

Two loop $\mathcal{O}(G_F^2 m_t^4)$ correction to Higgs decay into bottom quarks

Mathias Butenschön

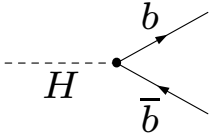
II. Institut für Theoretische Physik
Universität Hamburg

In Collaboration with F. Fugel and B. A. Kniehl

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Higgs Decay into Bottom and Antibottom

$M_H < 140$ GeV: **Higgs** decays almost **exclusively** into $b\bar{b}$.
EW precision tests: Expect M_H to lie in that range!

- Born level: 
- **One loop** corrections are known exactly:
 - $\mathcal{O}(\alpha_s)$: Braaten and Leveille, 1980
 - $\mathcal{O}(G_F)$: Kniehl, 1992
- Known **higher loop** corrections: (Expansions assuming $M_W^2 \ll M_H^2 \ll m_t^2$)
 - $\mathcal{O}(\alpha_s^2)$: Various papers, 1991-1996
 - $\mathcal{O}(\alpha_s^3)$: Chetyrkin and Steinhauser, 1997
 - $\mathcal{O}(\alpha_s G_F m_t^2)$: Kniehl and Spira, 1994
 - $\mathcal{O}(\alpha_s^2 G_F m_t^2)$: Chetyrkin, Kniehl and Steinhauser, 1997

Overview: Calculation of $\mathcal{O}(G_F^2 m_t^4)$ Correction

- **Asymptotic Expansion** in small momenta and masses ($M_W^2 \ll M_H^2 \ll m_t^2$), keep only **leading term** in m_t .
- All non-naive contributions cancel \Rightarrow No large logarithms of scalar boson masses
- Calculation of diagrams with MATAD (based on FORM)
- **Genuine two loop renormalisation** in On-Shell-Scheme
 \Rightarrow Need general expressions for Counterterms (not only one loop)
- Use **counterterm vertex** method **only** for **Higgs mass**- and **Tadpole**-Counterterms

Two contributions, which are **seperately finite**:

1. **Universal Counterterm**: Appears in all decays into $f\bar{f}$.
(Was calculated before [Djouadi *et al.*, 1998]. Disagreement \Rightarrow We could find their mistake.)
2. **Non-universal Contributions**: Special to decay into $b\bar{b}$, as b is EW-isospin partner of t .

Tadpole Renormalization (1)

Higgs part of the SM Lagrangian: $\mathcal{L}_{\text{Higgs}} = (D_\mu \Phi)^\dagger (D^\mu \Phi) - \frac{\lambda}{4} (\Phi^\dagger \Phi)^2 + \mu^2 \Phi^\dagger \Phi$

Complex Higgs doublet: $\Phi(x) = \begin{pmatrix} \phi^+(x) \\ \frac{1}{\sqrt{2}} (v + H(x) + i\chi(x)) \end{pmatrix}$ with $\phi^- := (\phi^+)^\dagger$

Substitution: $M_H^2 := -\mu^2 + \frac{3\lambda v^2}{4}$ and $t := v \left(\mu^2 - \frac{\lambda v^2}{4} \right)$

$$\begin{aligned} \mathcal{L}_{\text{Higgs}} = & \frac{1}{2} (\partial_\mu H) (\partial^\mu H) + \frac{1}{2} (\partial_\mu \chi) (\partial^\mu \chi) + (\partial_\mu \phi^-) (\partial^\mu \phi^+) + t H - \frac{M_H^2}{2} H^2 + \frac{t}{2v} \chi^2 + \frac{t}{v} \phi^- \phi^+ \\ & - \left(\frac{M_H^2}{8v^2} + \frac{t}{8v^3} \right) (H^4 + \chi^4 + 2H^2 \chi^2 + 4H^2 \phi^- \phi^+ + 4\chi^2 \phi^- \phi^+ + 4(\phi^- \phi^+)^2) \\ & - \left(\frac{M_H^2}{2v} + \frac{t}{2v^2} \right) (H^3 + H\chi^2 + 2H\phi^- \phi^+) \end{aligned}$$

