

# Results for Charged-Current Deep-Inelastic Scattering at three loops

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# Content

- Introduction to the Deep-Inelastic Scattering

- ▲ Charged-current deep-inelastic scattering at three loops.

S. Moch and M. R. *Nucl. Phys. B* **782**, 51 (2007)

- ▲ Differences between CC coefficient functions

S. Moch, M. R. and A. Vogt. *arXiv:0708.3731v1 [hep-ph]*

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- ▲ Charged-current deep-inelastic scattering at three loops.

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- Related experiments
- Perturbative QCD corrections at three loops in QCD
- Results: its analysis and applications

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▲ Differences between CC coefficient functions

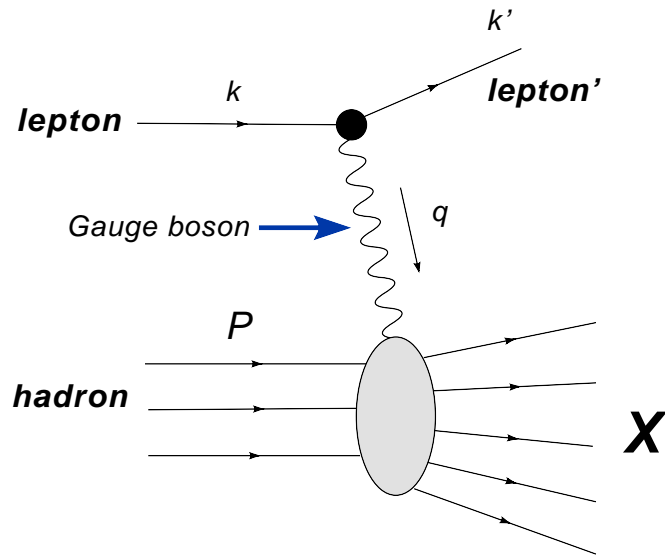
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# Introduction

- Deep-inelastic lepton-hadron scattering ( $e^\pm p$ ,  $e^\pm n$ ,  $\nu p$ ,  $\bar{\nu} p$ , ... - collisions)

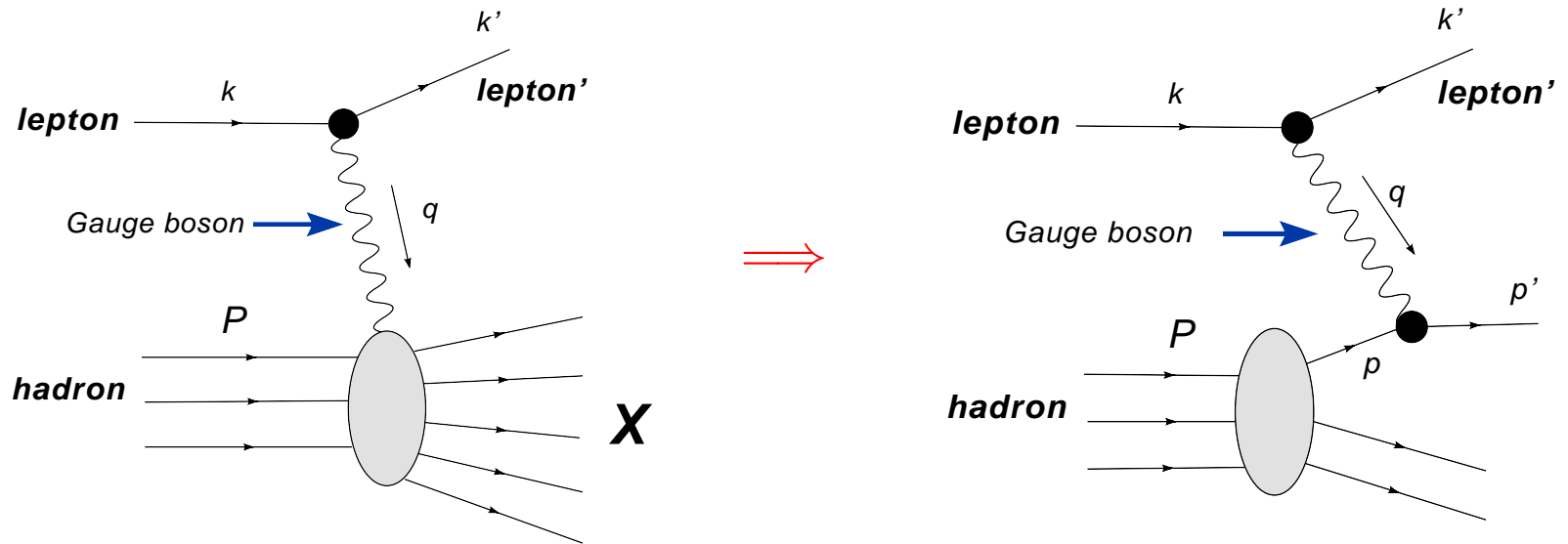
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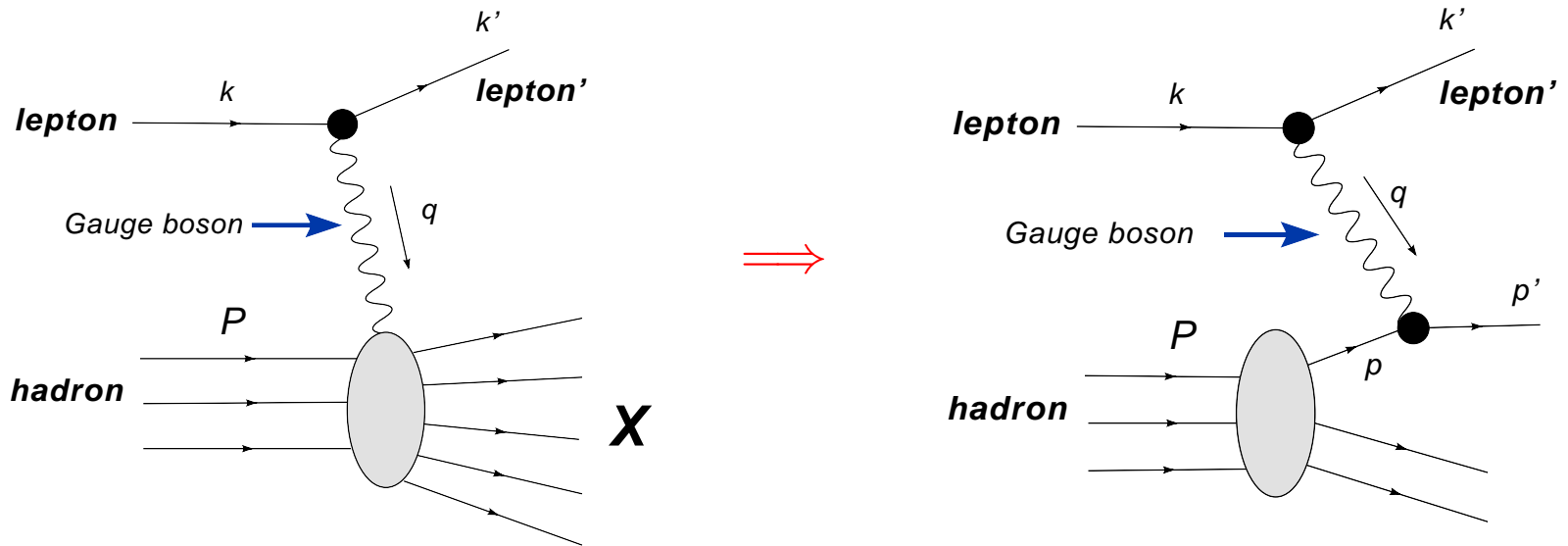
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# Introduction

