Ideas for alternative Cavity Geometries for $\geq$ MHz GWs

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Upper Limits for GW-Cavity Coupling

Coupling Coeff. of GW to vibration with displacement field $\xi^i(\boldsymbol{x})$

Cavity Wall Density

$$\Gamma^{ij} = \frac{1}{MV^{1/3}} \int_V d^3x\, \rho(\boldsymbol{x})\, \xi^i(\boldsymbol{x})\, x^j$$

Cavity Mass & Volume

Normalized Displacement Field of Vibration

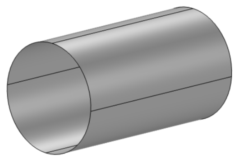
Plus Polarization: $\Gamma_+ = \Gamma^{11} - \Gamma^{22}$

Cross Polarization: $\Gamma_\times = \Gamma^{12} + \Gamma^{21}$

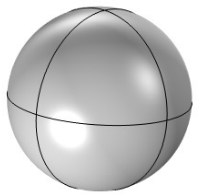
Can show in general (for GWs in z direction):

$$\Gamma^{ij} h_{ij} \le \frac{1}{V^{1/3}} \sqrt{\frac{1}{M} \int_V d^3x\, \rho(\boldsymbol{x})\, (x^2 + y^2)}$$

- For hollow cylinder: $\Gamma_{+,\times} \le \frac{R}{V^{1/3}}$



- For sphere: $\Gamma_{+,\times} \le \sqrt{\frac{2}{3}} \frac{R}{V^{1/3}} \approx 0.82 \frac{R}{V^{1/3}}$

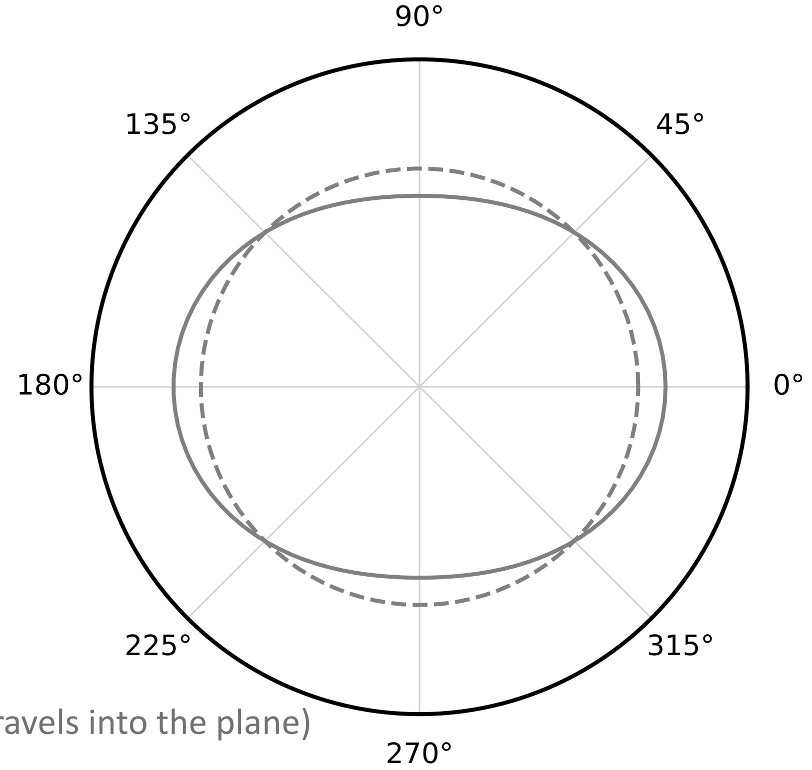


The $\boldsymbol{\xi}$ that maximizes $\Gamma_{+,\times}$ needs $(\xi_x, \xi_y) \propto (x, -y)$ → Exists for a cylinder without ends

⇒ **Cylindrical cavities can couple stronger to GWs than spheres** (but are directional)

