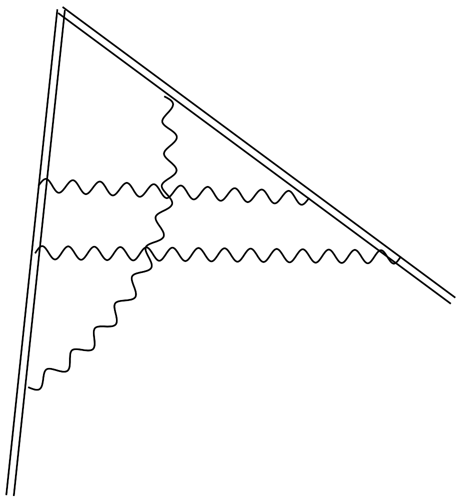
Exact results for cusped Wilson loops



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with D. Correa, A. Sever, J. Maldacena

based on

arXiv:1202.4455 [hep-th]

arXiv:1203.1019 [hep-th]

$$\mathcal{L} = \frac{1}{4}\mathrm{Tr}\int F_{\mu\nu}F^{\mu\nu}\,, \qquad F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu + ig[A^\mu, A^\nu]$$

$$A^\mu = \sum_{a=1}^{N^2-1} A_a^\mu\, t_{ij}^a$$ gauge group SU(N)

Wilson loops:

required for gauge invariance of non-local objects

$\bar{\Psi}(x_2)$

$W_C[x_1, x_2]$

$\Psi(x_1)$

$$\mathcal{O} = \Psi(x_1) W_C[x_1, x_2] \bar{\Psi}(x_2)$$

$$W_C[x_1, x_2] = P e^{\int_C dx_\mu A^\mu}$$

P: path ordering

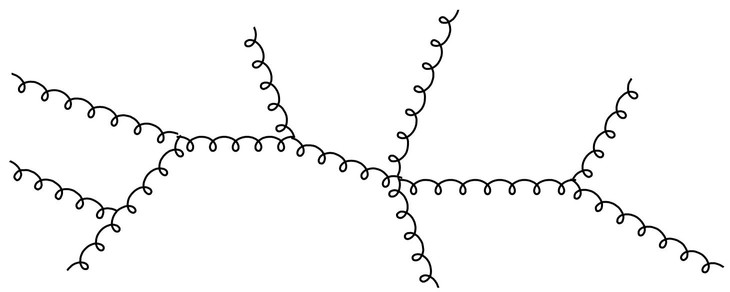
contain local operators

$$\sim 1 + \sigma^{\mu\nu} F_{\mu\nu} + \dots$$

gauge dynamics - Wilson loops of arbitrary shapes

QCD and supersymmetric Yang-Mills theories

- technically very similar
- elegant supersymmetry methods



e.g. all Yang-Mills tree amplitudes

Drummond, JMH 2008
Dixon, Plefka, JMH, Schuster, 2010

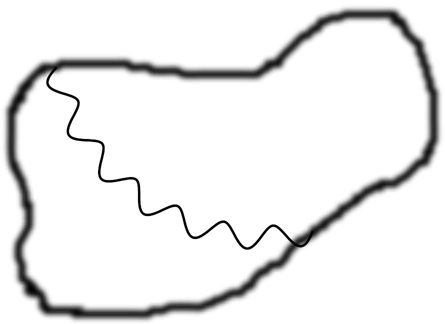
Dual conformal / Yangian symmetry

Understanding of structure of Yang-Mills
Exactly solvable gauge theory in four dimensions?
Dualities with string theories