Vacuum expectation value: $v^2 = \frac{4\mu^2}{\lambda} \implies t = 0$

End up with these counterterm vertices:

$$\begin{array}{llll}
\mathbf{H}: i\delta t & \mathbf{HHH}: -3iv^{-2}\delta t & \mathbf{HH}\chi\chi: -iv^{-3}\delta t & \mathbf{HHHH}: -3iv^{-3}\delta t \\
\chi\chi: iv^{-1}\delta t & \mathbf{H}\chi\chi: -iv^{-2}\delta t & \mathbf{HH}\phi\phi: -iv^{-3}\delta t & \chi\chi\chi\chi: -3iv^{-3}\delta t \\
\phi\phi: iv^{-1}\delta t & \mathbf{H}\phi\phi: -iv^{-2}\delta t & \chi\chi\phi\phi: -iv^{-3}\delta t & \phi\phi\phi\phi: -2iv^{-3}\delta t
\end{array}$$

Renormalization condition: $\delta t \stackrel{!}{=} -T$, where $i T := -\frac{H}{1-\text{PI}}$

The one-point counterterm vertex $\boxed{\mathbf{H} : i\delta t}$ then cancels all tadpole diagrams.

Effect:

- We do not need to consider tadpole diagrams.
- Instead: **Additional counterterms!**

On-Shell Mass Renormalization

On-Shell Condition: Pole of propagator including radiative corrections at renormalized mass
 \implies Renormalized mass equals physical mass.

Higgs Mass Counterterm: Write $M_{H,0}^2 = M_H^2 + \delta M_H^2$ and end up with:

$$\delta M_H^2 = \Sigma^H(M_H^2), \quad \text{where}$$

$$-\frac{H}{q} \text{---} \textcircled{1\text{-PI}} \text{---} \frac{H}{q} =: i \Sigma^H(q^2)$$

W Mass Counterterm: Write $M_{W,0}^2 = M_W^2 + \delta M_W^2$ and end up with:

$$\delta M_W^2 = \Sigma_T^W(M_W^2), \quad \text{where}$$

$$\frac{W_\mu}{q} \text{---} \textcircled{1\text{-PI}} \text{---} \frac{W_\nu}{q} =: -i \left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2} \right) \Sigma_T^W(q^2) - i \frac{q^\mu q^\nu}{q^2} \Sigma_L^W(q^2)$$

Fermion Mass Renormalization Constant (1)

In the SM right- and lefthanded fermions interact differently. Treat them as different particles.
 \implies Resum two-point-functions of **righthanded** field $r = \omega_+ f$ and **lefthanded** field $l = \omega_- f$.

Lagrangian: $\mathcal{L} = \bar{f} (i\not{\partial} - m_{f,0}) f = i\bar{l} \not{\partial} l + i\bar{r} \not{\partial} r - m_{f,0} \bar{r} l - m_{f,0} \bar{l} r$

Propagators: $\frac{l}{\underline{q}} = \frac{r}{\underline{q}} = \frac{i}{\not{q}}$ **Vertices:** $\frac{l}{\underline{q}} \bullet \frac{r}{\underline{q}} = \frac{r}{\underline{q}} \bullet \frac{l}{\underline{q}} = -i m_{f,0}$

Two Point Function: $S_l(q) = \frac{l}{\underline{q}} + \frac{l}{\underline{q}} \bullet \frac{r}{\underline{q}} \bullet \frac{l}{\underline{q}} + \frac{l}{\underline{q}} \bullet \frac{r}{\underline{q}} \bullet \frac{l}{\underline{q}} \bullet \frac{r}{\underline{q}} \bullet \frac{l}{\underline{q}} + \dots$

Notation: $\frac{l}{\underline{q}} := \frac{l}{\underline{q}} + \frac{l}{\underline{q}} \textcircled{1\text{-PI}} \frac{l}{\underline{q}} + \frac{l}{\underline{q}} \textcircled{1\text{-PI}} \frac{l}{\underline{q}} \textcircled{1\text{-PI}} \frac{l}{\underline{q}} + \dots$ (propagator-type)