- Deep-inelastic lepton-hadron scattering ( $e^\pm p, e^\pm n, \nu p, \bar{\nu} p, \dots$  - collisions)



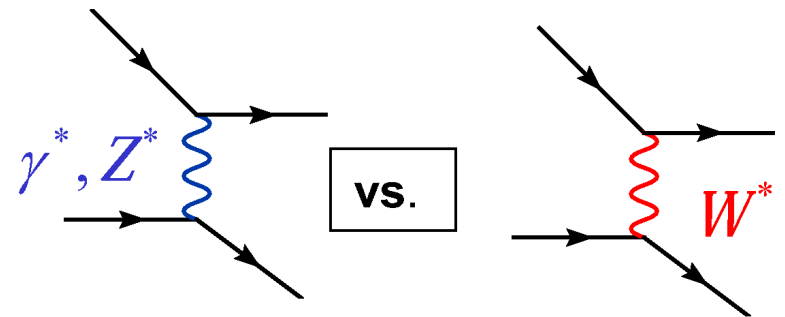
- Gauge boson:
  - $\gamma, Z^0$  - NC
  - $W^\pm$  - CC

## Kinematic variables

- momentum transfer  $Q^2 = -q^2 > 0$
- Bjorken variable  $x = Q^2 / (2P \cdot q)$
- Inelasticity  $y = (P \cdot q) / (P \cdot k)$

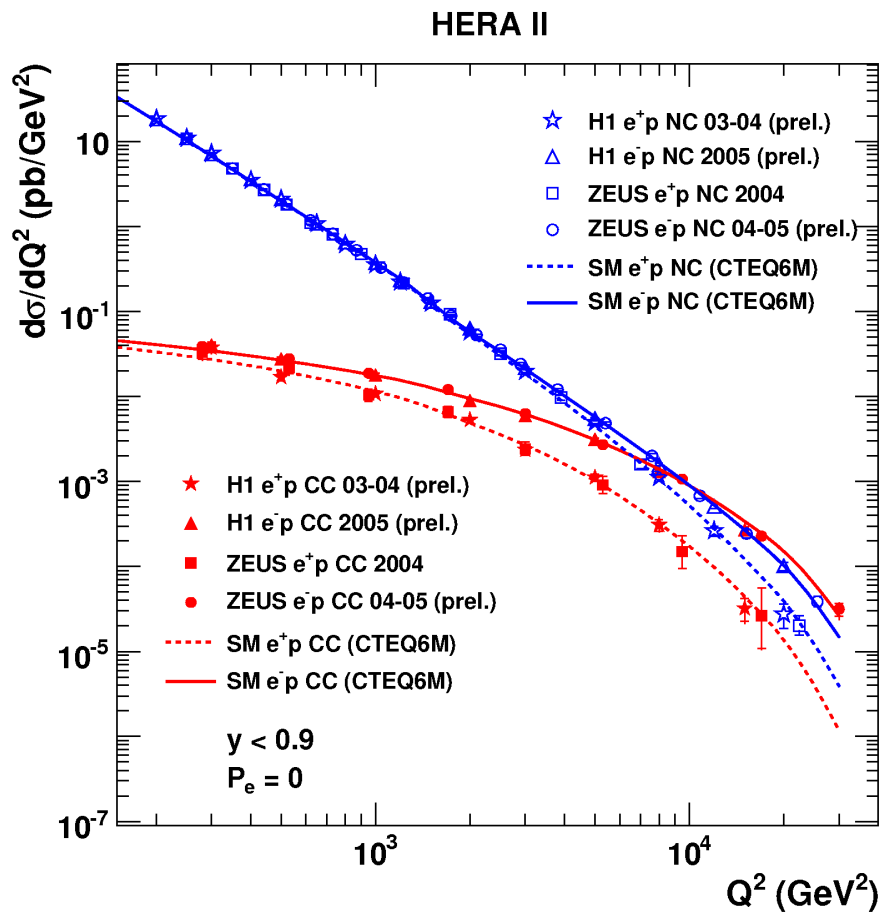
# DIS experiments

- EW unification at HERA:  
neutral vs . charged current

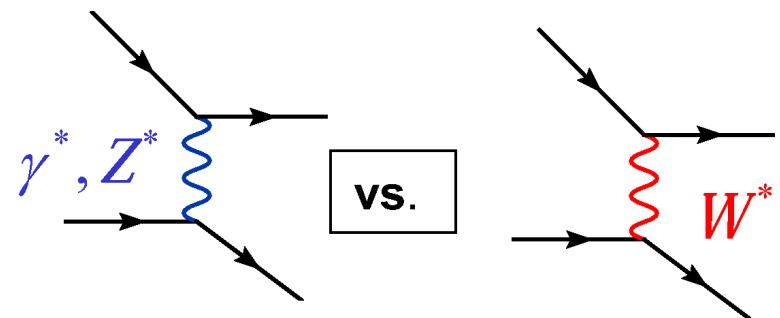


# DIS experiments

- EW unification at HERA:  
neutral vs . charged current



Charged and neutral deep inelastic scattering cross sections become comparable when  $Q^2$  reaches the electroweak scale



