DESY Theory Workshop 2007
26 September 2007

QCD: Phases and T_c

Akira Ukawa
Center for Computational Sciences
University of Tsukuba

- Overview
- Finite-temperature phases and Tc
- Dynamical look into QGP
- Future Prospects

Overview

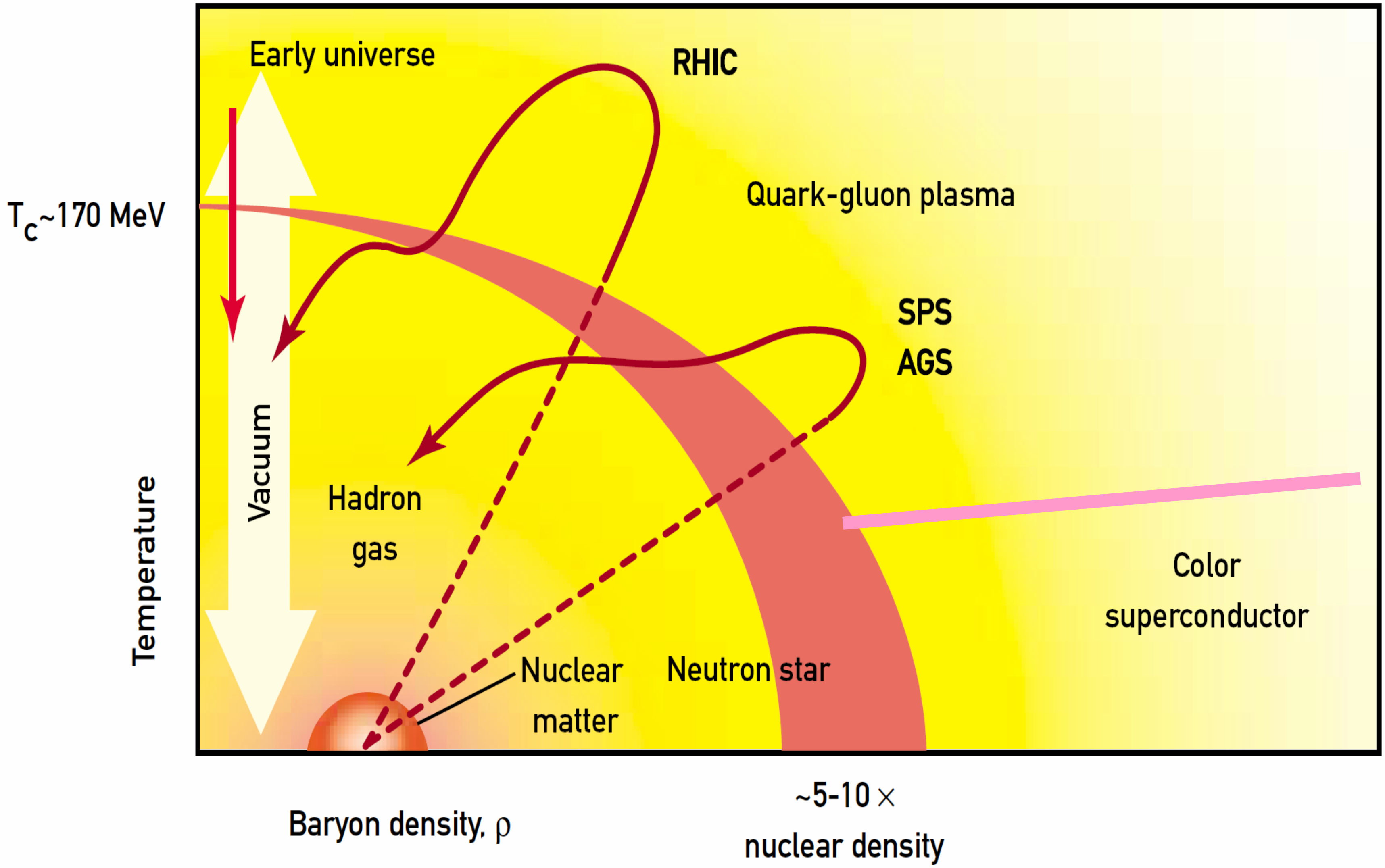
Now is a very interesting period for hot QCD:

- Experimental input from RHIC
  - Strong correlation above Tc
- Analytical insights from string theory
  - Thermodynamics and transport of QCD-like theories
- Progress in lattice QCD itself
  - Full QCD simulations with physical up, down, and strange quark masses using multi-time step algorithm
  - Simulations with chirally symmetric lattice quark action using domain wall or overlap fermion method (so far mostly at T=0)

This review:

- Finite-temperature phases and Tc
  - Phase transition for physical up, down and strange quark masses?
  - Tc
  - Equation of state
- Dynamics of the QGP phase
  - Fate of charmonium above Tc
  - Transport
- Future prospects
  - Simulations with chiral symmetry

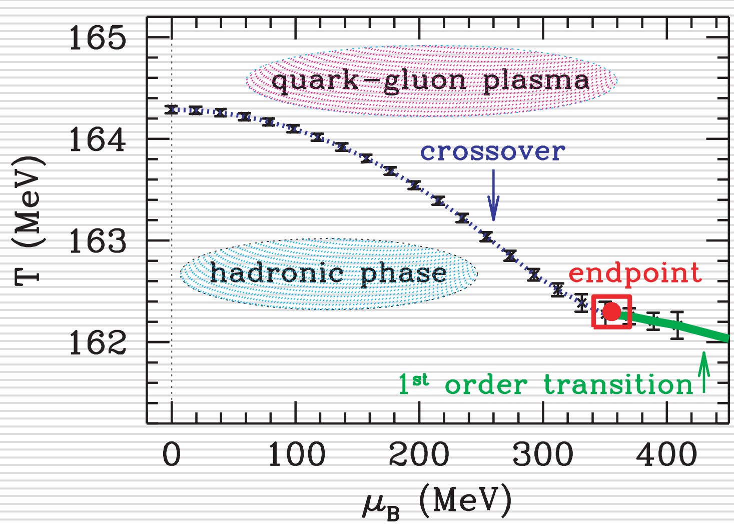
Schematic QCD phase diagram



Comments on finite-density QCD

- The *"sign problem"*, i.e., large phase fluctuation of the quark determinant detD for non-zero density

$$Z_{QCD} = \int \prod dU_{n\mu} \det D[U] \exp\left(-S_{gluon}[U]\right)$$



2-parameter reweigting method:
Z. Fodor, S. Katz, JHEP 0404 (2004) 050
Nf=2+1, Nt=4

