Examination of k_t-factorization in a Yukawa theory

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Motivation

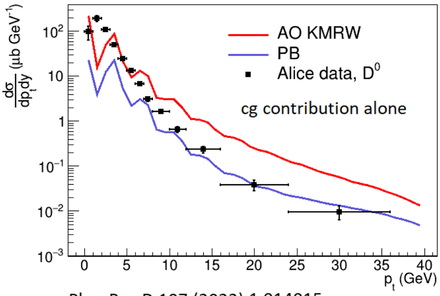
$$F_2(x_{\text{bj}}, Q) = \int_{Q^2/s}^{1} dx \int_0^{k_{\text{max}}^2} d^2k_t F_q(x, k_t; Q) \hat{F}_2(x, Q, k_t) \tag{1}$$

with

$$\int_0^{\mu^2} dk_t^2 F(x, k_t; \mu) = f(x; \mu) \tag{2}$$

1. k_{max} generally not discussed in the litterature.
2. If $\mu = Q$, UPDFs constrained on $[0, Q]$ but integration up to k_{max}. UPDFs built **only** from (2) can in principle have any value for $k_t \in [Q, k_{\text{max}}]$, giving any result for the cross section.
3. Overestimation of the D-meson cross section using KaTie. Not the case if $\int_0^{\infty} dk_t^2 F(x, k_t; \mu) = f(x; \mu)$.

D-meson cross section



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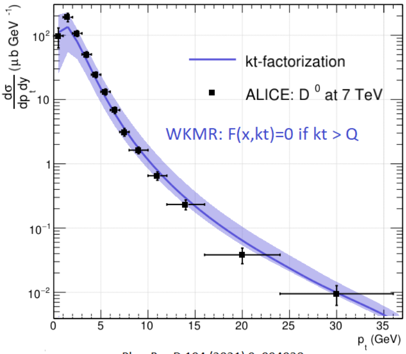

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All these results will be explained by the fact that $k_{\text{max}} = \mu_F$.

Interesting to perform similar calculations for F_2 in a Yukawa theory.

The Yukawa theory

Interest: Everything can be calculated perturbatively, including the exact (not factorized) F_2.

$$\mathscr{L} = \sum_j \frac{i}{2}\left[\overline{\psi}_j \gamma^\mu \partial_\mu \psi_j - (\partial_\mu \overline{\psi}_j)\gamma^\mu \psi_j\right] - M_j \overline{\psi}_j \psi_j + \frac{1}{2}(\partial\phi)^2 - \frac{m_s^2}{2}\phi^2$$
$$- \lambda\left(\overline{\psi}_1 \psi_2 \phi + \overline{\psi}_2 \psi_1 \phi\right), \tag{3}$$

$j = 1,2$, $\psi_1 = \psi_N$ represents the target "nucleon" with mass $M_1 = m_p$, $\psi_2 = \psi_q$ is the "quark" field with mass m_q, and ϕ is the "scalar gluon".

- The expression for the exact $\mathscr{O}(\lambda^2)$ F_2 given in *Aslan et al., Phys. Rev. D 107 (2023) 7 074031*

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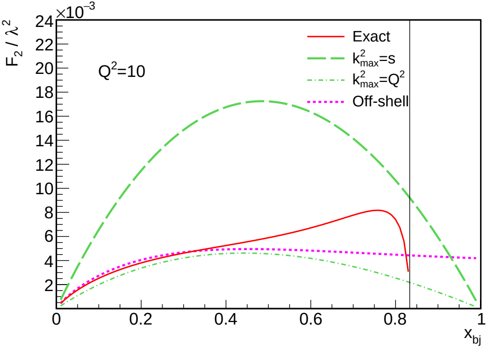


$s = 3 \times 10^4$ GeV^2 : Mandelstam variable for the EIC kinematics.

Use the WKMR UPDFs computed in the Yukawa theory (shown later).

Green: LO on-shell hard coefficient.

Pink: Off-shell hard coefficient. Here, $k_t \lesssim Q$ in the dominant region ($x_{\mathrm{bj}} \sim x$).

Detour by the TMD factorization

$F_2 = W + Y$. The W term includes the TMD PDF

$$\int_0^\infty d^2k_t f(x,k_t;\mu)^{(1)} \otimes F_2^{(0)} + CT = \int_0^{\mu^2} d^2k_t f(x,k_t;\mu)^{(1)} \otimes F_2^{(0)} \quad (4)$$

The Y term includes the collinear contribution and a subtraction term to avoid double counting

$$f(x;\mu)^{(0)} \otimes F_2^{(1)} - \text{sub.} = a_\lambda (1 - x_{\text{bj}}) \left(\int_0^{\hat{k}_m^2(x_{\text{bj}})} \frac{dk_t^2}{\kappa(x_{\text{bj}}) k_t^2} - \int_0^{\mu^2} \frac{dk_t^2}{k_t^2} \right), \quad (5)$$

With $\hat{k}_m^2(x) = \frac{(1-x)Q^2}{4x}$ the max k_t allowed by the (on-shell and massless) kinematics.

