Large N – the view from the lattice

Michael Teper (Oxford)

DESY 2007

Preamble

Is $N = 3$ close to $N = \infty$? Is large-N confining?

The closed string spectrum in $D = 3$ and $D = 4$

also (maybe)

Hot SU(N) gauge theory

k-strings

Topology and interlacing θ-vacua

$D = 3$: comparing with Karabali-Nair

Twisted Eguchi-Kawai : space-time reduction

'Physical' lattice strong coupling

- ‘Oxford’ group’

Bringoltz, Bursa, Liddle, Lucini, Meyer, Teper, Vairinhos, Wenger, ...

some other groups:

- ‘Pisa’

Del Debbio, Panagopoulos, Rossi, Vicari, ...
Campostrini, ...

- ‘Rutgers’

Narayanan, Neuberger, ...

- ‘Torino’

D’Adda, Caselle, Gliozzi, Hasenbusch, Panero, Rago, ...

- calculating masses from Euclidean correlators:

$\Phi(t)$ a gauge invariant operator

$$\langle \Phi^\dagger(t = an_t)\Phi(0)\rangle = \sum_i |c_i|^2 e^{-aE_i n_t} \stackrel{t\to\infty}{\simeq} |c|^2 e^{-man_t}$$

where am is lightest mass with quantum numbers of Φ in lattice units

- continuum limit :

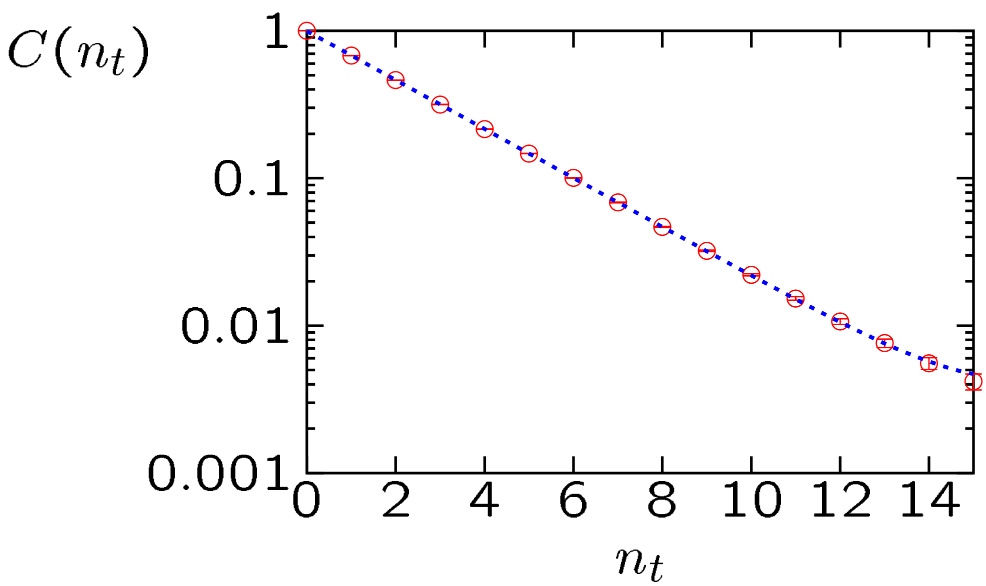
$$\frac{am(a)}{a\sqrt{\sigma(a)}} = \frac{m(a)}{\sqrt{\sigma(a)}} = \frac{m(0)}{\sqrt{\sigma(0)}} + c_0 a^2\sigma + O(a^4)$$

- large N limit :

$$\frac{m(N)}{\sqrt{\sigma(N)}} = \frac{m(\infty)}{\sqrt{\sigma(\infty)}} + \frac{c}{N^2} + O\left(\frac{1}{N^4}\right)$$

Can we do accurate calculations?

SU(3), 32^4, $a \simeq 0.046$ 'fm'

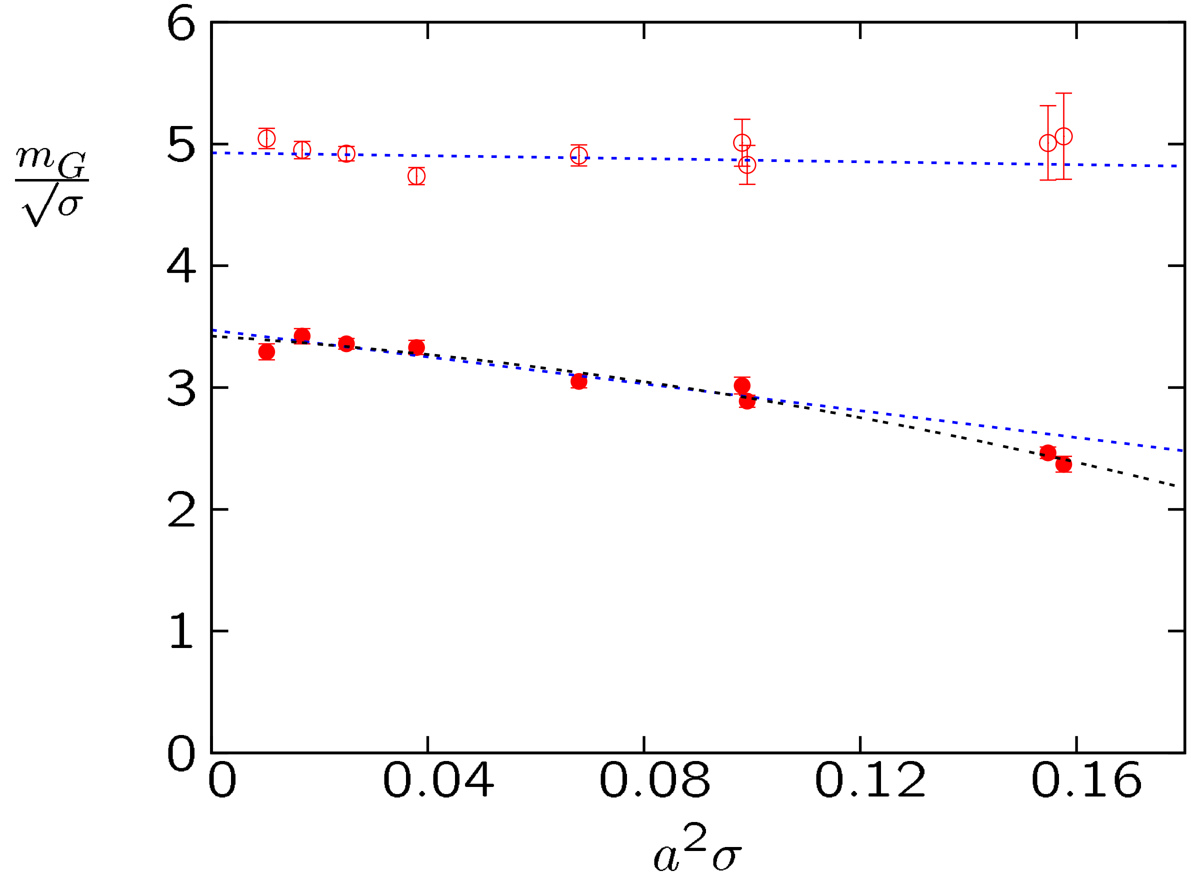


$$C(t = an_t) \stackrel{t\uparrow}{\simeq} |c|^2 e^{-man_t}$$

$\Rightarrow$

fit : $am_{0^{++}} = 0.330(7)$

Continuum limit mass spectrum: SU(3)



