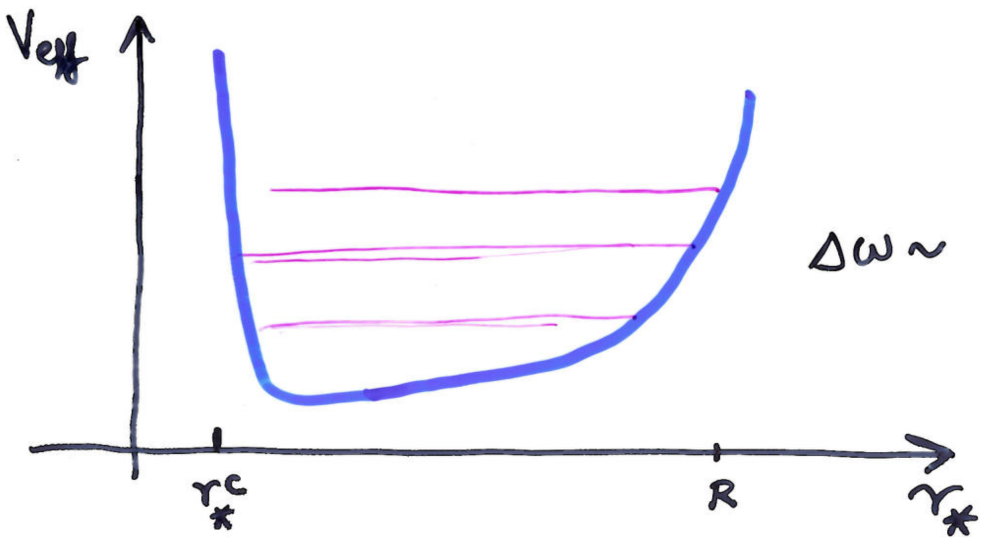
CONFINING PHASE



$r_c \sim R^2 \Lambda_c$

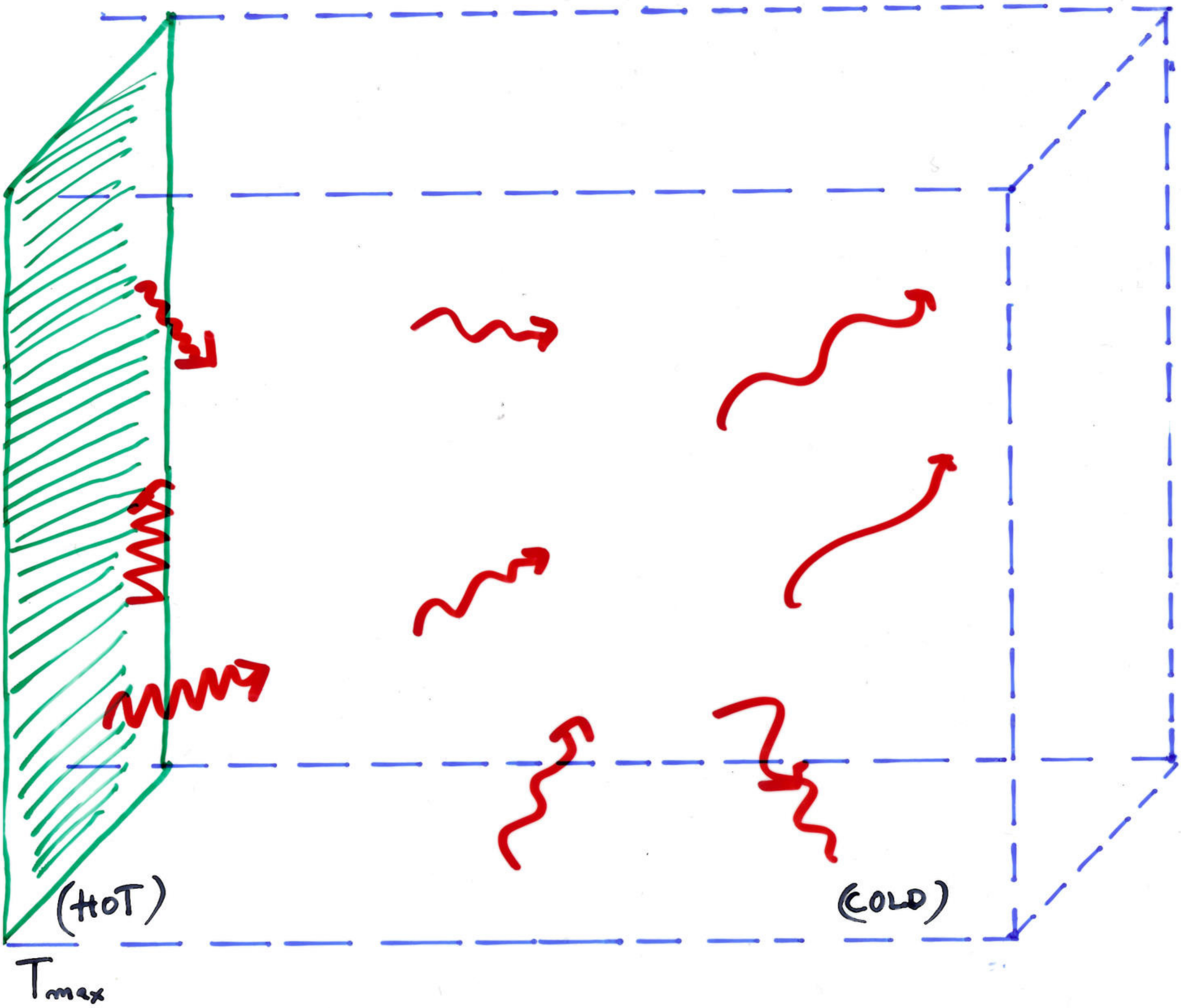


MASS GAP



$\Delta\omega \sim \Lambda_c^{-1}$

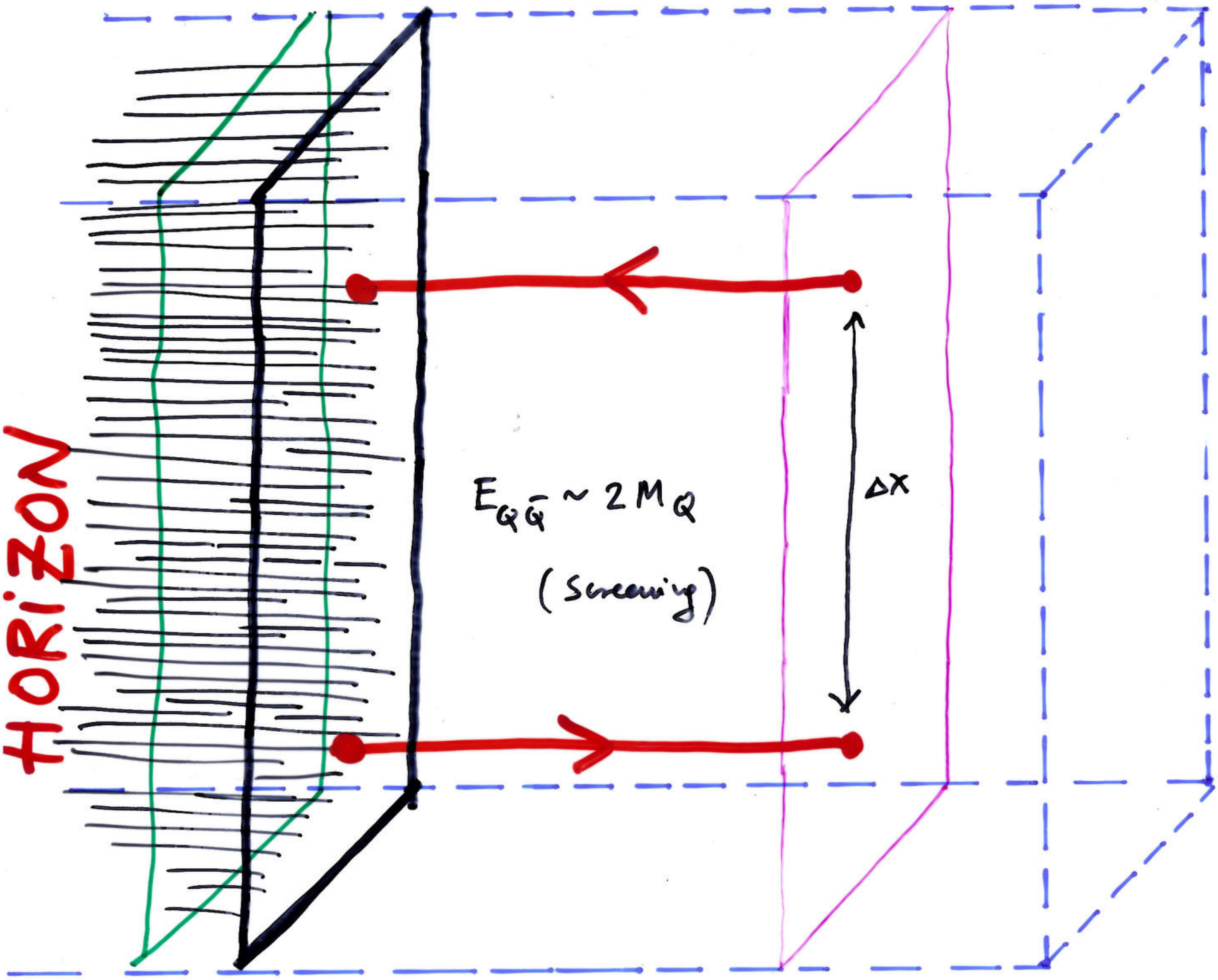