At small x, the Y term can be written as

$$Y \simeq a_\lambda (1 - x_{\text{bj}}) \int_{\mu^2}^{\hat{k}_m^2(x_{\text{bj}})} \frac{dk_t^2}{\kappa(x_{\text{bj}}) k_t^2} \quad (6)$$

Some comments

- The k_t-fact. formalism includes only the W term.
- The μ dependence cancels between the W and Y terms.
- TMD formalism cuts the k_t integral at $\mu^2 \sim Q^2$. In other words, $k_{\mathrm{max}} \sim Q$ (remember the plots for D mesons).
- Small $k_t < Q$ in TMD PDFs, large $k_t > Q$ in the hard coefficient.

Main claim: $k_{\text{max}} = \mu_F$

Definition of the UPDFs (first try).

(Early time) UPDFs product of the BFKL factorization of the cross section

$$F_N(k_t;\mu) = f_N(\mu) C_N(k_t,\mu); \quad C_N(k_t,\mu) = \frac{\gamma_N(\alpha_s)}{k_t^2}\left(\frac{k_t^2}{\mu^2}\right)^{\gamma_N(\alpha_s)} \tag{7}$$

The resummation factor obeys

$$\int_0^{\mu} dk_t^2 C_N(k_t,\mu) = 1, \tag{8}$$

and is solution of the BFKL equation.

Where does the scale μ in C_N comes from?

Main claim: $k_{\text{max}} = \mu_F$

- From the subtraction of small k_t, to define an infrared-safe impact factor.
- "μ" appears in the modified BFKL equation in Collins' paper. Let's call it μ_F, to differentiate with μ in collinear PDFs.
- Then, the cross section is k_t factorized, with small $k_t < \mu_F$ in UPDFs and large $k_t > \mu_F$ in the impact factor $\Rightarrow k_{\text{max}} = \mu_F$.

Definition of the UPDFs (second try).

$$F_N(k_t;\mu,\mu_F) = f_N(\mu) C_N(k_t,\mu_F); \quad C_N(k_t,\mu_F) = \frac{\gamma_N(\alpha_s)}{k_t^2}\left(\frac{k_t^2}{\mu_F^2}\right)^{\gamma_N(\alpha_s)} \tag{9}$$

$$\int_0^{\mu_F} dk_t^2 C_N(k_t,\mu_F) = 1, \text{ or } \int_0^{\mu_F^2 = k_{\text{max}}^2} dk_t^2 F(x,k_t;\mu,\mu_F) = f(x,\mu) \tag{10}$$

Factorization scale dependence

Check that F_2, with the on-shell hard coefficient $\propto \delta(1-x_{\mathrm{bj}})$ and $k_{\mathrm{max}} = \mu_F$, is μ_F independent. For $\mu_F = Q$ and $\mu_F = \sqrt{s} \gg Q$

$$\frac{F_2}{x_{\mathrm{bj}}} = \mathrm{TM}^{-1}\left\{ f_N(\mu) \int_0^{Q^2} dk_t^2 C_N(k_t, Q) \right\} = f(x_{\mathrm{bj}}; \mu) \qquad (11)$$

$$\frac{F_2}{x_{\mathrm{bj}}} = \mathrm{TM}^{-1}\left\{ f_N(\mu) \int_0^{s} dk_t^2 C_N(k_t, s) \right\} = f(x_{\mathrm{bj}}; \mu) \qquad (12)$$

Usually, $\int_0^{\mu^2 \sim Q^2} dk_t^2 F(x, k_t; \mu) = f(x, \mu)$. It implies $\mu_F \sim Q$. At the same time k_t integrated above Q, say, up to $k_{\mathrm{max}}^2 = s = \mu_F^2$.