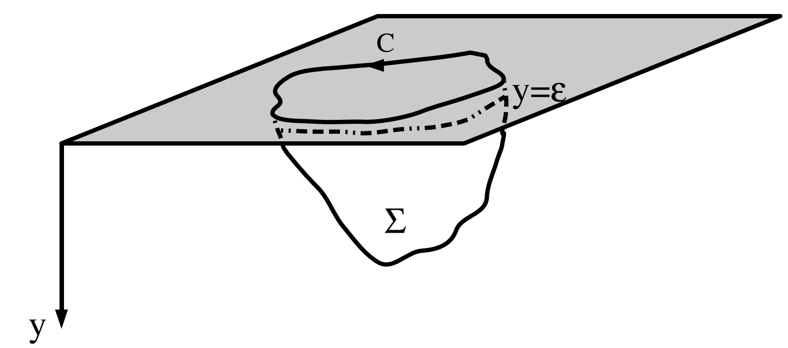
AdS/CFT correspondence

N=4 SYM
SU(N) gauge theory
scalars+fermions
conformal

dual string theory
description on AdS_5

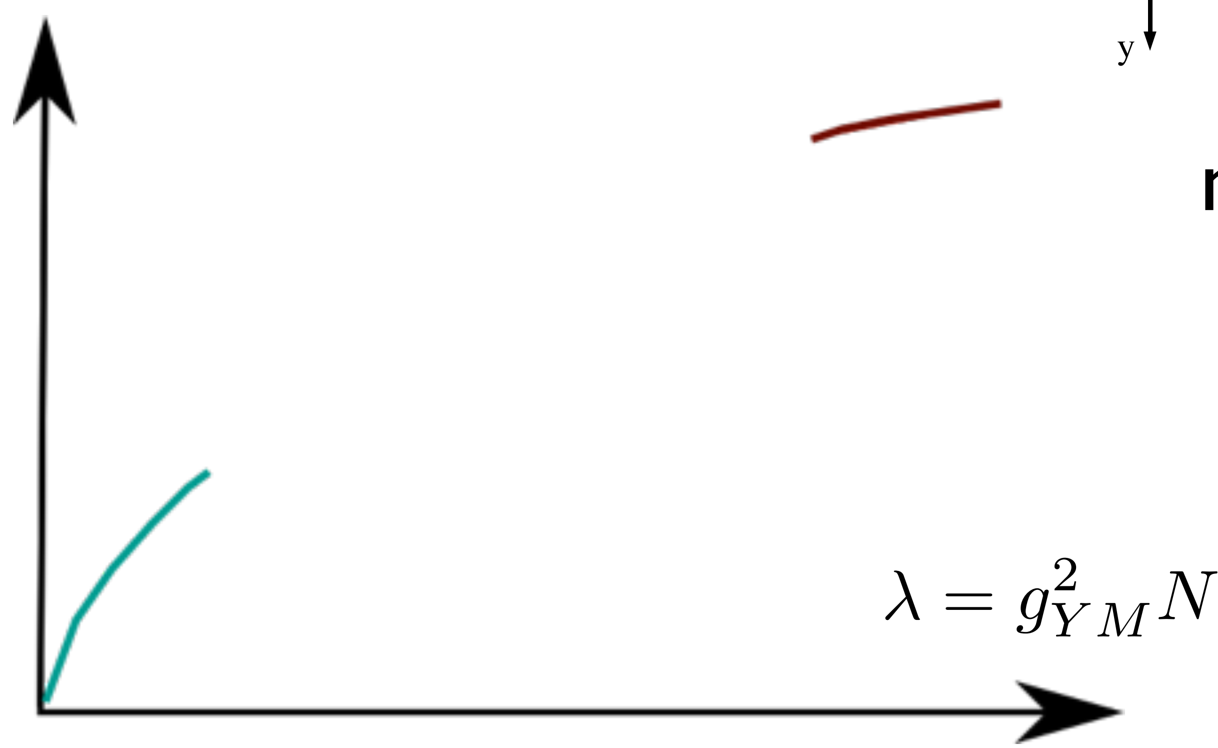


Wilson loop $W[C]$



Feynman diagrams
$\lambda \ll 1$

minimal surface
$\lambda \gg 1$



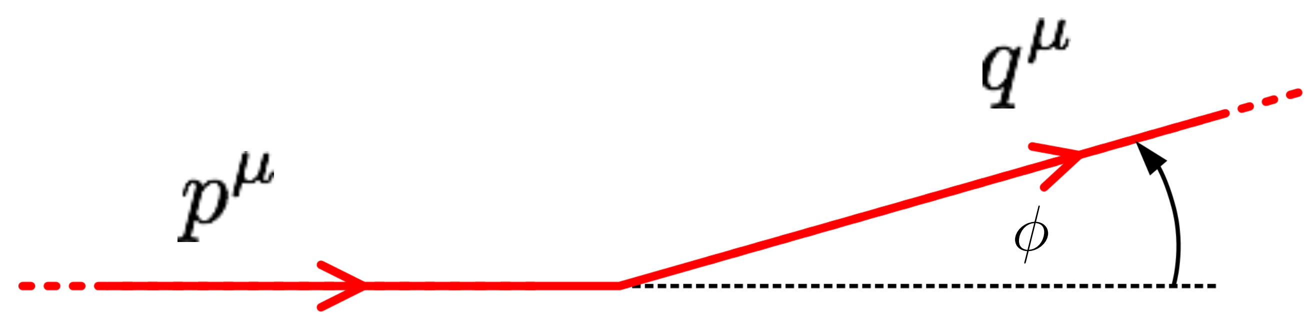
Outline of talk

- Introduction: cusped anomalous dimension Γ_{cusp}
  and physical motivation
- Part 1: Exact result at small angles
- Part 2: Three-loop result,