- Slow but steady progress over the years for *not too large baryon density:*
  - Estimate of the end point of the 1st order line on the T-μ plane

$$(T_E, \mu_E) = (162 \pm 2, 360 \pm 40) MeV$$

Taylor expansion method:
C. Allton etal, Phys.Rev. D71 (2005) 054508
Nf=2, Lt=4

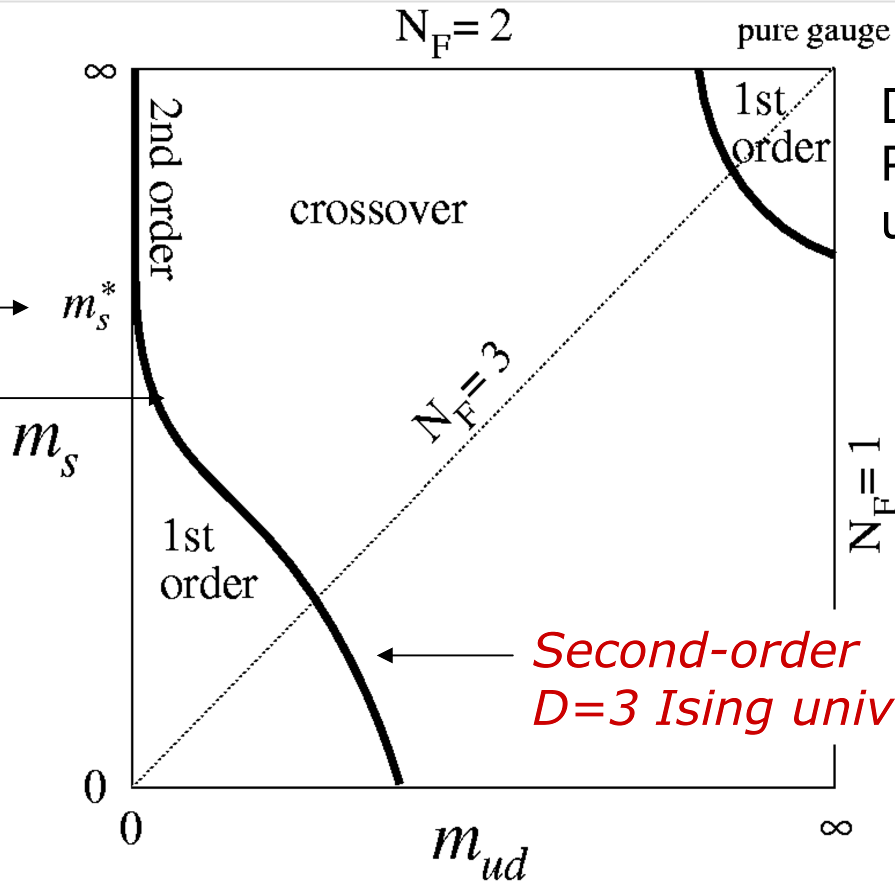
- Still no real prospect for large baryon number density

Finite-temperature phases and Tc at μ_B=0

$$N_f = 2+1\,QCD$$

Tricritical point

$$m_{ud} \propto (m_s^* - m_s)^{5/2}$$



D=3 Z(3) Potts universality

Second-order D=3 Ising universality

Consistent with simulations so far;need full confirmation. And where is the physical point?

Phase transition analyses

- Long-standing issue addressed by a large number of simulations since mid 1980's
  - Bielefeld group, MILC Collaboration, JLQCD Collaboration etc
- Significant methodological developments over the years
  - finite-size scaling techniques
  - incorporation of dynamical quarks, etc
- Recent work employs dynamical staggered up, down, strange quark with realistically small quark masses
  - Wuppertal group, Y. Aoki, G. Endrodi, Z. Fodor, S.D. Katz, K. Szabo,
    - Nature 443 (2006) 675-678;
    - Phys.Lett. B643 (2006) 46-54;
    - JHEP 0601 (2006) 089
  - Bielefeld-RBRC-BNL Collaboration, M. Cheng et al,
    - Phys.Rev. D75 (2007) 034506 ,
    - Phys.Rev. D74 (2006) 054507

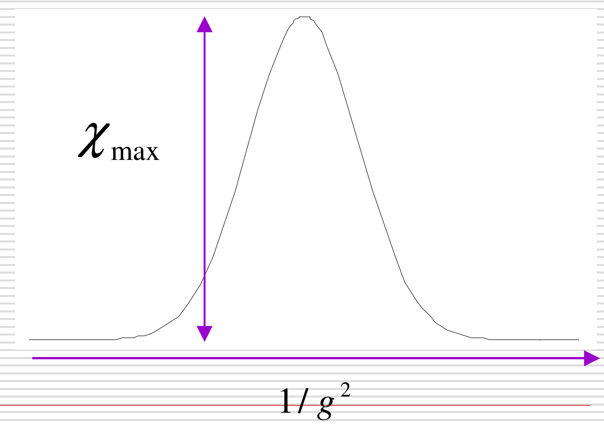
strategy of the Wuppertal Group

Y. Aoki et al, Nature 443 (2006) 675-678

- Concentrate at the physical point
  - Full QCD simulation using staggered quark action for up, down, and strange quark
  - Quark masses tuned the physical values with a separate T=0 simulation
  - Vary coupling $1/g^2(a)$ to change temperature $T=1/N_t a$ and locate the transition
  - Finite-size scaling analysis of susceptibilities to identify the nature of transition

$$\chi = \int dx \left(\langle \bar{q}q(x)\bar{q}q(0) \rangle - \langle \bar{q}q(x) \rangle \langle \bar{q}q(0) \rangle \right)$$

$$\chi_{\max} \xrightarrow{V\to\infty} \begin{cases} \text{constant} & \text{crossover} \\ V^{\alpha} & \text{2nd order} \\ V & \text{1st order} \end{cases}$$