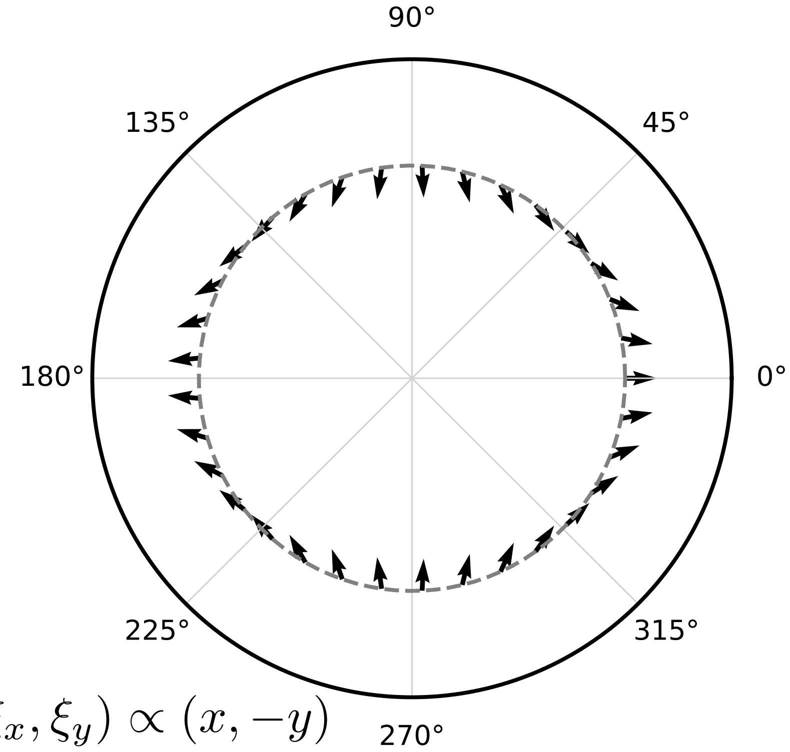
Cylinders Match GW Quadrupole Pattern

Effect of a GW on a circle of test masses

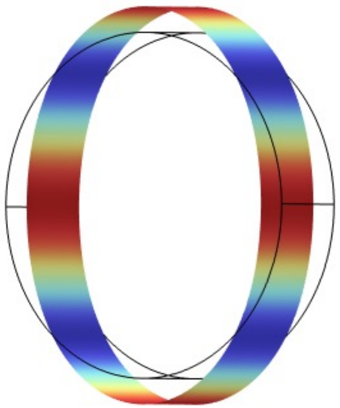


(GW travels into the plane)

Lowest Vibrational Quadrupole Mode of a Ring

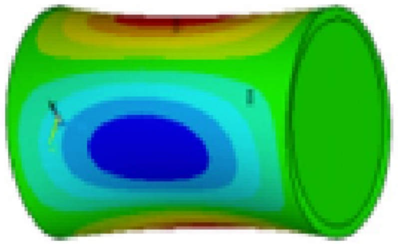


$(\xi_x, \xi_y) \propto (x, -y)$



Also exists for longer cylinders

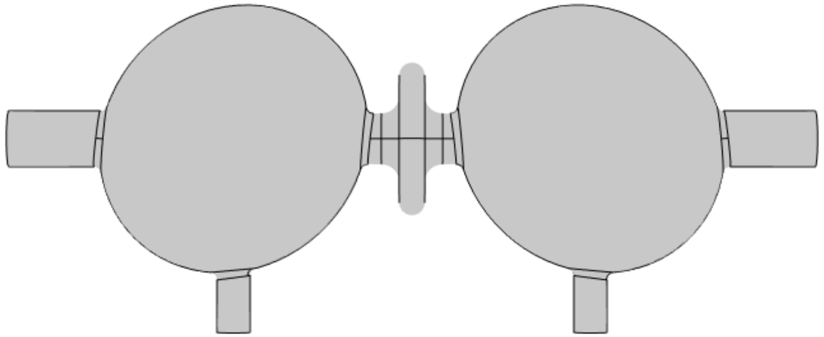
Quadrupole mode of closed cylinder looks more like this:



→ Slightly worse coupling

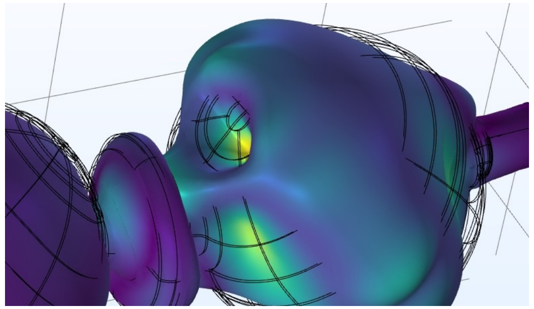
A MAGO-sized cylinder (R=0.1 m, L=0.4 m) has $\Gamma_{+,\times} \approx \Gamma_{+,\times}^{MAGO} \approx 0.5$

Cylinders have more predictable vibrations

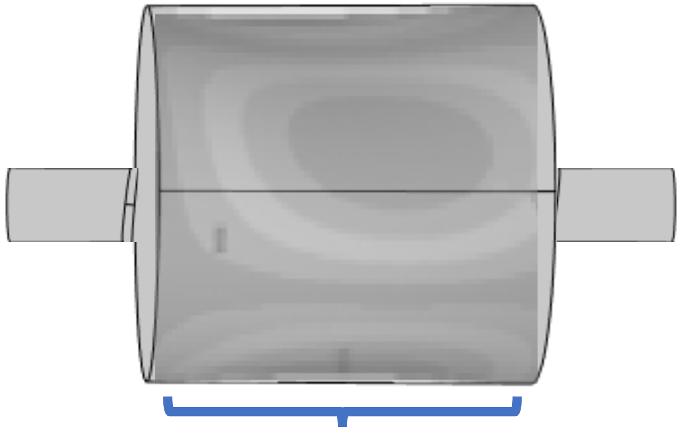


Pure quadrupole vibration does not exist due to complicated shape

Remember:



Predicting precise Vibrations of Cavity + Flanges is nearly impossible

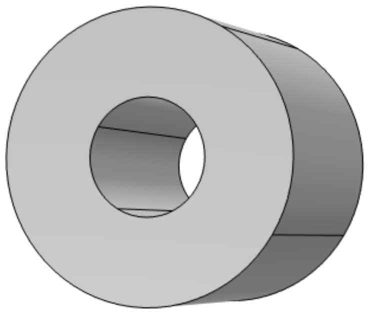


Main Body could still oscillate freely despite connections

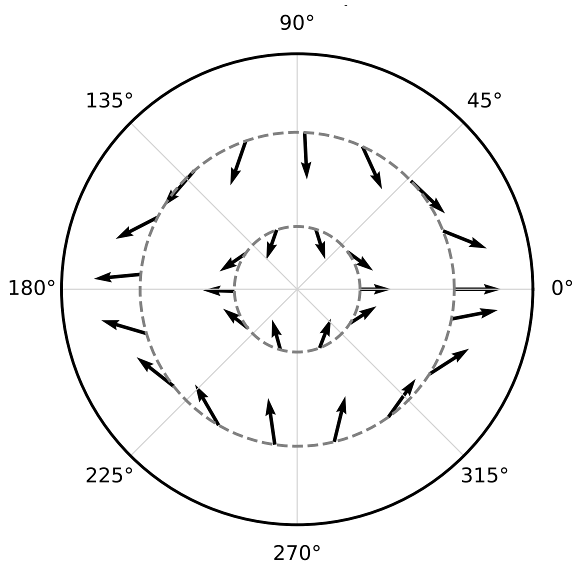
→ More control over vibrational modes of cavity

MHz EM-Mode Transition for Modified Shape

The quadrupole vibration induces **no strong** EM-mode transition in a cylinder
→ Cutting out a cylinder in the middle **does** gives us a good mode pair

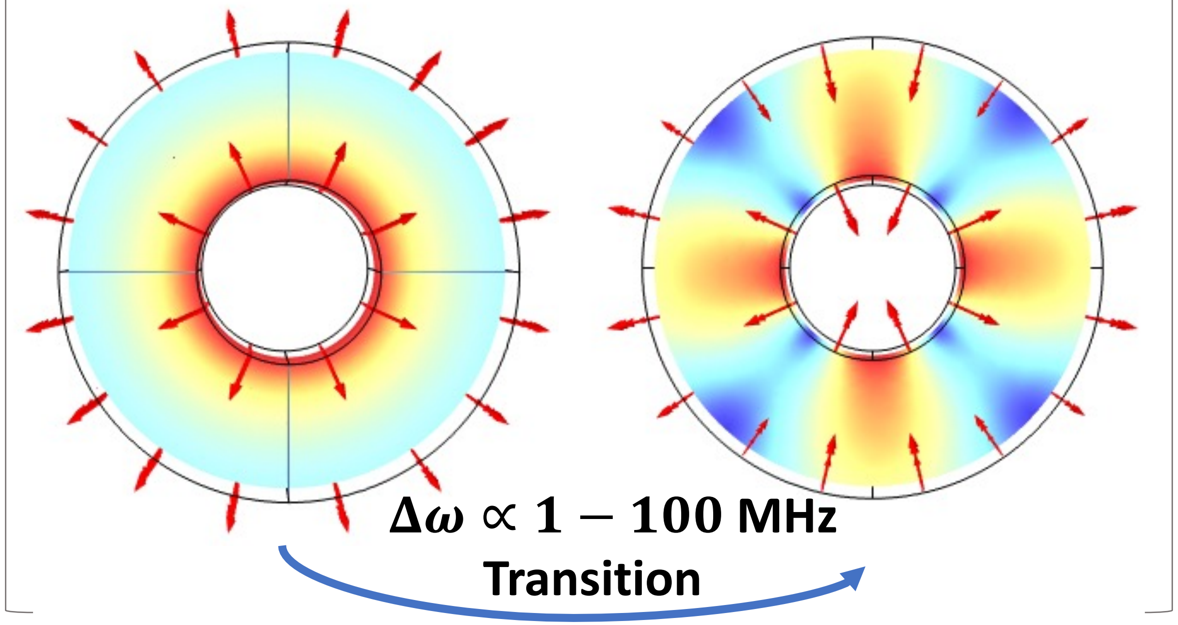


Mechanical Vibration



induces

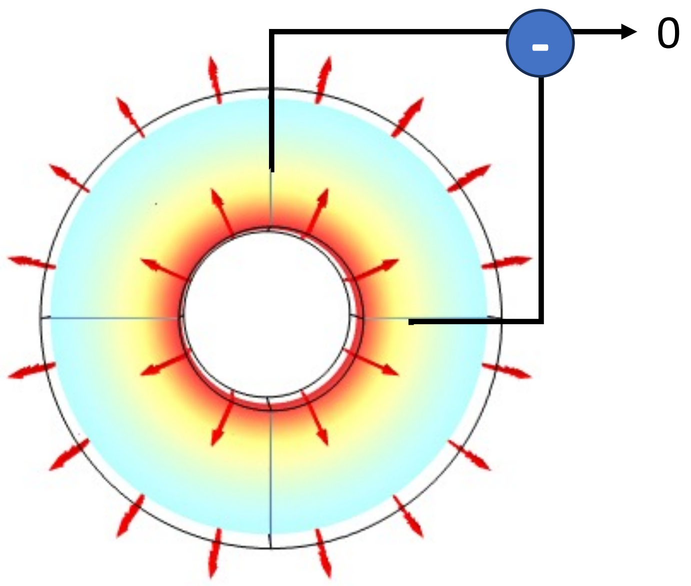
Pump Mode TEM$_{001}$ **Signal Mode TE$_{021}$**



