Efficient Reduced Magnetic Vector Potential Method for Superconducting Accelerator Magnets

TECHNISCHE UNIVERSITÄT DARMSTADT

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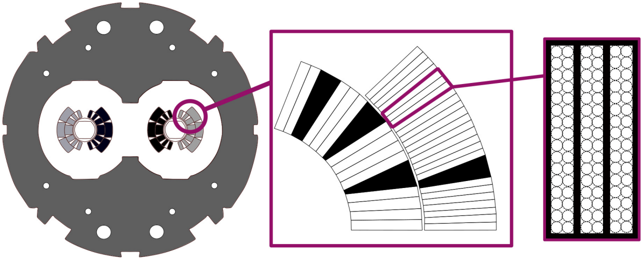
Superconducting Accelerator Magnet = Multi-Scale Problem



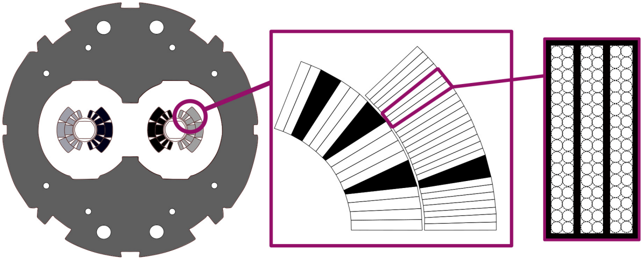
(a) (b) (c)

Challenges for the simulation:

- **Multi-scale** problem
- Multi-physical problem
- Three-dimensional effects
- Nonlinear material characteristics

⇒ **Meshing everything in full 3D resolution is expensive!**

Superconducting Accelerator Magnet = Multi-Scale Problem



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$\Rightarrow$ **Meshing everything in full 3D resolution is expensive!**

Solution idea:
Do not resolve the coils/cables/wires in the finite-element mesh, discretize by (macro-)wires

Outline

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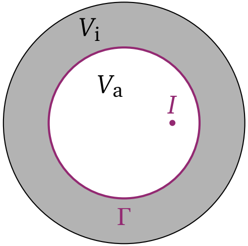
Reduced Magnetic Vector Potential (RMVP) Ansatz

Split the magnetic vector potential (MVP):

$$\vec{A}(\vec{r},t) = \underbrace{\vec{A}_\mathrm{s}(\vec{r},t)}_{\text{source MVP}} + \underbrace{\vec{A}_\mathrm{r}(\vec{r},t)}_{\text{reduced MVP}}$$

Biot-Savart integral for each wire $\mathscr{L}'$:

$$\vec{A}_\mathrm{s}(\vec{r}) = \frac{\mu_0}{4\pi}\int_{\mathscr{L}'} \frac{I}{|\vec{r}-\vec{r}'|}\,\mathrm{d}\vec{r}'$$



Split the total domain V into

- V_a: air domain with source currents
- V_i: source-free domain, typically iron yoke
- Γ: interface surface between V_a and V_i

Standard RMVP Formulation (Christian Paul 1997)

1. Evaluate the source MVP $\vec{A}_\mathrm{s}$ via Biot-Savart for each point $\vec{r} \in V$.

2. Find the reduced MVP $\vec{A}_\mathrm{r} \in H_\mathrm{r}(\mathrm{curl}; V) = \left\{ \vec{A}_\mathrm{r} \in H(\mathrm{curl}; V) \; : \; \vec{n} \times \vec{A}_\mathrm{r} = -\vec{n} \times \vec{A}_\mathrm{s} \text{ on } \partial V \right\}$, s.t.

$$\left(\nu \nabla \times \vec{A}_\mathrm{r}, \nabla \times \vec{A}_\mathrm{r}'\right)_V = -\left(\nu \nabla \times \vec{A}_\mathrm{s}, \nabla \times \vec{A}_\mathrm{r}'\right)_{V_\mathrm{i}} \qquad \forall \vec{A}_\mathrm{r}' \in H_\mathrm{r}(\mathrm{curl}; \mathrm{V})$$

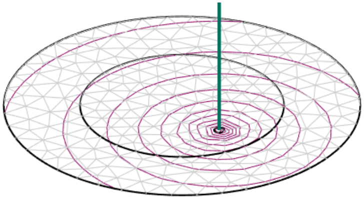
3. Compose total MVP: $\vec{A} = \vec{A}_\mathrm{s} + \vec{A}_\mathrm{r}$ in V.