  new limit, Schrödinger problem
- Part 3: relation to Regge limit of massive amplitudes

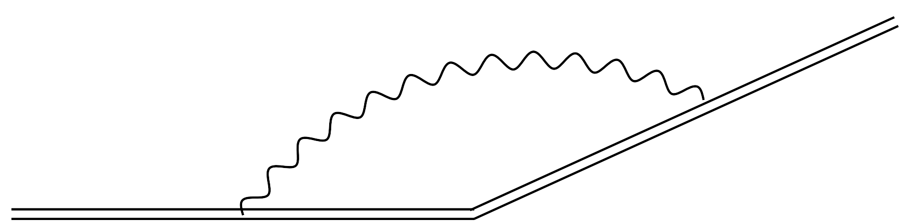
Cusp anomalous dimension

Wilson loop with cusp

$$\cos(\phi) = \frac{p \cdot q}{\sqrt{p^2 q^2}}$$



Γ_{cusp} governs ultraviolet (UV) divergences at cusp



Polyakov; Brandt, Neri, Sato
Korchemsky & Radyushkin '87

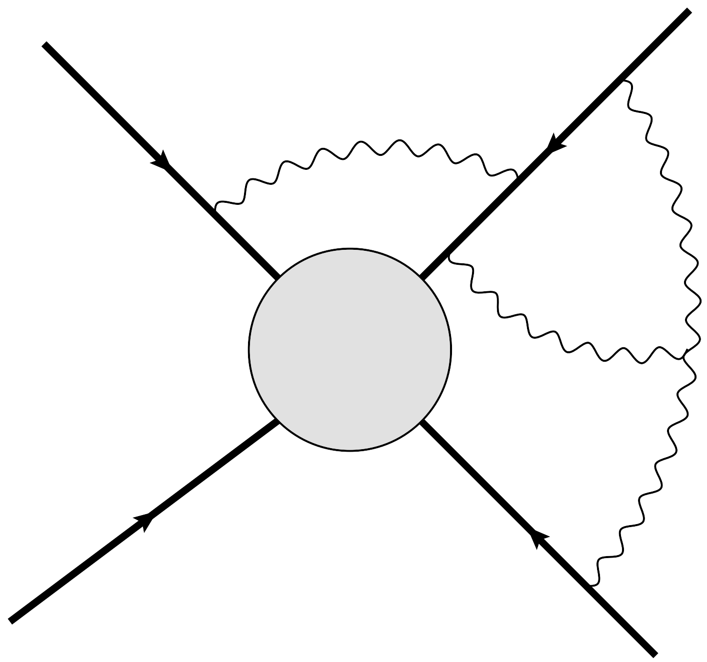
$$\langle W \rangle \sim e^{-|\ln \frac{\mu_{UV}}{\mu_{IR}}|\, \Gamma_{\text{cusp}}}$$