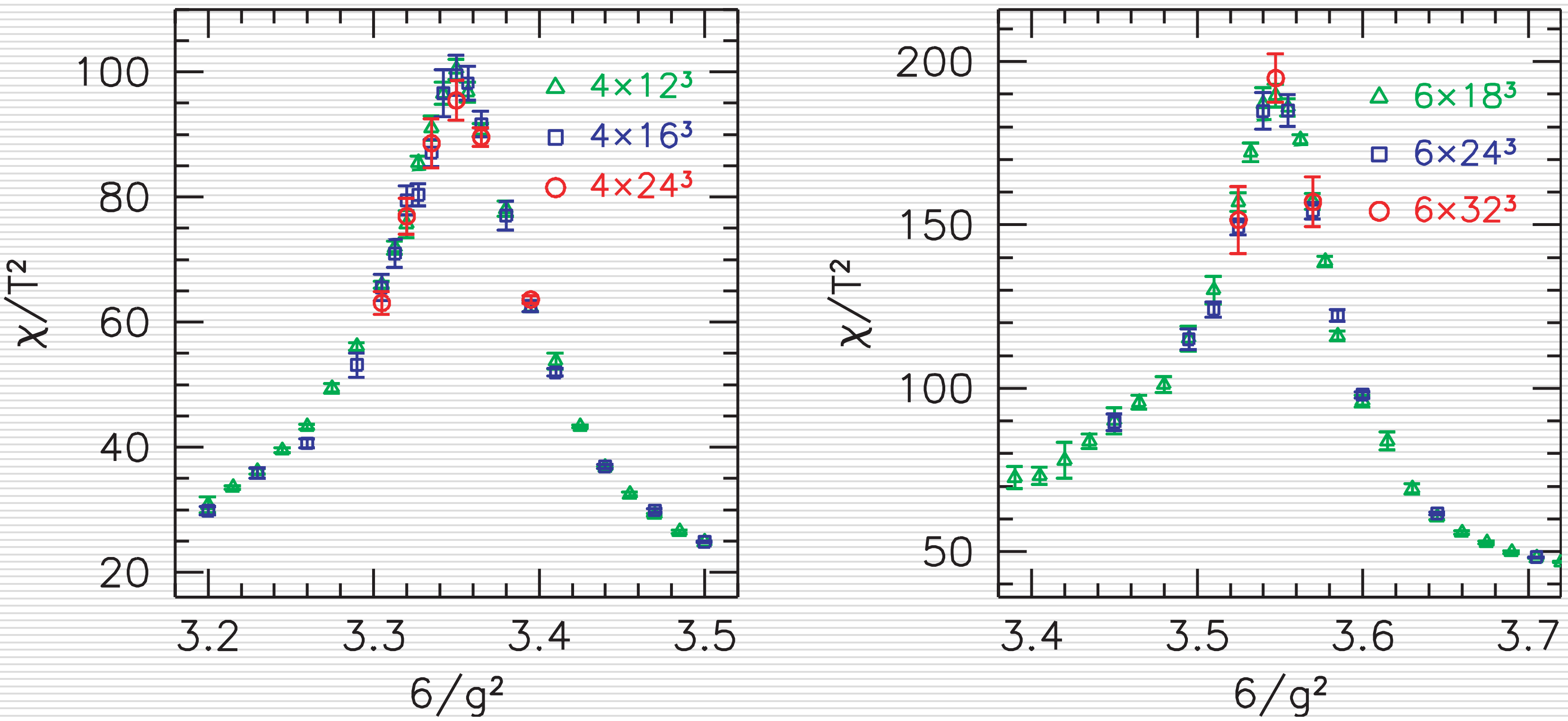


  - Continuum limit extrapolation

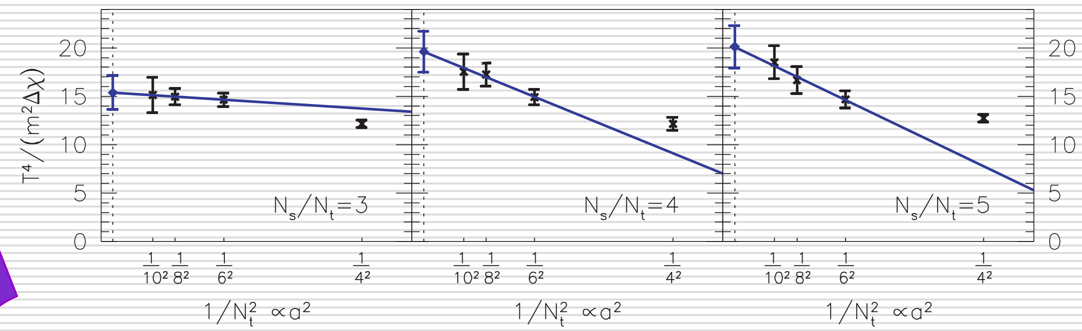
Results for chiral susceptibility

- Constant behavior for increasing volume shows a crossover at finite lattice spacing

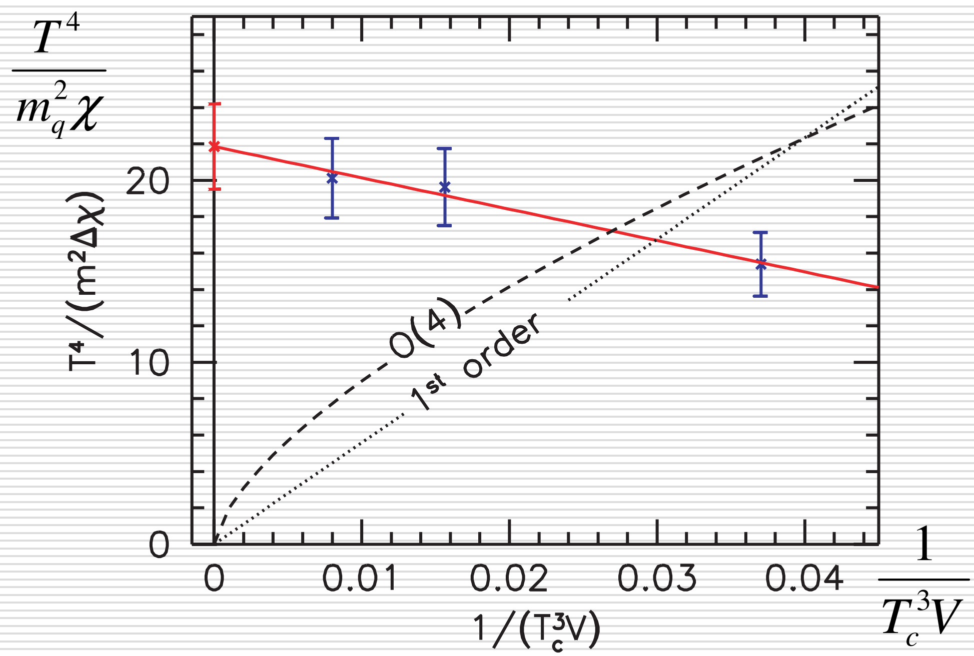
Y. Aoki et al, Nature 443 (2006) 675-678



Continuum limit extrapolation



Extrapolation to the continuum



Finite-size scaling in the continuum shows that the transition is *a crossover*, at least for the Staggered quark formalism

Comments

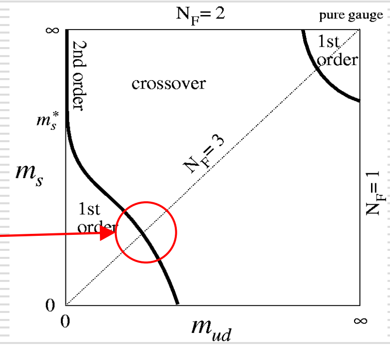
- Theoretical uncertainties with the staggered simulations
  - Only U(1)xU(1) chiral symmetry out of $SU(N_f)xSU(N_f)$
  - Fractional power of quark determinant $[detD(U)]^{Nf/4}$ to “adjust” the #flavor
- Does it converge to the correct QCD in the continuum limit?
  - OK perturbatively, but is it at the non-perturbative level?
  - Lots of discussions in the the community, not yet settled:
    - Lattice06
      S. Sharpe, “Rooted staggered fermions: good, bad, or ugly?”
    - Lattice07
      M. Creutz, “Why rooting fails”
      A. Kronfeld, “Lattice QCD with Staggered Quarks: Why, Where, and How”
- Clearly desirable to work with chiral action:
  - Domain-wall
  - Overlap

Much work and many simulations already done at T=0, so hot/dense QCD is the next natural target.

