Trace-Anomaly Matching

Hamburg/Golm

8.1.2024

Trace Anomaly Matching

S. Theisen

Albert-Einstein-Institut Potsdam

based on

A. Schwimmer + S. Theisen 2307.14957

Outline

- Relevant properties of CFTs — any d
- their trace or Weyl or conformal anomalies — $d=2n$
    - type A
    - type B
- Anomaly matching — $d=4$
    - what is matched?
    - why?
- Simple illustrative example: free massless scalar — $d=4$

NB: I will not give any references. They can be found in the paper.

But I should mention interesting and relevant work for our analysis by Niarchos, Papageorgakis, (Pini) & Pomoni

which prompted us to reconsider the problem of Weyl anomaly matching.

For a very nice and thorough discussion of many aspects of Weyl anomalies I also recommend Baume, Karen-Zur, Rattazzi & Vitale

Conformal Field Theories $(d>2)$

classical: action invariant under conformal transformations

$SO(d,2)$ invariance of light-cone in M^d_{ink}

Poincaré $\quad x^\mu \to \Lambda^\mu{}_\nu x^\nu + a^\mu$

+ dilations / scale transf. $\quad x^\mu \to \lambda x^\mu$

+ special conf. transf. $\quad x^\mu \to \dfrac{x^\mu + x^2 b^\mu}{1 + 2\, b\cdot x + x^2 b^2}$

Due to the scale symmetry, CFTs do not have an intrinsic length/energy/mass scale.

Simplest example: free massless scalar in any d

A useful connection:

conf. inv. in Mink $\eta_{\mu\nu}$ $\longleftrightarrow$ Weyl inv. in curved space $g_{\mu\nu}$

e.g. free scalar

$$S(\phi, g) = \int d^dx \sqrt{g} \left(g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + \xi R(g) \phi^2 \right) \qquad \xi = \frac{d-2}{4(d-1)}$$

in special dims. potential is also conf. inv. e.g. ϕ^4 in $d=4$

is invariant under

Weyl tranf.

$$g_{\mu\nu} \to e^{2\sigma} g_{\mu\nu} \qquad \sigma = \sigma(x) \quad \text{Weyl parameter}$$

$$\phi \to e^{-\Delta\sigma} \phi \qquad \Delta = \frac{d-2}{2} \quad \text{conf. weight of } \phi$$

in addition to inv. under diffeos

$$\delta g_{\mu\nu} = \nabla_\mu \epsilon_\nu + \nabla_\nu \epsilon_\mu$$

$$\delta \phi = \epsilon^\mu \partial_\mu \phi$$

Those special combinations of Weyl + diffeo transf., which leave the Minkowski metric invariant,

$$\partial_\mu \varepsilon_\nu + \partial_\nu \varepsilon_\mu + 2\sigma \eta_{\mu\nu} = 0$$

$$\epsilon^\mu = a^\mu + \omega^\mu{}_\nu x^\nu + \lambda x^\mu + x^2 b^\mu - 2 b\cdot x \, x^\mu$$

$$\sigma = -\lambda + 2 b\cdot x$$

generate conf. transf. of ϕ (or any other primary field)

"tensors" under conf. group

$$\delta\phi = \lambda(x^\mu \partial_\mu + \Delta)\phi + b^\mu (x^2 \partial_\mu - 2x_\mu \, x\cdot\partial - 2\Delta x_\mu)\phi + \text{Poincaré}$$

Once we have the coupling to $g_{\mu\nu}$ we can define a symmetric energy-momentum tensor via

$$T_{\mu\nu} = -\frac{2}{\sqrt{g}} \frac{\delta}{\delta g^{\mu\nu}} S$$

with the following on-shell properties $\frac{\delta S}{\delta \phi} = 0$

- $\nabla^\mu T_{\mu\nu} = 0$ diff inv. of S
- $T^\mu{}_\mu = 0$ Weyl inv. of S

These properties suffice to show that the Noether currents for conf. symm. are conserved.

Comments

- All interesting classical conf. inv. field theories can be coupled in a Weyl inv. way to gravity

  in particular those which lead upon quantization to a unitary QFT

- Here the metric is not dynamical, it rather acts as a source for the energy-momentum tensor.

so much for classical CFTs ...