$\frac{l}{\underline{q}} \bullet \frac{r}{\underline{q}} := \frac{l}{\underline{q}} \bullet \frac{r}{\underline{q}} + \frac{l}{\underline{q}} \textcircled{1\text{-PI}} \frac{r}{\underline{q}}$ (vertex-type)

Fermion Mass Renormalization Constant (2)

Result: Resummed two point function of left-/righthanded fermion field:

$$S_{l/r}(q) = \frac{\not{q}}{\left(q^2 - m_{f,0}^2 \frac{(1 - \Sigma_S^f(q^2))^2}{(1 + \Sigma_L^f(q^2))(1 + \Sigma_R^f(q^2))}\right) \left(1 + \Sigma_{L/R}^f(q^2)\right)}$$

Here, $\Sigma_S^f, \Sigma_L^f, \Sigma_R^f$ are the scalar, left- and righthanded parts of the **one-particle-irreducible** fermion self energy:

$$\frac{f}{q} \rightarrow \text{1-PI} \rightarrow f =: i m_{f,0} \Sigma_S^f(q^2) + i \not{q} \omega_+ \Sigma_R^f(q^2) + i \not{q} \omega_- \Sigma_L^f(q^2)$$

Write $m_{f,0} = m_f + \delta m_f$. Then the **on-shell condition** yields:

$$\frac{\delta m_f}{m_f} = \frac{\sqrt{(1 + \Sigma_L^f(m_f^2)) (1 + \Sigma_R^f(m_f^2))}}{1 - \Sigma_S^f(m_f^2)} - 1$$

Fermion Mass Renormalization Constant (3)

Expanding the mass counterterm we get:

$$\frac{\delta m_f^{(1)}}{m_f} = \Sigma_S^{f(1)}(m_f^2) + \frac{1}{2} \Sigma_L^{f(1)}(m_f^2) + \frac{1}{2} \Sigma_R^{f(1)}(m_f^2)$$

$$\begin{aligned} \frac{\delta m_f^{(2)}}{m_f} = & \Sigma_S^{f(2)}(m_f^2) + \frac{1}{2} \Sigma_L^{f(2)}(m_f^2) + \frac{1}{2} \Sigma_R^{f(2)}(m_f^2) + \left(\Sigma_S^{f(1)}(m_f^2) \right)^2 \\ & + \frac{1}{2} \Sigma_S^{f(1)}(m_f^2) \cdot \Sigma_L^{f(1)}(m_f^2) + \frac{1}{2} \Sigma_S^{f(1)}(m_f^2) \cdot \Sigma_R^{f(1)}(m_f^2) \\ & - \frac{1}{8} \left(\Sigma_L^{f(1)}(m_f^2) \right)^2 + \frac{1}{4} \Sigma_L^{f(1)}(m_f^2) \cdot \Sigma_R^{f(1)}(m_f^2) - \frac{1}{8} \left(\Sigma_R^{f(1)}(m_f^2) \right)^2 \end{aligned}$$

Wave Function Renormalization Constants (1)

- The residues of the resummed two point functions at the one-particle poles are called **wave function renormalization constants**. We need WFRC for Higgs (Z_H) and for the left-/righthanded fermion ($Z_{f,l/r}$):

$$S_H(q) \xrightarrow{q^2 \rightarrow M_H^2} \frac{Z_H}{q^2 - M_H^2} \quad \text{resp.} \quad S_{f,l/r}(q) \xrightarrow{q^2 \rightarrow m_f^2} \frac{\not{q} Z_{f,l/r}}{q^2 - m_f^2}$$

- From the calculated two point functions we derive:

$$Z_H = \frac{1}{1 + \Sigma'_H(M_H^2)}$$

$$Z_{f,l/r} = \frac{1}{\left(1 + \Sigma_{L/R}^f(m_f^2)\right) \left(1 + m_f^2 \frac{f'(m_f^2)}{f(m_f^2)}\right)} \quad \text{with} \quad f(q^2) = \frac{\left(1 - \Sigma_S^f(q^2)\right)^2}{\left(1 + \Sigma_L^f(q^2)\right) \left(1 + \Sigma_R^f(q^2)\right)}$$