EM-Mechanical Coupling Coefficient: $C_{01} \approx 0.8$ (preliminary result), $\Gamma_{+,\times} \approx \frac{R}{V^{1/3}}$ still holds for inner & outer cylinder

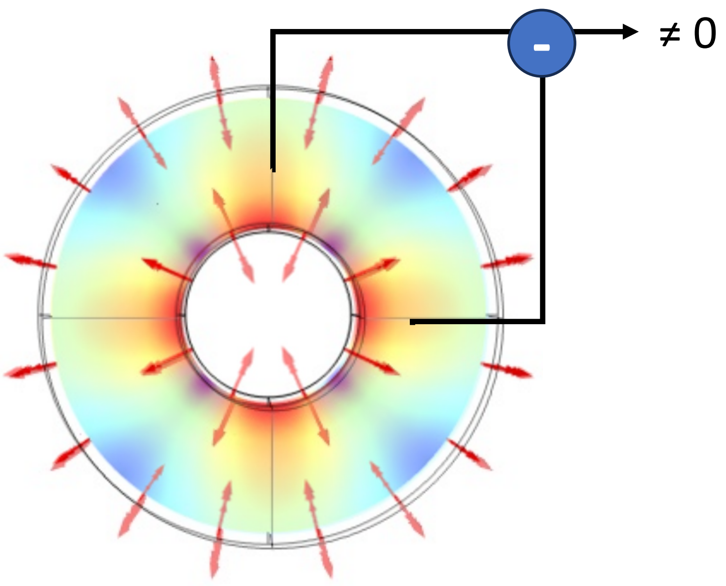
Signal Readout/Geometric Rejection

No Signal



TEM_{001}

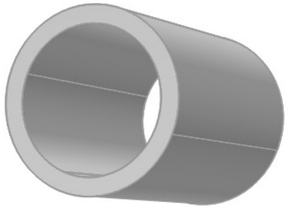
Signal

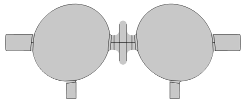


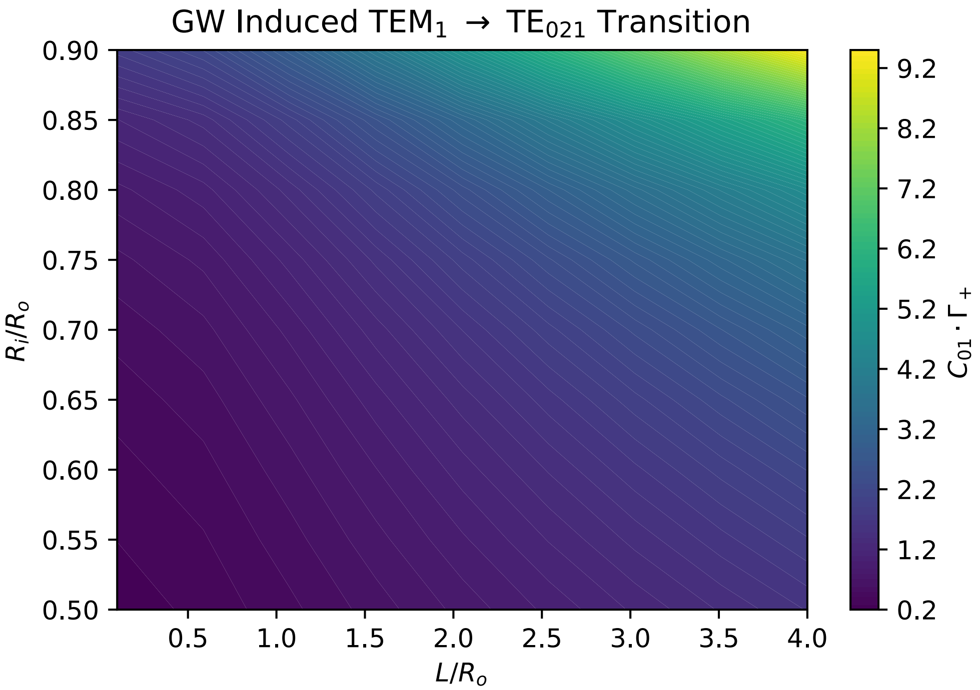
TEM_{001} + TE_{021}

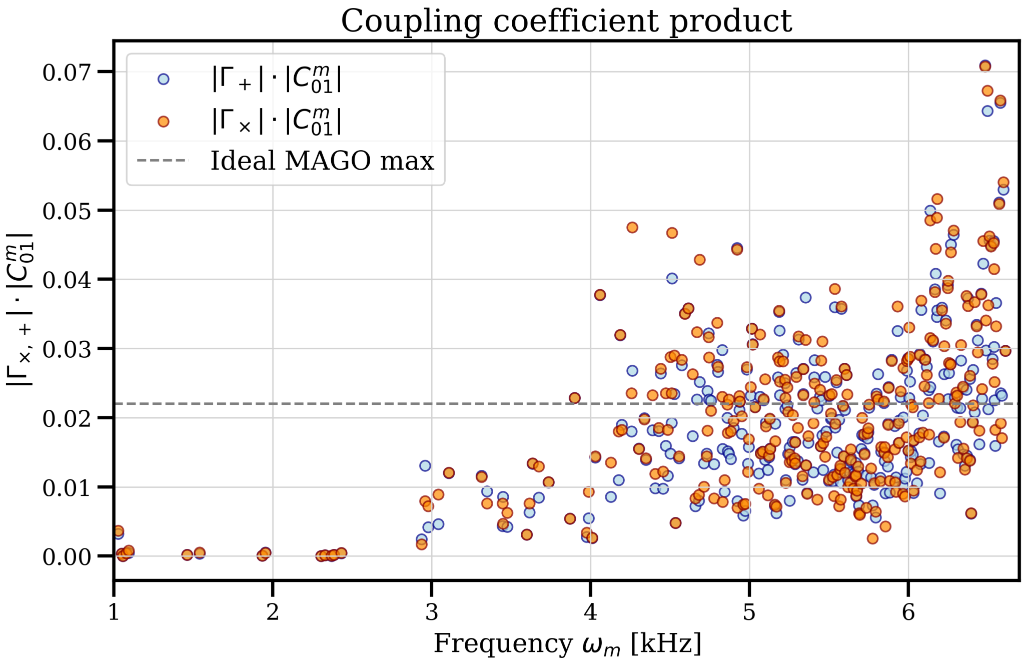
Coupling Coefficients

$\boldsymbol{C_{01}\Gamma_+ \approx 4}$
for a geometry like this:



comparison with  **:**





=> $\mathcal{O}(10^2)$ **improvement** (in one direction)

Size constraint: $L,\ R \ll \lambda_{GW} \approx 30\ \mathrm{m}\ \frac{1}{f_{GW}/10\ \mathrm{MHz}}$