Conformal Field Theories (d>2)

quantum:

Symmetries → restrictions on general form of correlation functions of local operators

conservation laws → Ward identities of correlators involving the symmetry currents

Of course a central issue are anomalies, here of conf. transfs.
As they are global (space-time) symmetries, they are no problem but rather a blessing. (exception: string theory)

In the following we will assume that the QFT has among its local ops an energy-momentum tensor. $T_{\mu\nu}$

If the QFT arises from a classical FT upon quantization, we couple it to an external metric and define

$$e^{W[g]} = \int D\phi \; e^{-S[\phi,g]}$$

$W[g]$: generating functional for connected correlation functions of $T_{\mu\nu}$

e.g. $$\langle T_{\mu\nu}(x) \rangle_g = -\frac{2}{\sqrt{g}} \frac{\delta}{\delta g^{\mu\nu}(x)} W(g) \quad , \ldots$$

or

$$W = \frac{1}{2!\,4} \int dx\, dy\, \Gamma^{(2)}_{\mu\nu,\rho\sigma}(x,y)\, h^{\mu\nu}(x)\, h^{\rho\sigma}(y)$$

expand around $g_{\mu\nu} = \eta_{\mu\nu}$

$$- \frac{1}{3!\,8} \int dx\, dy\, dz\, \Gamma^{(3)}_{\mu\nu,\rho\sigma,\alpha\beta}(x,y,z)\, h^{\mu\nu}(x)\, h^{\rho\sigma}(y)\, h^{\alpha\beta}(z) + \dots$$

where

- $$h^{\mu\nu} = g^{\mu\nu} - \eta^{\mu\nu}$$

- $$\Gamma^{(2)}_{\mu\nu,\rho\sigma}(x,y) = \langle T_{\mu\nu}(x)\, T_{\rho\sigma}(y) \rangle_\eta \quad , \text{ etc. for } \Gamma^{(n)}_{\dots}(\dots)$$

vevs of T-product

i.e. W sums up the correlation functions of $T_{\mu\nu}$ and as such is defined for any QFT with local $T_{\mu\nu}$.

If we are interested in correlation functions of other operators $\mathcal{O}_\Delta$ we introduce sources J and obtain

$$e^{W[g,J]} = \int D\phi \, e^{-S[\phi,g,J]}$$

In the absense of anomalies W has the same symmetries as the classical action.

For conformal $\longrightarrow$ diffeo + Weyl this implies

$$\delta_\sigma W = 0 \quad \text{under} \quad \delta_\sigma g_{\mu\nu} = 2\sigma g_{\mu\nu}$$

$$\delta_\sigma J = (\Delta - d)\sigma J \qquad \delta_\sigma \mathcal{O}_\Delta = -\Delta \sigma \mathcal{O}_\Delta$$

s.t. $\int \sqrt{g}\, J\, \mathcal{O}_\Delta$ inv.

This leads to Ward identities for the $\Gamma^{(n)}$s :

e.g. $$W(g,J) = \frac{1}{2}\int dx\, dy\, \langle O_\Delta(x)\, O_\Delta(y)\rangle\, J(x) J(y)$$

$$- \frac{1}{4}\int dx\, dy\, dz\, \langle T_{\mu\nu}(x)\, O_\Delta(y)\, O_\Delta(z)\rangle\, h^{\mu\nu}(x)\, J(y)\, J(z) + \dots$$

- diffeo : $\delta J = \epsilon^\mu \partial_\mu J \qquad \delta h^{\mu\nu} = -\partial^\mu \epsilon^\nu - \partial^\nu \epsilon^\mu + O(h)$

$$\delta_\epsilon W = 0 \Rightarrow \partial^\mu_x\, \Gamma^{(3)}_{\mu\nu}(x,y,z) = \partial^x_\nu\, \delta(x-y)\, \Gamma^{(2)}(x,z) + \partial^x_\nu\, \delta(x-z)\, \Gamma^{(2)}(x,y)$$

....

- Weyl: $\delta J = (\Delta - d)\sigma J, \quad \delta h^{\mu\nu} = -2\sigma\eta^{\mu\nu} + O(h)$

$$\delta_\sigma W = 0 \Rightarrow \eta^{\mu\nu}\, \Gamma^{(3)}_{\mu\nu}(x,y,z) = (d-\Delta)\left(\delta(x-y) + \delta(x-z)\right)\Gamma^{(2)}(y,z)$$

...

They relate n- to (n-1)-pt fctns.

However it is well known (c.f. Duff review) that the Weyl and diffeo Ward identities are not always satisfied but are anomalous.