similar to anomalous dimensions of composite operators

$\Gamma_{\text{cusp}}(\phi, \lambda, N)$ $\qquad$ $\lambda = g_{YM}^2 N$

Physical relevance of Γ_{cusp}

- IR divergences of massive amplitudes



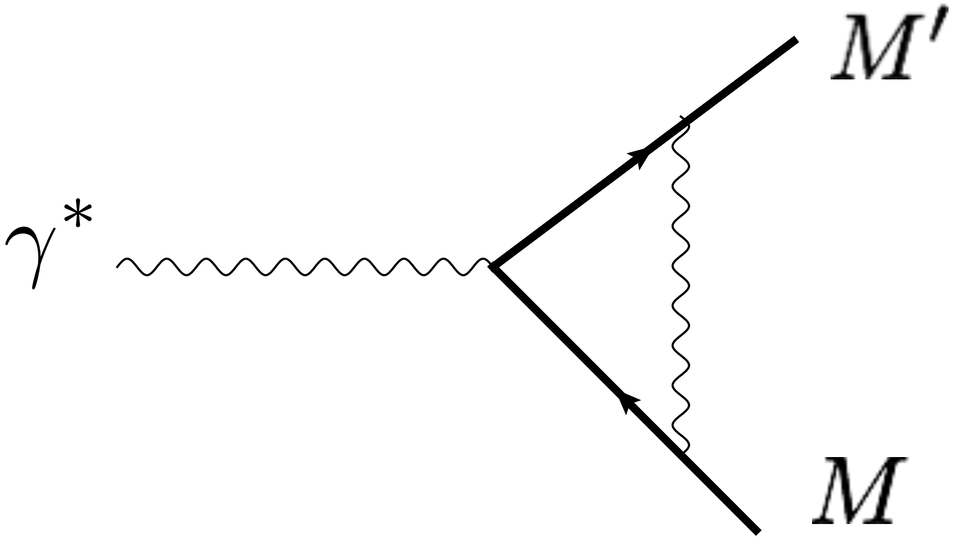
$$\mathcal{A} \sim e^{-|\log \mu_{IR}|\,\Gamma_{\text{cusp}}}$$

Korchemsky, Radyushkin;
Becher, Neubert

See Magnea's talk

resummation of soft divergences

- similarly for massive form factors (e.g Isgur-Wise)



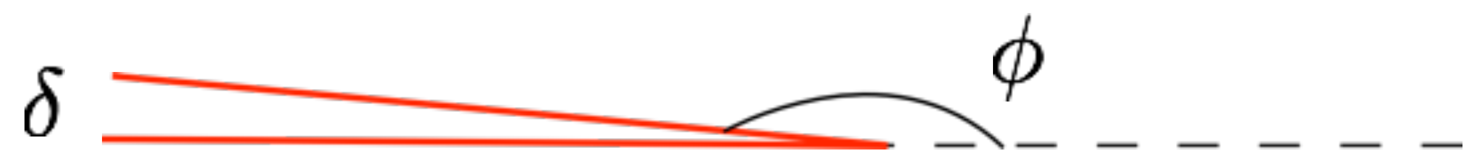
massless 3-loop form factors:
see Huber's talk

Limits and relations of $\Gamma_{\text{cusp}}(\phi)$

- vanishes at zero angle (straight line) $\quad \Gamma_{\text{cusp}}(\phi = 0, \lambda) \;=\; 0$