Wave Function Renormalization Constants (2)

Expand the WFRC: $\mathbf{Z} = 1 + \delta\mathbf{Z}^{(1)} + \delta\mathbf{Z}^{(2)} + \dots$

$$\delta Z_H^{(1)} = -\Sigma_H^{(1)'}(M_H^2)$$

$$\delta Z_H^{(2)} = -\Sigma_H^{(2)'}(M_H^2) + \left(\Sigma_H^{(1)'}(M_H^2)\right)^2$$

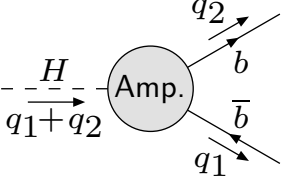
$$\delta Z_{f,l}^{(1)} = -\Sigma_L^{(1)} - \Sigma_L'^{(1)} - \Sigma_R'^{(1)} - 2\Sigma_S'^{(1)}$$

$$\begin{aligned} \delta Z_{f,l}^{(2)} = & -\Sigma_L^{(2)} - \Sigma_L'^{(2)} - \Sigma_R'^{(2)} - 2\Sigma_S'^{(2)} + \left(\Sigma_L^{(1)}\right)^2 + \left(\Sigma_L'^{(1)}\right)^2 + \left(\Sigma_R'^{(1)}\right)^2 + 4\left(\Sigma_S'^{(1)}\right)^2 + 2\Sigma_L^{(1)}\Sigma_L'^{(1)} \\ & + \Sigma_L^{(1)}\Sigma_R'^{(1)} + \Sigma_R^{(1)}\Sigma_R'^{(1)} + 2\Sigma_L^{(1)}\Sigma_S'^{(1)} - 2\Sigma_S^{(1)}\Sigma_S'^{(1)} + 2\Sigma_L'^{(1)}\Sigma_R'^{(1)} + 4\Sigma_L'^{(1)}\Sigma_S'^{(1)} + 4\Sigma_R'^{(1)}\Sigma_S'^{(1)} \end{aligned}$$

Used short notation: $\Sigma_X'^{(n)} = m_f^2 \frac{\partial}{\partial q^2} \Sigma_X^{f(n)}(q^2) \Big|_{q^2=m_f^2}$

Calculation of the Transition Matrix Element

- To the orders we calculated the **amputated** $H \rightarrow b\bar{b}$ diagrams have the following structure:



$$- \frac{H}{q_1 + q_2} \text{Amp.} = i \mathcal{A} = i \mathcal{A}_A + i \mathcal{A}_B \omega_+(\not{q}_2 - \not{q}_1)$$

- The transition matrix element according to the LSZ reduction formula:

$$\begin{aligned} \mathcal{T} &= \sqrt{Z_H} \left(\sqrt{Z_{b,r}} \bar{u}_r(q_2) + \sqrt{Z_{b,l}} \bar{u}_l(q_2) \right) \mathcal{A} \left(\sqrt{Z_{b,r}} v_r(q_1) + \sqrt{Z_{b,l}} v_l(q_1) \right) \\ &= \left(\mathcal{A}_A \sqrt{Z_{b,l} Z_{b,r}} + m_b \mathcal{A}_B Z_{b,l} \right) \sqrt{Z_H} \bar{u}(q_2) v(q_1) \end{aligned}$$

- Expand** the \mathcal{A} 's and Z 's perturbatively. Then replace **bare** masses by **renormalized** masses plus mass **counterterms** (also the bare masses in the counterterms themselves).