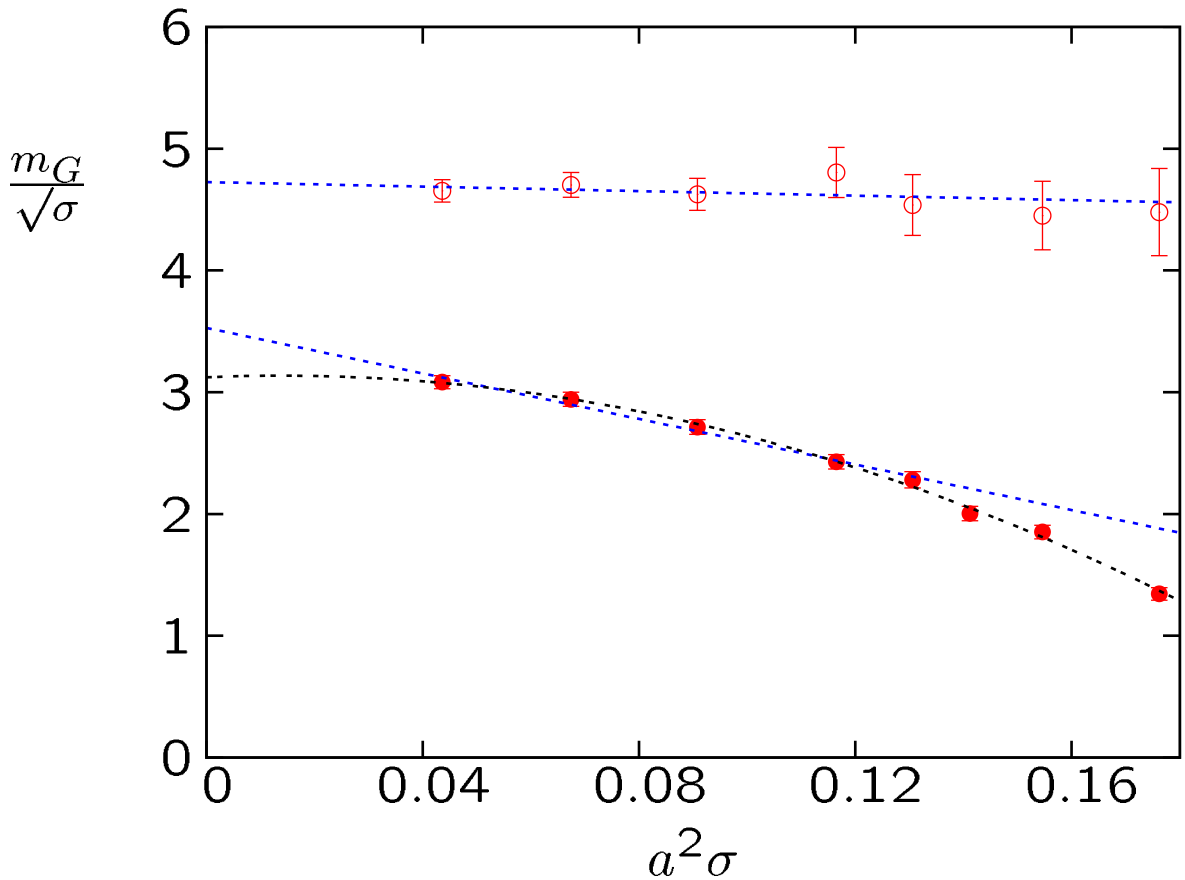
$O(a^2)$ continuum extrapolations:

$$\frac{m_{0^{++}}}{\sqrt{\sigma}} = 3.47(4) - 5.52(75)a^2\sigma$$

$$\frac{m_{2^{++}}}{\sqrt{\sigma}} = 4.93(5) - 0.61(1.36)a^2\sigma$$

$O(a^4)$ continuum extrapolation very similar

Continuum limit mass spectrum: SU(8)



$O(a^2)$ continuum extrapolation:

$$\frac{m_{0^{++}}}{\sqrt{\sigma}} = 3.53(8) - 9.3(1.0)a^2\sigma$$

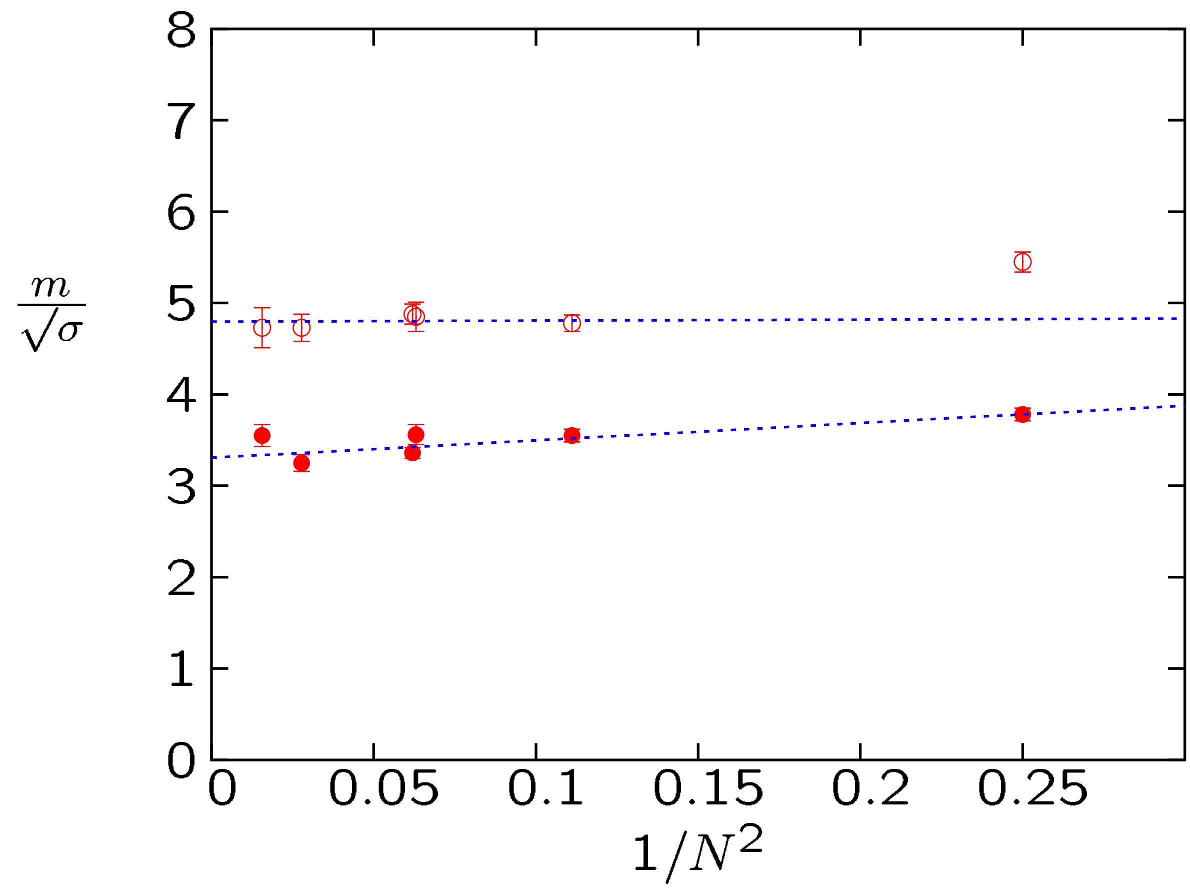
$O(a^4)$ extrapolation

$$\frac{m_{0^{++}}}{\sqrt{\sigma}} = 3.13(25) + 1.66a^2\sigma - 66.0(a^2\sigma)^2$$

this systematic error $\sim 5 \pm 3\times$ naive $O(a^2)$ statistical error !

Mass spectrum: large-N limit

B.Lucini, M.Teper, U.Wenger: hep-lat/0404008



$O(1/N^2)$ extrapolations to $N = \infty$:

$$\frac{m_{0^{++}}}{\sqrt{\sigma}}\Big|_N = 3.31 + \frac{1.90}{N^2}$$

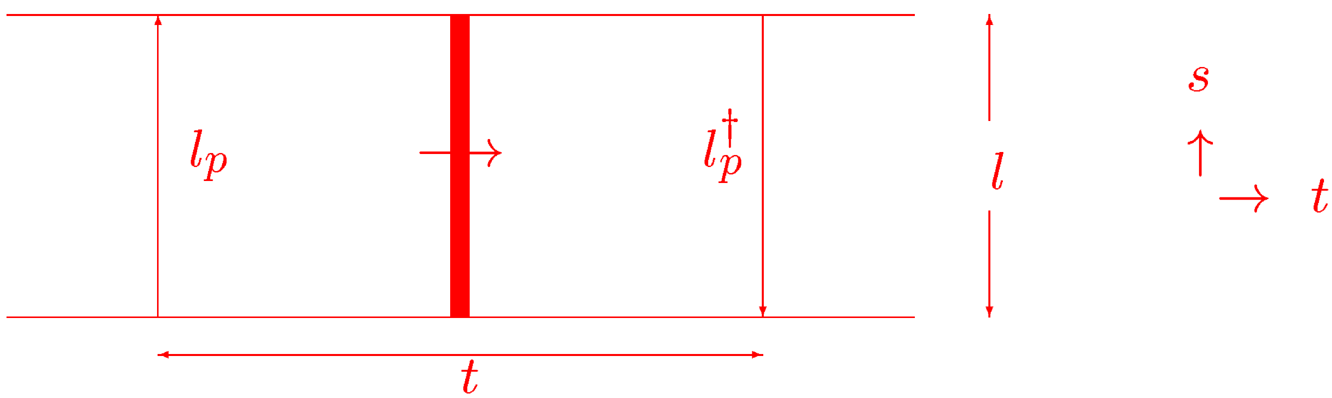
$$\frac{m_{2^{++}}}{\sqrt{\sigma}}\Big|_N = 4.80 + \frac{0.11}{N^2}$$

Linear confinement in SU($N \to \infty$)?