- quark-antiquark potential $\quad \delta = \pi - \phi \qquad \delta \ll 1$

$$\Gamma_{\text{cusp}} \sim \frac{C}{\delta}$$



- anomalous dimensions of large spin operators $\quad \lim_{\varphi \to \infty} \Gamma_{\text{cusp}}(i\varphi, \lambda) \;\sim\; \varphi\, \gamma_{\text{cusp}}(\lambda)$

known due to Beisert, Eden, Staudacher eq
integrability!

$$\mathcal{O}_J \sim \operatorname{Tr}(Z\mathcal{D}^J Z)\,, \qquad d = 2 + J + \gamma(J, \lambda)$$

$$\lim_{J \to \infty} \gamma(J, \lambda) \sim \log J \;\gamma_{\text{cusp}}(\lambda)$$

Wilson loops in supersymmetric theories

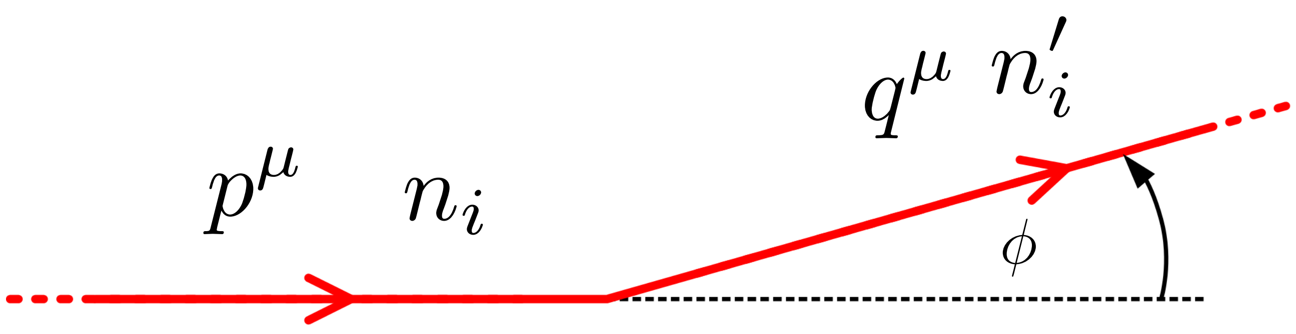
- loop couples to scalars $Tr(Pe^{\int ds A^\mu \dot{x}_\mu + ds\, n_i \Phi^i})$

  Maldacena; Rey

  six scalars Φ^i

- path-dependent coupling

$$\cos(\phi) = \frac{p \cdot q}{\sqrt{p^2 q^2}}$$



$$\cos(\theta) = n \cdot n' \,, \quad n^2 = n'^2 = 1$$

e.g. $n = (1,0,0,0,0,0)\,, \quad n' = (\cos(\theta), \sin(\theta), 0,0,0,0)$

$$\Gamma_{\rm cusp}(\phi, \theta, \lambda, N)$$

- supersymmetry $\Gamma_{\rm cusp}(\phi = \pm\theta) = 0$

  Zarembo

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Part1: Exact result for Γ_{cusp} at small angles

Correa, JMH, Maldacena, Sever

$$\Gamma_{\text{cusp}}(\phi = \pm\theta) = 0$$

$$\Gamma_{\text{cusp}} = (\phi^2 - \theta^2) H(\phi, \lambda, N) + \dots$$

- let us start with $B(\lambda, N) = H(0, \lambda, N)$
- need the coefficient of θ^2 : insertion of scalars

$$\langle\langle \phi(t_1)\phi(t_2) \rangle\rangle_{line} = \frac{\gamma(\lambda, N)}{(t_1 - t_2)^2}$$

- idea: obtain $\gamma(\lambda, N)$ from known circular Wilson loop!