The Transition Matrix Element To $\mathcal{O}(G_F^2 m_t^4)$

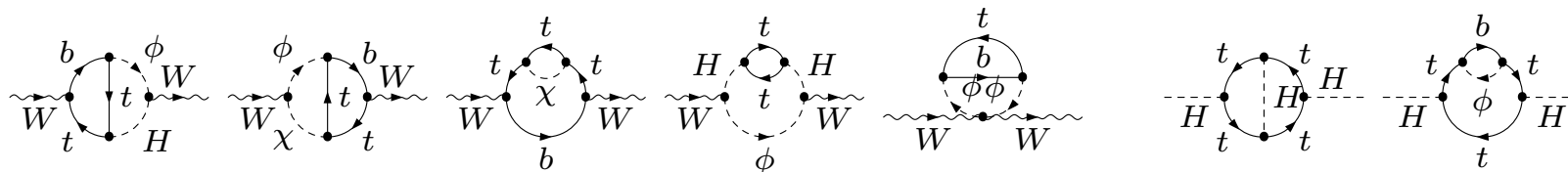
$$\begin{aligned}
\mathcal{T}^{(2)} = \mathcal{A}_A^{(0)} \bigg\{ & \delta_u^{(2)} + \frac{\delta m_b^{(2)}}{m_b} + \frac{1}{2} \delta Z_{b,l}^{(2)} + \frac{1}{2} \delta Z_{b,r}^{(2)} - \frac{1}{8} \left(\delta Z_{b,l}^{(1)} - \delta Z_{b,r}^{(1)} \right)^2 + \frac{1}{2} \frac{\delta m_b^{(1)}}{m_b} \left(\delta Z_{b,l}^{(1)} + \delta Z_{b,r}^{(1)} \right) \\
& + \left(\frac{1}{2} \delta Z_{b,r}^{(1)} + \frac{1}{2} \delta Z_{b,l}^{(1)} + \frac{\delta m_b^{(1)}}{m_b} \right) \left(\frac{1}{2} \delta Z_H^{(1)} - \frac{3}{2} \frac{\delta M_W^{2(1)}}{M_W^2} + (2 - 2\epsilon) \frac{\delta m_t^{(1)}}{m_t} \right) \bigg\} \\
& + \left(\mathcal{A}_A^{(1)} + m_b \mathcal{A}_B^{(1)} \right) \left(\frac{1}{2} \delta Z_H^{(1)} - \frac{3}{2} \frac{\delta M_W^{2(1)}}{M_W^2} + (2 - 2\epsilon) \frac{\delta m_t^{(1)}}{m_t} \right) + m_b \mathcal{A}_B^{(1)} \delta Z_{b,l}^{(1)} \\
& + \mathcal{A}_A^{(1)} \left(\frac{1}{2} \delta Z_{b,r}^{(1)} + \frac{1}{2} \delta Z_{b,l}^{(1)} + \frac{\delta m_b^{(1)}}{m_b} \right) + \mathcal{A}_A^{(2)} + m_b \mathcal{A}_B^{(2)}
\end{aligned}$$

Here, δ_u denotes the **universal counterterm**, which is finite by itself:

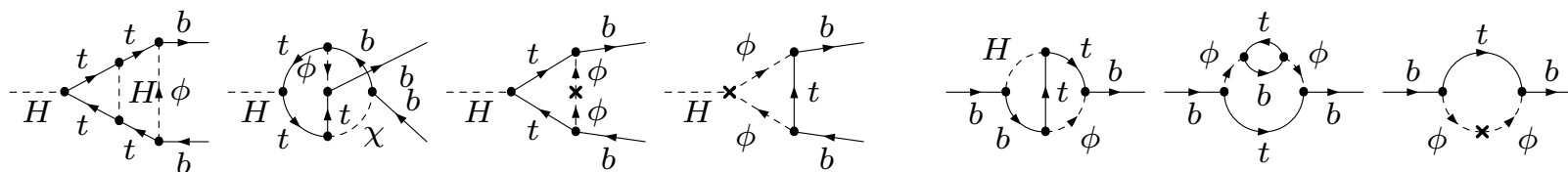
$$\delta_u^{(2)} = \frac{1}{2} \delta Z_H^{(2)} - \frac{1}{2} \frac{\delta M_W^{2(2)}}{M_W^2} + \frac{3}{8} \left(\frac{\delta M_W^{2(1)}}{M_W^2} \right)^2 - \frac{3}{4} \frac{\delta M_W^{2(1)}}{M_W^2} \delta Z_H^{(1)} - \frac{1}{8} \delta Z_H^{(1)2} + (1 - \epsilon) \frac{\delta m_t^{(1)}}{m_t} \left(\delta Z_H^{(1)} - \frac{\delta M_W^{2(1)}}{M_W^2} \right)$$

Calculated Diagrams

Universal Counterterm: Need Higgs and W self energies, e.g.



