

Ideas for alternative Cavity Geometries for $\geq$ MHz GWs

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Upper Limits for GW-Cavity Coupling

Coupling Coeff. of GW to vibration with displacement field $\xi^i(\mathbf{x})$

Plus Polarization: $\Gamma_+ = \Gamma^{11} - \Gamma^{22}$

Cross Polarization: $\Gamma_\times = \Gamma^{12} + \Gamma^{21}$

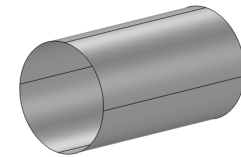
$$\Gamma^{ij} = \frac{1}{MV^{1/3}} \int_V d^3x \rho(\mathbf{x}) \xi^i(\mathbf{x}) x^j$$

Cavity Wall Density
Cavity Mass & Volume
Normalized Displacement Field of Vibration

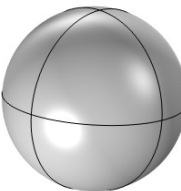
Can show in general (for GWs in z direction):

$$\Gamma^{ij} h_{ij} \leq \frac{1}{V^{1/3}} \sqrt{\frac{1}{M} \int_V d^3x \rho(\mathbf{x}) (x^2 + y^2)}$$

- For hollow cylinder: $\Gamma_{+,\times} \leq \frac{R}{V^{1/3}}$



- For sphere: $\Gamma_{+,\times} \leq \sqrt{\frac{2}{3}} \frac{R}{V^{1/3}} \approx 0.82 \frac{R}{V^{1/3}}$

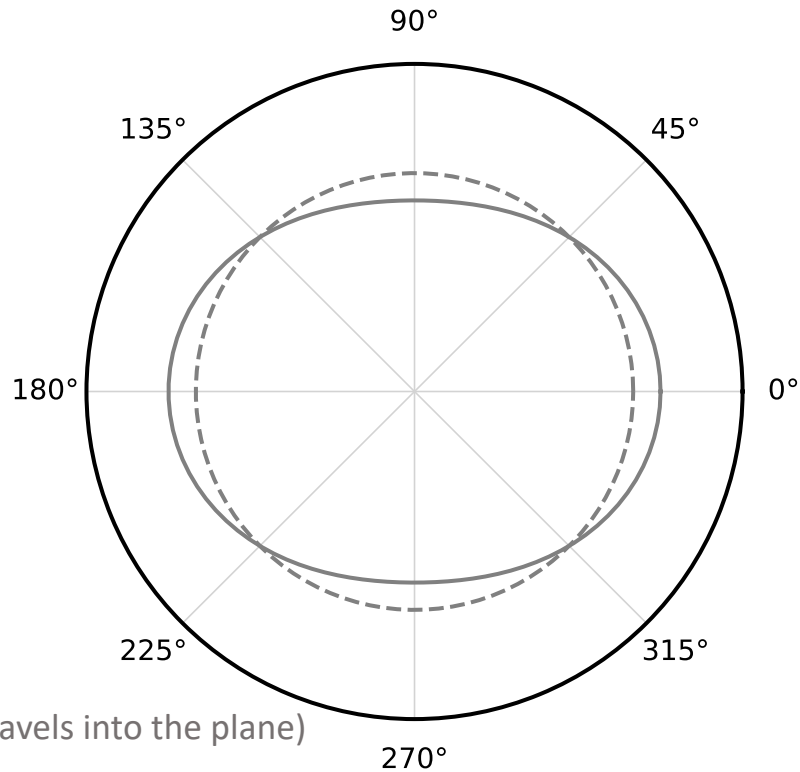


The ξ that maximizes $\Gamma_{+,\times}$ needs $(\xi_x, \xi_y) \propto (x, -y) \rightarrow$ Exists for a cylinder without ends