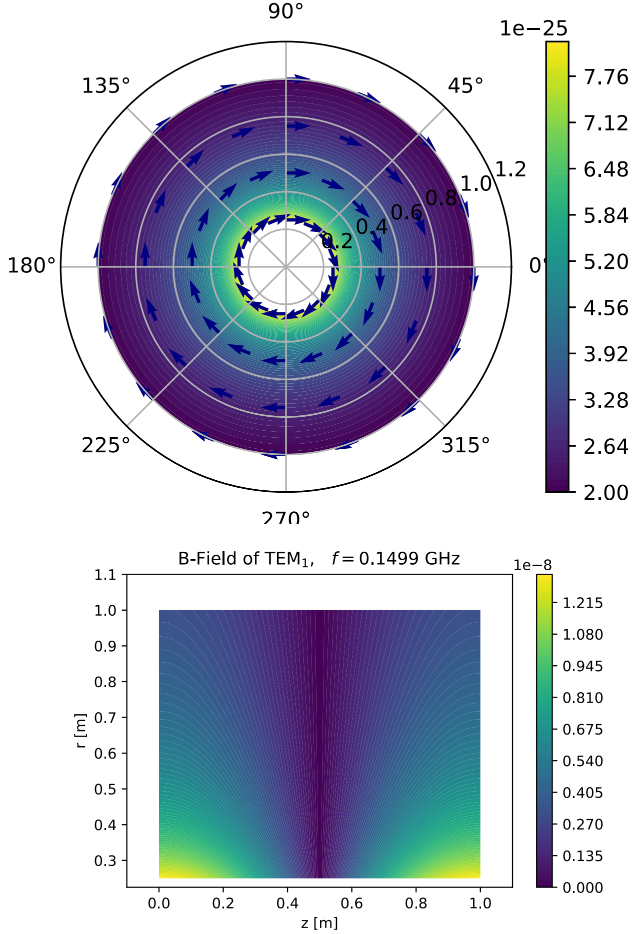
Simultaneous Axion Sensitivity

$$\frac{R_i}{R_o} = 0.8$$

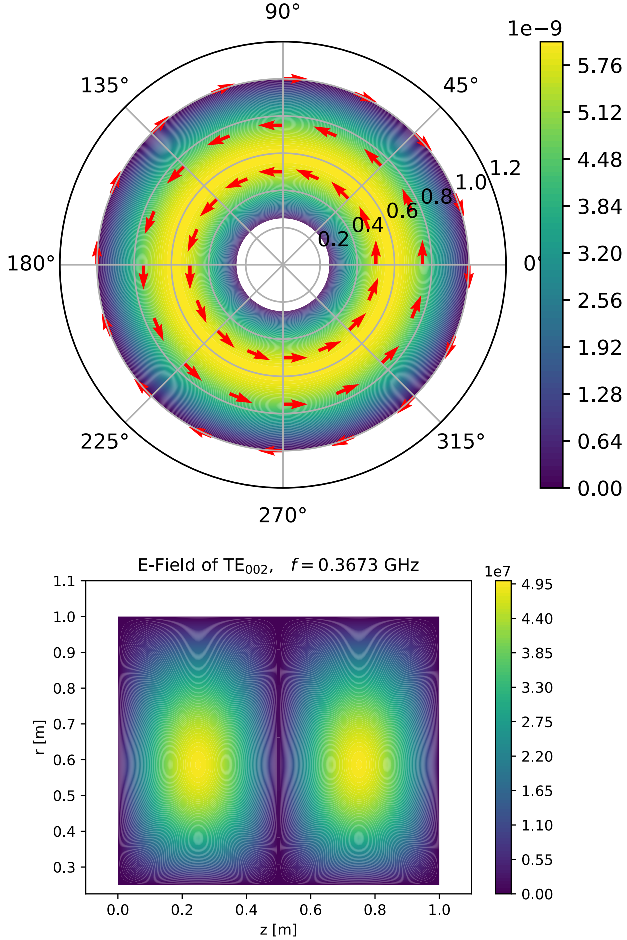
$$B_0 \equiv \sqrt{\frac{1}{V}\int_V |\tilde{\mathbf{B}}_0|^2} = 0.18\ \mathrm{T}\sqrt{\frac{\mathrm{Ln}(R_o/R_i)}{(R_o/R_i)^2 - 1}} = 0.14\ \mathrm{T}$$

$$\eta_{10} \equiv \frac{\left|\int_V \tilde{\mathbf{E}}_1^* \cdot \tilde{\mathbf{B}}_0\right|}{\sqrt{\int_V |\tilde{\mathbf{E}}_1|^2}\sqrt{\int_V |\tilde{\mathbf{B}}_0|^2}} = 0.71$$

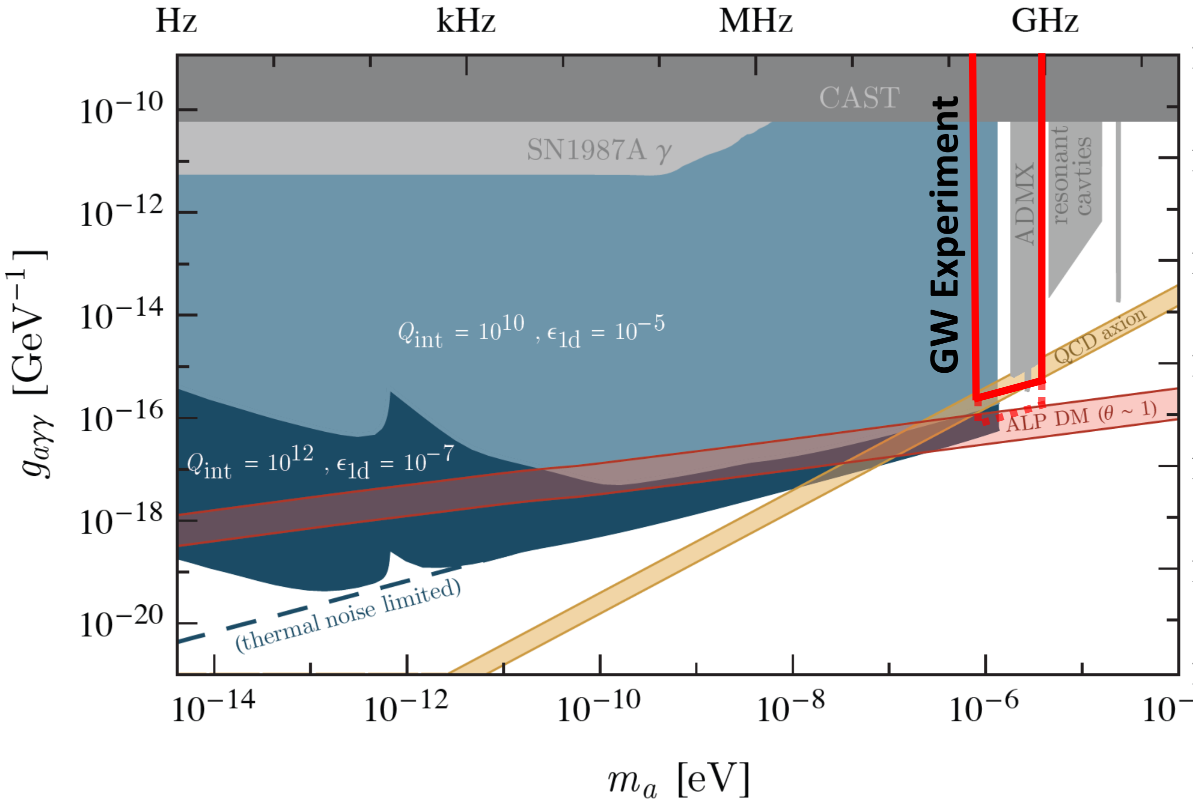
Pump Mode



Signal Mode



frequency $= m_a/2\pi$



Downside: **Frequency spacing ≈ 100 MHz – 1 GHz**

$g_{\min}$ reduced by $\dfrac{\eta_{01}/1 \cdot B_0/0.2}{\eta_{01}/0.71 \cdot B_0/0.14} \approx \dfrac{1}{2}$