More specifically:

- The anomaly is an unavoidable clash between diffeo & Weyl. But one of the two W.I.s can always be imposed. Which one is up to us. One usually chooses diffeo to be preserved.
- whether the other one is broken depends on d and Δ for even d there is always an anomaly in $\langle T_{\mu\nu} \dots T_{\rho\sigma} \rangle$
- the anomaly is local, i.e. the W.I.s are broken by pure contact terms

$$\sim \delta(x-y)\,\delta(x-z)\ \dots$$

Anomalies are constrained by the Wess-Zumino consistency condition:

$$\delta_\sigma W = \int \sqrt{g}\, \sigma \mathcal{A} \qquad \mathcal{A} \text{ local functional of sources}$$

$$[\delta_{\sigma_1}, \delta_{\sigma_2}] W = 0$$

$$\Longrightarrow \quad \delta_{\sigma_2} \int \sqrt{g}\, \sigma_1 \mathcal{A} - \delta_{\sigma_1} \int \sqrt{g}\, \sigma_2 \mathcal{A} = 0$$

$$\text{modulo} \qquad \int \sqrt{g}\, \sigma \mathcal{A} \neq \delta_\sigma \int \sqrt{g} \text{ local}$$

(W.Z. cohomology problem)

I will now be more specific and restrict to

$d = 4, \quad \Delta = 2$

i.e. we have an operator $\mathcal{O}$ of dimension 2 in the CFT

with source J $\quad \delta_\sigma J = -2\sigma J$

Solutions to W.Z.

$$\delta_\sigma W[g, J] = \int \sqrt{g}\, \sigma \left\{ \bar{c}\, J^2 + c\, C^2 - a\, E_4 \right\}$$

where:

- $\bar{c}, c, a$ constants $\quad a, c, \bar{c}$ Weyl anomaly coefficients characteristic of a given CFT

$$\delta_\sigma W[g, J] = \int \sqrt{g}\,\sigma \left\{ \tilde{c}\, J^2 + c\, C^2 - a\, E_4 \right\}$$ *

- $C^2 = C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma}$ square of Weyl tensor

  $E_4 = R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} - 4 R^{\mu\nu} R_{\mu\nu} + R^2$ (topological) Euler density

Weyl anomalies come in two types: Type A & Type B — Deser & Schwimmer

The first two in * are Type B, the last one is Type A.

In which ways do they differ?

Type A vs Type B

- **Type B:**
  - originate from a log-divergence in correlation functs.
  - WZ is trivially satisfied, i.e. anomaly is itself Weyl inv.
  - does not vanish for $\sigma = \text{const}$.
  - Anomaly coefficient related to normalization of a 2pt. fct.

- **Type A:**
  - do not originate from divergence but from $\frac{0}{0}$ structure in dim. reg.
  - WZ non-trivially satisfied; i.e. anomaly itself is not Weyl inv.
  - for $\sigma = \text{const}$ it vanishes in top. trivial backgrd.

$$\sqrt{g}\, E_4 = \partial^\mu k_\mu$$

In any even d, there is always one Type A (E_d), but a growing (with d) # of type B's.

Comment:

Recently there has been a lot of activity, also by mathematicians, on chiral ('t Hooft) anomalies (including discrete, higher form, ...)
It is not clear how much of this applies to Weyl anomalies.

The difference being that in Euclidean space chiral anomalies are phases in the generating functional, while Weyl anomalies are real.

Is there a descent procedure for Weyl anomalies? (Holography?)
Our treatment of Weyl anomalies can also be applied to continuous chiral anomalies.

We will now turn to anomaly matching ...

Anomaly matching

general remarks:

- well known for chiral anomalies ('t Hooft)
  useful tool to constrain the IR-theory to which a given UV-theory flows.

- anomaly matching for Type A Weyl anomaly (in the sense described below) was proven some time ago...

- ... and was an essential ingredient in the Komargodski-Schwimmer proof of the a-theorem

- matching of type B Weyl anomalies remained an open question

but we need to explain first what we mean by matching.