Nature of the 2nd order endline

- Existence of the endline well established around 2000

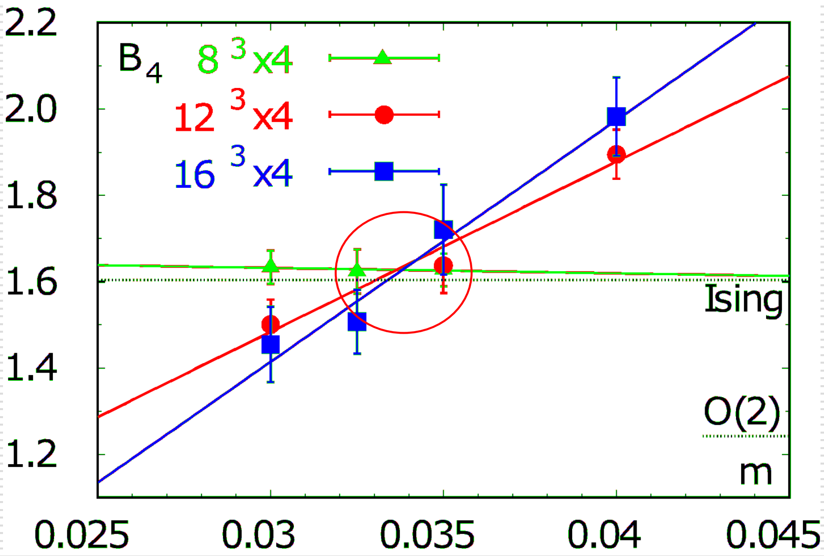
  JLQCD/Bielefeld/Columbia
- Binder cumulant test to distinguish universality class



$$B_4 = \frac{\left\langle (\delta\overline{\psi}\psi)^4 \right\rangle}{\left\langle (\delta\overline{\psi}\psi)^2 \right\rangle^2}$$

  - Clear evidence of Ising universality as predicted by S. Gavin et al

  S. Gavin et al
  Phys.Rev. D49 (1994) 3079-3082

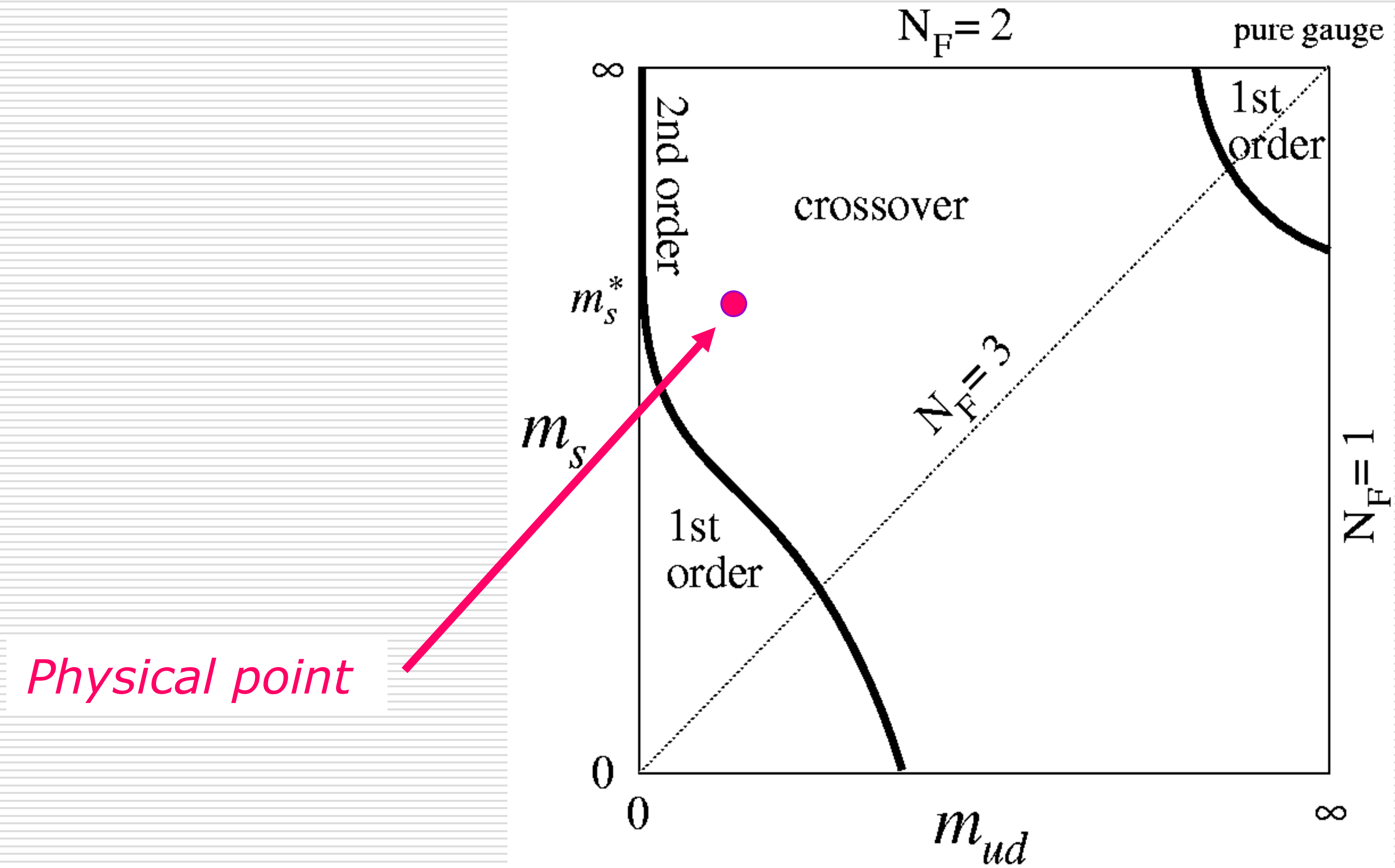


Karsch et al Phys.Lett. B520 (2001) 41-49

- Recent analyses confirm these results

  G. Endrodi etal, Lattice2007

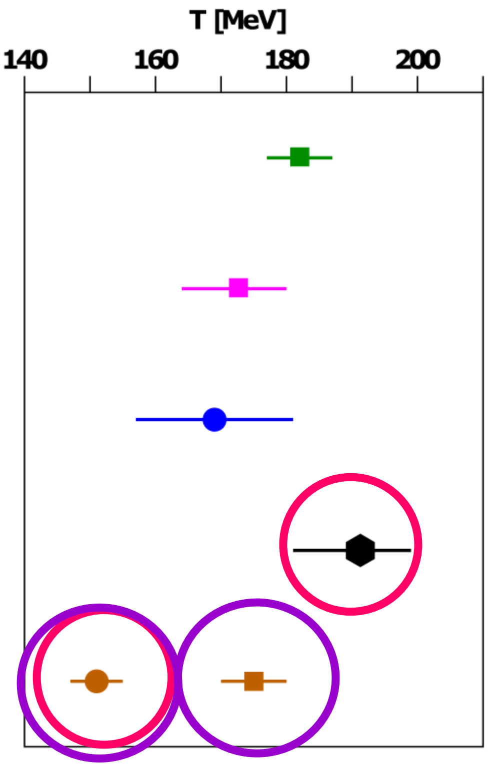
Phase diagram according to lattice QCD with staggered quark action



Transition temperature Tc at the physical point

Summary of recent results on T_c

use T=0 scale: r0=0.469fm



Nf=2:

V.G. Bornyakov et al, POS Lat2005, 157 (2006)
(improved Wilson, Nt=8, 10; input: r0=0.5 fm)
(added Nt=12, Lattice'07) (rescaled to r0)

Y. Maezawa et al., hep-lat/0702005 (QM'2006)
(improved Wilson, Nt=4, 6; input: m-rho)
(no cont. exp. yet)

Nf=2+1:

C. Bernard et al., Phys.Rev. D71, 034504 (2005)
(improved staggered (asqtad), Nt=4,6,8, input r1)
(rescaled to r0)

M. Cheng et al., Phys.Rev D74, 054507 (2006)
(improved staggered (p4), Nt=4,6; input r0)

Y. Aoki et al., Phys. Lett. B643, 46 (2006)
(staggered (stout), Nt=4,6,8,10; input fK)
(converted to r0)

● chiral ■ deconfinement ⬢ chiral+deconfinement

F. Karsch, Lattice 2007, Regensburg, July 2007 – p. 4/33

Summary
by F. Karsch
at Lattice07

Two notable features

- Disagreement using the same quantity
- Disagreement using different quantities

Disagreement using different quantities?