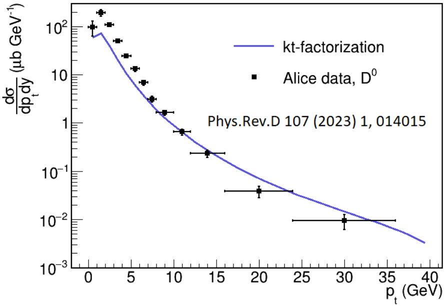
Inconsistent, leads to an overestimation

$$\frac{F_2}{x_{\mathrm{bj}}} = \mathrm{TM}^{-1}\left\{ f_N(\mu) \int_0^{s} dk_t^2 C_N(k_t, \alpha_s; \mu) \right\}$$

$$= f(x_{\mathrm{bj}}; \mu) + \mathrm{TM}^{-1}\left\{ f_N(\mu) \int_{\mu^2}^{s} dk_t^2 C_N(k_t, \alpha_s; \mu) \right\} \qquad (13)$$

Possible choices: partial answer

1. Use $\mu_F = \mu \sim Q$ and stop the k_t integration at Q.
   - WKMR UPDFs or similar are fine and have the correct perturbative behavior at large k_t (i.e. $1/k_t^2$).
   - But the transverse-momentum is usually integrated to far away.
2. (Not recommended) use $\mu_F = \sqrt{s} \gg Q$ and integrate k_t above Q.
   - Then, UPDFs should obey $\int_0^s dk_t^2 F(x, k_t; \mu; s) = f(x, \mu)$.



$k_{\max} = \mu_F$: No k_t integration in a region where the UPDFs are not constrained!

More on UPDFs

$f(x,\mu) = \int_0^\infty dk_t^2 \mathscr{F}(x,k_t;\mu)$: Why is this quantity not divergent?

The Feynman rules for collinear PDFs give UV divergences, renormalized with a counterterm.

In the Yukawa theory, the Feynman rules for collinear PDFs are the same as for TMDs integrated over k_t.

$$f(x;\mu) = \int d^2k_t f(x,k_t;\mu) + \text{C.T.} \tag{14}$$

Unusual, but we can write

$$(2\pi\mu)^{2\varepsilon} \int d^{2-2\varepsilon}k_t \ \text{c.t.}(\mu) = \ \text{C.T.} \tag{15}$$

and

$$\mathscr{F}(x,k_t;\mu) = F(x,k_t;\mu) + \ \text{c.t.}(\mu) \tag{16}$$

Illustration with the KMRW UPDFs

Starting with the usual definition

$$F_q(x, k_t; \mu) = \frac{x}{\pi} \frac{\partial}{\partial k_t^2} \left[T_q(k_t, \mu) f_q(x, k_t) \right], \quad k_t \geq \mu_0 \tag{17}$$

$$F_q(x, k_t; \mu^2) = \frac{x}{\mu_0^2 \pi} T_q(\mu_0, \mu) f_q(x, \mu_0), \quad k_t < \mu_0, \tag{18}$$

and using the expression for the $\mathcal{O}(\lambda^2)$ collinear PDF, we find

$$F_q(x, k_t; \mu)/x = \frac{a_\lambda}{\pi k_t^2} (1-x), \quad k_t \geq \mu_0 \tag{19}$$

$$F_q(x, k_t; \mu^2)/x = \frac{1}{\mu_0^2 \pi} a_\lambda (1-x) \left(\frac{\chi(x)^2}{\Delta(x)^2} + \ln\left(\frac{\mu_0^2}{\Delta(x)^2} \right) - 1 \right), \quad k_t < \mu_0, \tag{20}$$

At this order, the Sudakov factor is 1.

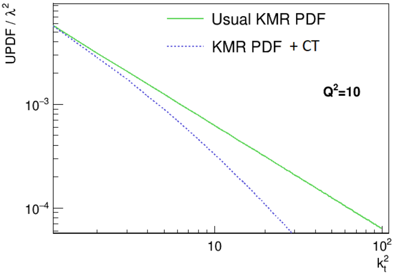
Illustration with the KMRW UPDFs

The $\overline{\text{MS}}$ C.T. is

$$a_\lambda (1-x)\left(\frac{1}{\varepsilon}-\gamma_E+\ln 4\pi\right)+\mathscr{O}(\varepsilon)=(2\pi\mu)^\varepsilon\int d^{2-2\varepsilon}k_t\,\frac{a_\lambda}{\pi}\frac{1-x}{(k_t^2+\mu^2)}. \tag{21}$$

Including the integrand to the KMRW UPDF leads to the modification

$$\frac{a_\lambda}{\pi k_t^2}(1-x)\to\frac{a_\lambda}{\pi}(1-x)\frac{\mu^2}{k_t^2(k_t^2+\mu^2)} \tag{22}$$



Final comments

A negligible value in the region $k_t > \mu$ is typical of UPDFs obeying $\int_0^\infty dk_t^2 \mathscr{F}(x, k_t; \mu) = f(x; \mu)$.

In this case, any $k_{\mathrm{max}} > Q$ is fine (this is the third "recommendation").

Otherwise, the simpler choice is probably $\mu_F = \mu \sim Q$.

Setting $k_{\mathrm{max}} \neq \mu_F$ does not lead necesarily to an overestimation, depending on the kinematics.