This best illustrated by the example of the anomaly in

$$\langle T_{\mu\nu}(-q)\, O(k_1)\, O(k_2)\rangle \equiv \Gamma^{(3)}_{\mu\nu}(q, k_1, k_2)$$

$\dim O = 2$

$q = k_1 + k_2$

We recall the W.I., written in momentum space

diffeo $$q^{\mu}\, \Gamma^{(3)}_{\mu\nu}(q, k_1, k_2) = k_{1\nu}\, \Gamma^{(2)}(k_2^2) + k_{2\nu}\, \Gamma^{(2)}(k_1^2)$$

Weyl $$\eta^{\mu\nu}\, \Gamma^{(3)}_{\mu\nu}(q, k_1, k_2) = \Gamma^{(2)}(k_1^2) + \Gamma^{(2)}(k_2^2) + 2\bar{c}$$

where we have included the anomaly

$\bar{c}$ is the normalization of the 2pt function :

$$\Gamma^{(2)}(k) = \langle O(k)\, O(-k)\rangle = -\bar{c}\, \log k^2/\Lambda^2$$

We now expand $\Gamma^{(3)}_{\mu\nu}$ in invariant amplitudes $A(q^2, k_1^2, k_2^2), \ldots$

(*) $$\Gamma^{(3)}_{\mu\nu}(q, k_1, k_2) = A\,\eta_{\mu\nu} + B\,q_\mu q_\nu + C\,(q_\mu r_\nu + q_\nu r_\mu) + D\,r_\mu r_\nu$$

$q = k_1 + k_2$
$r = k_1 - k_2$

$\dim A = 0$ $\quad\sim \log \Lambda^2$ dep.

$\dim B, C, D = -2$ $\quad$ finite, scheme indep.

Insert in W.I.s $\Rightarrow$ 3 eqs. 2 from diff W.I. + 1 from Weyl W.I.

They can be used to arrive at

1 anomalous relation between finite amplitudes:

$$\boxed{-3q^2 B - 2\,q\cdot r\, C + r^2 D = 2\bar{c}}$$

This is the central eq and if we rewrite it in the general form ...

(*) $$\boxed{s_1 E_1 + s_2 E_2 + s_3 E_3 = c}$$

$s_i = (q_i^2, k_1^2, k_2^2)$

$c =$ const.

$E_i(s_1, s_2, s_3)$ dim - 2 amplitudes

... it generalizes to other cases.

here $E_1 = -3B - D$ $E_2 = 2(D - C)$ $E_3 = 2(D + C)$

Now comes a crucial remark: nowhere in the derivation of the WIs and therefore of (*) the conformal invariance of the vacuum was used. Only Poincaré inv. was used.

Therefore let us assume there exists another Poincaré inv. vacuum s.t.

$$\langle 0|\mathcal{O}|0\rangle = v^{\Delta} \neq 0 \qquad \dim \mathcal{O} = \Delta > 0$$

$$[v] = \text{mass}$$

i.e. SSB conf. sym $\longrightarrow$ Poincaré sym.

with the usual consequences, e.g. massless Goldstone boson: dilaton.

+ massive states $m^2 \sim v^2$

But most importantly for us:

- all operatorial relations are unmodified e.g. $T^{\mu}_{\mu} = 0$
- the Ward identitie still hold

In particular

$$\boxed{S_1 E_1^B + S_2 E_2^B + S_3 E_3^B = c^B}$$

- $E_i^B(S_1, S_2, S_3, v^2)$ they are quite different from the amplitudes in unbroken phase
- $c^B = \text{const.}$ (indep. of v.) the solutions to WZ consistency are as before

But

$$c^B \overset{?}{=} c$$

This is the issue of Weyl <u>anomaly matching</u>

Matching now follows from the following general fact about amplitudes in a SSB phase

$$\lim_{\lambda \to \infty} \frac{A(\lambda q_1^2, \lambda q_2^2, \dots)}{A^B(\lambda q_1^2, \lambda q_2^2, \dots)} = 1$$

As the kinematical invariants are sent to ∞ the signs of SSB (e.g. $m^2 \sim v^2$) disappear.

This a consequence of the fact that the OPEs are unchanged
$\Rightarrow$ UV - structure of correlators remains the same
(while for finite values of the s_i they are quite different, as the spectrum is now massive, etc.)