Y. Aoki etal Phys.Lett. B643 (2006) 46-54

- Chiral susceptibility

  $T_c = 151 \pm 3 \pm 3 MeV$

  $\Delta T_c = 28 \pm 5 \pm 1 MeV$

- Quark number susceptibility

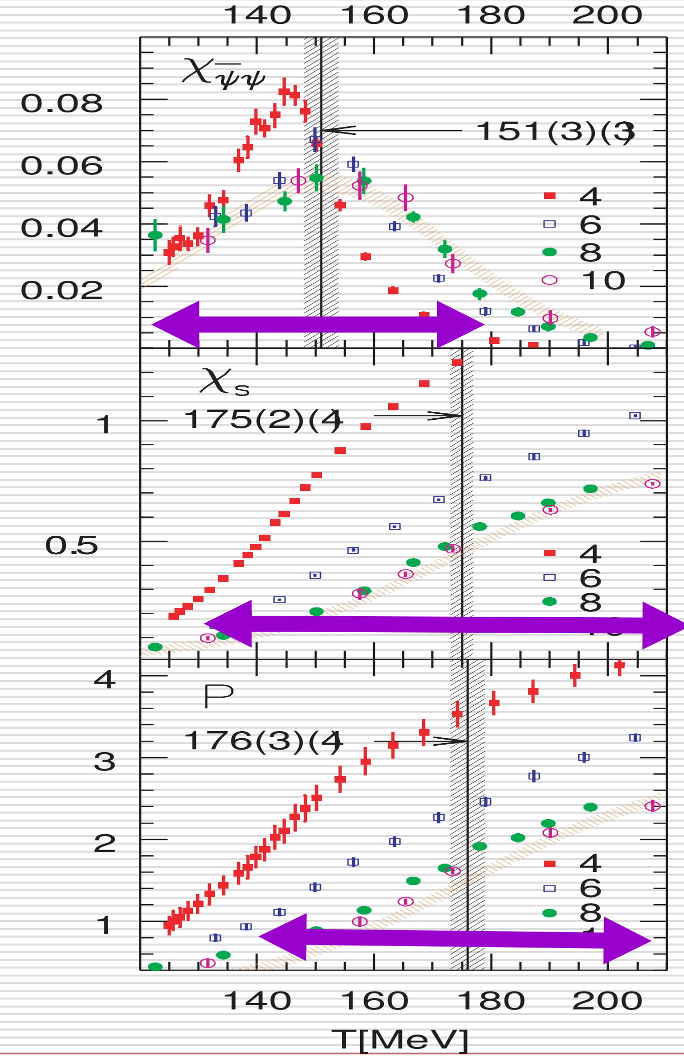
  $T_c = 172 \pm 2 \pm 4 MeV$

  $\Delta T_c = 42 \pm 4 \pm 1 MeV$

- Polyakov loop susceptibility

  $T_c = 176 \pm 3 \pm 4 MeV$

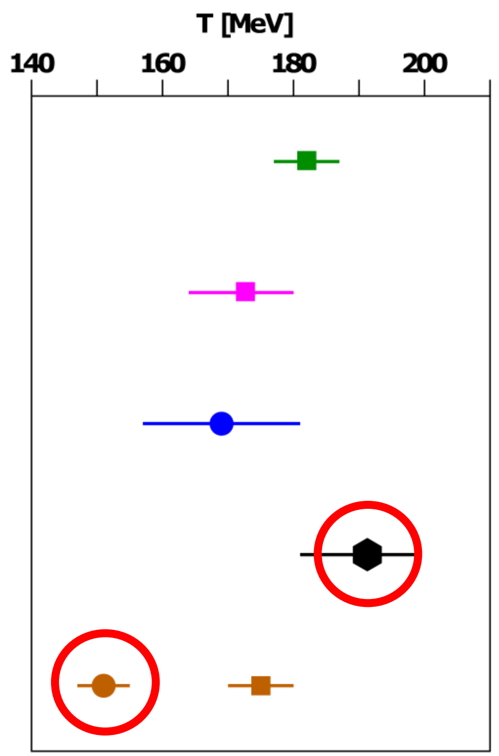
  $\Delta T_c = 38 \pm 5 \pm 1 MeV$



- Possible for a crossover
- Inflection point for χ_s and P do not seem clearcut . large and mutually overlapping width
- No cause for worry

Disagreement using the same quantity?

- Too different beyond error; has to be resolved
- Possible origin of the difference
  - Choice of action
    - Both staggered but improved in a different manner; “p4fat3” vs “stout-improved”
  - Scale setting
    - Using r_0 vs f_K
  - Temporal lattice size and continuum extrapolation
    - N_t =4, 6 vs 4,6,8,10



M. Cheng et al., Phys.Rev D74, 054507 (2006)
(improved staggered (p4), Nt=4,6; input r0)

Y. Aoki et al., Phys. Lett. B643, 46 (2006)
(staggered (stout), Nt=4,6,8,10; input fK)
(converted to r0)

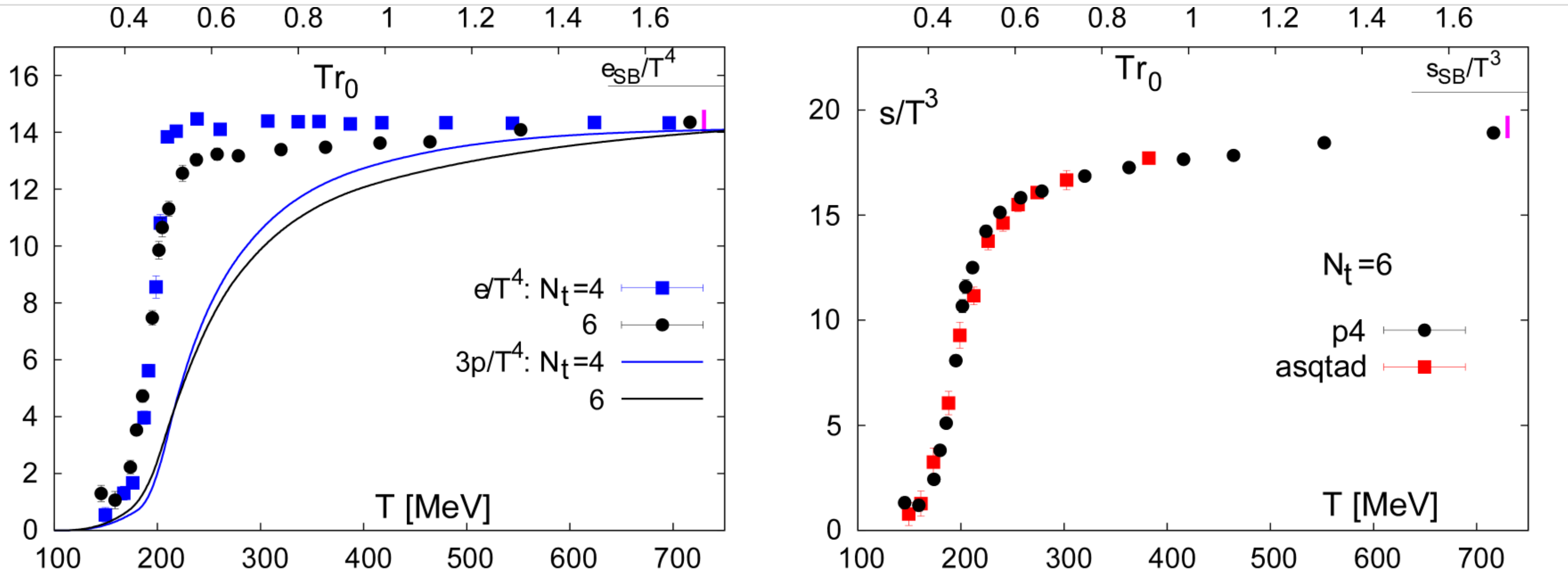
chiral deconfinement chiral+deconfinement