Non universal contributions: Also need $Hb\bar{b}$ diagrams and b self energies, e.g.



Here, tadpole counterterms and Higgs-mass counterterms occur:

$$H\phi\phi: iv^{-1}\delta M_H^2 + iv^{-2}\delta t$$

$$\phi\phi: iv^{-1}\delta t$$

Tadpole contributions do not cancel! They are necessary to get a UV-finite result.

The Final Result (1)

Universal Counterterm: $\delta_u^{(G_F^2 m_t^4)} = \left(\frac{G_F m_t^2}{8\sqrt{2} \pi^2} \right)^2 \left\{ N_C \left(\frac{29}{2} - 6 \zeta(2) \right) + N_C^2 \frac{49}{24} \right\}$

(Disagreement with [Djouadi *et al.*, 1998] \Rightarrow We could track down their error.)

Complete correction to the decay rate:

$$\frac{\Gamma(G_F^2 m_t^4)}{\Gamma^{(0)}} = \left(\frac{G_F m_t^2}{8\sqrt{2} \pi^2} \right)^2 \left\{ -20 + N_C \left(29 - 12 \zeta(2) \right) + N_C^2 \frac{49}{9} \right\} = + \mathbf{0.047\%}$$

The correction is **larger** than the correction of $\mathcal{O}(\alpha_s G_F m_t^2)$, which we recalculated:

$$\frac{\Gamma(\alpha_s G_F m_t^2)}{\Gamma^{(0)}} = \frac{G_F m_t^2}{8\sqrt{2} \pi^2} \frac{C_F \alpha_s}{\pi} \left\{ -36 + N_C \left(\frac{157}{12} - \zeta(2) \right) \right\} = - \mathbf{0.022\%}$$

The Final Result (2)

Corrections to the **decay rates** into Tauon, Charm and Bottom:

Correction	$\Gamma_\tau/\Gamma_\tau^{(0)}$	$\Gamma_c/\Gamma_c^{(0)}$	$\Gamma_b/\Gamma_b^{(0)}$
$\mathcal{O}(G_F m_t^2)$	+2.021%	+2.021%	+0.289%
$\mathcal{O}(G_F^2 m_t^4)$	+0.064%	+0.064%	+0.047%
$\mathcal{O}(\alpha_s G_F m_t^2)$	+0.060%	+0.452%	-0.022%

$$\frac{\Gamma_\tau}{\Gamma_\tau^{(0)}} = (1 + \delta_u)^2 - 1 \quad (\text{universal counterterm})$$

$$\frac{\Gamma_c}{\Gamma_c^{(0)}} = (1 + \Delta_{\text{QCD}})(1 + \delta_u)^2 - 1 \quad (\text{universal counterterm and 1-loop QCD correction})$$

Summary

- $H \rightarrow b\bar{b}$ by far **dominant decay channel** for $M_H < 140$ GeV.
(region favoured by electroweak precision tests)
- We calculated a **new** correction: $\mathcal{O}(G_F^2 m_t^4)$, assuming $M_H \ll m_t$.
- Result also applies to $b\bar{b} \rightarrow H$.
- Performed a **genuine two loop** renormalisation.
- Derived **general expressions** for mass counterterms and WFRC,
valid at an arbitrary number of loops.
- Our calculation is an example, where Higgs **tadpole contributions** do not cancel.
- The correction is **surprisingly large**, even larger than the $\mathcal{O}(\alpha_s G_F m_t^2)$ correction!