Final result:

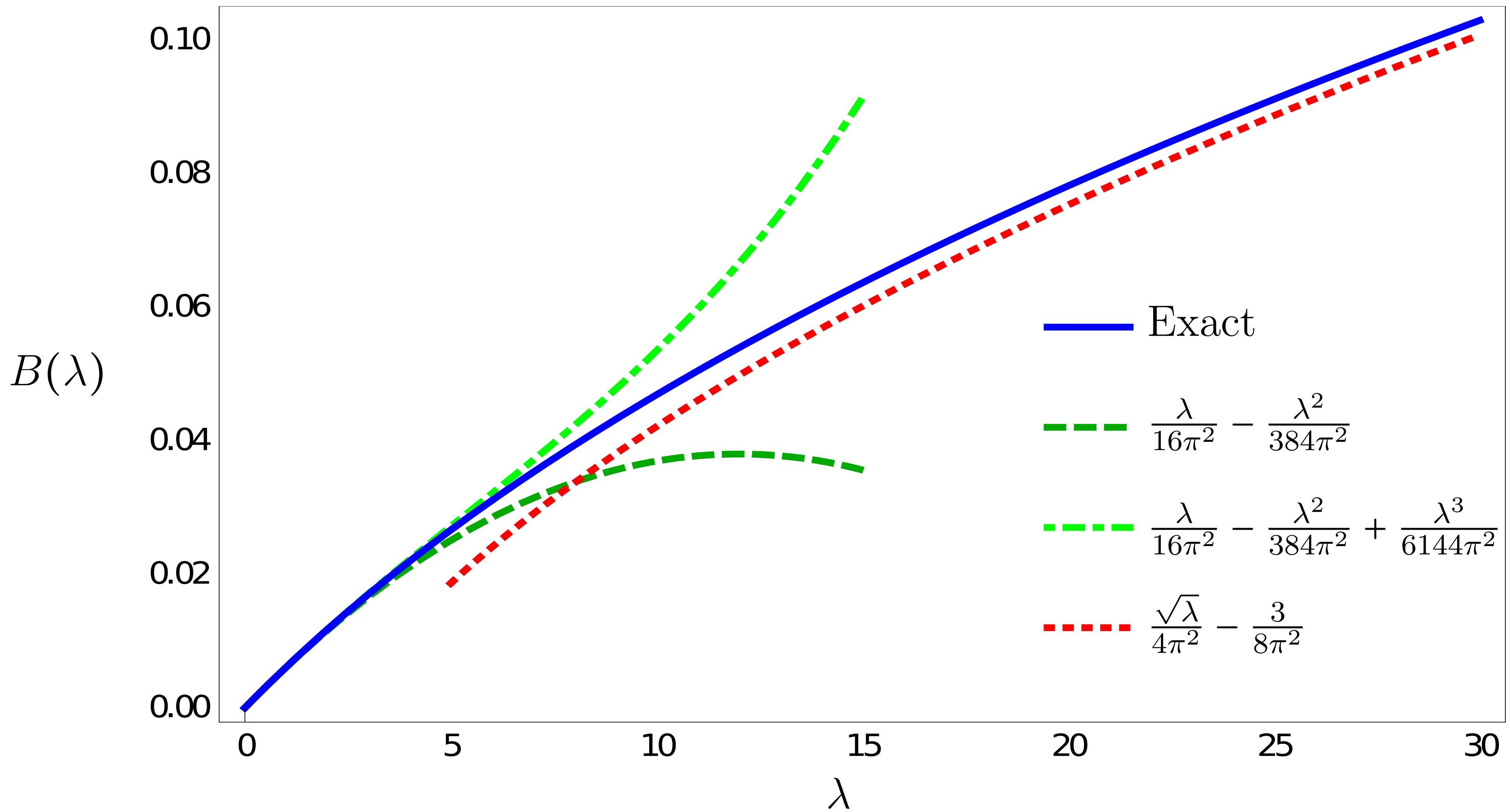
$$B = \frac{1}{4\pi^2}\sqrt{\lambda}\,\frac{I_2(\sqrt{\lambda})}{I_1(\sqrt{\lambda})} + o(1/N^2)$$

I_a :modified Bessel function

Exact result interpolating between weak and strong coupling!