Therefore

$$\lim_{\lambda\to\infty} \frac{\lambda s_1 E_1(\lambda s_1, \lambda s_2, \lambda s_3) + \lambda s_2 E_2(\lambda s_1, \lambda s_2, \lambda s_3) + \lambda s_3 E_3(\lambda s_1, \lambda s_2, \lambda s_3)}{\lambda s_1 E_1^B(\lambda s_1, \lambda s_2, \lambda s_3) + \lambda s_2 E_2^B(\lambda s_1, \lambda s_2, \lambda s_3) + \lambda s_3 E_3^B(\lambda s_1, \lambda s_2, \lambda s_3)} = \frac{c}{c^B} = 1$$

i.e.

$$\boxed{c^B = c}$$

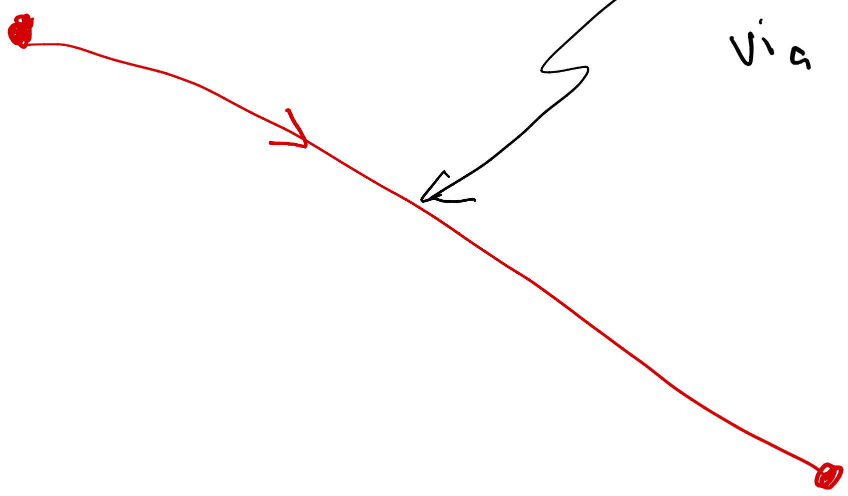
This establishes Weyl anomaly matching:

The l.h.s. of the anomalous WIs evaluate to the same constant in the two phases. for any value of the breaking scale.

i.e. for a flow triggered by SBB

CFT_{UV}

all anomaly coefficients defined via Weyl WI are const.



CFT_{IR} incl. dilaton

Comment:

The analysis of the anomaly in the correlator

$$\langle T_{\mu\nu} T_{\rho\sigma} T_{\alpha\beta} \rangle$$

is considerably more involved (136 inv. amplitudes of dims. 4, 2, 0, −2)

Nevertheless one finds WIs involving only dim −2 amplitudes

$$\boxed{\begin{aligned} s_1 \tilde{E}_1 + s_2 \tilde{E}_2 + s_3 \tilde{E}_3 &= a \\ s_1 E_1 + s_2 E_2 + s_3 E_3 &= c \end{aligned}}$$

$E_i, \tilde{E}_i$ linear comb.s of the 27 dim −2 amplitudes

This guarantees the matching of the two anomaly coefficients

We can also use our master eq.

$$s_1 E_1 + s_2 E_2 + s_3 E_3 = c \qquad (*)$$

to derive a sum rule for each of the invariant amplitudes E_i:

- As the E_i have dimension -2, they satisfy unsubtracted disp. rels.

$$E_i(s_i, s_j, s_k) = \frac{1}{\pi}\int dx_i \, \frac{\mathrm{Im}_i E_i(x_i, s_j, s_k)}{x_i - s_i} \qquad s_j, s_k \text{ fixed}$$

- large s behaviour of a dim -2 inv. amplitude in a CFT is generically

$$\frac{1}{s}\left[\log s\right]^p \qquad \text{for any of the } s_i$$

However if it satisfies $(*)$, it also satisfies

$s_1 E_1 + s_2 E_2 + s_3 E_3 = c$

$$s_i \, \mathrm{Im}_i E_i + s_j \, \mathrm{Im}_i E_j + s_k \, \mathrm{Im}_i E_k = 0 \qquad i \neq j \neq k$$

which implies

$$\mathrm{Im}_i E_i \longrightarrow \frac{1}{s_i^2} \left[\log s_i\right]^p$$

$(*)$

$$\Longrightarrow \qquad E_i \xrightarrow[s_i \to \infty]{} \frac{c}{s_i} + O\left(\frac{s_j, s_k}{s_i^2} (\log s_i)^p \right)$$

and from the dispersion relation we obtain the sum rules

$$E_i(s_i, s_j, s_k) = \frac{1}{\pi} \int dx_i \, \frac{\mathrm{Im}_i E_i(x_i, s_j, s_k)}{x_i - s_i}$$

$$-\frac{1}{\pi} \int ds_i \, \mathrm{Im}_i E_i(s_i, s_j, s_k) = c$$

for each E_i in $(*)$

s_j, s_k fixed

An identical analysis in the broken phase gives

$$-\frac{1}{\pi}\int ds_i \; \mathrm{Im}_i \, E_i^B(s_i, s_j, s_k) = c^B = c \quad \text{for each } E_i^B$$

use matching

These sum rules are valid for all $0 \leq v < \infty$ and exist for type A and type B Weyl anomalies. They are a consequence of the Ward identities.