● Polarized charged current DIS at HERA

CC cross section modified by polarization:

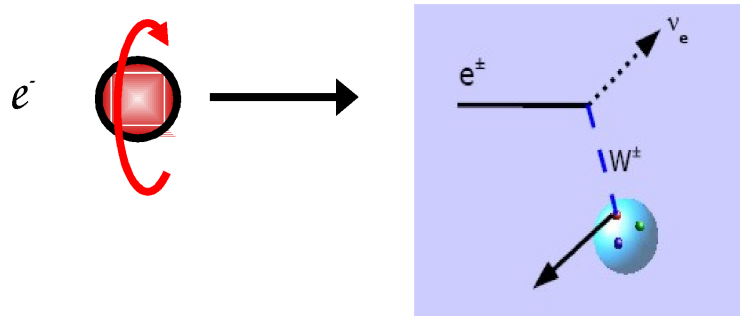
$$\sigma_{CC}^{e^\pm p}(P_e) = (1 \pm P_e) \cdot \sigma_{CC}^{e^\pm p}(P_e = 0)$$

$$P_e = \frac{N_R - N_L}{N_R + N_L}$$

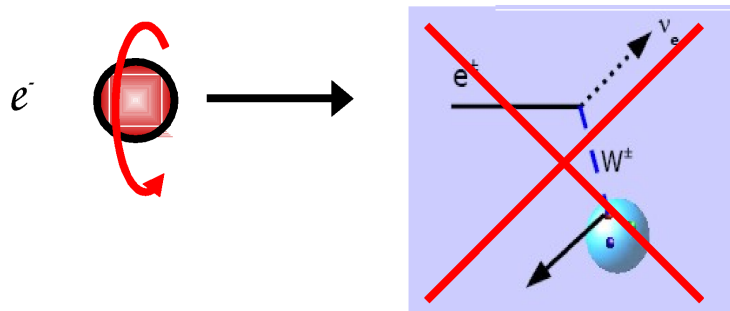
● Cross section is linearly proportional to polarization  $P_e$

● **Standard model prediction:** vanishing cross section for  $P_e = +1(-1)$  in  $e^{-(+)}$  scattering