Correa, JMH, Maldacena, Sever

``Bremsstrahlung function'' $B(\lambda)\,, \quad \lambda = g_{YM}^2 N$



(exact N dependence also known)

First deviation from supersymmetric case:

Correa, JMH, Maldacena, Sever

$$\Gamma_{\text{cusp}} = (\phi^2 - \theta^2) H(\phi, \lambda, N) + \ldots$$

H obtained by similar methods
relating it to Wilson loops on S^2

$$H(\phi, \lambda) = \frac{2\phi}{1 - \frac{\phi^2}{\pi^2}} B(\tilde{\lambda}) \,, \qquad \tilde{\lambda} = \lambda \left(1 - \frac{\phi^2}{\pi^2}\right)$$

Comments:

- perturbatively, H is a polynomial in ϕ, π
- strong coupling $H = \frac{\sqrt{\lambda}}{2} \frac{\phi}{\sqrt{1 - \frac{\phi^2}{\pi^2}}}$

agrees with formula extracted from
Drukker, Forini

Part 2 : structure of perturbative results and ladder limit

- full 3-loop result: $\xi = \dfrac{\cos\theta - \cos\phi}{\sin\phi}$

Correa, JMH, Maldacena, Sever

$$\begin{aligned}\Gamma_{\text{cusp}} = &\; \lambda\, \xi\, \phi \\ &+ \lambda^2 \left[\xi\, \phi(\pi^2 - \phi^2) + \xi^2 \left(\mathrm{Li}_3(e^{2i\phi}) + \ldots\right)\right] \\ &+ \lambda^3 \left[\xi\phi(\pi^2 - \phi^2)^2 + \xi^2 \left(\mathrm{Li}_5(e^{2i\phi}) + \ldots\right) + \xi^3 \left(\mathrm{HPL}(e^{2i\phi}) + \ldots\right)\right]\end{aligned}$$

- uniform transcendentality

cf. QCD/N=4 SYM transcendentality principle (KLOV)

$$\phi = i \log x \qquad \mathrm{Li}_n(x) = \int_0^x \frac{dy}{y} \mathrm{Li}_{n-1}(y)\,, \quad \mathrm{Li}_1(x) = -\log(1-x)$$

HPL: harmonic polylogarithms: kernels $\dfrac{1}{y}, \dfrac{1}{1-y}, \dfrac{1}{1+y}$

- linear in ξ - exactly known
- highest term $\lambda^L \xi^L$ from new limit $\theta \to i\theta\,, \quad \theta \to \infty$

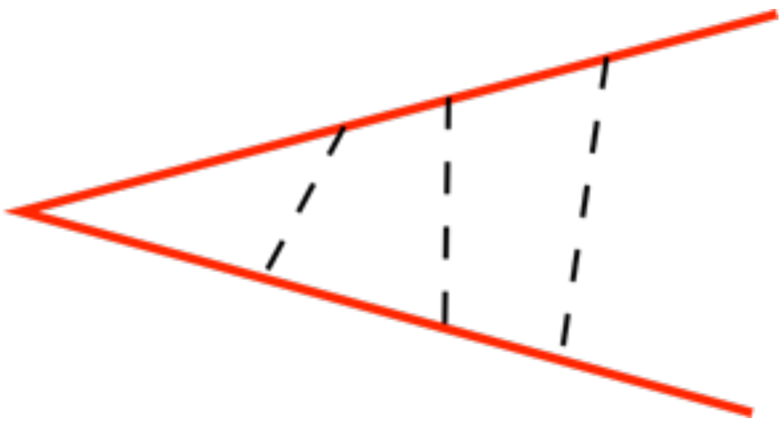
limit selects ladders

Ladders limit

Correa, JMH, Maldacena, Sever

- highest term $\lambda^L \xi^L$ from new limit $\theta \to i\theta\,, \quad \theta \to \infty$

limit selects ladders



Bethe-Salpeter equation

- Γ_{cusp} from ground-state energy of Schrödinger problem

$$\left[-\partial_x^2 - \frac{\hat{\lambda}}{8\pi^2}\frac{1}{\cosh x + \cos\phi}\right]\Psi(x) = -\frac{\Gamma_{\text{cusp}}^2}{4}\Psi(x) \qquad \hat{\lambda} \sim \lambda\,\xi$$