FINITE T



$$T(r) \cdot r = \text{constant}$$

EVENTUALY, RADIATION COLLAPSES INTO A BLACK HOLE HORIZON THAT CLOAKES ALL IR STUFF, INCLUDING THE CONFINEMENT WALL

DECONFINED PHASE



$r_H \sim R^2 T$

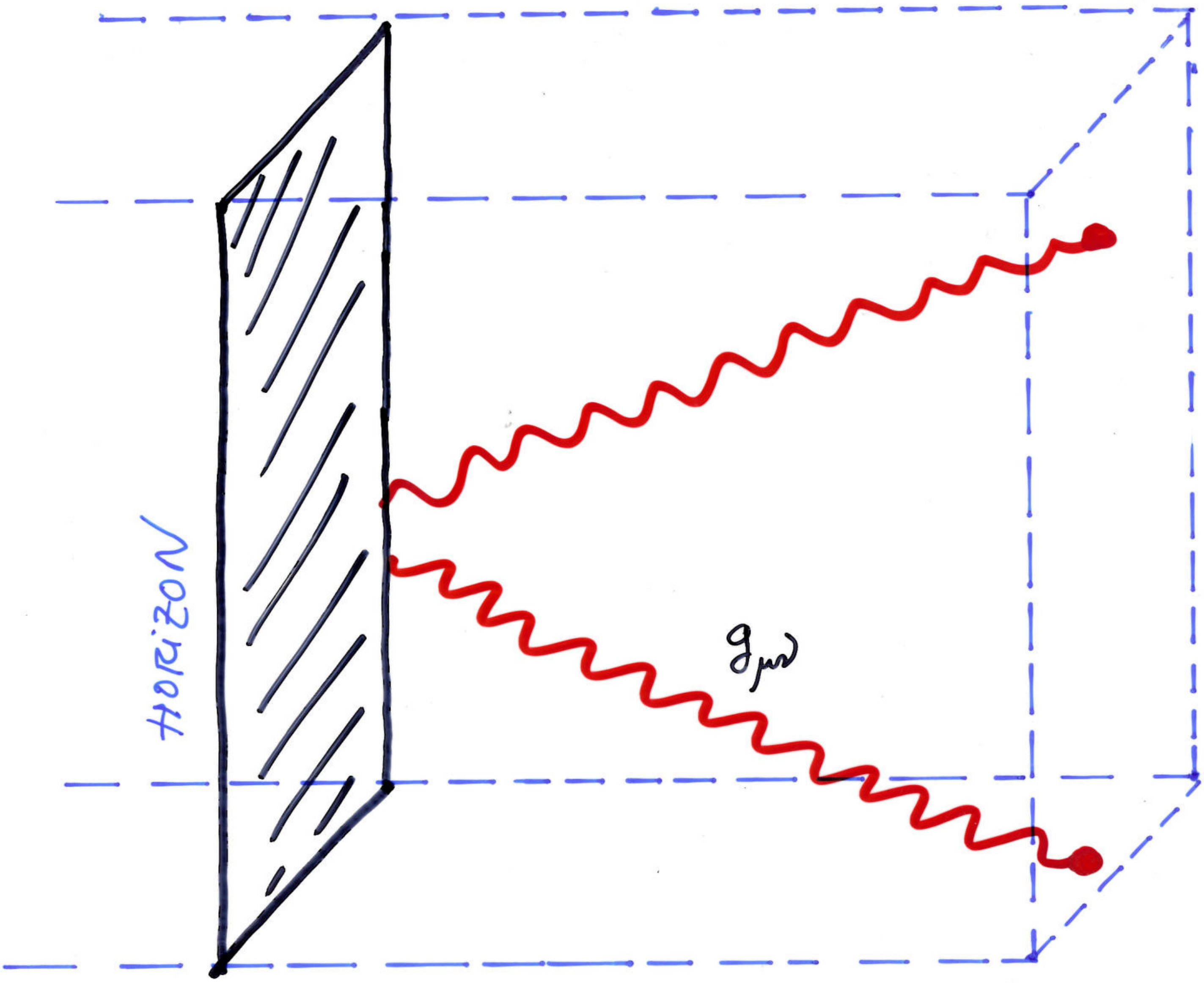
$T > \Lambda_c$

$T < T_c \rightarrow$ $\boxed{S \sim \text{gravitons} \sim VT^3}$

$T > T_c \rightarrow$ $\boxed{S \sim \frac{A_{HOR}}{4G} \sim O(g_s^{-2}) \sim \underline{N}^2 VT^3}$