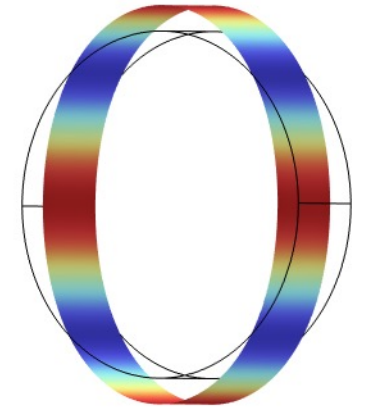
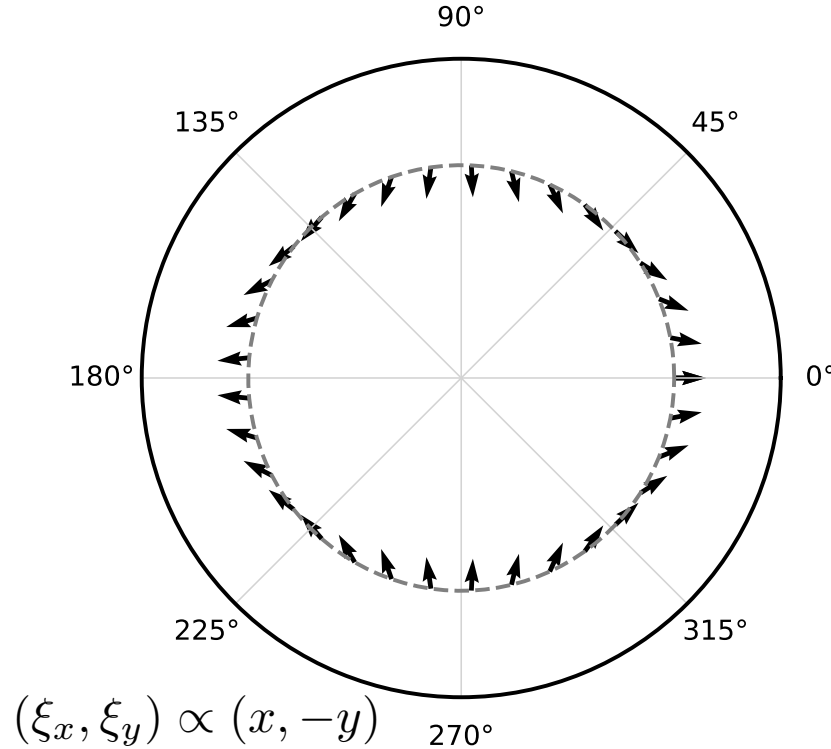
$\Rightarrow$ Cylindrical cavities can couple stronger to GWs than spheres (but are directional)

Cylinders Match GW Quadrupole Pattern

Effect of a GW on a circle of test masses

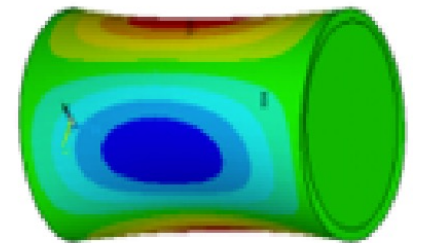


Lowest Vibrational Quadrupole Mode of a Ring



Also exists for longer cylinders

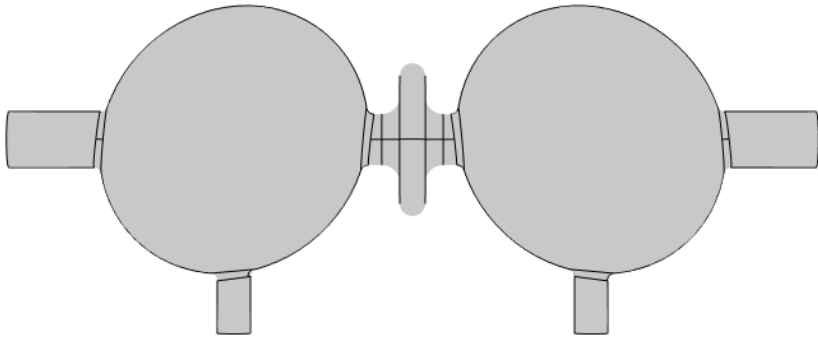
Quadrupole mode of closed cylinder looks more like this:



→ Slightly worse coupling

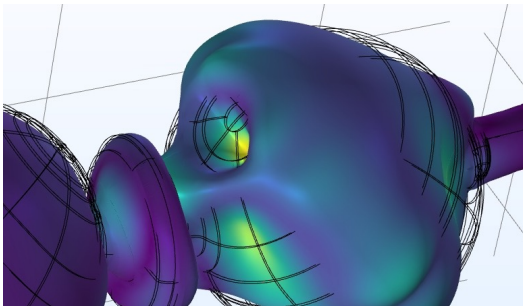
A MAGO-sized cylinder ($R=0.1$ m, $L=0.4$ m) has $\Gamma_{+,x} \approx \Gamma_{+,x}^{MAGO} \approx 0.5$

Cylinders have more predictable vibrations

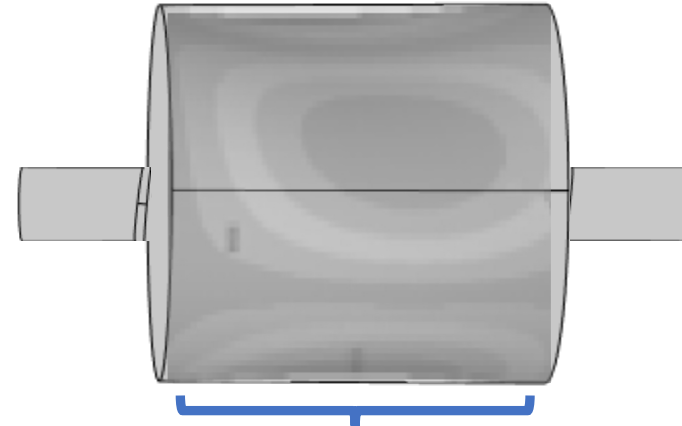


Pure quadrupole vibration does not exist due to complicated shape

Remember:



Predicting precise Vibrations of Cavity + Flanges is nearly impossible

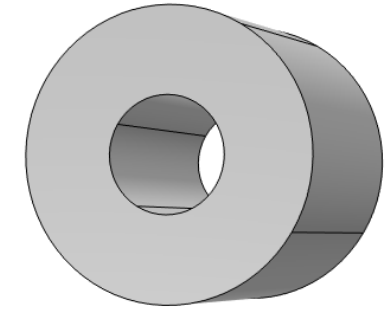


Main Body could still oscillate freely despite connections

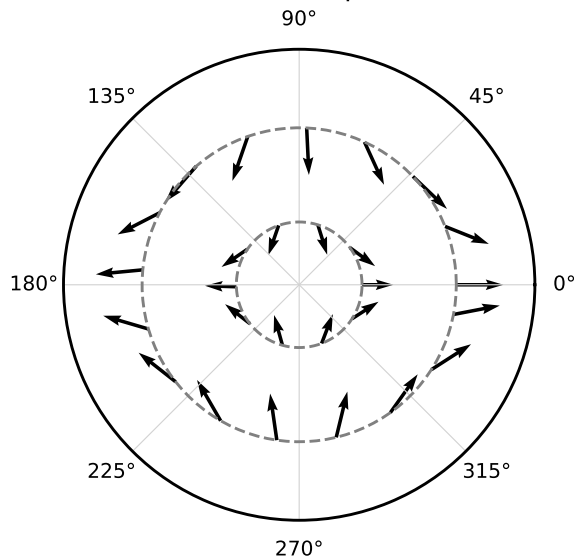
→ More control over vibrational modes of cavity

MHz EM-Mode Transition for Modified Shape

The quadrupole vibration induces **no strong** EM-mode transition in a cylinder
 → Cutting out a cylinder in the middle **does** gives us a good mode pair

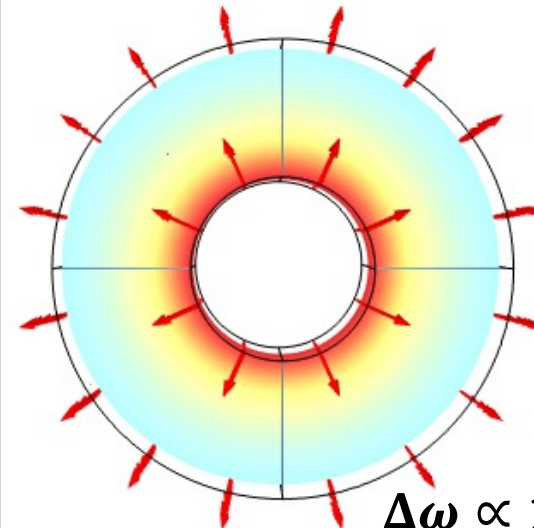


Mechanical Vibration

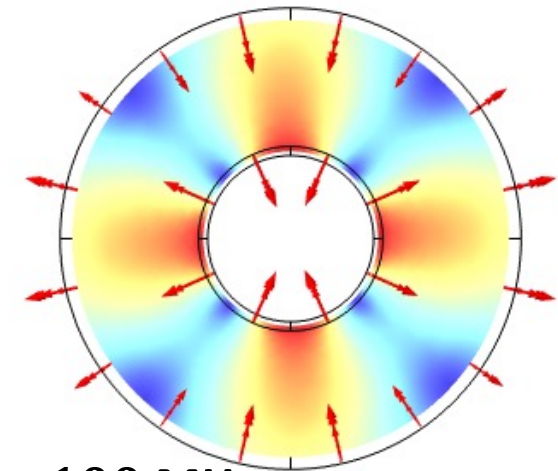


induces

Pump Mode TEM_{001}



Signal Mode TE_{021}



$\Delta\omega \propto 1 - 100 \text{ MHz}$

Transition

EM-Mechanical Coupling Coefficient: $C_{01} \approx 0.8$ (preliminary result), $\Gamma_{+, \times} \approx \frac{R}{V^{1/3}}$ still holds for inner & outer cylinder

Further thoughts

- Isolate the inner cylinder from the outer one
 - Reduced vibrational noise, (mostly) freely oscillating cylinder
 - Inner cylinder could be used to tune the resonant frequencies
- Round off edges to prevent strong fields
- Consider other geometries with 'holes'
e.g. ellipsoids, doughnuts, ...