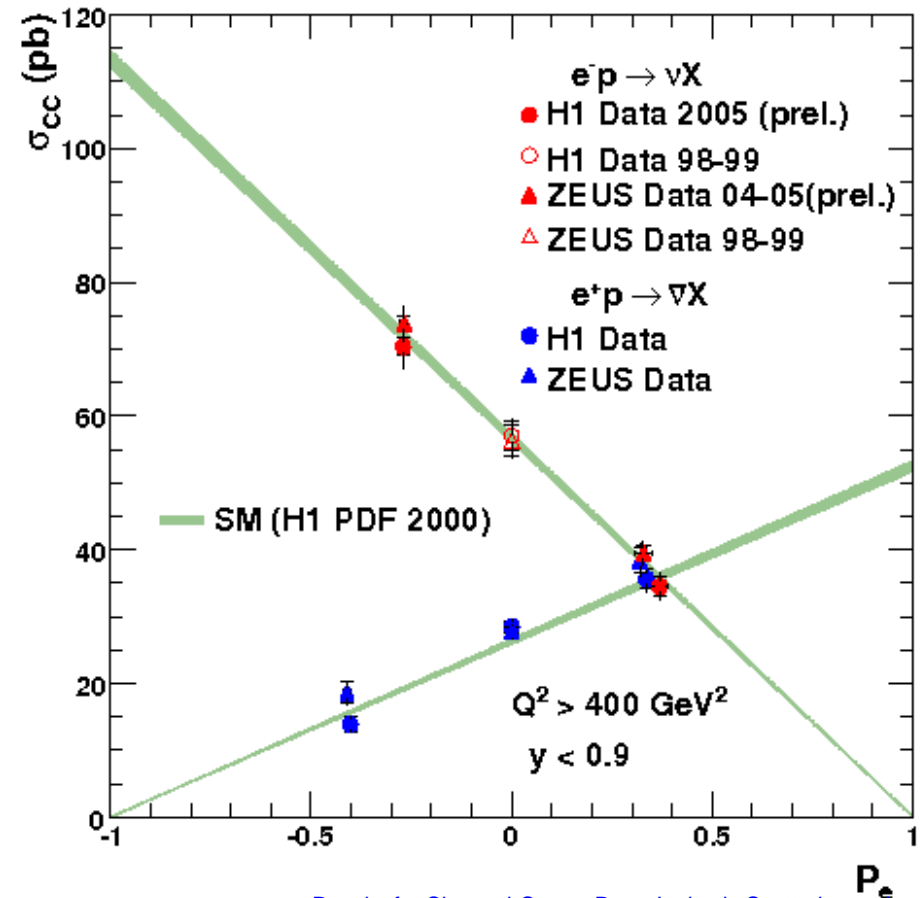
■ *lefthanded electrons interact (CC)*



■ *righthanded electrons do not!*



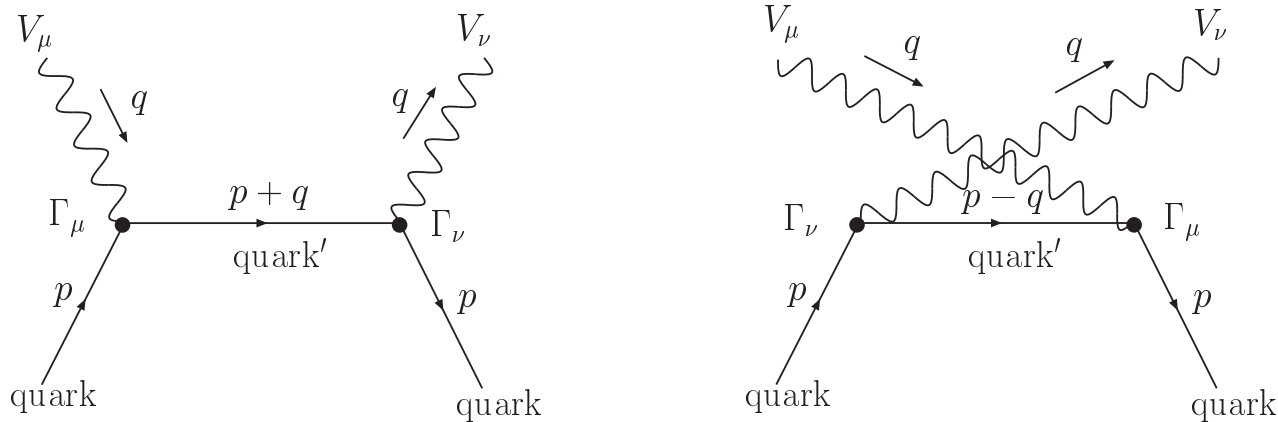
Charged Current  $e^\pm p$  Scattering



# Calculation

- Leading order diagrams at parton level

- Vector and axial-vector interaction  $a\gamma^\mu + b\gamma^\mu\gamma^5$



- Mellin moments with definite symmetry properties

- process dependent distinction even/odd  $N$  (from OPE)

$$F_i(N, Q^2) = \int_0^1 dx x^{N-2} F_i(x, Q^2), \quad i = 2, L$$

$$F_3(N, Q^2) = \int_0^1 dx x^{N-1} F_3(x, Q^2)$$

## Known

- NC (exchange via  $\gamma$  gauge boson)  $\longrightarrow F_2^{eP}$
- CC (exchange via  $W^\pm$  gauge boson)  $\longrightarrow F_2^{\nu p + \bar{\nu} p}, F_3^{\nu p + \bar{\nu} p}$

even  $N$  for  $F_2$ , odd  $N$  for  $F_3$

- NLO Bardeen, Buras, Duke, Muta '78
- N<sup>2</sup>LO Zijlstra, van Neerven '92
- N<sup>3</sup>LO Moch, Vermaseren, Vogt '05/'06

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## New

- NC  $\gamma - Z$  interference at N<sup>3</sup>LO still missing
- CC (exchange via  $W^\pm$  gauge boson)  $\longrightarrow F_2^{\nu p - \bar{\nu} p}, F_3^{\nu p - \bar{\nu} p}$

odd  $N$  for  $F_2$ , even  $N$  for  $F_3$

- order N<sup>3</sup>LO *already known* Moch, M. R. '07  
*best use: difference “even-odd”* Moch, M. R. and Vogt. '07

### The calculation

- Big number of diagrams  $\Rightarrow$  need of automatization  
e.g. DIS structure functions  $F_{2,L}^{\nu p \pm \bar{\nu} p}$  - 1076 diagrams,  $F_3^{\nu p \pm \bar{\nu} p}$  - 1314 diagrams up to **3** loops



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- latest version of FORM and TFORM (multi-threaded version)  
Vermaseren, FORM version 3.2 (Apr 16 2007); Tentyukov, Vermaseren '07  
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Calculation of diagrams  $\mapsto$

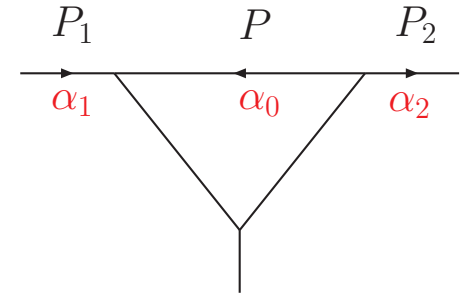
- MINCER in FORM Larin, Tkachev, Vermaseren '91

## Mincer

- $\int dP \frac{\partial}{\partial P^\mu} [(P - l_j)^\mu \times I(l_1, \dots, P, \dots)] = 0$  - integration by part identities  
t'Hooft, Veltman '72; Chetyrkin, Tkachov '81  
Leibniz, Newton :-)

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## Triangle rule

Define


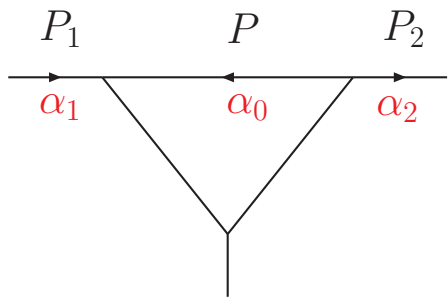
$$I(\alpha_0, \beta_1, \beta_2, \alpha_1, \alpha_2) = \int d^D P \frac{1}{(P^2)^{\alpha_0} ((P + P_1)^2)^{\beta_1} (P_1^2)^{\alpha_1} ((P + P_2)^2)^{\beta_2} (P_2^2)^{\alpha_2}}$$

and act the integrand with  $\frac{\partial}{\partial P_\mu} P_\mu = D + P_\mu \frac{\partial}{\partial P_\mu}$ . Result  $\Rightarrow$

