Quantum storage: storing patterns & state preparation:

recall: efficient representation of the feature data into

basis state $\to$ basis encoding

For sake of simplification the algorithm presented last time will be rewritten in a slightly different method:

recall: this algorithm offers a polynomial number in length and number of patterns

Definitions:

Let

$$\hat{S}^p = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & \\ 0 & 0 & \sqrt{(p-1)/p} & -1/\sqrt{p} \\ 0 & 0 & 1/\sqrt{p} & \sqrt{(p-1)/p} \end{bmatrix}$$

$1 \leq p \leq m$

m: nb of pattens
n: nb of bits per pattern

$\{\hat{S}^p\}$: conditional transforms $\longrightarrow$ bring the set of patterns into coherent quantum states

$$\hat{F} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \longrightarrow$$ flips the state of a qubit

$$\hat{F}^0 = \begin{bmatrix} \hat{F} & \hat{0} \\ \hat{0} & \hat{I}_2 \end{bmatrix} \longrightarrow$$ 2 qubit CNOT

↳ flips the state of the 2nd qubit if the first qubit is in $|0\rangle$ state

$$\hat{F}^1 = \begin{bmatrix} \hat{I}_2 & \hat{0} \\ \hat{0} & \hat{F} \end{bmatrix}$$ → same as $\hat{F}^0$ but flips if the first qubit is in $|1\rangle$ state

$$\hat{A}^{00} = \begin{bmatrix} \hat{F} & \hat{0} \\ \hat{0} & \hat{I}_6 \end{bmatrix}$$ → 3-qubit operator

→ flips the state of the 3rd qubit iff the first 2 are in state $|00\rangle$

Fredkin gate

other operators are also used: $\hat{A}^{01}$, $\hat{A}^{10}$ & $\hat{A}^{11}$

→ same as $\hat{A}^{00}$ but $\hat{F}$ appears in the other 3 possible location along the main diagonal

Let P be the set of m binary patterns of length n to be memorized

→ the algorithm requires $2n+1$ qubits as:

$|x, g, c\rangle$ with

x: Storage register of length n

g: temp storage of length $n-1$

c: control qbits of length 2

* the algorithm holds a superposition of the patterns
  1 qubit $\rightarrow$ 1 bit in the pattern
* the g & c registers are restored into $|0\rangle$ after every iteration

The algorithm:

- The quantum system is initialized into the $|0\rangle$ state
- The qubits in the x register are selectively flipped so that their states correspond to the inputs of the first pattern.
- the state in the superposition is broken into 2 pieces smaller & larger
- The status of the smaller is made permanent in the c- register
- The x register of the larger piece is selectively flipped to match the input of the second pattern

- The pattern to be stored is composed of the smaller pieces of the same size, i.e. a coherent superposition of states is created that corresponds to the patterns where the amplitude of the states in the superposition are all equal

* The algorithm requires $O(mn)$ steps to encode the pattern as a quantum superposition over n quantum neuron

Example: Let $P = \{01, 10, 11\}$

1) $|00, 0, 00\rangle$
2) $|00, 0, 00\rangle \xrightarrow{F} |01, 0, 10\rangle$
3) $|01, 0, 10\rangle \xrightarrow{\hat{S}^3} \frac{1}{\sqrt{3}} |01, 0, 11\rangle + \sqrt{\frac{2}{3}} |01, 0, 10\rangle$

$S^{P=3} = S^3$; P is the number of patterns (including curr)

4) $\frac{1}{\sqrt{3}}|01,0,11\rangle + \sqrt{\frac{2}{3}}|01,0,10\rangle \xrightarrow{S} \frac{1}{\sqrt{3}}|01,0,01\rangle + \sqrt{\frac{2}{3}}|01,0,00\rangle$

2^{nd} it. 5) $\frac{1}{\sqrt{3}}|01,0,01\rangle + \sqrt{\frac{2}{3}}|0,0,00\rangle \xrightarrow{F} \frac{1}{\sqrt{3}}|01,0,01\rangle + \sqrt{\frac{2}{3}}|10,0,10\rangle$

now (p is 2)

6) $\xrightarrow{S_2} \frac{1}{\sqrt{3}}|01,0,01\rangle + \frac{1}{\sqrt{2}} \cdot \sqrt{\frac{2}{3}}|10,0,11\rangle + \frac{1}{\sqrt{2}}\sqrt{\frac{2}{3}}|10,0,10\rangle$

7) $\xrightarrow{S} \frac{1}{\sqrt{3}}|10,0,01\rangle + \frac{1}{\sqrt{3}}|10,0,01\rangle + \frac{1}{\sqrt{3}}|10,0,00\rangle$

8) $\xrightarrow{F} \frac{1}{\sqrt{3}}|01,0,01\rangle + \frac{1}{\sqrt{3}}|10,0,01\rangle + \frac{1}{\sqrt{3}}|11,0,10\rangle$

9) (now $p=1$) $\xrightarrow{S'} \frac{1}{\sqrt{3}}|01,0,01\rangle + \frac{1}{\sqrt{3}}|10,0,01\rangle + \frac{1}{\sqrt{3}}|11,0,11\rangle$

$+ \frac{1}{\sqrt{3}}|11,0,10\rangle$

10) $\xrightarrow{S} |01,0,01\rangle + \frac{1}{\sqrt{3}}|10,0,01\rangle + \frac{1}{\sqrt{3}}|11,0,01\rangle$

11) g & c are the same for all the states in the superposition
g & c cannot be entangled with x register

$$\Rightarrow \quad \frac{1}{\sqrt{3}}|01\rangle + \frac{1}{\sqrt{3}}|10\rangle + \frac{1}{\sqrt{3}}|11\rangle$$

* the Flip operation:

loop over n
  if non-zero state
  apply $\hat{F}^{0}_{c_2 x_i}$
end loop

* the save operation

apply $\hat{A}^{z_1 z_2}_{x_1 x_2 g_1}$; $z_{n+1} = \{0\}^n$

loop 3 : n

apply $\hat{A}^{z_i 1}_{x_i g_{i-1} g_{i-2}}$

Apply $\hat{F}^{1}_{g_{i-1} c_1}$

Loop from n : 3

Apply $\hat{A}^{z_i 1}_{x_i g_{i-2} g_{i-1}}$

Apply $A^{z_1 z_2}_{x_1 x_2}$