Calculate the mass of a confining flux tube winding around a spatial torus of length l, using correlators of Polyakov loops:

$$\langle l_p^\dagger(t) l_p(0) \rangle \overset{t\to\infty}{\propto} \exp\{-m_p(l)t\}$$

in pictures



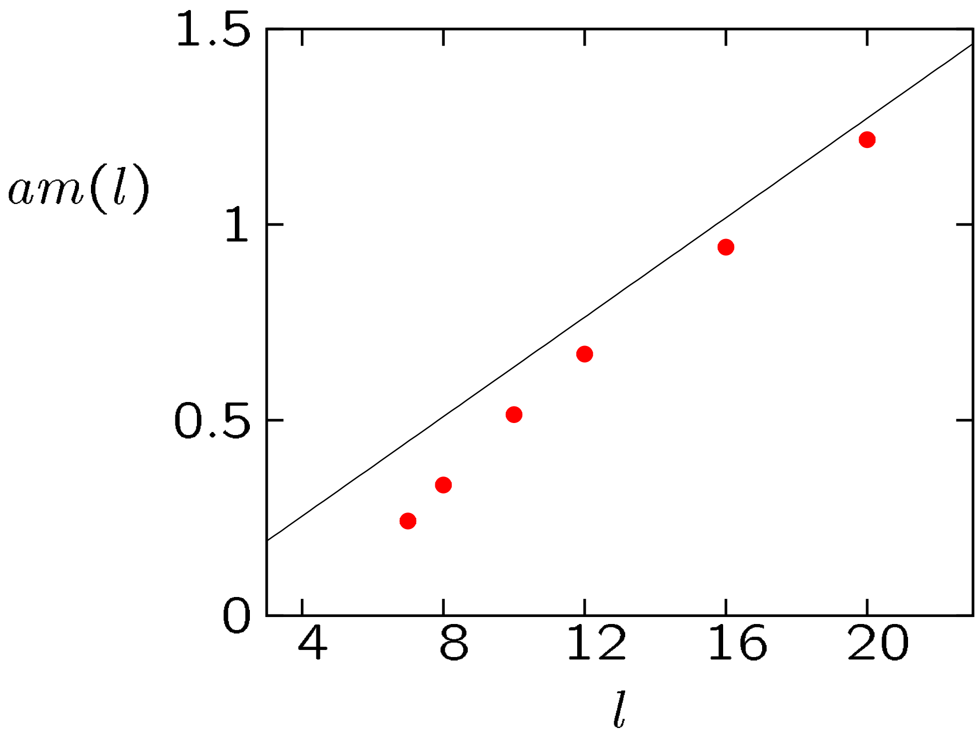
where we expect, for linear confinement,

$$m_p(l) = \sigma l - \frac{\pi(D-2)}{6l^2} + O\left(\frac{1}{l^4}\right)$$

- no sources, no Coulomb terms, flux tubes for $l \geq 1/T_c$

SU(6)

H. Meyer, M. Teper: hep-lat/0411039



indeed we find

$$am(l) \simeq \sigma l$$

over a range of 'string' lengths up to

$$l \simeq 5.0 \times \frac{1}{\sqrt{\sigma}}$$

surely large enough to be asymptotic ...

So :

- SU(3) $\sim$ SU(∞) for many quantities

- linear confinement persists at large N

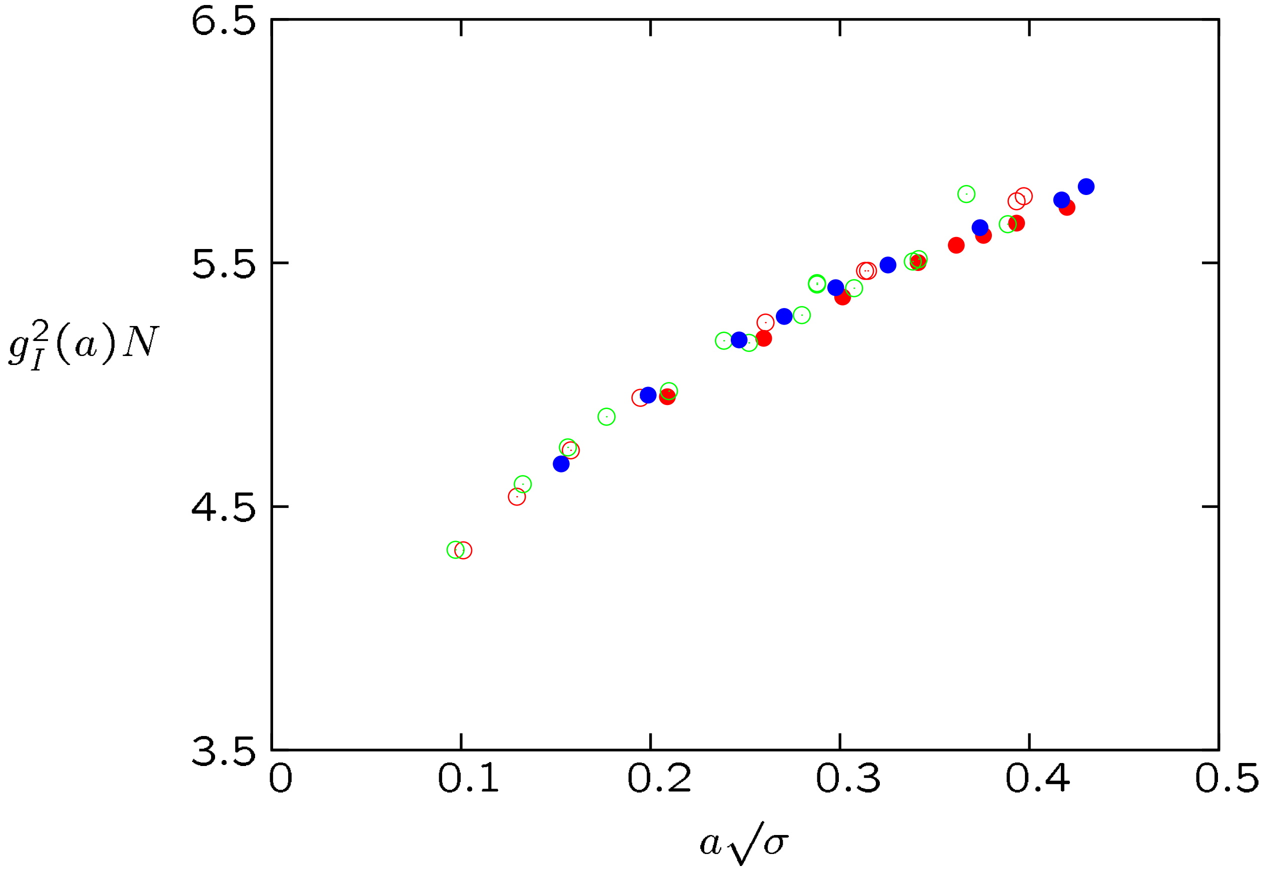
the apparent phenomenological relevance of large-N, provides the motivation for pursueing further the properties of this theory ...

$g^2 N$ fixed as $N \to \infty$?

Lucini,Teper,Wenger: hep-lat/0502003

- $g^2(l)$ $g^2(l)$ versus $\frac{l}{\xi}$ with $\xi = \frac{1}{\sqrt{\sigma}}$, $l = a$

and using $\beta = 2N/g_L^2(a)$ $\quad g_I^2 = g_L^2/u_p$



SU(2) ○ ; SU(3) ○; SU(4) ● ;
SU(6) ○ ; SU(8) ●

Strongly Coupled Gluon Plasma - at large N?

B. Bringoltz, M.Teper: hep-lat/0506034

Consider

$$Z(T,V) = \exp\left\{-\frac{F}{T}\right\} = \exp\left\{-\frac{fV}{T}\right\} = \int DU \exp\left(-\beta S_W\right).$$

now $p = T\frac{\partial}{\partial V}\log Z(T,V) = \frac{T}{V}\log Z(T,V) = \frac{T}{V}\int_{\beta_0}^{\beta} d\beta' \frac{\partial \log Z}{\partial \beta'}$

but $\frac{\partial \log Z}{\partial \beta} = -\langle S_W\rangle = N_p\langle u_p\rangle$