F. Karsch, Lattice 2007, Regensburg, July 2007 – p. 4/33

Further simulations being undertaken in USA to settle the difference, so we wait…

Equation of state

- Recent results from MILC and Bielefeld-RBRC
- Agreement between the two groups; qualitatively no change since previous results



p4: RBC-Bielefeld, preliminary

asqtad: C. Bernard et al., PRD75, 094505 (2007)

entropy density: p4 vs. asqtad

F. Karsch, Lattice 2007, Regensburg, July 2007 – p. 20/33

Dynamical look into QGP

- Non-trivial task for lattice QCD since real-time information needed
- Spectral density $\rho(\omega)$ used for extracting several quantities
  - Excitation spectrum
  - Transport coefficients via Kubo formula
- In practice,

$$G_E(\tau) \equiv \langle O_E(\tau) O_E(0) \rangle = \int_0^\infty \frac{d\omega}{2\pi} \frac{\cosh\left[\omega\left(\tau - \frac{1}{2T}\right)\right]}{\sinh\left[\frac{\omega}{2T}\right]} \rho(\omega)$$

Imaginary time Green's function directly calculated on the lattice

- assume a functional form or
- parameter free inference methods such as the Maximum entropy method

Charmonium above Tc (I)

- Important probe since the early days of QGP studies as pointed out by Matsui-Satz
- Quenched QCD simulations indicate

  J/Psi, η_c survive up to 1.5Tc, but χ_c states melt just above Tc

  Asakawa Hatsuda, PRL92 (2004) 012001

  Datta, Karsch, Petreczky, Wetzorke, PRD69 (2004) 094507

  Jakovac, Petreczky, Petrov, Velvtsky, PRD75 (2007) 014506

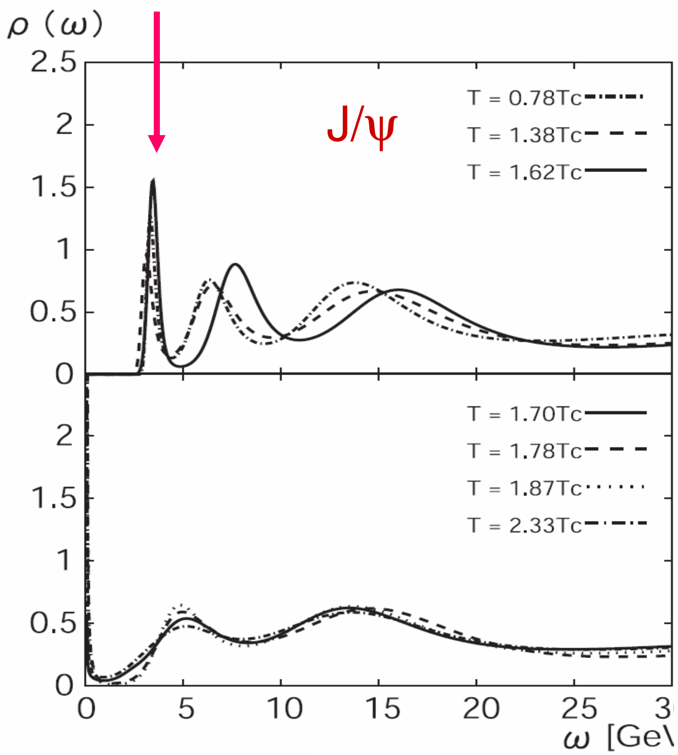
- Recent progress
  - Full QCD?
  - Constant mode and orbital states

MEM results for charmonium above Tc in quenched QCD

Asakawa & Hatsuda, PRL92 (2004) 012001

anisotropic lattice, 32^3 x (96-32)

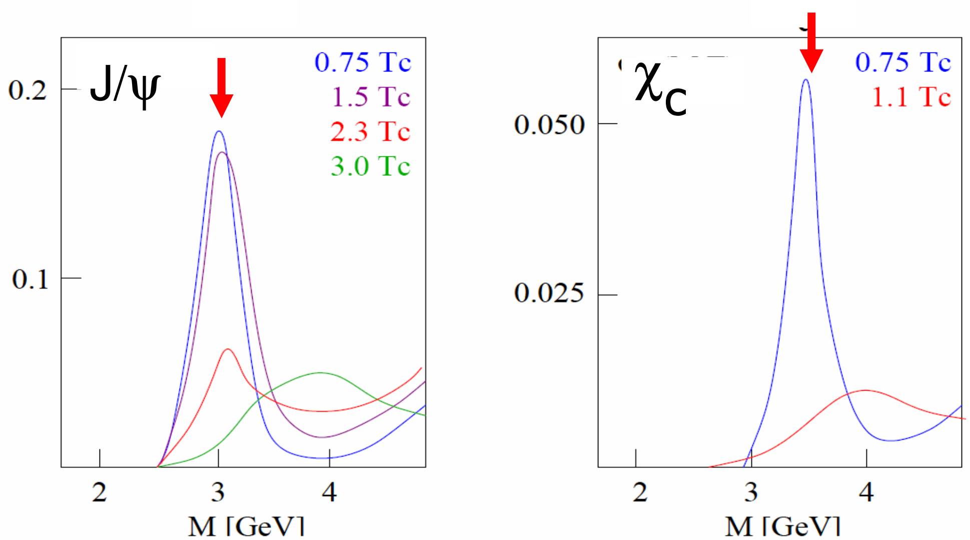
ξ=4.0, a_t=0.01 fm, (L_s=1.25fm)