**Recursion relation:**

$$\begin{aligned} I(\alpha_0, \beta_1, \beta_2, \alpha_1, \alpha_2) \times (D - 2\alpha_0 - \beta_1 - \beta_2) = \\ \beta_1 (I(\alpha_0 - 1, \beta_1 + 1, \beta_2, \alpha_1, \alpha_2) - I(\alpha_0, \beta_1 + 1, \beta_2, \alpha_1 - 1, \alpha_2)) \\ \beta_2 (I(\alpha_0 - 1, \beta_1, \beta_2 + 1, \alpha_1, \alpha_2) - I(\alpha_0, \beta_1, \beta_2 + 1, \alpha_1, \alpha_2 - 1)) \end{aligned}$$

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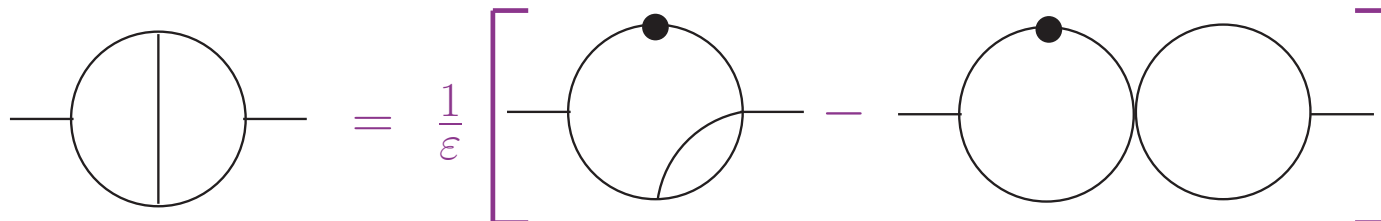
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 & \beta_2 (I(\alpha_0 - 1, \beta_1, \beta_2 + 1, \alpha_1, \alpha_2) - I(\alpha_0, \beta_1, \beta_2 + 1, \alpha_1, \alpha_2 - 1))
 \end{aligned}$$

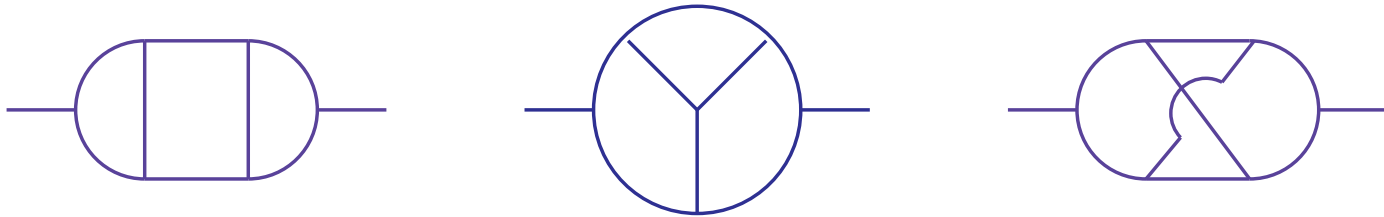
## In pictures



$$\text{Circle with vertical line} = \frac{1}{\epsilon} \left[ \text{Circle with dot and self-energy loop} - \text{Two adjacent circles with dot on first} \right]$$

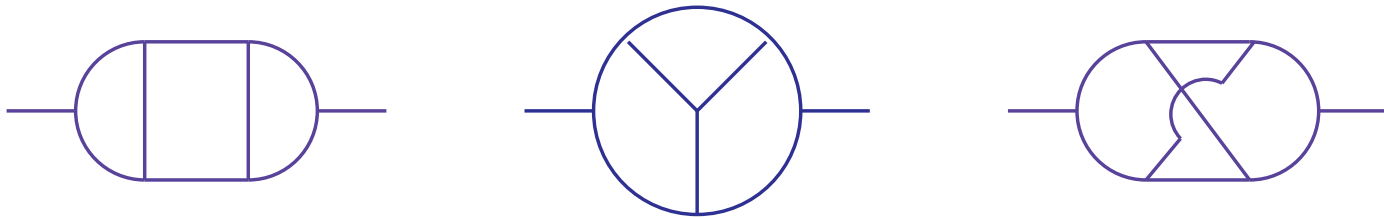
## Classification of loop integrals

- Classify according to topology of underlying two-point function
  - top-level topology types ladder, benz, non-planar  $\Rightarrow$

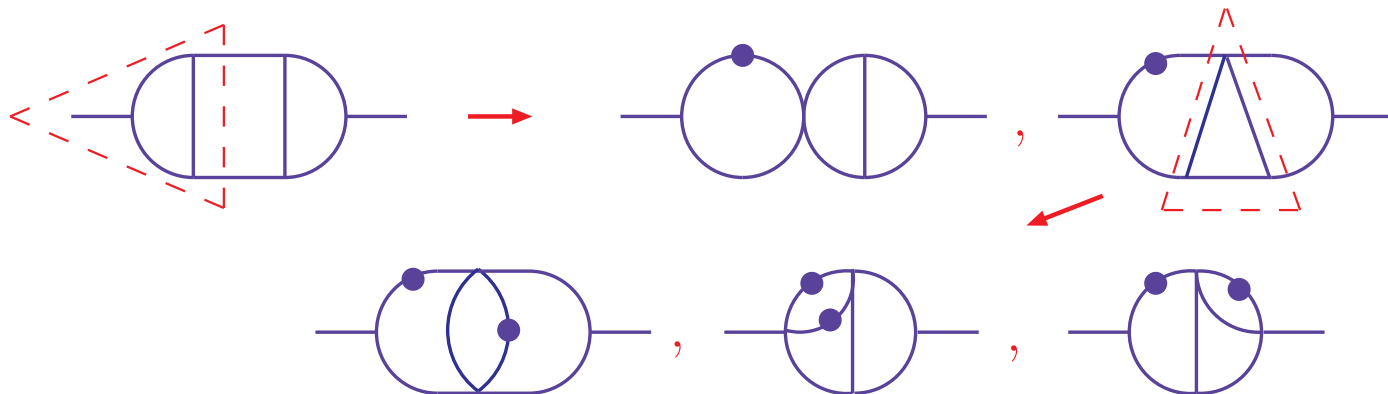


# Classification of loop integrals

- Classify according to topology of underlying two-point function
  - top-level topology types ladder, benz, non-planar  $\Rightarrow$



