Taxonomy of Infinite-Distance Limits

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Based on 2206.04063 with Muldrow Etheredge, Ben Heidenreich, Sami Kaya, and Yue Qiu
2306.16440, work to appear with Muldrow Etheredge, Ben Heidenreich,
Jacob McNamara, Ignacio Ruiz Garcia, Irene Valenzuela
2309.10024, 2312.00120
Many figures adapted from Ben Heidenreich's talk at String Pheno 2023

Outline

I. Review: Some Swampland Conjectures
II. Classification of Infinite-Distance Polytopes
III. Evidence from Supergravity
IV. Applications
V. Conclusions
VI. Bonus: Persistence of the “Pattern” in 5d Moduli Spaces

Review: Some Swampland Conjectures

The Distance Conjecture

Massless scalar fields parametrize a “moduli space” of vacua.

At large distances in moduli space, a tower of particles becomes light exponentially quickly with increasing distance:

$$m(\phi) \sim e^{-\alpha\phi}$$

Ooguri, Vafa ‘06

The Sharpened Distance Conjecture

In a given infinite-distance limit, at least one tower satisfies the Distance Conjecture with a coefficient α that satisfies

$$\boxed{\alpha \geq 1/\sqrt{d-2}}$$

Etheredge, Heidenreich, Kaya, Qiu, TR ’22

The Emergent String Conjecture

Any infinite-distance limit in moduli space is either a decompactification limit (accompanied by a tower of Kaluza-Klein modes) or an emergent string limit (accompanied by a tower of string oscillator modes)

In known examples, decompactification limits strictly satisfy the Sharpened Distance Conjecture ($\alpha > 1/\sqrt{d-2}$), emergent string limits saturate it ($\alpha = 1/\sqrt{d-2}$)

Lee, Lerche, Weigand '19

Scalar Weak Gravity Conjecture (SWGC)

Given a (canonically normalized) massless scalar field, there exists a particle of mass *m* that satisfies

$$-\partial_\phi \log m \geq 1/\sqrt{d-2}$$

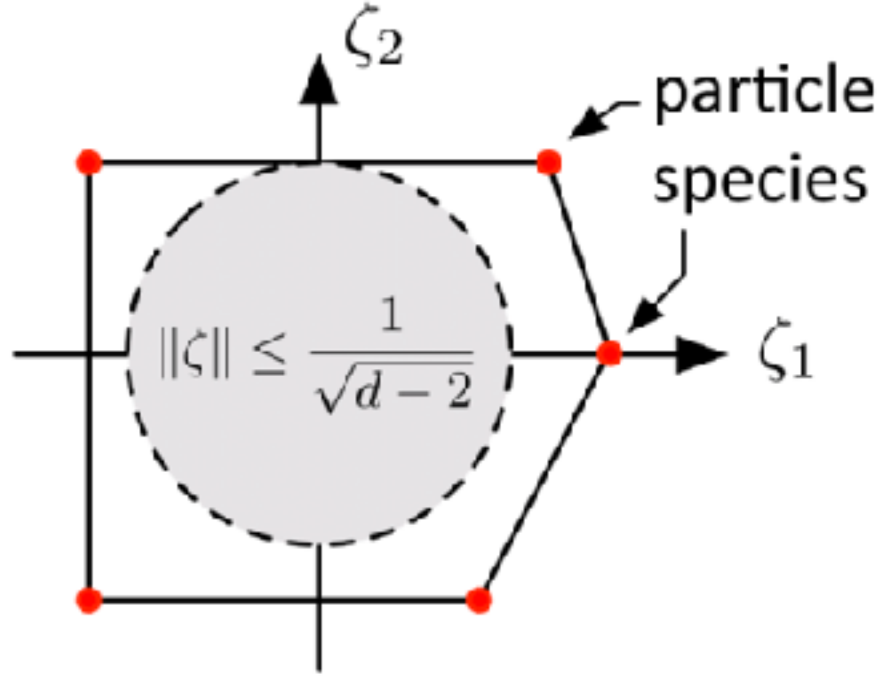
Etheredge, Heidenreich, Kaya, Qiu, TR ’22
building on Palti ’17, Calderon-Infante, Uranga, Valenzuela ‘20

SWGC: A Reformulation

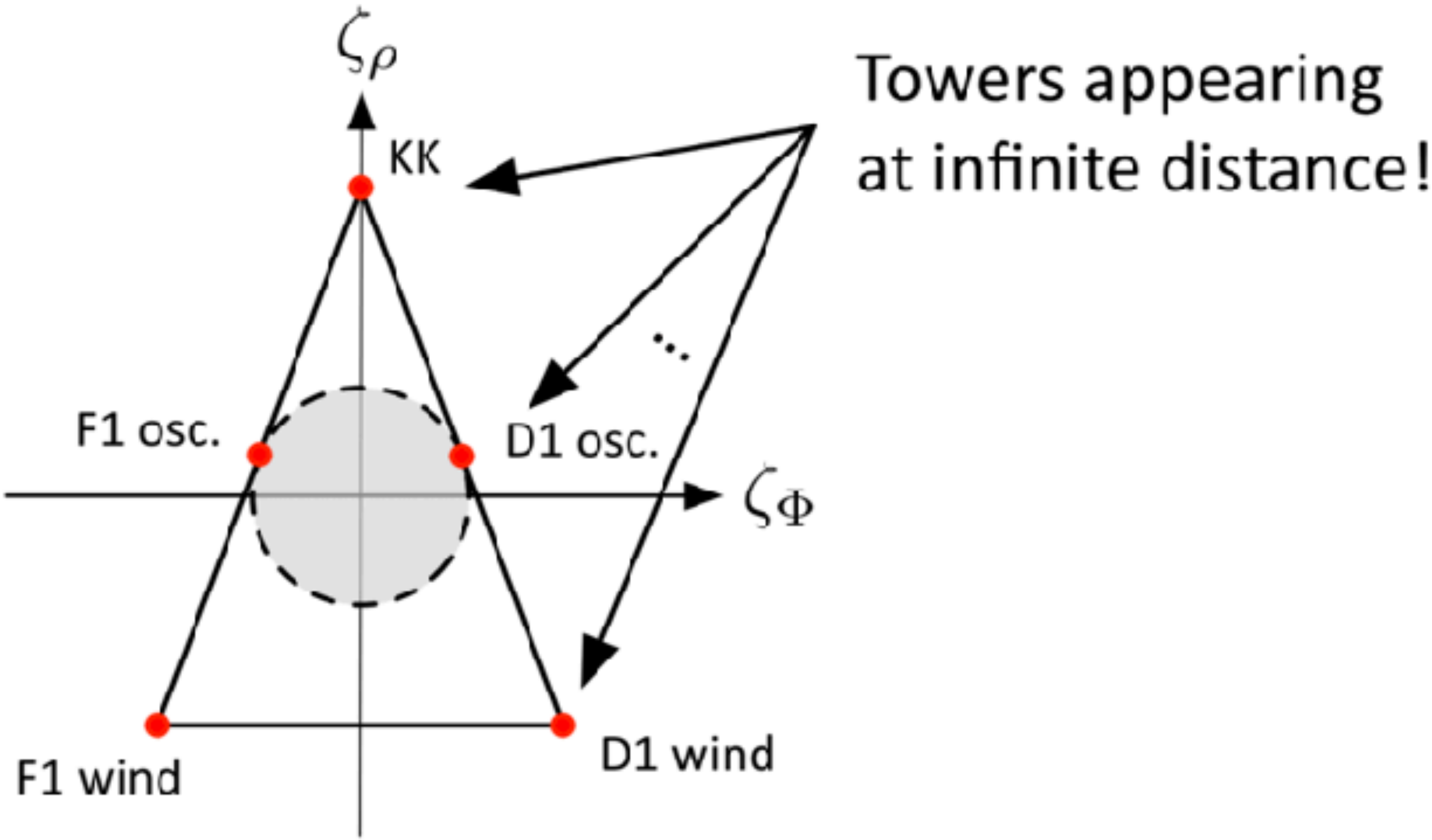
Given a collection of canonically normalized scalar fields ϕ_i and a particle of mass m, define the scalar charge-to-mass vector