Datta, Karsch, Petreczky & Wetzorke, PRD69 (2004) 094507

isotropic lattice, 48^3 x(24-12),

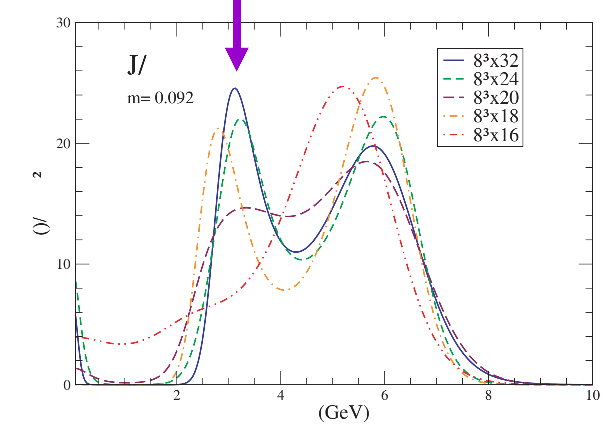
a=0.04 fm (L_s=1.9 fm)



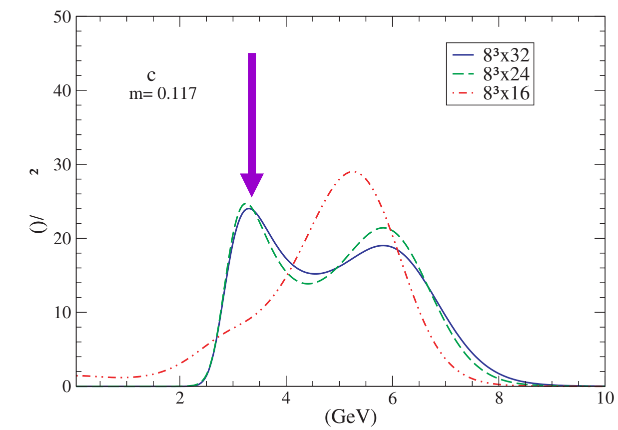
Charmonium results for 2-flavor QCD

Aarts, Allton, Bugrahan, Peardon, Skullerud, hep-lat/0705.2198

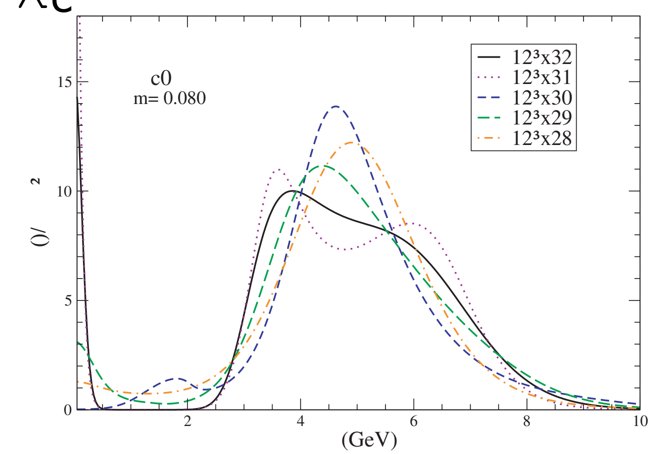
J/ψ $T_c \le T \le 2T_c$



η_c $T_c \le T \le 2T_c$



χ_c $T = (1.1 - 1.2)T_c$

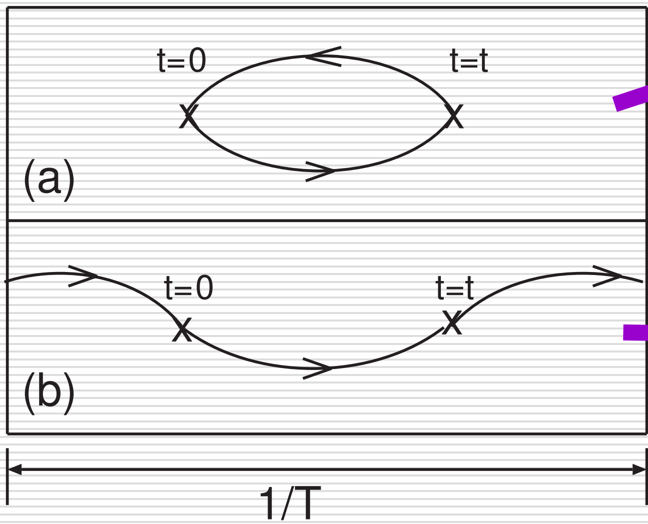


- J/ψ and η_c states survive at least to T $2T_c$
- χ_c states melt just above T_c

As in quenched QCD, however,…

Constant mode at finite T

T. Umeda, Phys.Rev.D75:094502,2007



$$\propto \exp(-m_q\tau)\cdot\exp(-m_q\tau)$$
$$=\exp(-2m_q\tau)$$

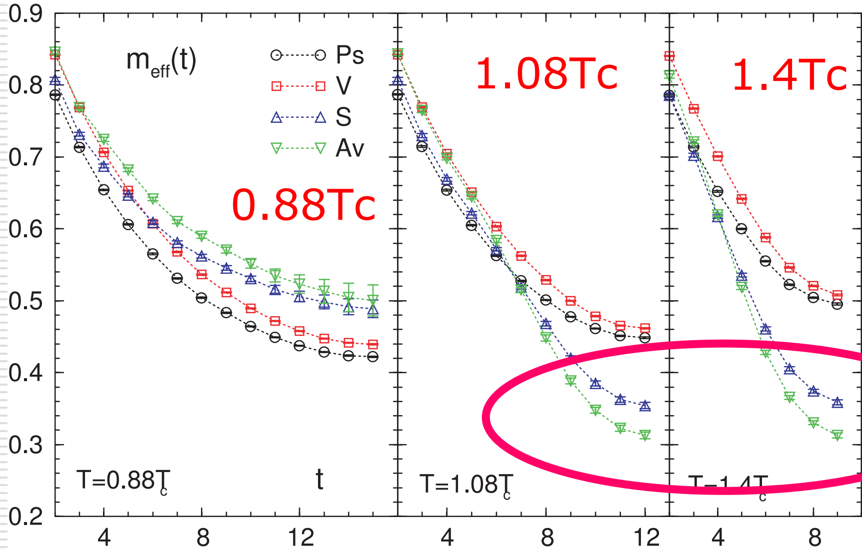
usual contribution
decaying exponentially in time

$$\propto \exp(-m_q\tau)\cdot\exp(-m_q(1/T-\tau))$$
$$=\exp(-m_q/T)$$

"wrap-around" contribution *constant in time*

- There is a constant contribution in meson correlators at finite T
- This needs to be subtracted for proper spectral analyses