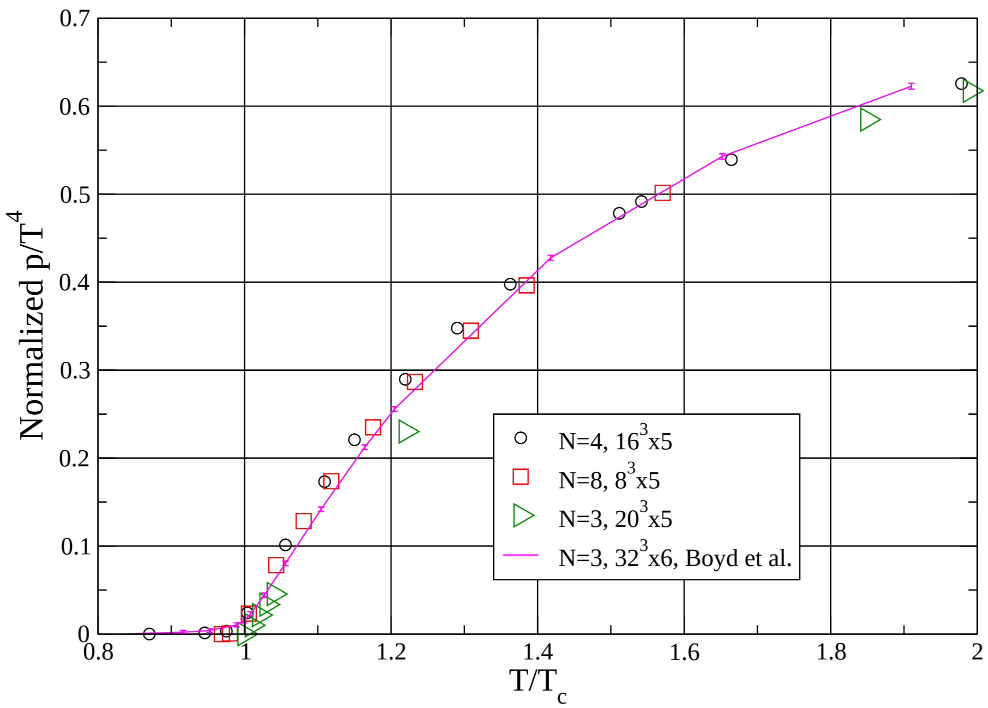
so $a^4[p(T) - p(0)] = 6\int_{\beta_0}^{\beta} d\beta'(\langle u_p\rangle_T - \langle u_p\rangle_0).$

i.e. $\frac{p(T)}{T^4} = 6L_t^4\int_{\beta_0}^{\beta} d\beta'(\langle u_p\rangle_T - \langle u_p\rangle_0).$

similarly $(\epsilon - 3p)/T^4 = 6L_t^4(\langle u_p(\beta)\rangle_0 - \langle u_p(\beta)\rangle_T) \times \frac{\partial \beta}{\partial \log(a(\beta))}.$

Strong Gluon Plasma - high-T pressure anomaly

B. Bringoltz, M.Teper: hep-lat/0506034



$\Rightarrow$

SGP is a large-N phenomenon: dynamics must survive at $N = \infty$

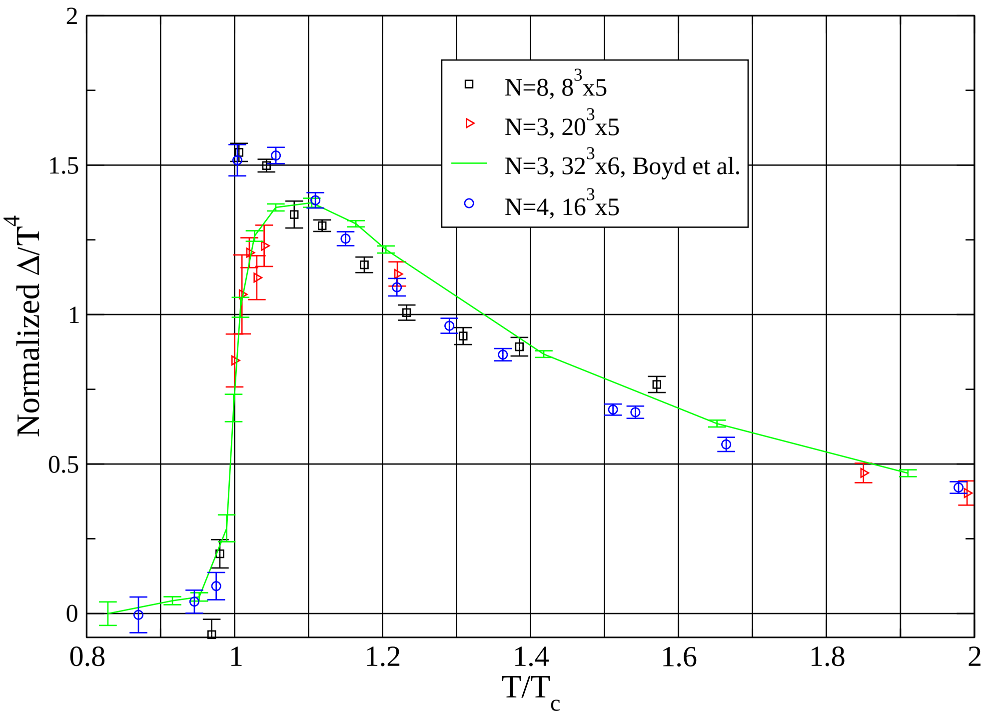
$\Rightarrow$

- not (colour singlet) hadrons above T_c
- not topology (instantons)

$$\Delta \equiv \epsilon - 3p$$

B. Bringoltz, M.Teper: hep-lat/0506034

$\Delta = 0$ in Stefan-Boltzman gas



analysing effective string theories

• field theory approach (non-covariant 'gauge fixing' of the string theory)
M. Luscher, K. Symanzik, P. Weisz : Nucl. Phys. B173 (1980) 365; M. Luscher : Nucl. Phys. B180 (1981) 317;
M. Luscher, P. Weisz : JHEP 0407 (2004) 014

• covariant effective string approach
J. Polchinski, A. Strominger : Phys. Rev. Lett. 67 (1991) 1681;
J. Drummond : hep-th/0411017; N. Hari Dass, P. Matlock : hep-th/0612291

In both approaches the starting point is to consider a long (open or closed) string of length r and to consider those corrections allowed by symmetry in powers of $1/r$ – the corrections being to the spectrum of the free Nambu-Goto string theory

coordinate invariance not anomalous
$\Rightarrow \qquad \beta = \frac{D-26}{12}$
which translates into the usual expression for the Luscher string correction

we can now continue to one higher order and we find
J. Drummond : hep-th/0411017; N. Hari Dass, P. Matlock : hep-th/0612291

$$E_n = \sigma l + \frac{\pi}{l}\left(n - \frac{D-2}{6}\right) - \frac{\pi^2}{2\sigma l^3}\left(n - \frac{D-2}{6}\right)^2 + O(l^{-4})$$

i.e. identical to Nambu-Goto to this order in $1/l$ for both $D = 2+1$ *and* $D = 3+1$

Note:
the effective action is only valid for very long strings – $l\sqrt{\sigma} \gg 1$ – as is obvious from the denominators in the effective action.
$\Rightarrow$ it tells us nothing about light glueballs since these are composed of small closed loops
it tells us nothing about k-strings or other multiple strings, since the interaction between these (at the origin of their binding) will in general involve the exchange of small closed loops

$\Rightarrow$
what we learn about confining flux tubes with $l\sqrt{\sigma} \ggg 1$ will tell us whether what we have is just an effective string theory for very long flux tubes or possibly an effective string theory on all scales ...

Nambu-Goto free string theory

$$\int \mathcal{D}X e^{-\frac{i}{\sigma}\times \text{Area}}$$

a string breaks spontaneously the transverse translation invariance

$\rightarrow$

D-2 Goldstone bosons – massless transverse oscillations of frequencies quantised by the string length

$\rightarrow$

these massless modes determine the effective action of long strings

spectrum of a string of length l winding once around a spatial torus with zero transverse momentum

$$E^2(l) = (\sigma\, l)^2 + 8\pi\sigma\left(\frac{N_L+N_R}{2} - \frac{D-2}{24}\right) + \left(\frac{2\pi q}{l}\right)^2 .$$

for states with total momentum $2\pi q/l$ along the string and with left and right oscillators summing to N_L and N_R

$$N_L = \sum_{k>0} n_L(k)\, k, \qquad N_R = \sum_{k'>0} n_R(k')\, k'$$
$$N_L - N_R = q, \qquad \prod_k a_k^{+n_k}|0\rangle$$

J. Arvis, Phys. Lett. 127B(1983)106

I will focus on the spectrum of strings that are closed around a spatial torus of length l :

- the winding states are flux 'tubes' for all l down to the phase transition at $l = l_c = 1/T_c$ at which one loses confinement

- this phase transition is first order for $N \geq 3$ in $D = 4$ and for $N \geq 4$ in $D = 3$

- thus it is possible that we may have a simple string description of the closed string spectrum for all possible lengths (at large N)

- such a simple string description is most likely at $N \to \infty$ where complications such as mixing, e.g string $\to$ string + glueball, will go away