$$\zeta_i = -\partial_{\phi_i} \log m$$

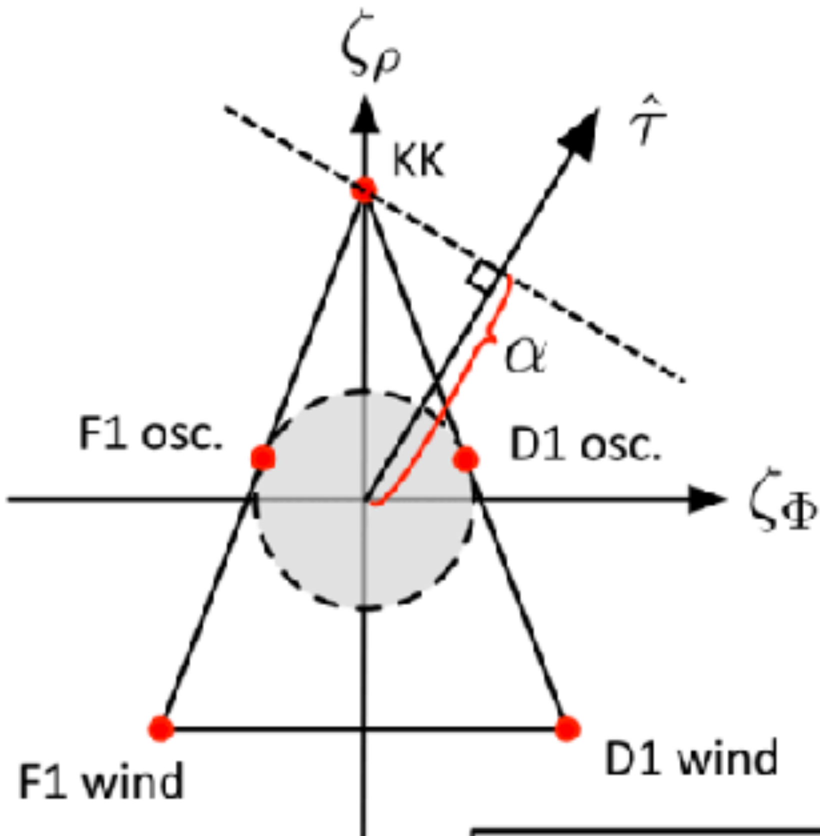
SWGC ⇔ Convex hull condition



Example: IIB in 9d



Example: IIB in 9d



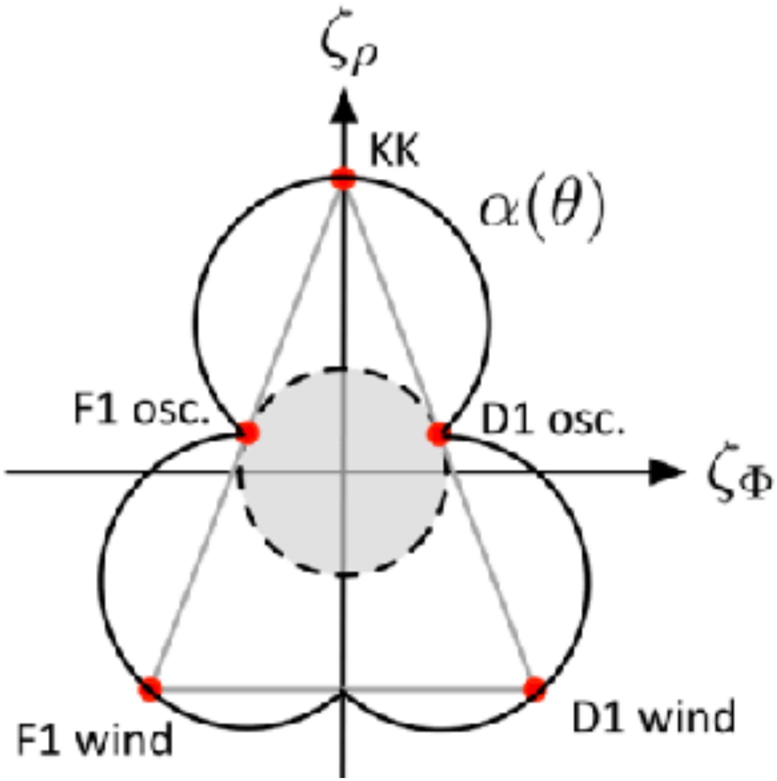
$$m_{\text{tower}} \propto e^{-\alpha\tau}$$ ← geodesic distance

$$\hat{\tau}^i = \frac{d\phi^i}{d\tau}$$ tangent to geodesic $(\|\hat{\tau}\| = 1)$

$$\alpha = -\frac{d\log m}{d\tau} = \vec{\zeta}\cdot\hat{\tau}$$

Convex hull determines $\alpha(\theta)$!