An illustration in quenched QCD



$$m_{eff}(\tau) = -\log \frac{C(\tau+1)}{C(\tau)}$$

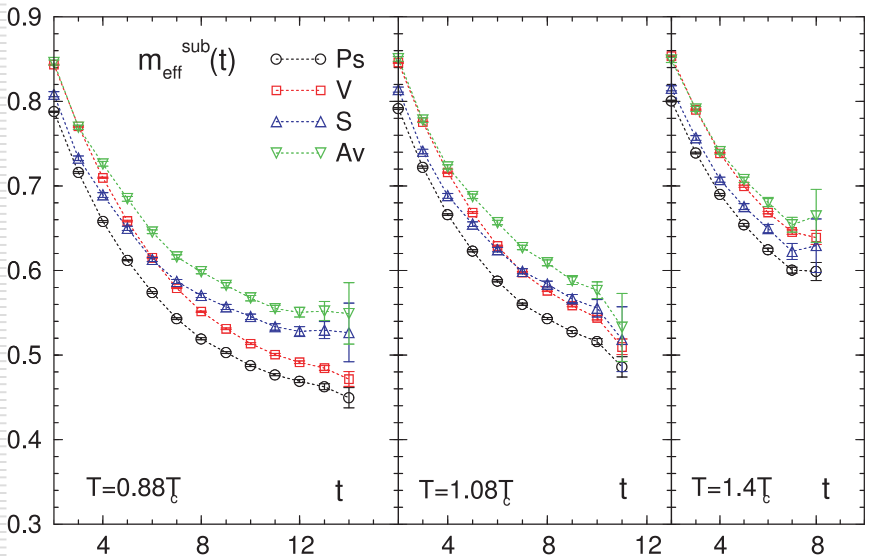
J/ψ

η_c

χ_c

Without subtraction

May erroneously conlude that p-wave states do not exist above Tc



With subtraction

Behavior of p-wave states significantly changed

Transport coefficients

- Very limited work over the years
  - Karsch, Wylde, PRD35 (1987)2518
  - S. Gupta, PLB597(2004)57
  - Nakamura, Sakai, PRL94 (2005)072305
  - Aarts, Allton. Foley, Hands, Kim, PRL99(2007)022002
  - H. B. Meyer, hep-lat/0704.1801
- resurgence of interest due to RHIC experiment

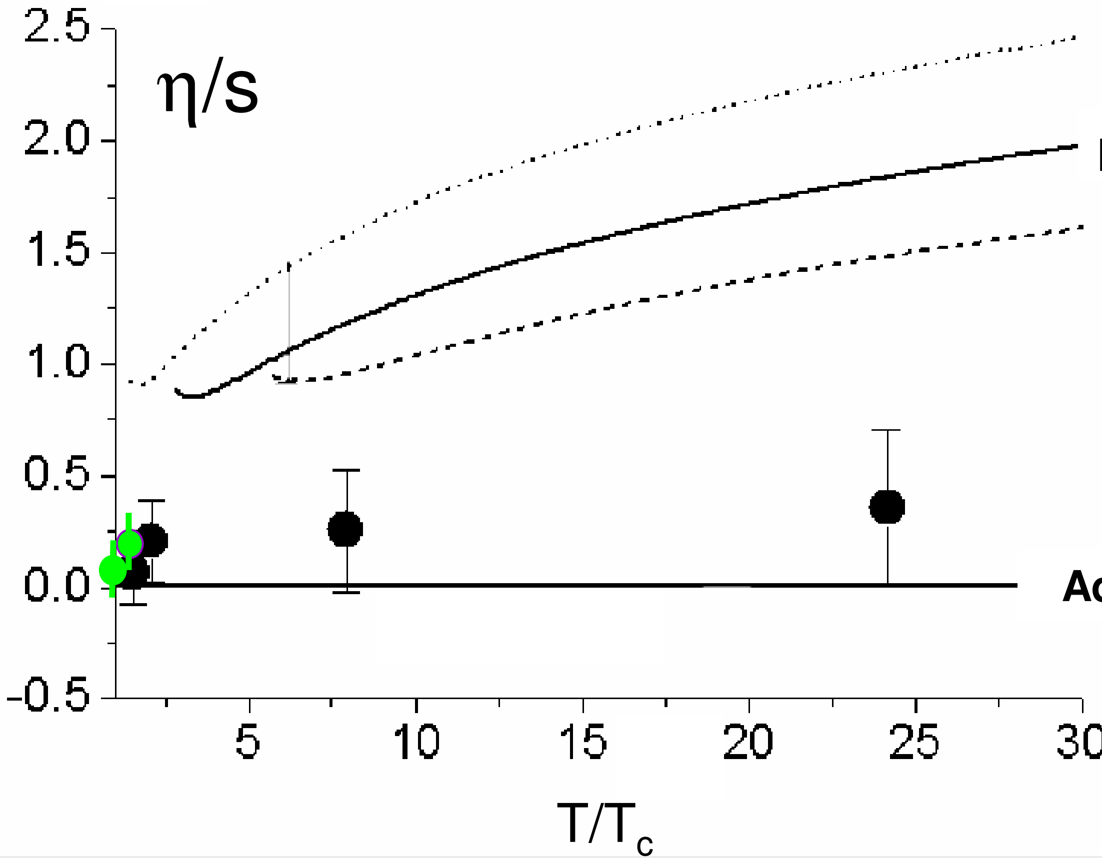
  e.g.,

  $$\lambda \approx \frac{\eta}{sT} \quad \text{mean free path} \qquad R \approx \left(\frac{\eta + \varsigma}{s} \frac{1}{T} \frac{1}{\tau} \right)^{-1} \quad \text{Reynolds number}$$

  η: shear viscosity, s: entropy density
- Application of the Kubo formula

$$\eta = \lim_{\omega \to 0} \frac{\rho_{12,12}(\omega, 0)}{2\omega} \qquad \rho_{\mu\nu,\rho\sigma}(\omega, \vec{p}) = \int d^4 x \exp(ip \cdot x) \langle T_{\mu\nu}(x) T_{\rho\sigma}(0) \rangle$$

Quenched result for shear viscosity



Baym, Monien, Pethick, Ravenhall, PRL64 (1990)1867.
Arnold, Moore & Yaffe, JHEP 0305(2003)051.

Kovtun, Son & Starinets, PRL94(2005)111601

● 24^3x8 Nakamura, Sakai, hep-lat/0510100

● 20^3x8,28^3x8 H. Myer, hep-lat/0704.1801

Future prospects

- Conditions for realistic lattice QCD simulations
  - ✓ Large enough volume
  - ✓ Small enough lattice spacing
  - ✓ Dyamical up, down, strange quark *with physical masses*
  - ◆ quark action with exact chiral symmetry

- ◆ Bulk of finite-temperature simulations still done with the *staggered* and *Wilson* fromalism which do not have exact chiral symmetry

- However, much progress since around 2000 with exactly chiral fermion simulations

Lattice fermion with exact chiral symmetry

- Theoretically based on the Ginsparg-Wilson relation:

$$D\gamma_5 + \gamma_5 D = 2aDR\gamma_5 D$$

  - Domain-wall fermion Kaplan('92)/Furman-Shamir('94)
  - Overlap formalism Neuberger-Narayanan('92,'97)
  - Fixed point action Hasenfratz-Neidermyer('94)

- Avoids the Nielesen-Ninomiya Theorem by using "infinitely" many fields , e.g.,

  Introduction of 5th dimension (domain-wall)

  Singular quark operator (overlap)

  hence needs more computer power

- Presently concentrated on T=0 simulations:
  - Current full QCD runs with O(10) Tflop computers

domain wall	24^3x48 m_{ud} $m_s/10$	RBRC-UKQCD	QCDOC
Overlap	16^3x32 m_{ud} $m_s/10$	JLQCD	BlueGene/L

- Next target should be hot/dense QCD simulations

conclusions

- *Very interesting period when experiment, theory, and simulations begin to meet*
- *Physical results with realistically light quark spectrum begin to be obtained for the bulk thermodynamical quantities*
  - *Phase diagram, Tc, equation of state, ...*
- *Effort toward dynamics spurred by RHIC experiement; will continue to increase*
- *Hope for exact chiral fermion simulations seem realistic in the near future*