- Using **IBP** identities more complicated topologies are reduced to simpler topologies





$$\begin{aligned}
C_{3,10}^{\text{ns}} = & 1 + a_s C_F \frac{1953379}{138600} + a_s^2 C_F n_f \left( -\frac{537659500957277}{15975002736000} \right) + a_s^2 C_F^2 \left( \frac{597399446375524589}{14760902528064000} \right. \\
& \left. + \frac{7202}{105} \zeta_3 \right) + a_s^2 C_A C_F \left( \frac{5832602058122267}{29045459520000} - \frac{99886}{1155} \zeta_3 \right) \\
& + a_s^3 C_F n_f^2 \left( \frac{51339756673194617191}{996360920644320000} + \frac{48220}{18711} \zeta_3 \right) \\
& + a_s^3 C_F^2 n_f \left( -\frac{125483817946055121351353}{209235793335307200000} - \frac{59829376}{3274425} \zeta_3 + \frac{24110}{693} \zeta_4 \right) \\
& + a_s^3 C_F^3 \left( -\frac{744474223606695878525401307}{7088908678200207936000000} + \frac{28630985464358}{24960941775} \zeta_3 \right. \\
& \left. + \frac{151796299}{8004150} \zeta_4 - \frac{53708}{99} \zeta_5 \right) \\
& + a_s^3 C_A C_F n_f \left( -\frac{185221350045507487753}{226445663782800000} + \frac{8071097}{39690} \zeta_3 - \frac{24110}{693} \zeta_4 \right) \\
& + a_s^3 C_A C_F^2 \left( \frac{19770078729338607732075449}{8369431733412288000000} - \frac{619383700181}{5546875950} \zeta_3 \right. \\
& \left. - \frac{151796299}{5336100} \zeta_4 - \frac{37322}{99} \zeta_5 \right) \\
& + a_s^3 C_A^2 C_F \left( \frac{93798719639056648125143}{36231306205248000000} - \frac{43202630363}{20582100} \zeta_3 \right. \\
& \left. + \frac{151796299}{16008300} \zeta_4 + \frac{195422}{231} \zeta_5 \right).
\end{aligned}$$

# Checks

- Known Mellin moments for  $F_{2,L}^{\nu P+\bar{\nu}P}$  (even) and  $F_3^{\nu P+\bar{\nu}P}$  (odd) recalculated

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- All calculations with gauge parameter  $\xi$  for gluon propagator (Up to 10'th MM)

$$i \frac{-g^{\mu\nu} + (1 - \xi)q^\mu q^\nu}{q^2}$$

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- All calculations with gauge parameter  $\xi$  for gluon propagator (Up to 10'th MM)

$$i \frac{-g^{\mu\nu} + (1 - \xi)q^\mu q^\nu}{q^2}$$

- Adler sum rule for DIS structure functions  $\longrightarrow C_{2,1}^{\text{ns}} = 1$

$$\int_0^1 \frac{dx}{x} (F_2^{\nu P}(x, Q^2) - F_2^{\nu N}(x, Q^2)) = 2$$

- measures isospin of the nucleon in the quark-parton model
- neither perturbative nor non-perturbative corrections in QCD

# Applications

- Gottfried sum rule (charged lepton( $l$ )-proton( $P$ ) or neutron( $N$ ) DIS)

$$\int_0^1 \frac{dx}{x} (F_2^{lP}(x, Q^2) - F_2^{lN}(x, Q^2))$$

- Conjecture: difference of non-singlet coefficient functions for even and odd Mellin moments **suppressed** by  $[C_F - C_A/2] \simeq 1/N_c$

Broadhurst, Kataev, Maxwell '04

•  $\delta C_{i,n}^{\text{ns}} = C_{i,n}^{\nu P+\bar{\nu}P} - C_{i,n}^{\nu P-\bar{\nu}P}$  with color coefficient  $[C_F - C_A/2]$

• e.g.

$$\begin{aligned}
\delta C_{2,3}^{\text{ns}} = & +a_s^2 C_F [C_F - C_A/2] \left( -\frac{4285}{96} - 122\zeta_3 + \frac{671}{9}\zeta_2 + \frac{128}{5}\zeta_2^2 \right) \\
& + a_s^3 C_F [C_F - C_A/2]^2 \left( \frac{1805677051}{466560} - \frac{2648}{9}\zeta_5 + \frac{10093427}{810}\zeta_3 - \frac{1472}{3}\zeta_3^2 \right. \\
& \quad \left. - \frac{7787113}{1944}\zeta_2 + \frac{55336}{9}\zeta_2\zeta_3 - \frac{378838}{45}\zeta_2^2 - \frac{8992}{63}\zeta_2^3 \right) \\
& + a_s^3 C_F^2 [C_F - C_A/2] \left( -\frac{5165481803}{1399680} + \frac{40648}{9}\zeta_5 - \frac{9321697}{810}\zeta_3 + \frac{1456}{3}\zeta_3^2 \right. \\
& \quad \left. + \frac{8046059}{1944}\zeta_2 - 4984\zeta_2\zeta_3 + \frac{798328}{135}\zeta_2^2 - \frac{56432}{315}\zeta_2^3 \right) \\
& + a_s^3 n_f C_F [C_F - C_A/2] \left( \frac{20396669}{116640} - \frac{1792}{9}\zeta_5 + \frac{405586}{405}\zeta_3 - \frac{139573}{486}\zeta_2 \right. \\
& \quad \left. + \frac{1408}{9}\zeta_2\zeta_3 - \frac{50392}{135}\zeta_2^2 \right).
\end{aligned}$$