LATENT HEAT $O(N^2)$. 1^{st} ORDER PHASE TR.

MORE ON PLASMA PHASE



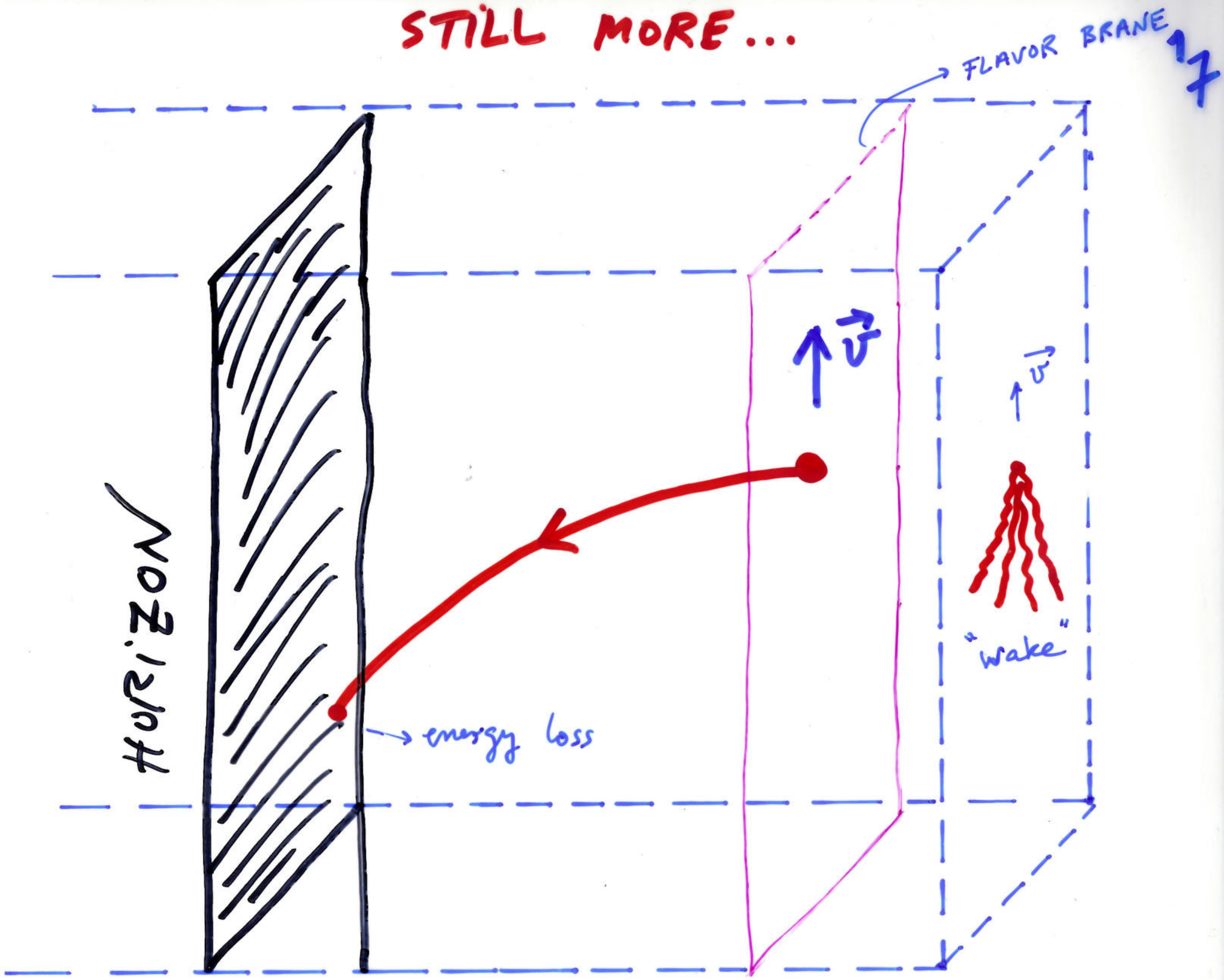
$\langle T_{\mu\nu} T_{\rho\sigma} \rangle_\beta$ CAN BE EXTRACTED FROM LINEAR PERTURBATION THEORY OF HORIZONS