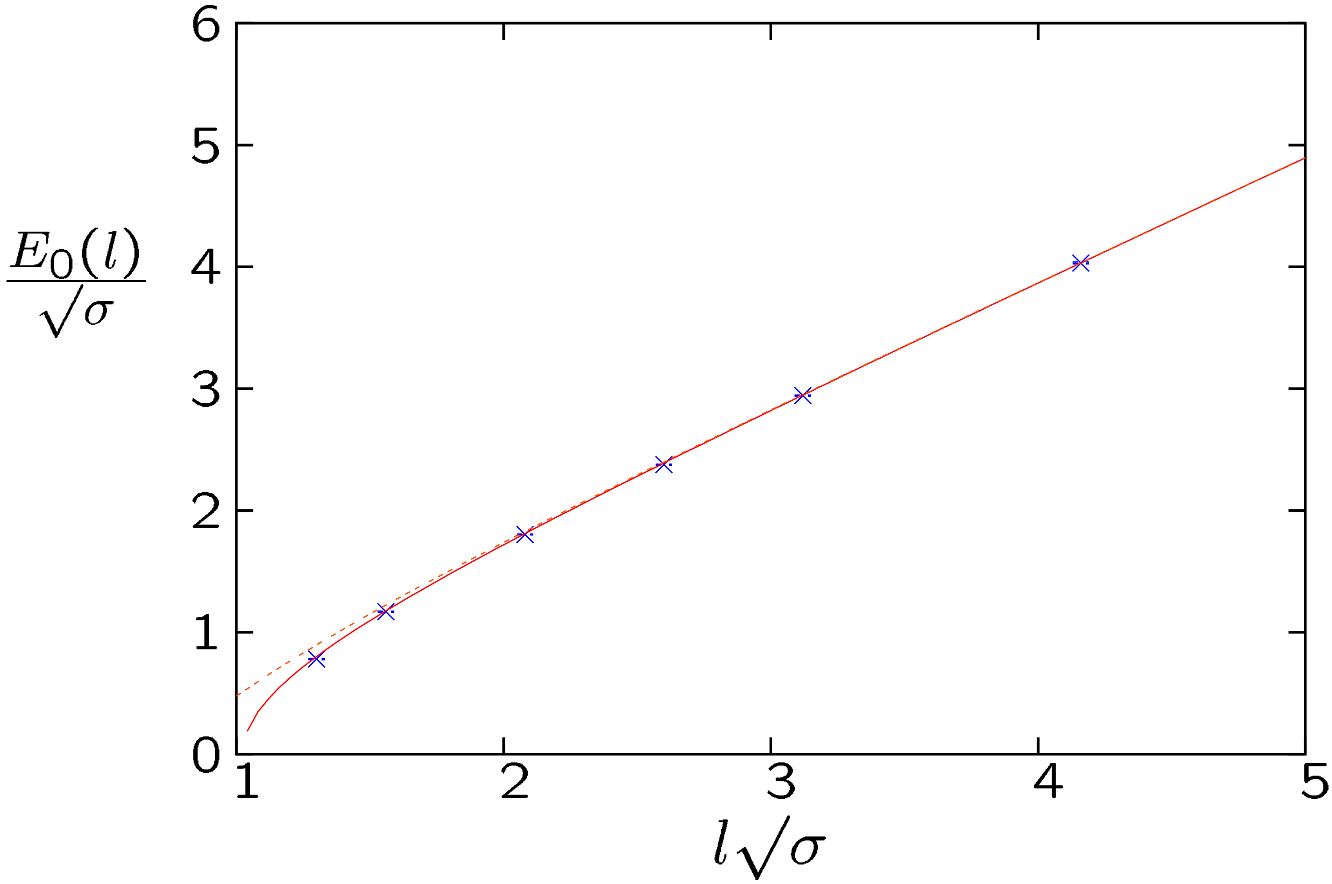
- by contrast, for the potential $V(r)$ between static sources there is a cross-over in r between flux tubes and a Coulomb potential, over some ill-determined distance, and so it is not straightforward to investigate the properties of shorter strings – although there may be a string theory description that includes the Coulomb potential, that is a much more challenging goal

and mostly D=3 ... from:
A.Athenodorou, B.Bringoltz, M.Teper arXiv:0709.0693
B.Bringoltz, M.Teper hep-th/0611286
A.Athenodorou, B.Bringoltz, M.Teper in progress

The spectrum of flux tubes that are closed around a spatial torus of lnegth l : SU(N) $D = 2 + 1$

- linear confinement?
- how good are our energy calculations?
- bosonic string universality class?
- what happens as $l \to l_c$?
- $E_n(l)$: expansion in $1/l$ or covariant Nambu-Goto?

D=2+1 ; SU(5)



Luscher:...

$$E_0(l) = \sigma l - \frac{\pi}{6l}$$

Nambu-Goto:-

$$E_0(l) = \sigma l \left(1 - \frac{\pi}{3\sigma l^2}\right)^{\frac{1}{2}}$$

effective string theory
– universality class?

central charge appears in the string 'Casimir' energy

$$E_0(l) = \sigma l - \frac{c\pi(D-2)}{6l} + O\left(\frac{1}{l^3}\right)$$

where

$$c = 1,\ 1.5,\ 0$$

for bosonic, Neveu-Schwartz, Ramond strings respectively

to determine the central charge numerically, calculate the ground state energy $E_0(l)$ for a sequence of increasing lengths, and fit an effective central charge, $c_{eff}(l)$, to neighbouring values of l, i.e.

$$c_{eff}(l,l') = \frac{6}{\pi(D-2)} \frac{\frac{E(l)}{l} - \frac{E(l')}{l'}}{\frac{1}{l'^2} - \frac{1}{l^2}}$$

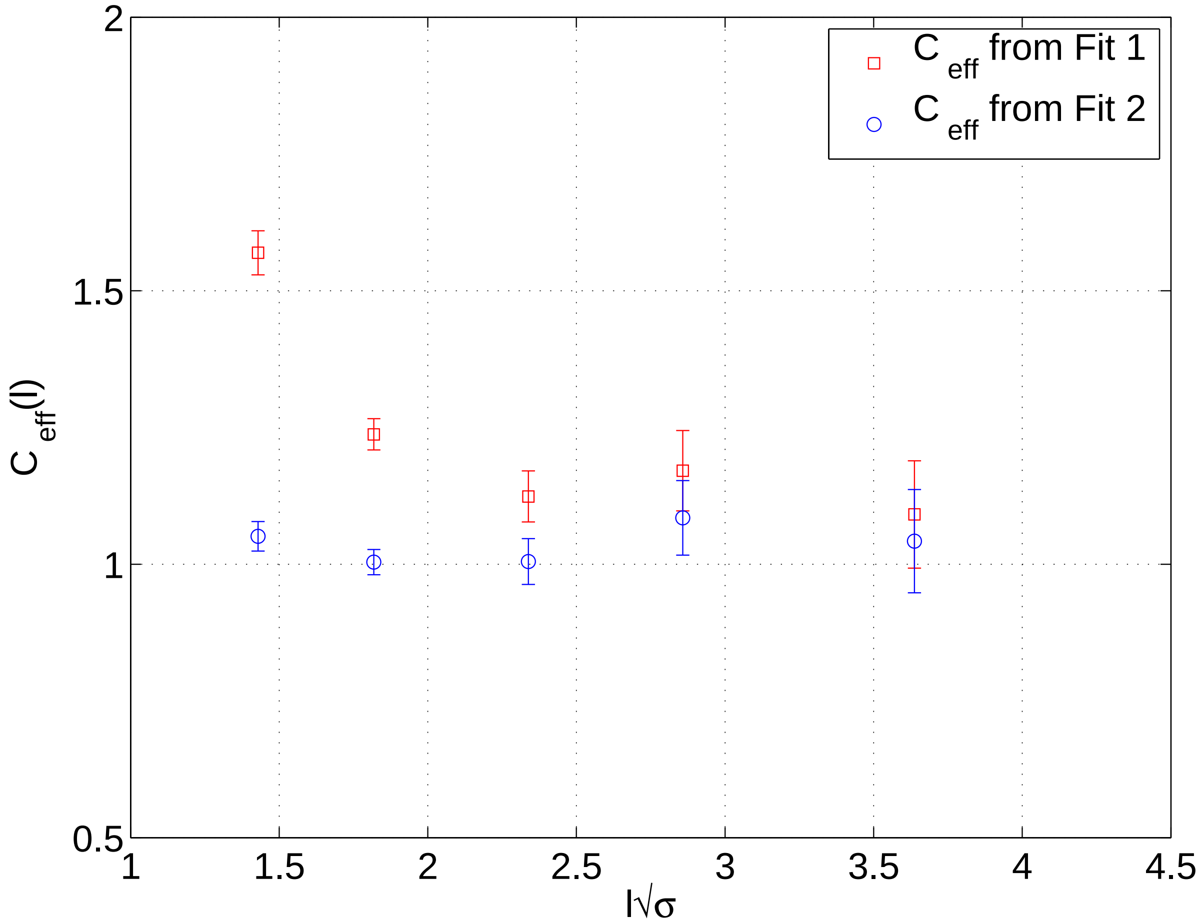
alternatively do the same for Nambu-Goto, solving

$$E_0(l) = \sigma l \left(1 - c_{eff}(l,l')\frac{\pi(D-2)}{3\sigma l^2}\right)^{\frac{1}{2}}$$

$$E_0(l') = \sigma l' \left(1 - c_{eff}(l,l')\frac{\pi(D-2)}{3\sigma l'^2}\right)^{\frac{1}{2}}$$