## Results in $x$ space

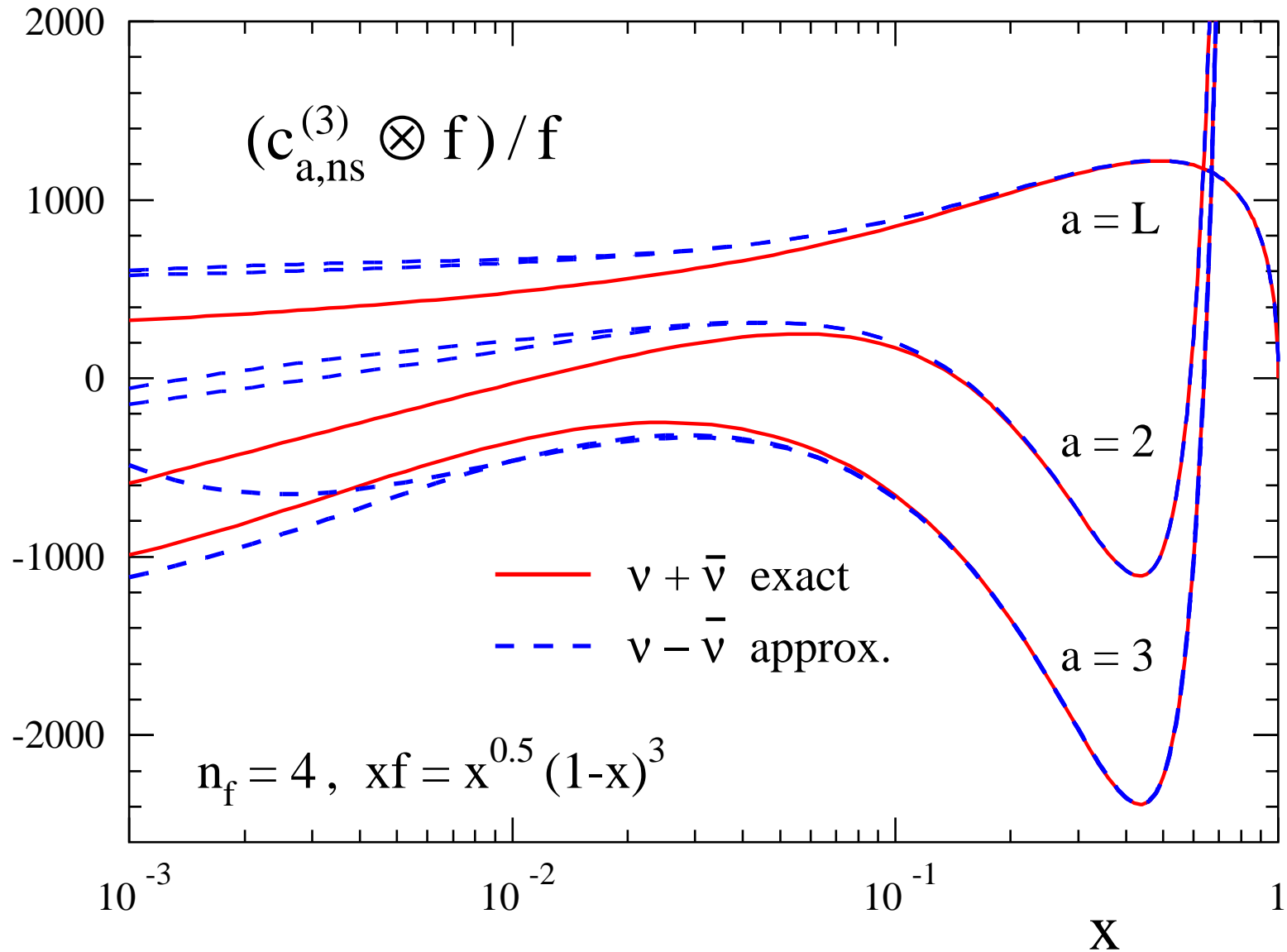
$$\begin{aligned} \delta c_{3,A}^{(3)}(x) &= (3.216 L_1^2 + 44.50 L_1 - 34.588) x_1 + 98.719 L_0^2 + 2.6208 L_0^5 \\ &\quad - n_f \{ (0.186 L_1 + 61.102 (1 + x)) x_1 + 122.51 x L_0 - 10.914 L_0^2 \\ &\quad - 2.748 L_0^3 \} , \end{aligned}$$

$$\begin{aligned} \delta c_{3,B}^{(3)}(x) &= -(46.72 L_1^2 + 267.26 L_1 + 719.49 x) x_1 - 171.98 L_0 + 9.470 L_0^3 \\ &\quad + n_f \{ (0.8489 L_1 + 67.928 (1 + \frac{x}{2})) x_1 + 97.922 x L_0 - 17.070 L_0^2 \\ &\quad - 3.132 L_0^3 \} , \end{aligned}$$