Example: IIB in 9d



$m_{\text{tower}} \propto e^{-\alpha\tau}$ ← geodesic distance

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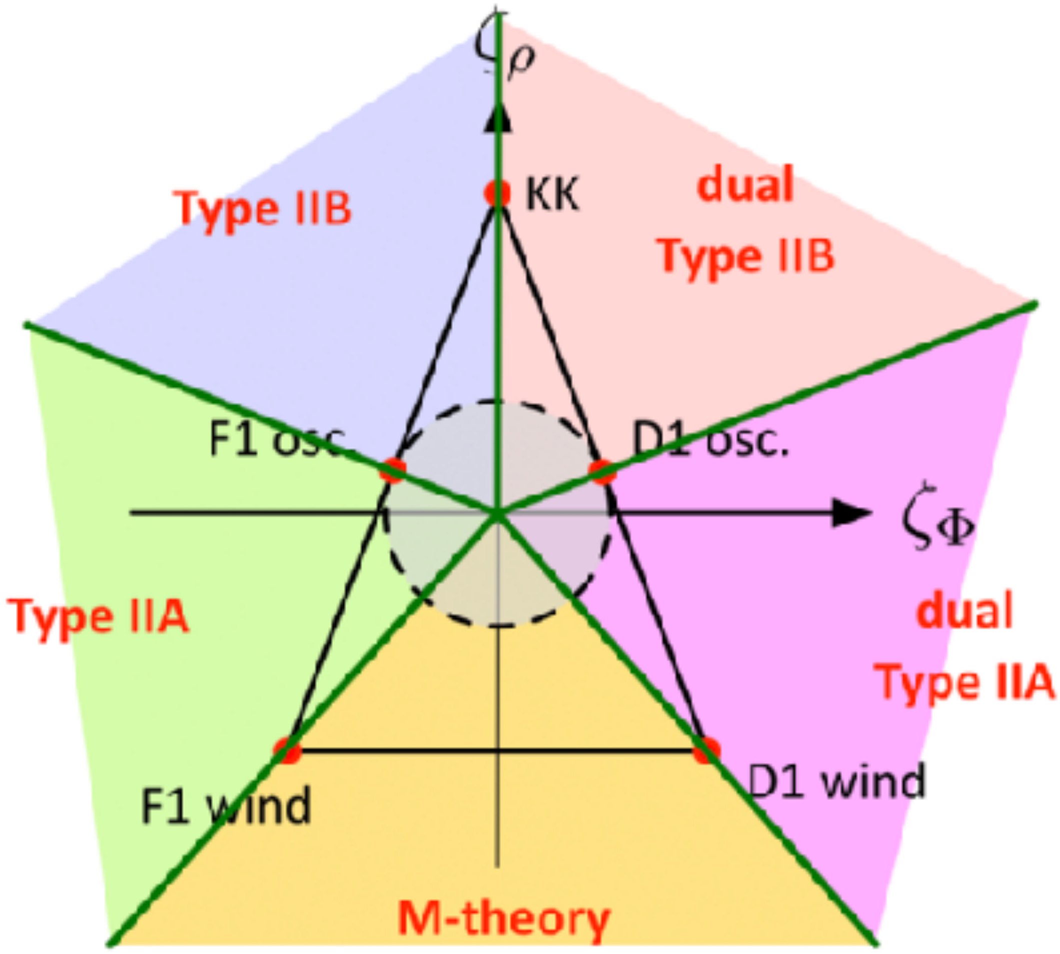
$\alpha = -\frac{d\log m}{d\tau} = \vec{\zeta} \cdot \hat{\tau}$

Convex hull determines $\alpha_{\max}(\theta)$!

CHC ⇒ Sharpened DC (typically)

Etheredge '23

Example: IIB in 9d



Related to dualities: each facet* represents a different phase

Classification of Infinite-Distance Polytopes

Assumptions

- The main ingredient that goes into this taxonomy is the Emergent String Conjecture.
- The taxonomy applies only in “generic” asymptotic limits of moduli space, which means that there are not multiple towers becoming light at the same exponential rate.
- Soon, I will show you an example where this classification does not apply in certain limits or in the interior of moduli space.

Classification of Vertices

- Fundamental string oscillation modes:

$$m_{\text{string}} \sim \exp(-\phi/\sqrt{d-2}) \;\; \Rightarrow \;\; ||\vec{\zeta}|| = \frac{1}{\sqrt{d-2}}$$

- KK modes for n-dimensional decompactification:

$$m_{\text{KK}} \sim \exp(-\sqrt{\frac{n+d-2}{n(d-2)}}\rho) \;\; \Rightarrow \;\; ||\vec{\zeta}|| = \sqrt{\frac{n+d-2}{n(d-2)}}$$

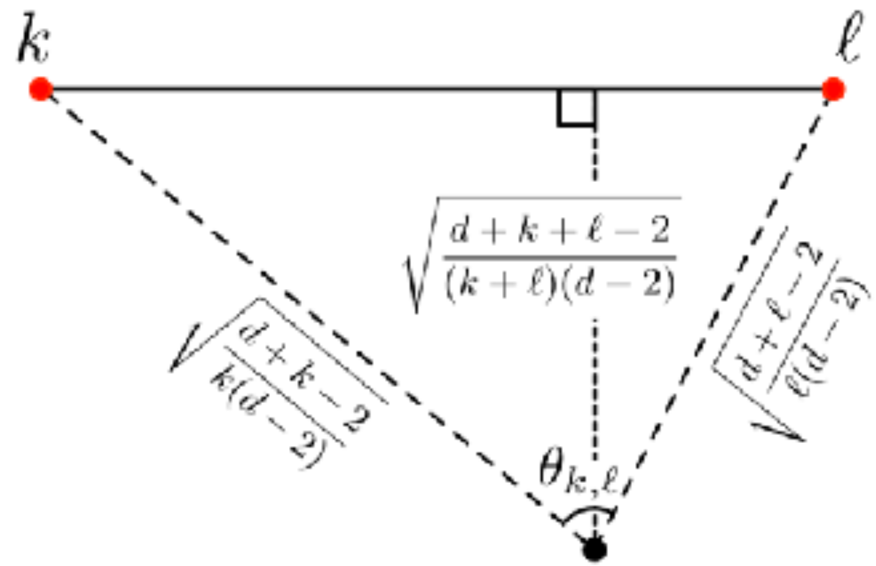
That's it!