we refer to these as Fit 1 and Fit 2 respectively

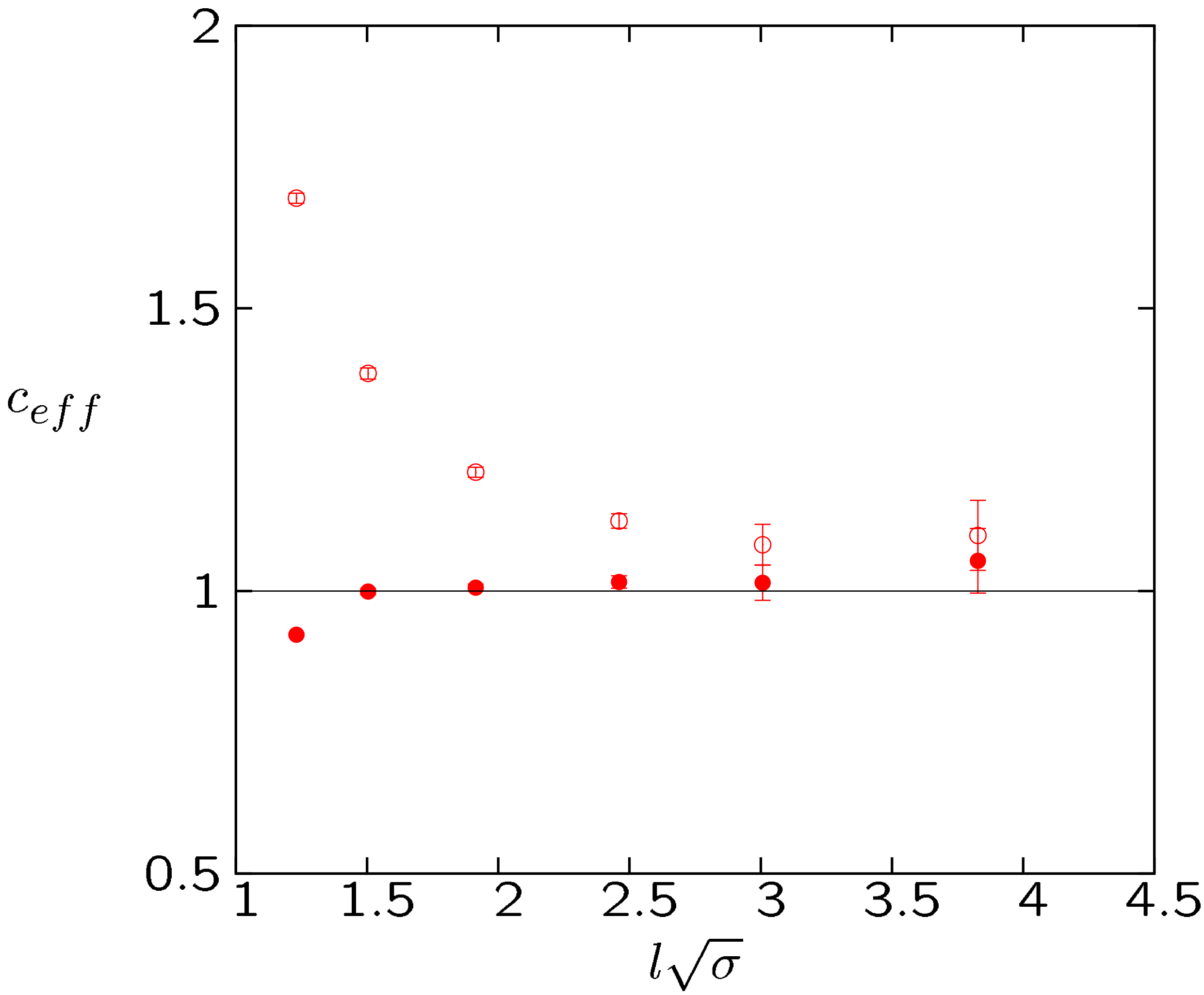
SU(5) : $l_c\sqrt{\sigma} \simeq 1.07$



○ : c_{eff} from Luscher

○ : c_{eff} from Nambu-Goto

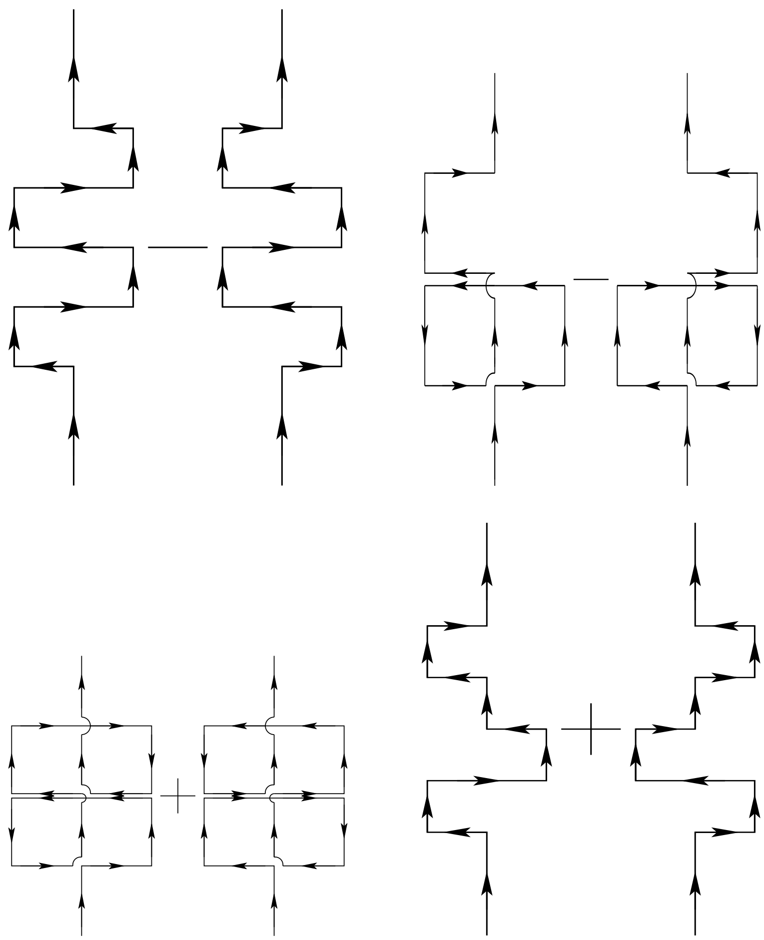
SU(2) : $l_c\sqrt{\sigma} \simeq 0.94$



○ : c_{eff} from Luscher
● : c_{eff} from Nambu-Goto

Excited States

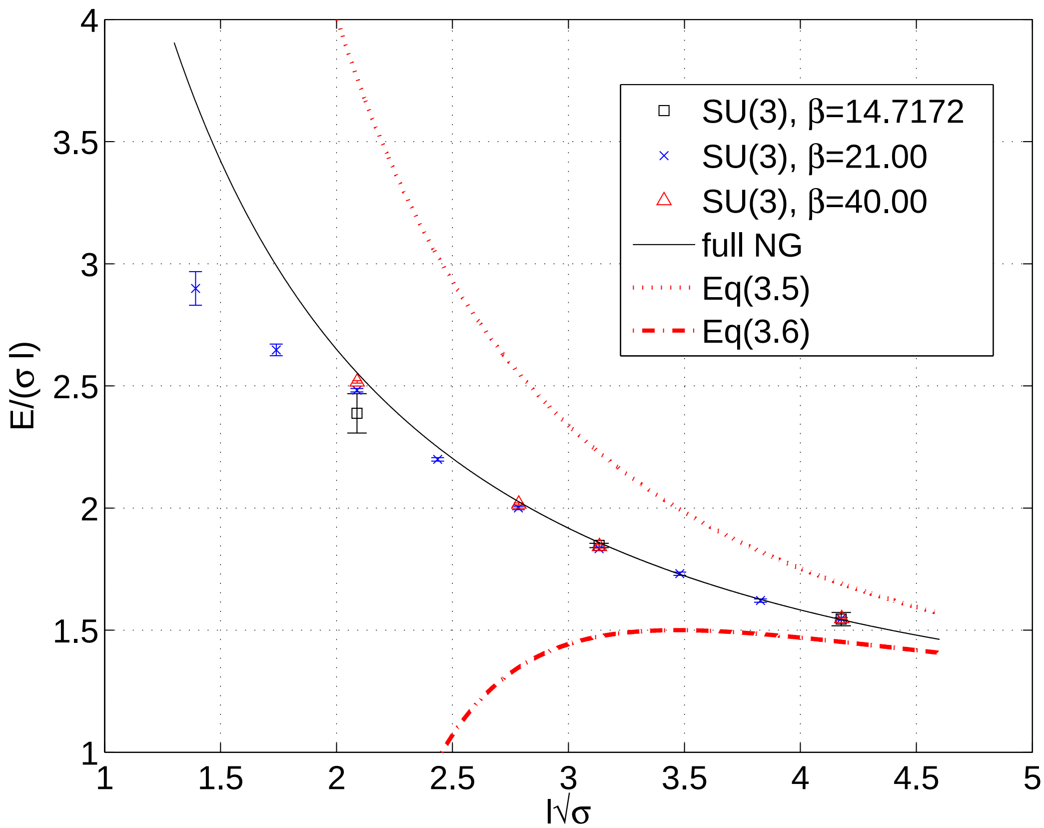
to have good overlaps onto excited string states, we need to include many mor operators in our variational basis – in particular operators that ‘look’ excited and ones that have an intrinsic handedness so that we can construct $P = -$ as well as $P = +$, e.g.