All features of the analysis presented so far can be realized on a simple toy model where all features can be explicitly verified

For the unbroken phase, consider a

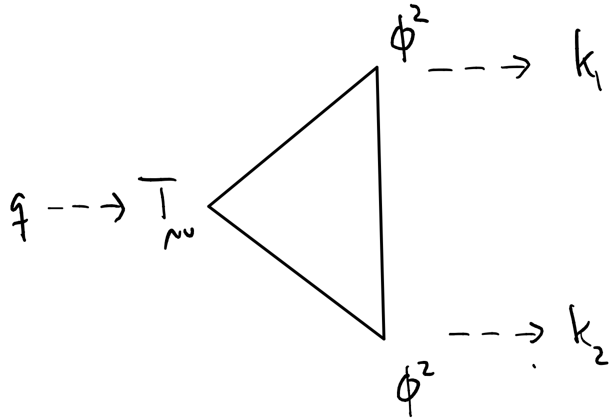
free massless scalar ϕ in $d=4$ with $\mathcal{O}_{\Delta=2} = \phi^2$

$$S = \frac{1}{2}\int d^4x \sqrt{g}\left(\nabla^\mu\phi\nabla_\mu\phi + \frac{1}{6}R\phi^2\right)$$

$$T_{\mu\nu} = \partial_\mu\phi\,\partial_\nu\phi - \frac{1}{2}\eta_{\mu\nu}\partial^\rho\phi\,\partial_\rho\phi + \frac{1}{6}\left(\eta_{\mu\nu}\Box - \partial_\mu\partial_\nu\right)\phi^2$$

$$\partial^\mu T_{\mu\nu} = 0 = T^\mu{}_\mu \qquad \text{on-shell} \qquad \Box\phi = 0$$

$$\langle T_{\mu\nu}(-q)\,\phi^2(k_1)\,\phi^2(k_2)\rangle$$



log divergent, but dim -2 amplitudes can be unambiguously identified by their tensor structure

recall the decomposition

$$\Gamma^{(3)}_{\mu\nu}(q,k_1,k_2) = A\,\eta_{\mu\nu} + \underline{B}\,q_\mu q_\nu + \underline{C}\,(q_\mu r_\nu + q_\nu r_\mu) + \underline{D}\,r_\mu r_\nu$$

and

$$E_1 = -3B - D \qquad E_2 = 2(D-C) \qquad E_3 = 2(D+C)$$

Indeed one finds that they are indep. of the RG-scale and they satisfy

(x) $$E_i \xrightarrow[s_i \to \infty]{} \frac{4}{s_i}$$ consistent with $E_i \to \frac{c}{s_i}$ in the normalizations chosen for the calculation

One can also compute, using Cutkosky rules, e.g.

$$\mathrm{Im}_{q^2}\, E_1(q^2, k_1^2, k_2^2)$$

and verify the sum rule

(xx) $$-\frac{1}{\pi}\int_0^\infty dq^2\, \mathrm{Im}_{q^2}\, E_1(q^2, k_1^2, k_2^2) = 4$$

Comment: (x) & (xx) also hold for a massive conf. coupled scalar

For the broken phase we consider two massless scalars ϕ, φ coupled through a marginal interaction, i.e.

$$L = \frac{1}{2}\partial^\mu\phi\partial_\mu\phi + \frac{1}{2}\partial^\mu\varphi\partial_\mu\varphi - g\,\phi^2\varphi^2$$

in flat space

In both phases we will eventually take $g \to 0$ to preserve conformality

Breaking : $\langle\varphi\rangle = v$

To compute in the broken vacuum we shift in L the field φ

$$\varphi = v + \tilde{\varphi}$$

and calculate with the usual Feynman rules for $\tilde{\varphi}$.