☛ Can get computationally expensive!

Updated RMVP Formulation (D'Angelo et al. 2024)

1. Evaluate the source MVP $\vec{A}_\mathrm{s}$ via Biot-Savart only at the interface $\Gamma = \partial V_\mathrm{a}$...
   ...and on every point of interest $\vec{r} \in V_\mathrm{a}$.

☛ Huge improvement in computational efficiency!



source MVP $\vec{A}_\mathrm{s}$ (everywhere) and source current

Updated RMVP Formulation (D’Angelo et al. 2024)

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2. Find the image MVP $\vec{A}_\mathrm{m} \in H(\mathrm{curl}; V_\mathrm{a})$, $\vec{n} \times \vec{H}_\mathrm{m} \in H^{-1/2}(\mathrm{curl}; \Gamma)$, s.t.

$$\left(\nu_0 \nabla \times \vec{A}_\mathrm{m}, \nabla \times \vec{A}'_\mathrm{m}\right)_{V_\mathrm{a}} + \left(\vec{n} \times \vec{H}_\mathrm{m}, \vec{A}'_\mathrm{m}\right)_\Gamma = 0 \qquad \forall \vec{A}'_\mathrm{m} \in H(\mathrm{curl}; V_\mathrm{a}),$$

$$\left(\vec{A}_\mathrm{m}, \vec{n} \times \vec{H}'_\mathrm{m}\right)_\Gamma + \left(\vec{A}_\mathrm{s}, \vec{n} \times \vec{H}'_\mathrm{m}\right)_\Gamma = 0 \qquad \forall \vec{n} \times \vec{H}'_\mathrm{m} \in H^{-1/2}(\mathrm{curl}; \Gamma).$$

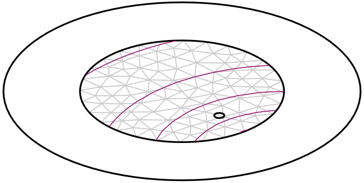

image MVP $\vec{A}_\mathrm{m}$ in V_a

Updated RMVP Formulation (D'Angelo et al. 2024)

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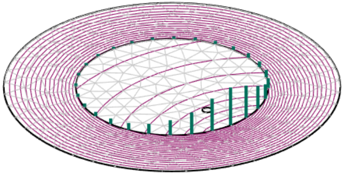
3. Find the reaction MVP $\vec{A}_\mathrm{g} \in H_0(\text{curl}; V)$ s.t.

$$\left(\nu\nabla\times\vec{A}_\mathrm{g}, \nabla\times\vec{A}'_\mathrm{g}\right)_V = \left(\vec{n}\times\vec{H}_\mathrm{s}, \vec{A}'_\mathrm{g}\right)_\Gamma + \left(\vec{n}\times\vec{H}_\mathrm{m}, \vec{A}'_\mathrm{g}\right)_\Gamma \quad \forall \vec{A}'_\mathrm{g} \in H_0(\text{curl}; V).$$



reaction MVP $\vec{A}_\mathrm{g}$ in V,
surface current density $\vec{K}_\mathrm{g} = \vec{n}\times(\vec{H}_\mathrm{s}+\vec{H}_\mathrm{m})$ on Γ

Updated RMVP Formulation (D'Angelo et al. 2024)

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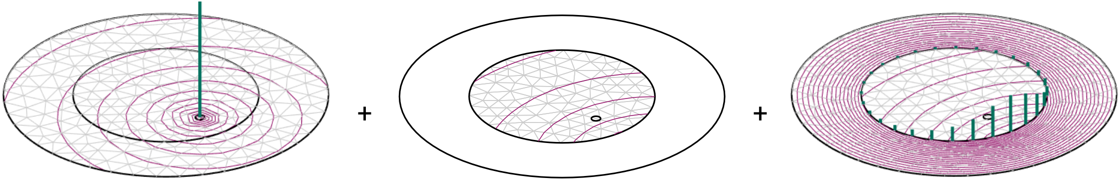

source MVP $\vec{A}_{\mathrm{s}}$ + image MVP $\vec{A}_{\mathrm{m}}$ + reaction MVP $\vec{A}_{\mathrm{g}}$