GET TRASPORT COEFFICIENTS, LIKE ~~SHEAR~~ SHEAR VISCOSITY

$$\boxed{\frac{\eta}{s} = \frac{1}{4\pi} + \cdots}$$

UNIVERSALLY!

STILL MORE...



* ENERGY LOSS OF FAST QUARK THROUGH THE PLASMA

* JET QUENCHING.

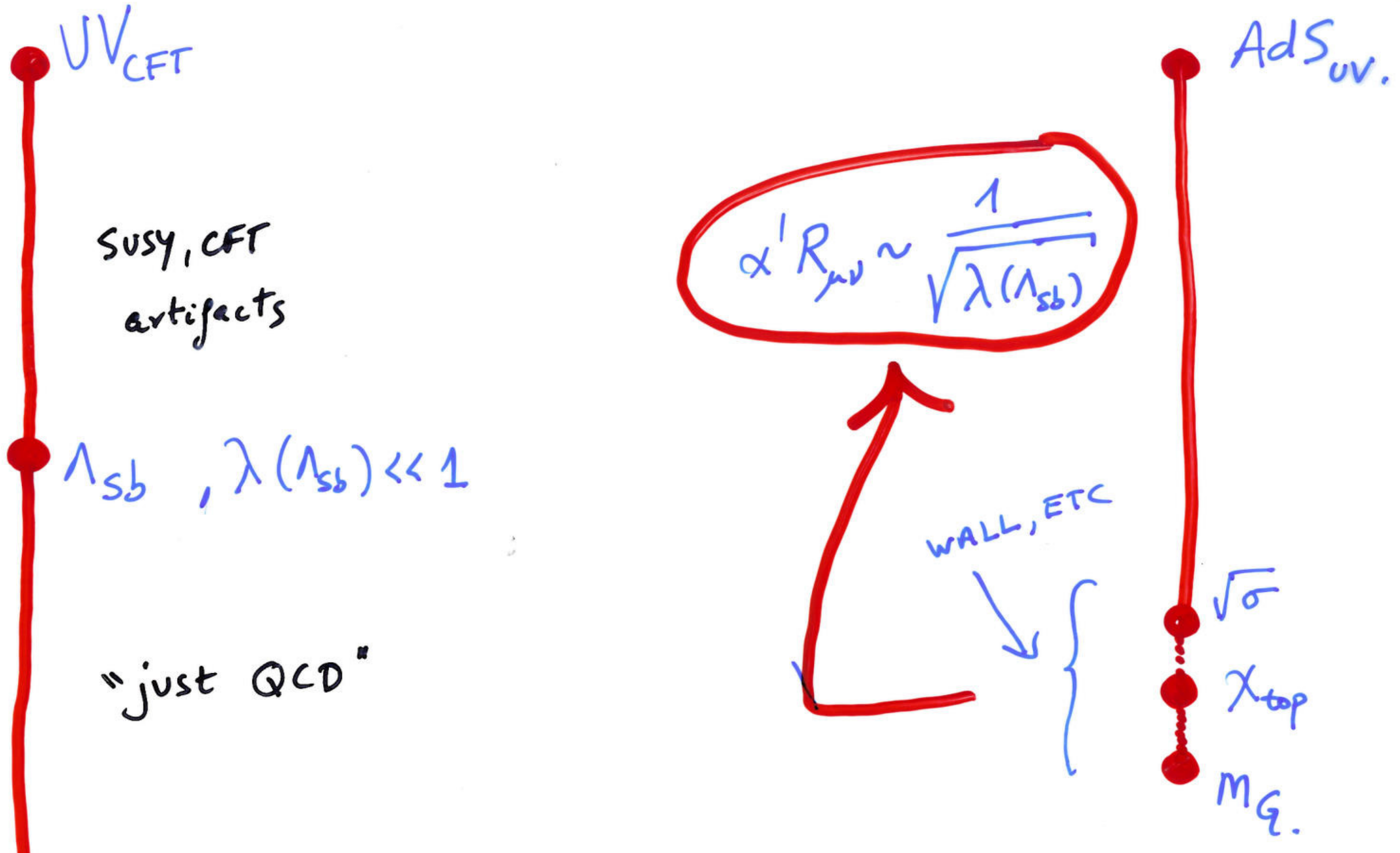
$$\boxed{\lambda_{eff}\Big|_{RHIC} \sim 1}$$

* THERE IS AN INTRINCATE CASUISTICS IN SUCH AdS/QCD MODELS. GOOD QUALITATIVE UNDERSTANDING OF QCD FACTS

- DECONFINEMENT PHASE TRANSITION
- MESON / GLUEBALL SPECTRUM
- χSB
- η' PHYSICS

AND THE NICE SURPRISE OF PLASMA PHYSICS.

HOWEVER ...



$$\Lambda_{QCD} \sim \Lambda_{sb}\, e^{-c/\lambda(\Lambda_{sb})}$$

$$\frac{m_G}{\sqrt{\sigma}} \sim \lambda^{\alpha}$$

$$\Lambda_{QCD} \sim m_G \sim \sqrt{\sigma} \sim (\chi_{top})^{1/4}$$

THE BASIC PROBLEM :

GEOMETRICAL INTUITION/ANALYSIS

REQUIRES $\alpha' R_{\mu\nu} \ll 1$

BUT THEN NO $\frac{\Lambda_{QCD}}{\Lambda_{sb}}$ HIERARCHY

RECENTLY, PROGRESS WAS OBTAINED IN THE INTERPOLATION PROBLEM

$$\lambda \ll 1 \longleftrightarrow \lambda \gg 1$$

FOR THE CFT CASE AND "THERMODYNAMIC" OPERATORS

$$\mathrm{Tr}\big(\Phi\, \underbrace{\nabla \nabla \nabla \cdots\cdots \nabla}_{S \gg 1}\, \Phi\big)$$

$$\Delta - S \sim \text{twist} \sim f(\lambda) \log S$$

$$\mathrm{Tr}\big(\underbrace{ZZZ \cdots WZW\cdots\cdots}_{J \gg 1} ZZZ\big)$$

$\Delta - J \sim$ Heisenberg spin chain Hamiltonian

INTEGRABILITY KEY TO INTERPOLATION

CONCLUSION

THE QCD STRING IS THIN AT $N=\infty$

SO THIN THAT IT REMAINS VERY DIFFICULT TO FIND ...