In the broken phase the system consists of

- massive field ϕ $\quad M^2 = 2g v^2$
- massless Goldstone boson $\tilde{\varphi}$ $\quad$ related to dilaton

with linear coupling in energy-momentum tensor

$$T_{\mu\nu} = \frac{1}{3} v \left(\eta_{\mu\nu} \Box - \partial_\mu \partial_\nu \right) \tilde{\varphi} + \dots$$

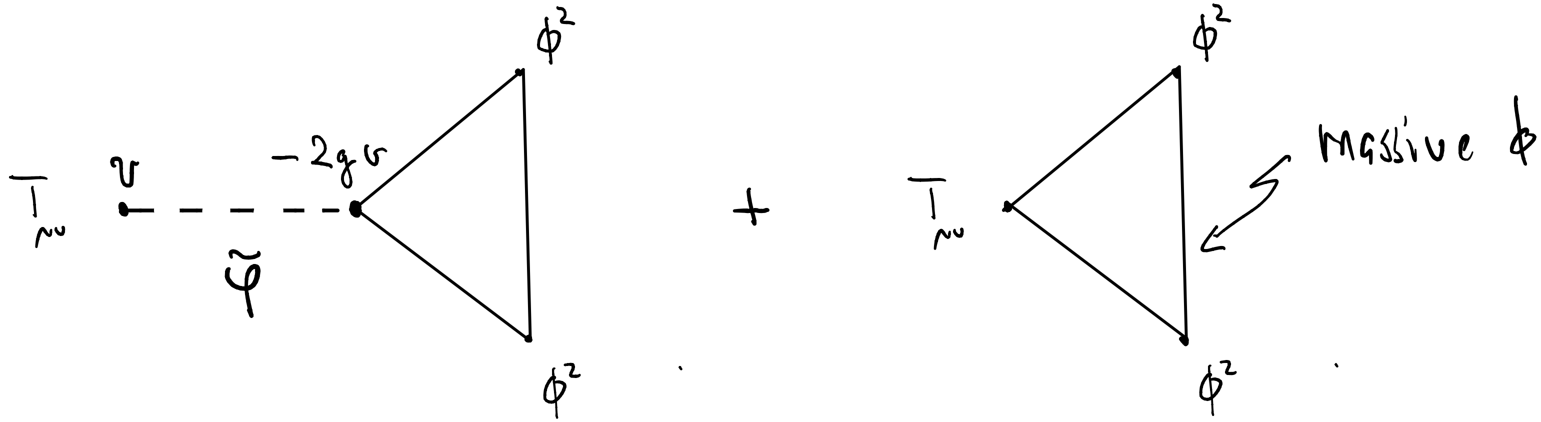
and cubic coupling

$$- 2 g v \phi^2 \tilde{\varphi}$$

We now consider the limit

$$g \to 0\,, \quad v \to \infty\,, \qquad M^2 = 2gv^2 \text{ fixed}$$

In this limit only two diagrams contribute to $\langle T_{\mu\nu}\, \phi^2\, \phi^2 \rangle$



- One checks that for generic M^2 both diagrams are needed to reproduce the anomaly coefficient.
- However the first diagram does not contribute to the sum rules, which is completely saturated by the massive triangle.

Two interesting limits:

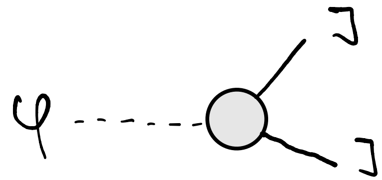
- $M^2 = 0$: the first, the dilaton diagram vanishes

  $\longrightarrow$ back to unbroken case

- $M^2 = \infty$: – the contribution of 2^{nd} diagram to dim - 2 amplitudes vanishes

  – the dilaton diagram has a finite limit

$$E_1 = \frac{4}{g^2} \,, \qquad E_2 = E_3 = 0$$



effective dilaton vertex in WZ action

i.e. it saturates the anomalous W.I. by itself.

note: anomaly matching $\Longrightarrow$ constraints on dilaton couplings

One could also discuss the $\langle T_{\mu\nu} T_{\rho\sigma} T_{\alpha\beta} \rangle$ for this toy model in the broken phase.

This is much more involved and will not be done here.

Instead ...

Summary

By writing the anomalous WI in terms of scheme indep. amplitudes (dim-2) one obtains a definition for the anomaly coefficients which is constant along RG-flows triggered by SSB.