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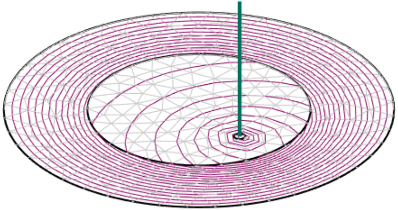

4. Compose final solution:

$$\vec{A} = \vec{A}_{\mathrm{s}} + \vec{A}_{\mathrm{m}} + \vec{A}_{\mathrm{g}} \quad \text{in } V_{\mathrm{a}},$$
$$\vec{A} = \vec{A}_{\mathrm{g}} \quad \text{in } V_{\mathrm{i}}.$$

Outline

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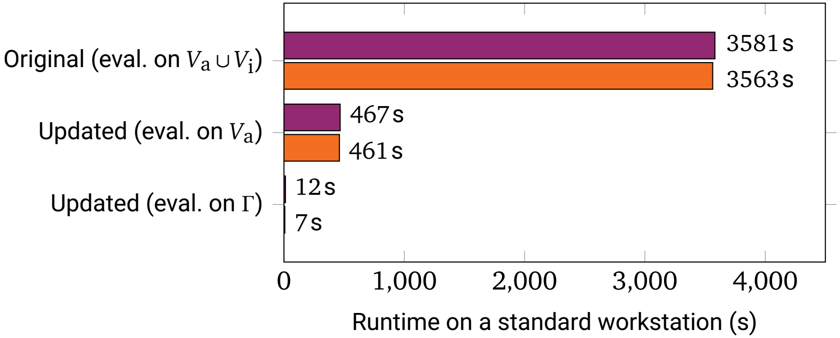
Performance Comparison

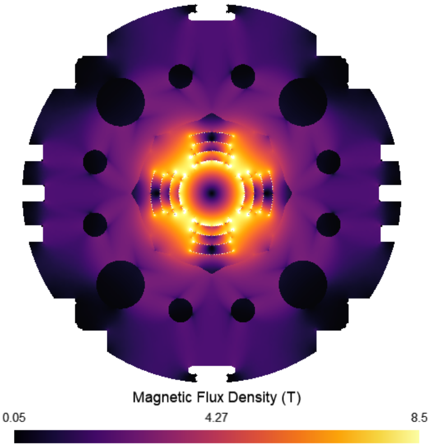
Benchmark Model with > 100,000 DoF and 18 wires

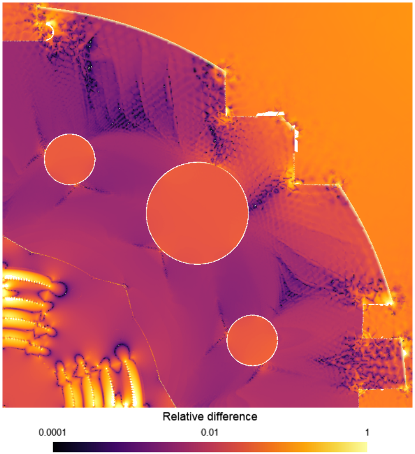


1. **Biot-Savart** computation is dominant ⇒ Parallelize and use fast-multipole methods
2. **Updated RMVP** formulation by far superior than standard one

Quadrupole Magnet Simulation: Magnetic Flux Density

MQXA low-beta quadrupole @LHC





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Summary

☛ **Goal:** Tackle the multi-scale problem of superconducting accelerator magnets

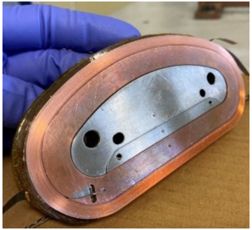
Updated RMVP Formulation:

- No explicit meshing of wires
- Biot-Savart's law for source field computation
- But evaluate only on $\Gamma \Rightarrow$ High efficiency increasement
- Superior efficiency to original RMVP

↪ QR code to paper

Outlook

- Expansion of the RMVP method for magnetization and superconducting effects
- Goal: Calculation of transient effects in high-temperature superconducting coils
- Collaborative research with Karlsruhe Institute of Technology (KIT)



Outlook

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Thank you for your attention!
Questions?