Etheredge, Heidenreich, Kaya, Qiu, TR '22

Classification of Edges

- Two possibilities: (a) KK modes and KK modes, (b) KK modes and string oscillation modes

(a)



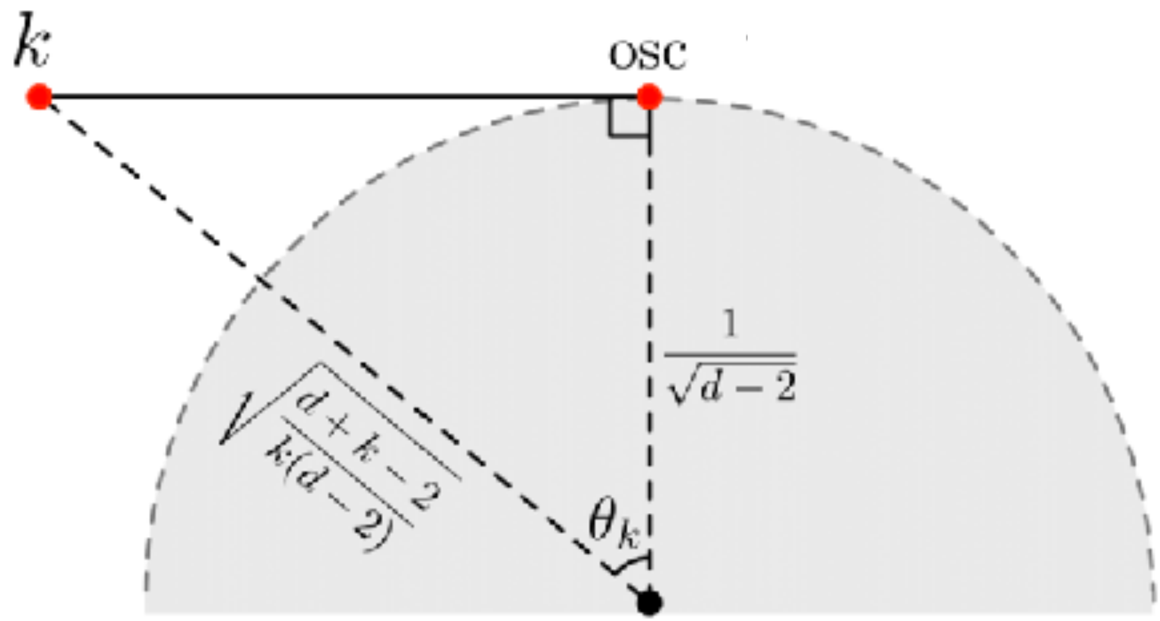
$$\cos\theta_{k,\ell} = \sqrt{\frac{k\ell}{(d+k-2)(d+\ell-2)}}$$

Etheredge, Heidenreich, TR, Ruiz, Valenzuela, to appear

Classification of Edges

- Two possibilities: (a) KK modes and KK modes, (b) KK modes and string oscillation modes

(b)

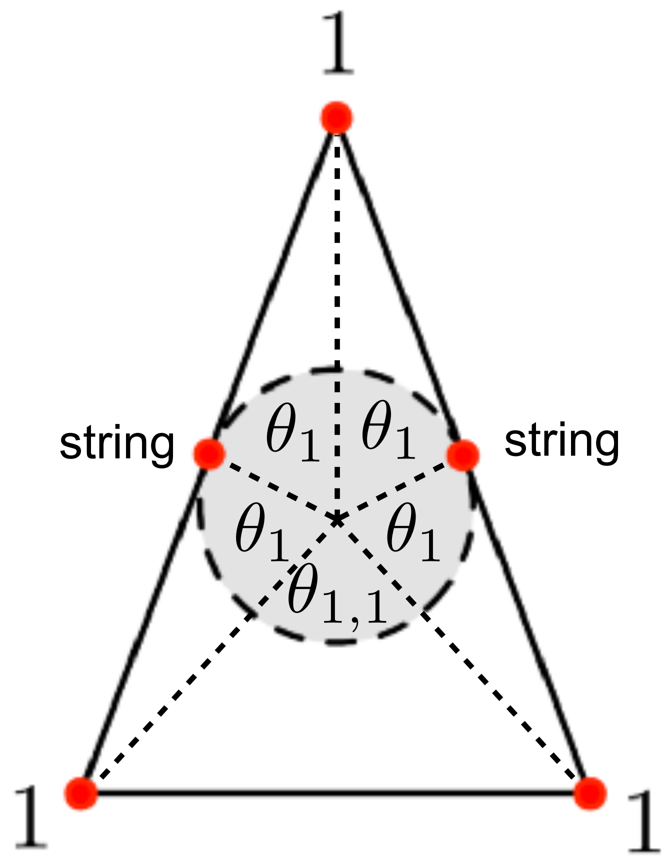


$$\cos\theta_k = \sqrt{\frac{k}{d+k-2}}$$

Etheredge, Heidenreich, TR, Ruiz, Valenzuela, to appear

Classification of Edges

- Example: IIB in 9d



Etheredge, Heidenreich, TR, Ruiz, Valenzuela, to appear

Classification of Facets

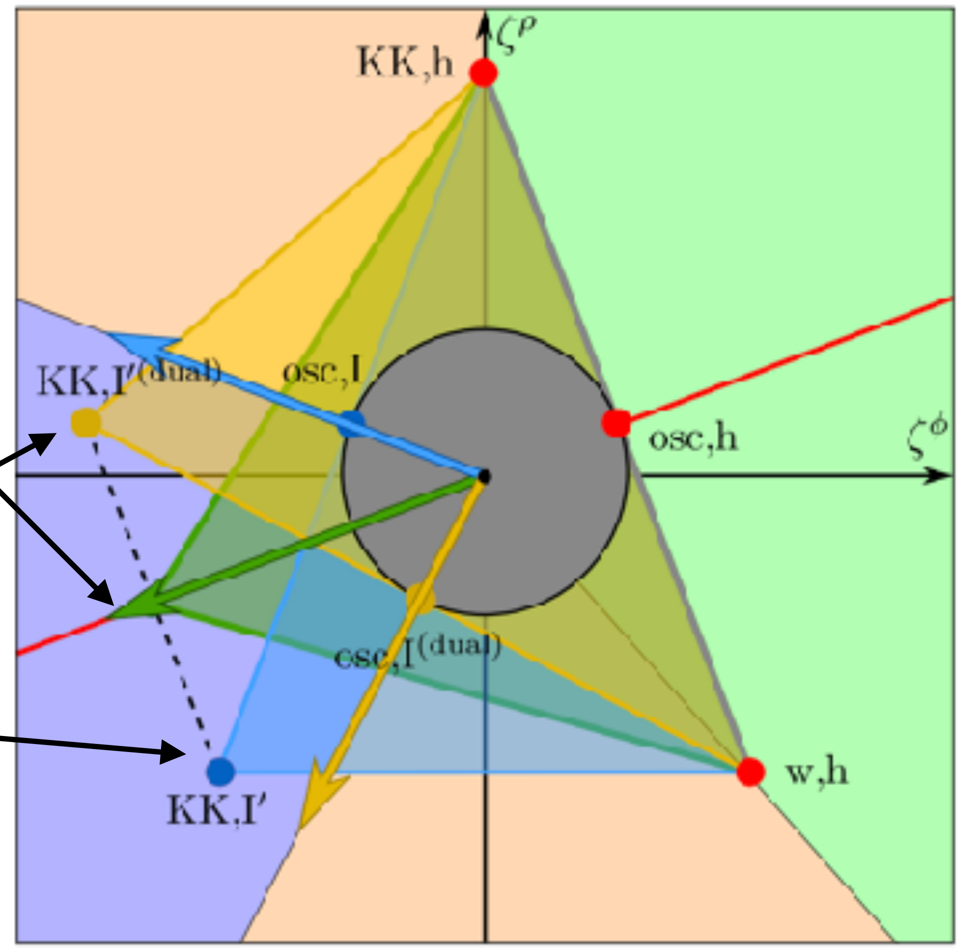
- Facets of the polytope represent weakly-coupled *phases* of the theory
- They fall into two categories:
  - String theory phases (one stringy vertex, rest KK vertices)
  - Geometric phases (no stringy vertices, all KK vertices)
- These two types of phases are distinguished by the nature of the species scale (string scale vs. higher-dimensional Planck scale, respectively)
- Similar quantization conditions apply to such facets

Sliding

A non-example: sliding in SO(32) heterotic string theory on a circle:

Polytope rules violated at generic point in Type I' phases (green)

But—in generic *asymptotic* regions of moduli space, rules are satisfied (yellow, blue)!