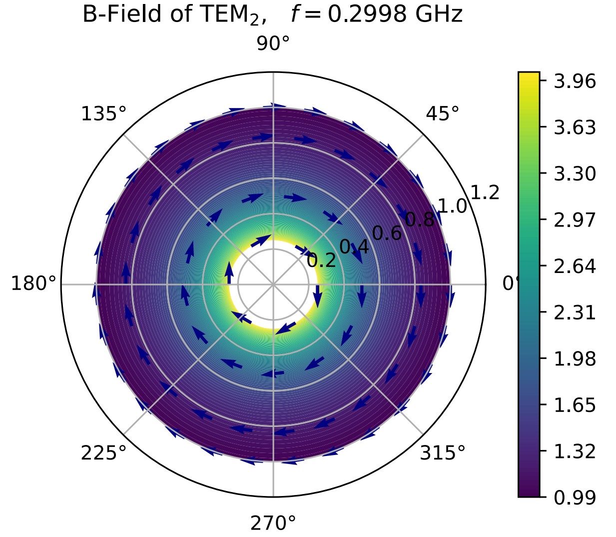
Axion Sensitivity (other mode)

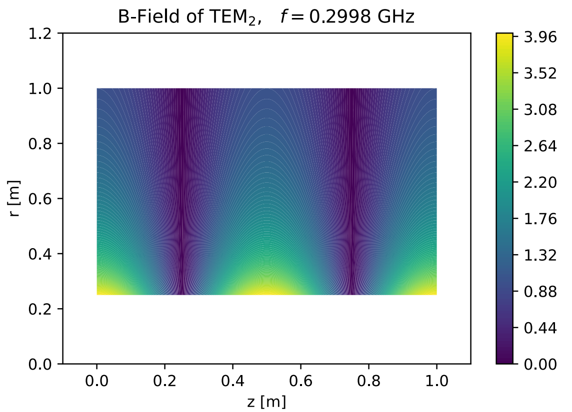
$$\frac{R_i}{R_o} = 0.8$$

$$B_0 \equiv \sqrt{\frac{1}{V}\int_V |\tilde{\mathbf{B}}_0|^2} = 0.18\,\mathrm{T}\sqrt{\frac{\mathrm{Ln}(R_o/R_i)}{(R_o/R_i)^2 - 1}} = 0.14\ \mathrm{T}$$

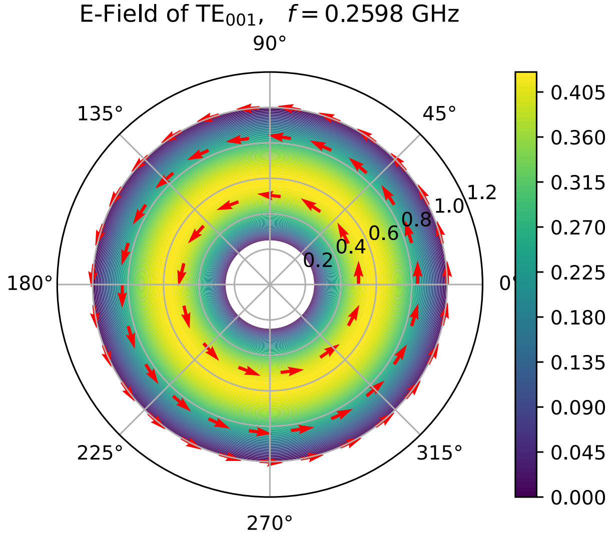
$$\eta_{10} \equiv \frac{\left|\int_V \tilde{\mathbf{E}}_1^* \cdot \tilde{\mathbf{B}}_0\right|}{\sqrt{\int_V |\tilde{\mathbf{E}}_1|^2}\sqrt{\int_V |\tilde{\mathbf{B}}_0|^2}} = 0.36$$