where

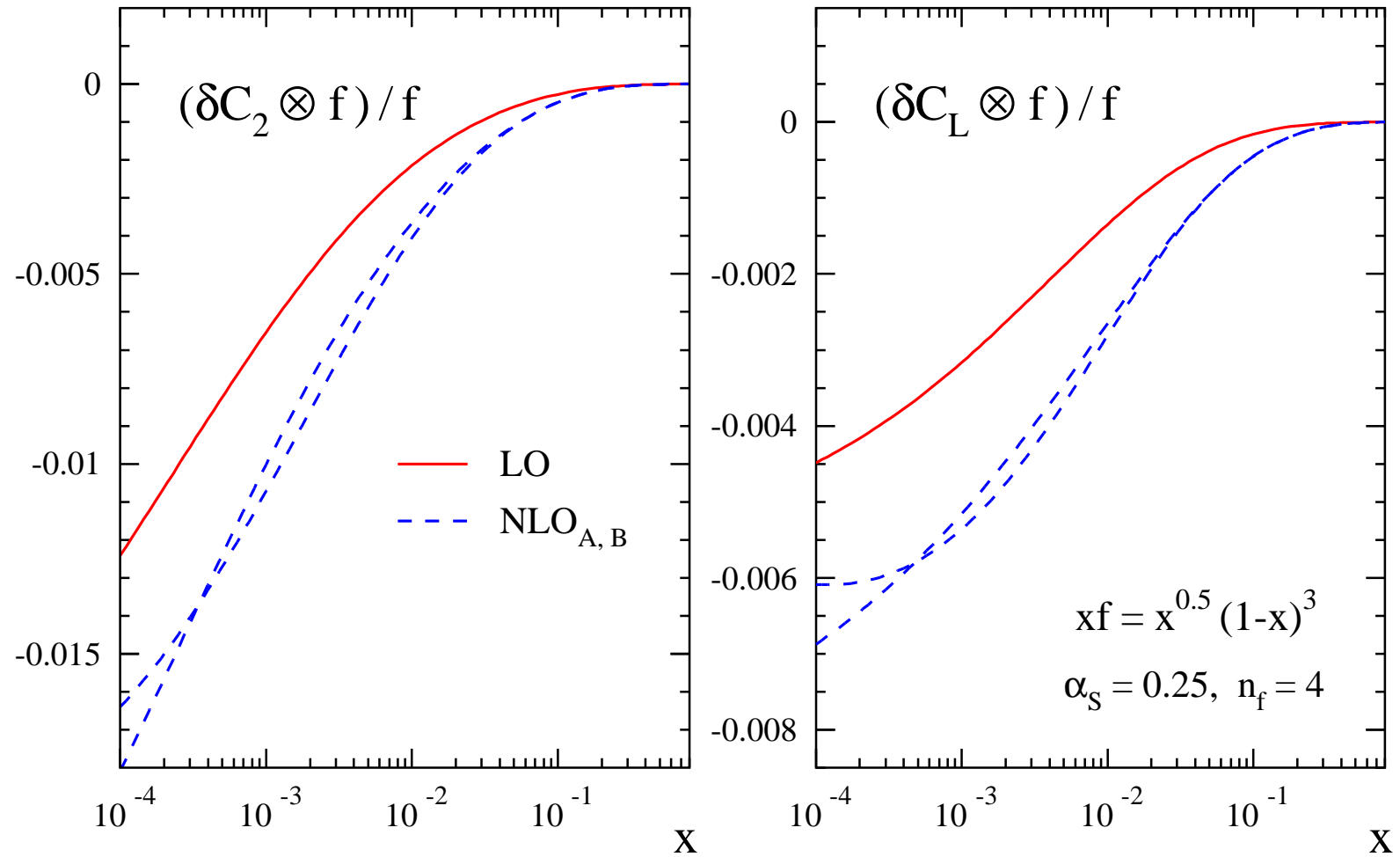
$$L_0 = \ln(x), \quad x_1 = (1 - x), \quad L_1 = \ln(x_1) .$$

# Convolution of the $\alpha_s^3$ order CC coefficient functions





# LO and NLO of the differences for $F_2$ and $F_L$ in CC DIS



## ● NuTeV experiment - Paschos-Wolfenstein relation

- Exact relation for massless quarks and isospin zero target

Paschos, Wolfenstein '73, Llewelin Smith '83

$$R^- = \frac{\sigma(\nu_\mu N \rightarrow \nu_\mu X) - \sigma(\bar{\nu}_\mu N \rightarrow \bar{\nu}_\mu X)}{\sigma(\nu_\mu N \rightarrow \mu^- X) - \sigma(\bar{\nu}_\mu N \rightarrow \mu^+ X)} = \frac{1}{2} - \sin^2 \theta_W$$

- measurement of  $\sin^2 \theta_W$  NuTeV '01 with large deviations from Standard model expectations

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## ● QCD corrections to the Paschos-Wolfenstein relation

- second moments of valence PDFs  $q^- = \int dx x(q - \bar{q})$
- expansion in isoscalar combination  $u^- + d^-$

Davidson, Forte, Gambino, Rius, Strumia '01; Dobrescu, Ellis '03; Moch, McFarland '03

$$R^- = \frac{1}{2} - \sin^2 \theta_W + \left[ 1 - \frac{7}{3} \sin^2 \theta_W + \frac{8\alpha_s}{9\pi} \{ 1 + \alpha_s 1.689 + \alpha_s^2 (3.661 \pm 0.002) \} \left( \frac{1}{2} - \sin^2 \theta_W \right) \right] \times \left( \frac{u^- - d^-}{u^- + d^-} - \frac{s^-}{u^- + d^-} + \frac{c^-}{u^- + d^-} \right)$$

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- expansion in isoscalar combination  $u^- + d^-$

Davidson, Forte, Gambino, Rius, Strumia '01; Dobrescu, Ellis '03; Moch, McFarland '03

$$R^- = \frac{1}{2} - \sin^2 \theta_W + \left[ 1 - \frac{7}{3} \sin^2 \theta_W + \frac{8\alpha_s}{9\pi} \{ 1 + \alpha_s 1.689 + \alpha_s^2 (3.661 \pm 0.002) \} \left( \frac{1}{2} - \sin^2 \theta_W \right) \right] \times \left( \frac{u^- - d^-}{u^- + d^-} - \frac{s^-}{u^- + d^-} + \frac{c^-}{u^- + d^-} \right)$$

- main uncertainties in  $s^-$

Martin, Roberts, Stirling, Thorne '04; Lai, Nadolsky, Pumplin, Stump, Tung, Yuan '07

- QCD corrections **under control, relevant** Moch, M. R., Vogt '07

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- New results for fixed Mellin moments at order  $\alpha_s^3$

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- and differences “even-odd”

▲ **interesting**: OPE based moments

$$[F_{2,L}^{\nu p - \bar{\nu} p} - 1, 3, 5, \dots ; F_3^{\nu p - \bar{\nu} p} - 2, 4, 6, \dots ]:$$

weight  $w$  of zeta functions up to  $2l - 1$  ( $l$  - number of loops)

$$\text{Intermediate integer } N [F_{2,L}^{\nu p + \bar{\nu} p} - 1, 3, 5, \dots ; F_3^{\nu p + \bar{\nu} p} - 2, 4, 6, \dots ]:$$

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⇒ sufficient for HERA-CC,  $\nu$  - DIS (e.g. **Alekhin** makes use of it)