Etheredge, Heidenreich, McNamara, TR, Ruiz, Valenzuela '23

Evidence from Supergravity

Aside: Strong Forms of the Weak Gravity Conjecture

The **Tower WGC** implies that any time a gauge coupling *g* vanishes in some infinite-distance limit, there is a tower of particles beginning at the mass scale

$$m \sim g M_{\mathrm{Pl};d}^{(d-2)/2}$$

The **WGC for strings** implies that any time a 2-form gauge coupling *g* vanishes in some infinite-distance limit, there is a tower of string oscillator modes beginning at the mass scale

$$m \sim \sqrt{g} M_{\mathrm{Pl};d}^{(d-2)/4}$$

Arkani-Hamed, Motl, Nicolis, Vafa ’06
Heidenreich, Reece, TR ’15, ’16,
Andriolo, Junghans, Noumi, Shiu ’18

Bottom-Up Evidence: Minimal Supergravity in $d = 5$

- Supergravity in 5d controlled largely by cubic prepotential:

$$\mathcal{F} = \frac{1}{6} C_{IJK} Y^I Y^J Y^K$$

- Here, Y^I are homogenous coordinates on vector multiplet moduli space, identified under simultaneous rescaling $Y^I \sim \lambda Y^I$
- Consider "straight-line" path in the space of these homogenous coordinates:

$$Y^I = Y_0^I + sY_1^I \,, s \in [0, 1]$$

Bottom-Up Evidence: Minimal Supergravity in $d = 5$

- Assume $s = 0$ is at infinite distance $\Rightarrow$ two cases to consider:

<u>Case 1: $\mathscr{F} \sim s$</u>

$\Rightarrow$ gauge couplings scale as

$$g_{\rm min} \sim \exp(-\frac{2}{\sqrt{3}}\rho), \quad 1/g_{\rm max} \sim \exp(-\frac{1}{\sqrt{3}}\rho)$$

Tower WGC $\Rightarrow$

$$m_{\rm KK} \lesssim g_{\rm min} \sim \exp(-\frac{2}{\sqrt{3}}\rho) \sim \exp(-\sqrt{\frac{d-1}{d-2}}\rho)$$

WGC for strings $\Rightarrow$

$$m_{\rm string} \sim \sqrt{T_{\rm string}} \lesssim 1/\sqrt{g_{\rm max}} \sim \exp(-\frac{1}{2\sqrt{3}}\rho) \sim \exp(-\frac{1}{\sqrt{(d-1)(d-2)}}\rho)$$

Expected scaling for decompactification limit!

$$||\vec{\zeta}_{\rm KK}|| = \sqrt{\frac{d-1}{d-2}}, \quad \vec{\zeta}_{\rm KK} \cdot \vec{\zeta}_{\rm string} = \frac{1}{d-2}$$

Etheredge, Heidenreich, Kaya, Qiu, TR '22

Bottom-Up Evidence: Minimal Supergravity in $d = 5$

- Assume $s = 0$ is at infinite distance $\Rightarrow$ two cases to consider:

<u>Case 2: $\mathscr{F} \sim s^2$</u>

$\Rightarrow$ gauge couplings scale as

$$g_{\text{min}} \sim \exp(-\frac{1}{\sqrt{3}}\phi), \quad 1/g_{\text{max}} \sim \exp(-\frac{2}{\sqrt{3}}\phi)$$

Tower WGC $\Rightarrow$

$$m \lesssim g_{\text{min}} \sim \exp(-\frac{1}{\sqrt{3}}\phi) \sim \exp(-\frac{1}{\sqrt{d-2}}\phi)$$

WGC for strings $\Rightarrow$

$$M_{\text{string}} \sim \sqrt{T_{\text{string}}} \lesssim 1/\sqrt{g_{\text{max}}} \sim \exp(-\frac{1}{\sqrt{3}}\phi) \sim \exp(-\frac{1}{\sqrt{d-2}}\phi)$$

Expected scaling for emergent string limit!

$$||\vec{\zeta}_{\text{string}}|| = \frac{1}{\sqrt{d-2}}, \quad \vec{\zeta}_{\text{string}} \cdot \vec{\zeta}_{\text{KK}} = \frac{1}{d-2}$$

Etheredge, Heidenreich, Kaya, Qiu, TR ’22

Bottom-Up Evidence: Minimal Supergravity in $d > 5$

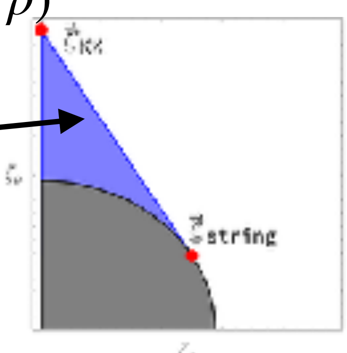
- Similar results apply to tensor multiplet moduli space in $d = 6$, moduli space in $d > 7$
- In all cases, find (assuming tower/string WGC) that infinite distance are characterized by either:
  - Charged tensionless strings with

$$M_{\text{string}} \sim \sqrt{T_{\text{string}}} \lesssim \exp\left(-\frac{1}{\sqrt{d-2}}\phi\right)$$

  - Towers of charged particles and charged strings

$$m_{\text{KK}} \lesssim \exp\left(-\sqrt{\frac{d-1}{d-2}}\rho\right), \quad M_{\text{string}} \lesssim \exp\left(-\frac{1}{\sqrt{(d-1)(d-2)}}\rho\right)$$

  - Some intermediate regime between the two TR ’23



- Fits perfectly with the classification of vertices and angles

Applications

Sharpened Distance Conjecture/SWGC

- String theory facet:

$$\min ||\vec{\zeta}|| = ||\vec{\zeta}_{\text{string}}|| = \frac{1}{\sqrt{d-2}}$$

- Geometric facet:

$$\min||\vec{\zeta}|| = ||\vec{\zeta}_D|| = \sqrt{\frac{D-2}{(D-d)(d-2)}} \quad , \qquad D = d + \sum_i n_i$$

$$> \frac{1}{\sqrt{d-2}}$$

- Sharpened Distance Conjecture, SWGC satisfied!