typically we have 100-200 operators in our basis ...

first excited state : $N = 3$

no parameter: σ from ground state

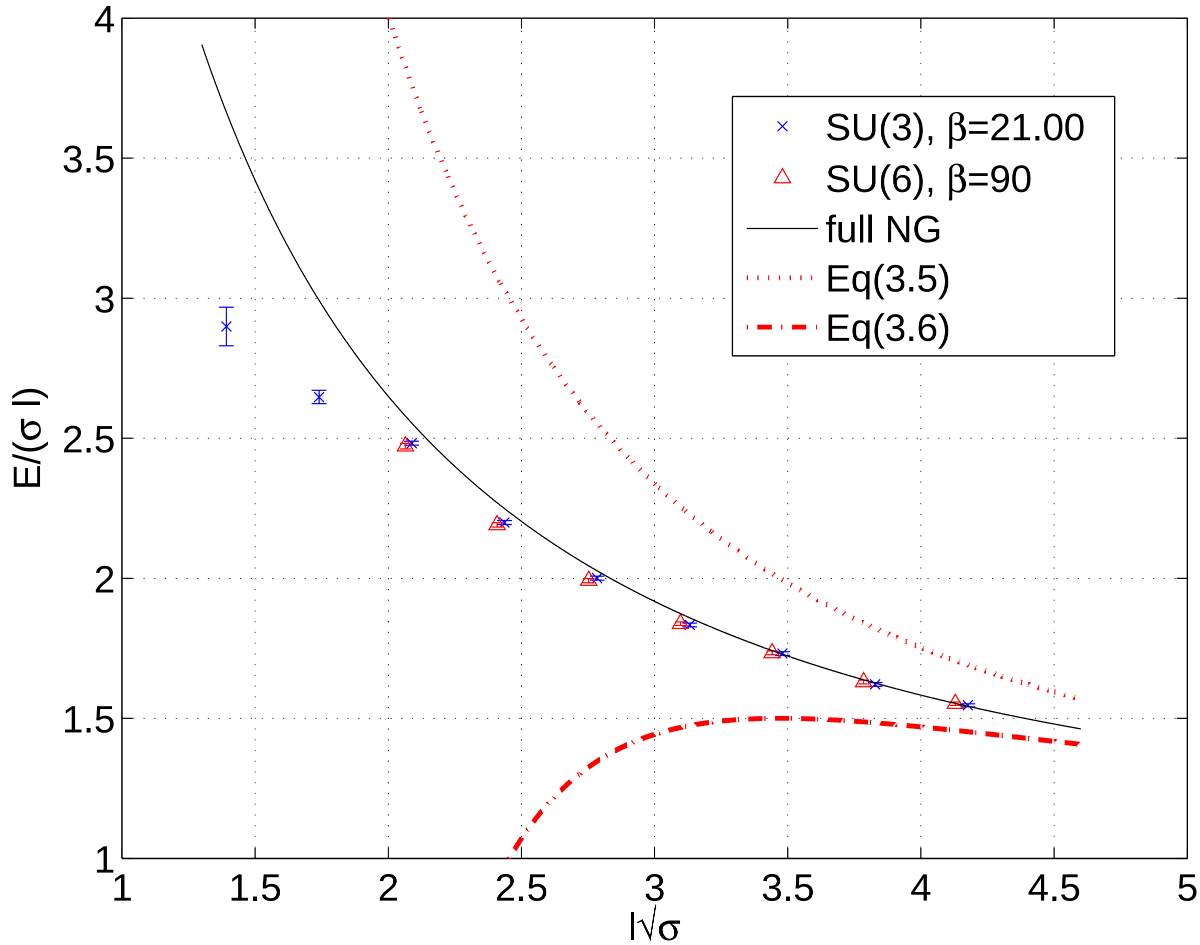


— Nambu-Goto : $E_n = \sigma l \sqrt{1 + \frac{8\pi}{\sigma l^2}\left(n - \frac{1}{24}\right)}$

... Luscher 1980: $E_n = \sigma l + \frac{4\pi}{l}\left(n - \frac{D-2}{24}\right)$

- - Luscher 2004: $E_n = \sigma l + \frac{4\pi}{l}\left(n - \frac{1}{24}\right) - \frac{8\pi^2}{\sigma\, l^3}\left(n - \frac{1}{24}\right)^2$

first excited state : N-dependence?



Why ?

the covariant Nambu-Goto expression e.g. for $q = 0$,

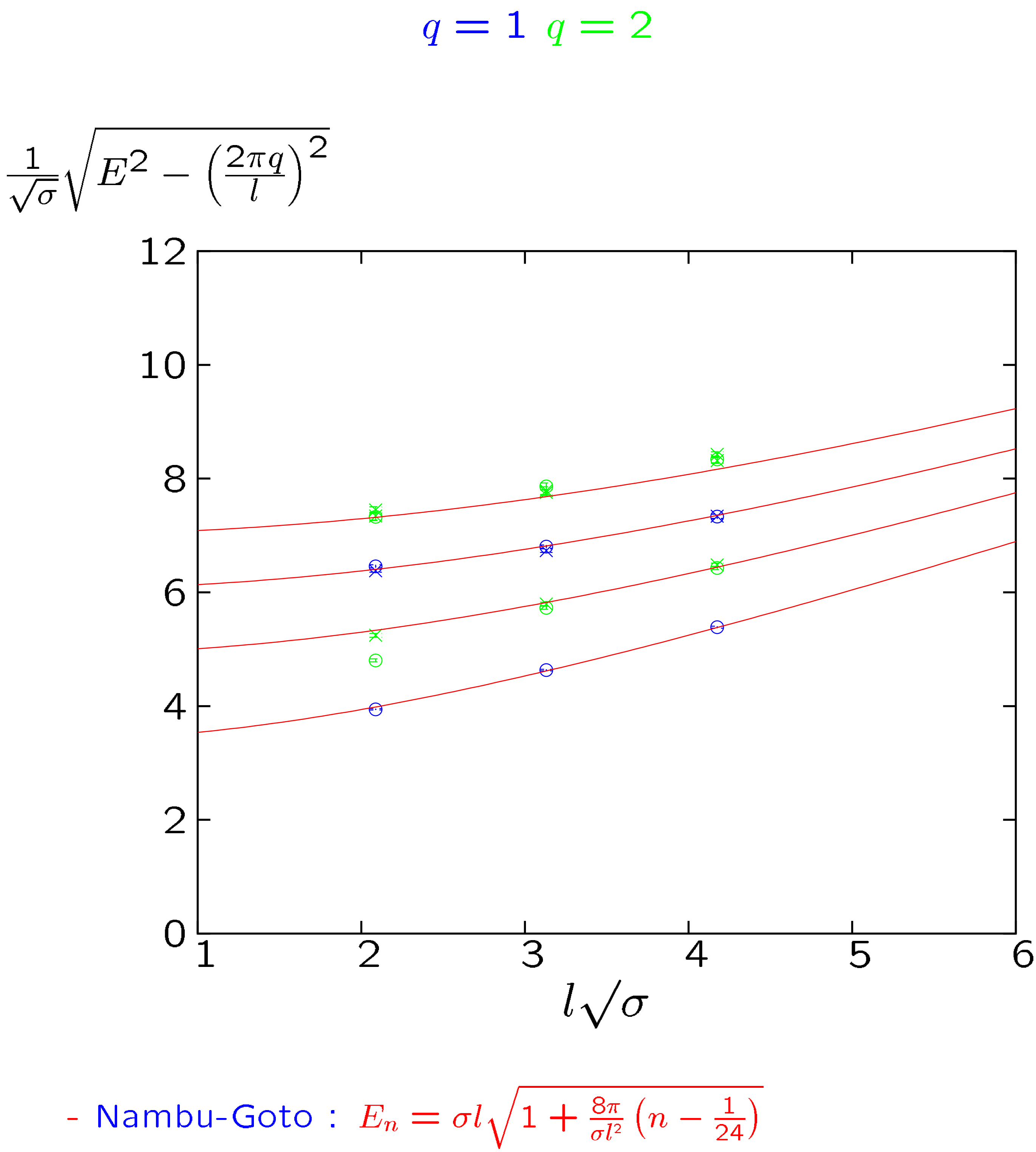
$$E(l) = \sigma l \left(1 + \frac{8\pi}{\sigma l^2}\left(n - \frac{D-2}{24}\right)\right)^{\frac{1}{2}}$$