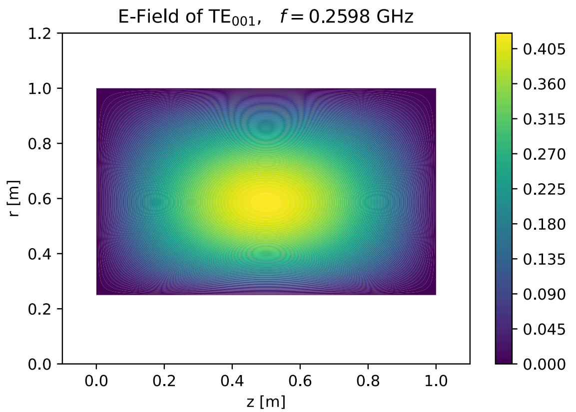
Pump Mode



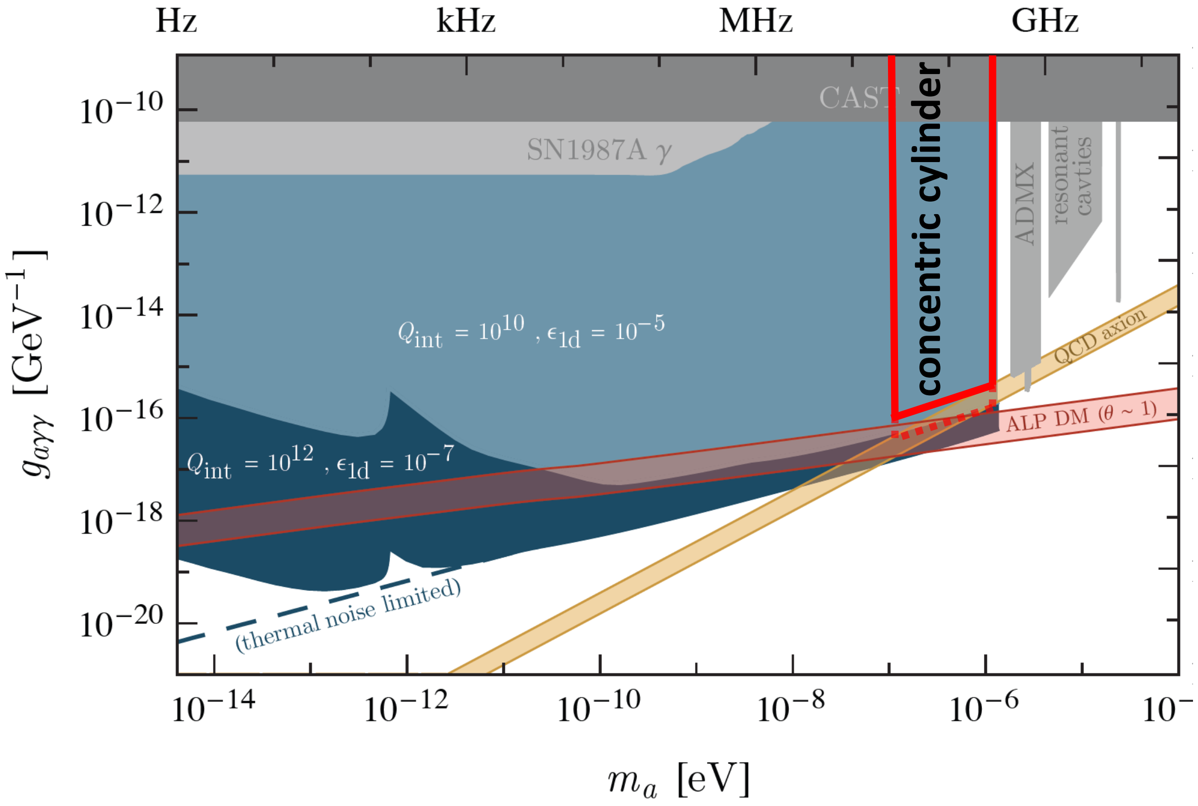


Signal Mode





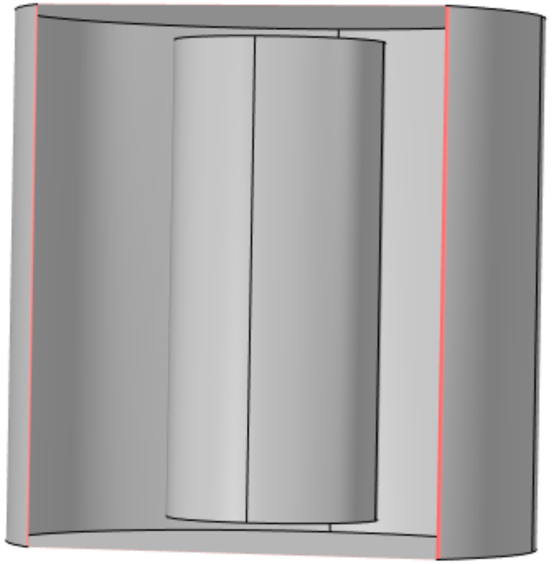
frequency $= m_a/2\pi$



$g_{\min}$ reduced by $\dfrac{\eta_{01}/1 \cdot B_0/0.2}{\eta_{01}/0.36 \cdot B_0/0.14} \approx \dfrac{1}{4}$

Downside: requires different pump mode from GW detection

Further thoughts



- Isolate the inner cylinder from the outer one
  → Reduced vibrational noise, (mostly) freely oscillating cylinder
  → Inner cylinder could be used to tune the resonant frequencies

- Round off edges to prevent strong fields

- Consider other geometries with 'holes'
  e.g. ellipsoids, doughnuts, …