Species Scale Relations

- Any vertex on a stringy facet satisfies

$$\vec{\zeta}_v \cdot \vec{\zeta}_{\rm str} = \frac{1}{d-2}$$

- For any point on the facet, can write

$$\vec{\zeta} = \sum_i x_i \vec{\zeta}_i\,, \quad \sum_i x_i = 1\,, \quad x_i \geq 0.$$

$$\Rightarrow \vec{\zeta} \cdot \vec{\zeta}_{\rm str} = \frac{1}{d-2}$$

- With this, can prove previously discussed relations:

$$||\vec{\zeta}||\lambda_{\rm QG}(\hat{\zeta}) = \frac{1}{d-2} \Rightarrow \lambda_{\rm QG}(\hat{\zeta}) \leq \frac{1}{\sqrt{d-2}} \qquad \vec{\zeta}_{\rm max} \cdot \vec{\lambda}_{\rm QG} = \frac{1}{d-2}$$

- Similar argument for geometric facet

Calderon-Infante, Castellaano, Herraez, Ibanez '23
van de Heisteeg, Vafa, Weisner '23
Castellano, Ruiz, Valenzuela '23

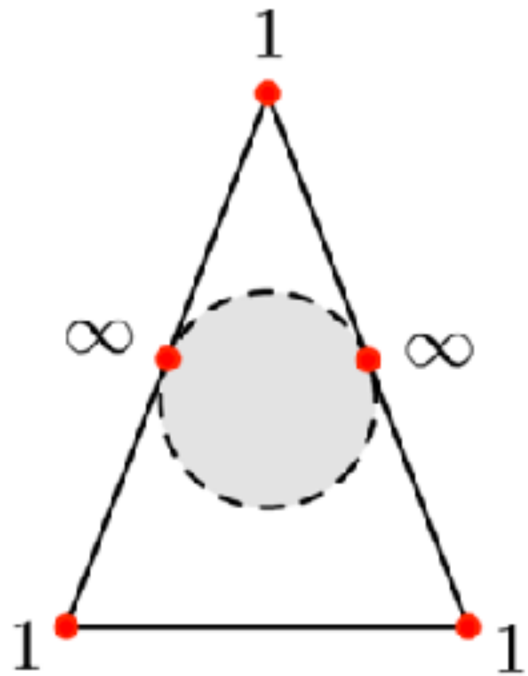
Classification of 2d Slices

- Assume $D = d + k \leq 11$ for every decompactification limits
- Assume $D \leq 10$ for decompactification limits adjacent to strings (no 11d strings)

$\Rightarrow$ Finite list of possible 2d slices!

Classification of 2d Slices: 9d Results

In $d = 9$, only **one** option:

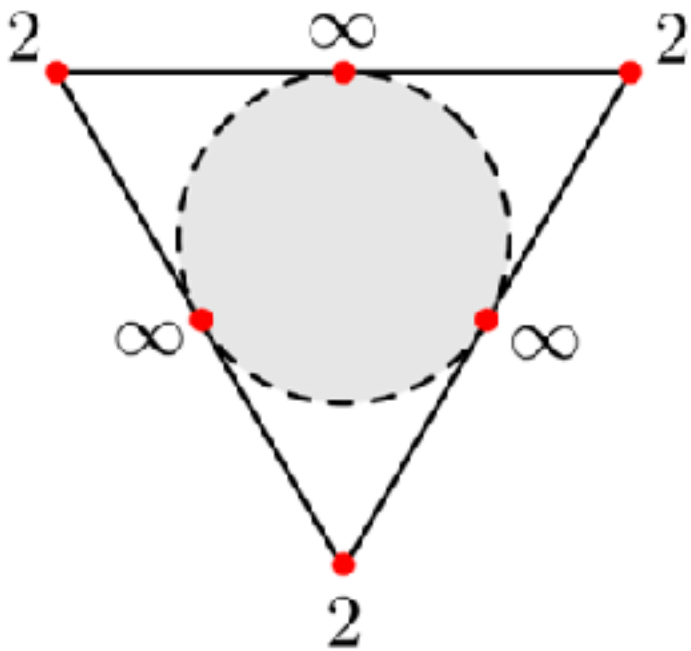


$(1, 1, \infty, 1, \infty)$

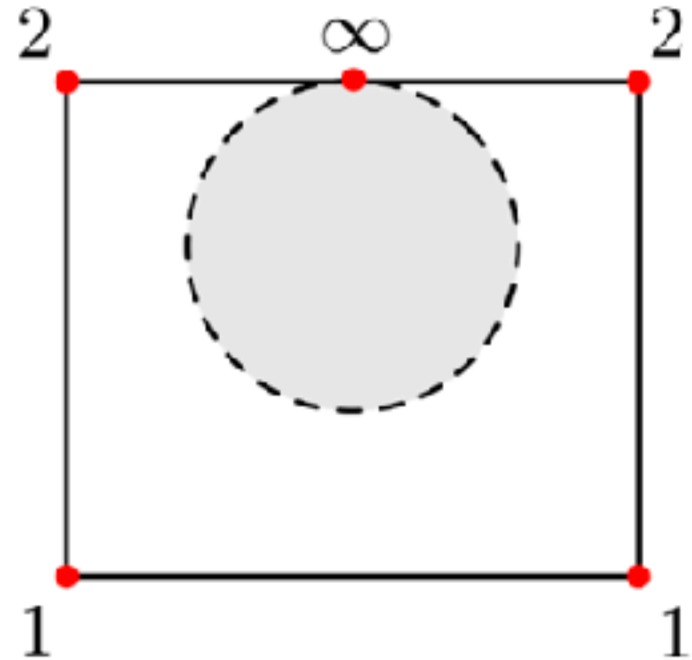
Same as maximal SUGRA!