can only be expanded as a power series in $1/l\sqrt{\sigma}$ when

$$x \equiv \frac{8\pi}{\sigma l^2}\left(n - \frac{1}{24}\right) \leq 1$$

whereas in practice we have a very good fit by Nambu-Goto even down to

$$x \sim 12 \qquad : \qquad l\sqrt{\sigma} \sim 2 \ , \ n = 2$$

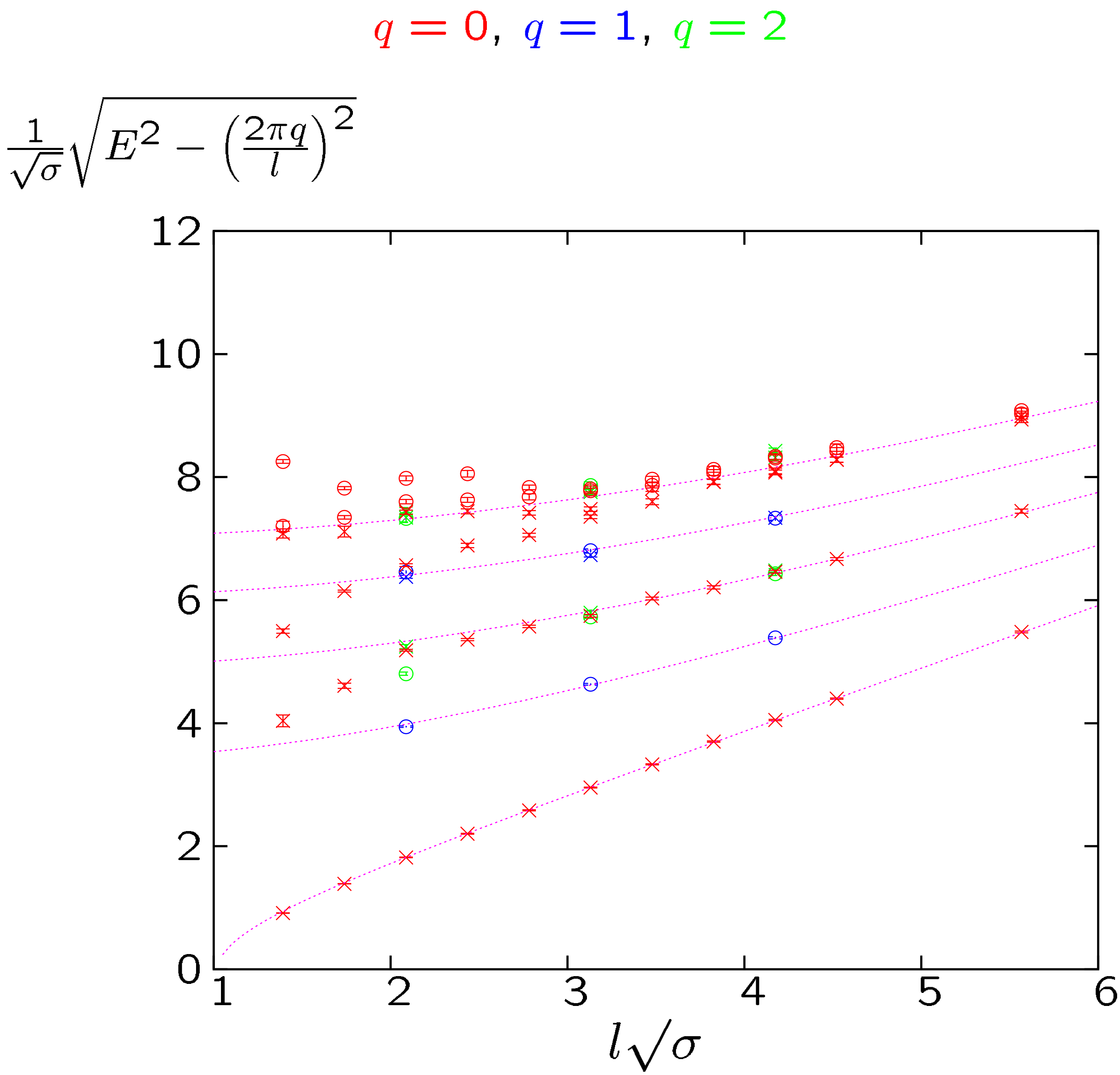

$q = 1$ $q = 2$
$\frac{1}{\sqrt{\sigma}}\sqrt{E^2 - \left(\frac{2\pi q}{l}\right)^2}$
12
10
8
6
4
2
0
1
2
3
4
5
6
$l\sqrt{\sigma}$
- Nambu-Goto : $E_n = \sigma l \sqrt{1 + \frac{8\pi}{\sigma l^2}\left(n - \frac{1}{24}\right)}$

content of NG states:

$a^R(k=1)\|0\rangle$	P=−, q=1
$a^R(k=2)\|0\rangle$	P=−, q=2
$a^R(k=1)a^R(k=1)\|0\rangle$	P=+, q=2
$a^R(k=2)a^L(k=1)\|0\rangle$	P=+, q=1
$a^R(k=1)a^R(k=1)a^L(k=1)\|0\rangle$	P=−, q=1
$a^R(k=3)a^L(k=1)\|0\rangle$	P=+, q=2
$a^R(k=2)a^R(k=1)a^L(k=1)\|0\rangle$	P=−, q=2
$a^R(k=1)a^R(k=1)a^R(k=1)a^L(k=1)\|0\rangle$	P=+, q=2

the individual sets of degeneracies tell us something specific about the interactions amongst the corresponding phonons – usually that they are very weak ... although there certainly should be more to be said than that

[also $q=-1, q=-2$ degenerate within errors]



$q = 0$, $q = 1$, $q = 2$
$\frac{1}{\sqrt{\sigma}}\sqrt{E^2 - \left(\frac{2\pi q}{l}\right)^2}$
12
10
8
6
4
2
0
1
2
3
4
5
6
$l\sqrt{\sigma}$

content of NG states:

$\|0\rangle$	P=+, q=0
$a^R(k=1)\|0\rangle$	P=−, q=1
$a^R(k=1)a^L(k=1)\|0\rangle$	P=+, q=0
$a^R(k=2)\|0\rangle$	P=−, q=2
$a^R(k=1)a^R(k=1)\|0\rangle$	P=+, q=2
$a^R(k=2)a^L(k=1)\|0\rangle$	P=+, q=1
$a^R(k=1)a^R(k=1)a^L(k=1)\|0\rangle$	P=−, q=1
$a^R(k=2)a^L(k=2)\|0\rangle$	P=+, q=0
$a^R(k=1)a^R(k=1)a^L(k=2)\|0\rangle$	P=−, q=0
$a^R(k=2)a^L(k=1)a^L(k=1)\|0\rangle$	P=−, q=0
$a^R(k=1)a^R(k=1)a^L(k=1)a^L(k=1)\|0\rangle$	P=+, q=0
$a^R(k=3)a^L(k=1)\|0\rangle$	P=+, q=2
$a^R(k=2)a^R(k=1)a^L(k=1)\|0\rangle$	P=−, q=2
$a^R(k=1)a^R(k=1)a^R(k=1)a^L(k=1)\|0\rangle$	P=+, q=2

observed near-degeneracies for $l \geq 2/\sqrt{\sigma} \sim 1\text{fm} \sim$ width flux tube!

• In D=2+1 SU(N) gauge theories, confining flux tubes belong to the universality class of a simple bosonic string theory

• More than that, the Nambu-Goto covariant free string spectrum

$$E^2(l) = (\sigma\, l)^2 + 8\pi\sigma\left(\frac{N_L+N_R}{2} - \frac{D-2}{24}\right) + \left(\frac{2\pi q}{l}\right)^2 .$$

is very accurate down to values of $l\sqrt{\sigma}$ where an effective string theory expansion, $x = l\sqrt{\sigma}$,

$$\frac{E_n}{\sqrt{\sigma}} = x\left(1 + \frac{c}{x^2}\right)^{\frac{1}{2}} = x + \frac{c}{2x} - \frac{c}{8x^3} + \cdots$$

makes no sense (is far past its range of convergence)

• So, since in the range of $l\sqrt{\sigma}$ where such a power expansion is relevant, any difference with Nambu-Goto is totally negligible, it is clear that there is a challenge here to incorporate string corrections to Nambu-Goto in some 'resummed' way ...

• The fact that the spectrum is very close to Nambu-Goto down to such small l, makes it reasonable to expect that one should be able to reconstruct the string interactions from the way the different states, with different oscillator occupation number content, are observed to deviate from the free string values as l decreases ...