Classification of 2d Slices: 8d Results

In $d = 8$, **two** options:



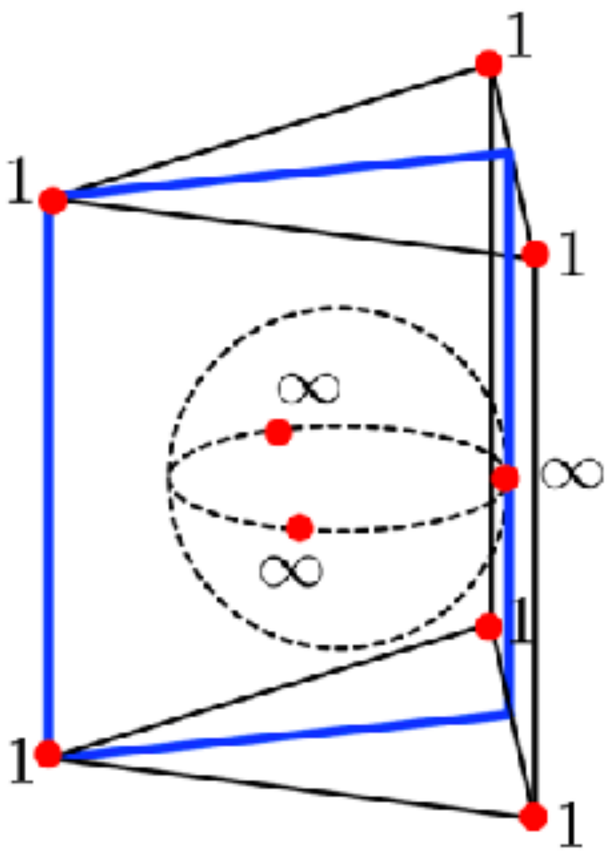
$(2, \infty, 2, \infty, 2, \infty)$

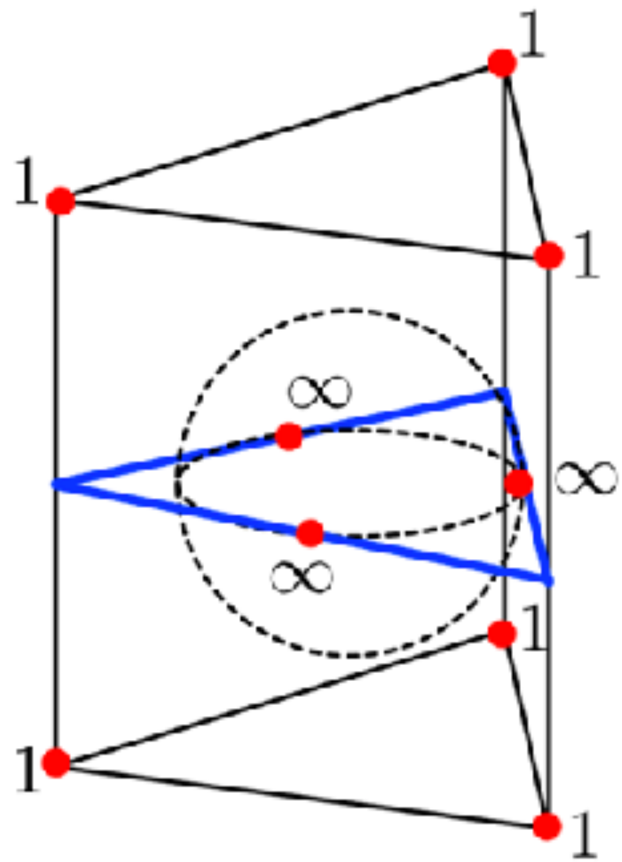


$(1, 1, 2, \infty, 2)$

Classification of 2d Slices: 8d Results

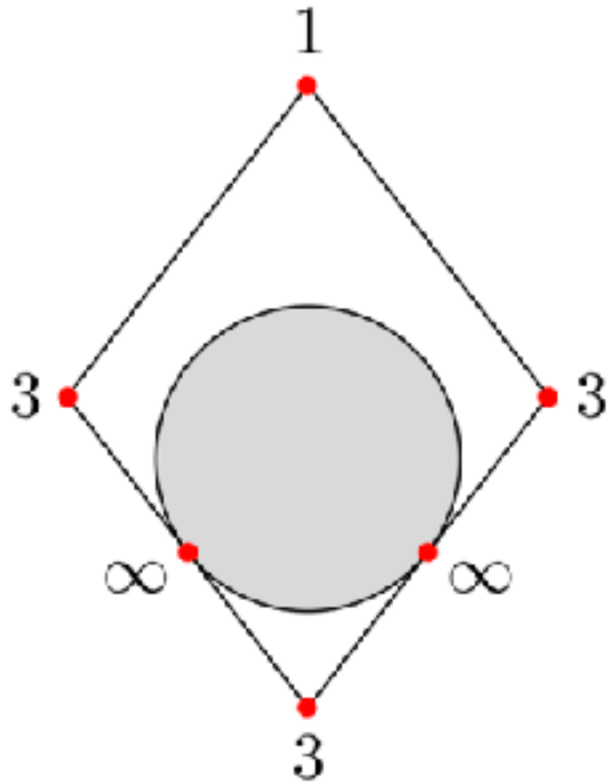
Both are slices of the 8d maximal SUGRA polytope!



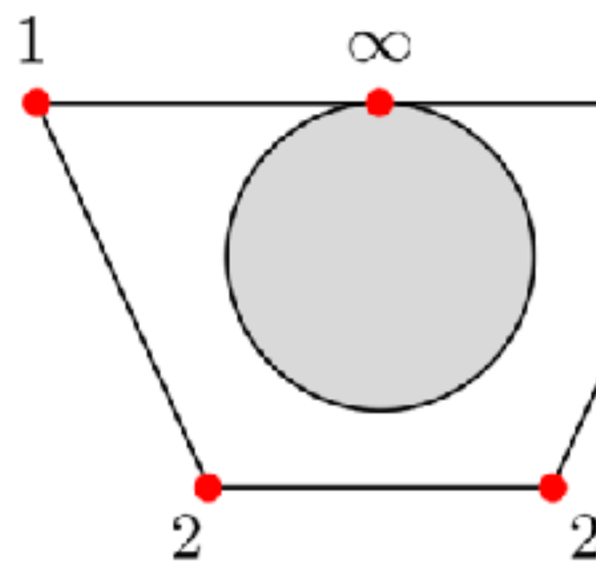


Classification of 2d Slices: 7d Results

In $d = 7$, **two** options:

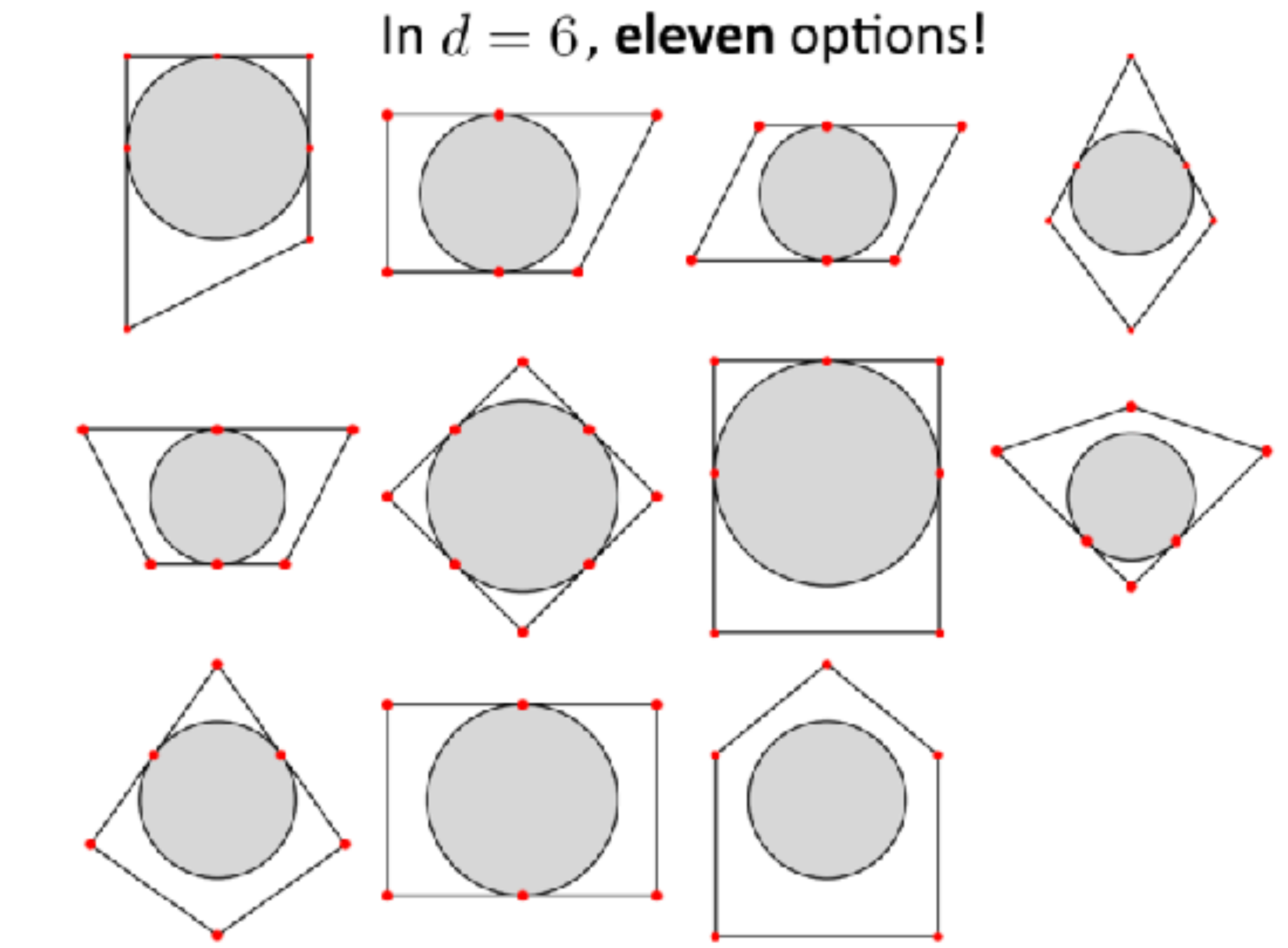


$(1, 3, \infty, 3, \infty, 3)$



$(1, 2, 2, 1, \infty)$

Classification of 2d Slices: 6d Results



Assessment

- In $d \geq 7$, these are all orthogonal slices of the maximal SUGRA polytope (could also be realized by other theories)
- Have some results for $n > 2$ slices; see forthcoming paper
- Answer for $n = 11 - d$ seems to be unique (except for $d = 10$): matches maximal SUGRA!

Conclusions

Summary

- Used Emergent String Conjecture + other assumptions to derive rules for the geometry of the convex hull generated by the scalar charge-to-mass vectors
- Sharpened Distance Conjecture, species scale relations follow from these rules
- Used these results to classify convex hull polytopes, found match with maximal SUGRA
- Results are quite general, but not completely general (ignored sliding, etc.)

To Do

- Understand cases with sliding, interior of moduli spaces
- Check in more sophisticated examples (esp. 4d, 5d)
- Prove the Emergent String Conjecture

Bonus: Persistence of the Pattern in 5d Moduli Spaces

The Species Scale "Pattern"

- Earlier, I mentioned a remarkable pattern observed by Castellano, Ruiz, and Valenzuela

$$\vec{\zeta}_{\mathrm{max}} \cdot \vec{\lambda}_{\mathrm{QG}} = \frac{\vec{\nabla} m}{m} \cdot \frac{\vec{\nabla} \Lambda_{\mathrm{QG}}}{\Lambda_{\mathrm{QG}}} = \frac{1}{d-2}$$

- This relation can be proven in generic asymptotic limits, where our taxonomic rules apply
- However, in the context of 5d supergravity, a version of it can be proven in full generality

5d supergravity

- Recall that 5d supergravity is controlled by a cubic prepotential:

$$\mathcal{F} = \frac{1}{6} C_{IJK} Y^I Y^J Y^K$$

- The vector multiplet moduli space is the $\mathscr{F} = 1$ slice
- Helpful to define:

$$\mathcal{F}_I = \partial_I \mathcal{F} \qquad\qquad \mathcal{F}_{IJ} = \partial_I \partial_J \mathcal{F}$$

Gauge and Scalar Couplings

- Gauge kinetic matrix is then

$$a_{IJ} = \mathcal{F}_I \mathcal{F}_J - \mathcal{F}_{IJ}$$

- Metric on moduli space is pullback to $\mathscr{F} = 1$ slice:

$$g_{ij} = \frac{1}{2} a_{IJ} \partial_i Y^I \partial_j Y^J$$

- Satisfies an important identity: Alim, Heidenreich, TR '21

$$a^{IJ} = \frac{1}{2} g^{ij} \partial_i Y^I \partial_j Y^J + \frac{1}{3} Y^I Y^J$$

BPS Bound in 5d

- BPS particles saturate the bound:

$$m(q_I) \geq (2\pi^2)^{1/6} |q_I Y^I|$$

- BPS strings saturate the bound:

$$T(\tilde{q}^I) \geq \frac{1}{2}(2\pi^2)^{-1/6} |\tilde{q}^I \mathcal{F}_I|$$

The Pattern

- Setting $M_s = \sqrt{2\pi T(\tilde{q}^I)}$, using identity, can prove that for any BPS particle and BPS string,

$$g^{ij}\,\frac{\partial_i m}{m}\,\frac{\partial_j M_s}{M_s} = \frac{1}{3} - \frac{q_I \tilde{q}^I}{(q_K Y^K)(\tilde{q}^L \mathcal{F}_L)}$$

- If string and particle become light in asymptotic limit, their Dirac pairing vanishes, $q_I \tilde{q}^I = 0$
- Setting $\Lambda_{\text{QG}} = M_s$, we find the pattern:

$$\frac{\vec{\nabla} m}{m} \cdot \frac{\vec{\nabla} \Lambda_{\text{QG}}}{\Lambda_{\text{QG}}} = \frac{1}{3}$$

Thank You