  - exactly solvable for zero angle (Pöschl-Teller)
  - iterative solution in coupling, or angle
  - numerical solution

Part 3 : Relation to Regge limit of massive amplitudes in N=4 SYM

- massive scattering amplitudes in N=4 SYM

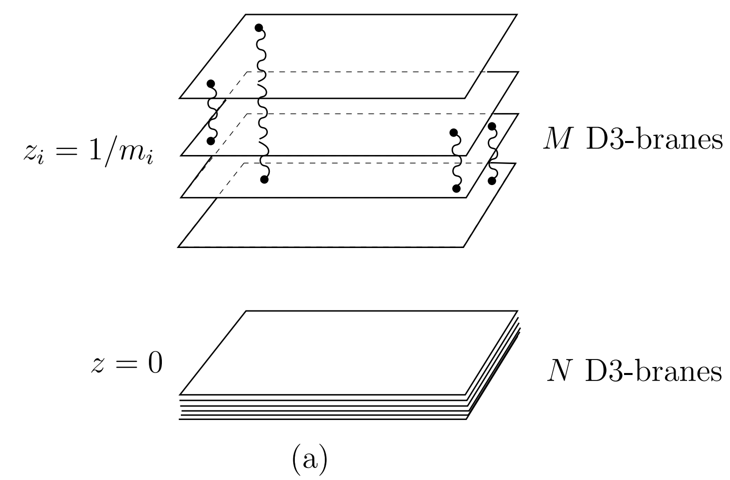
gauge theory

Higgs mechanism

$$\Phi \longrightarrow \langle \Phi \rangle + \varphi$$

$$U(N+M) \longrightarrow U(N) \times U(M)$$
$$\longrightarrow U(N) \times U(1)^M$$

string theory



Alday, JMH, Plefka, Schuster

- dual conformal symmetry (planar)

$$y_i^A \to \frac{y_i^A}{y_i^2} \qquad y_i^A = (x_i^\mu, m_i)$$

isometries of AdS_5 space
Poincare coordinates

$$p_i^\mu = x_i^\mu - x_{i+1}^\mu \qquad p_i^2 = -(m_i - m_{i+1})^2$$

- dual conformal symmetry $\quad s = (p_1+p_2)^2 \quad t = (p_2+p_3)^2$

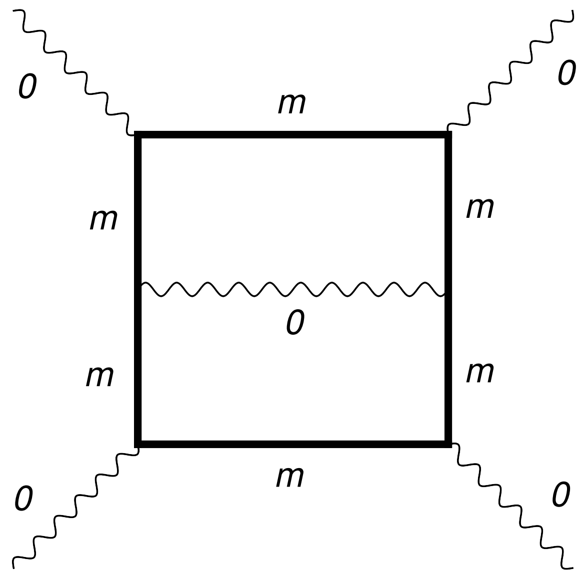
$$M_4(s,t;m_1,m_2,m_3,m_4) = M(u,v)$$

Alday, JMH, Plefka, Schuster

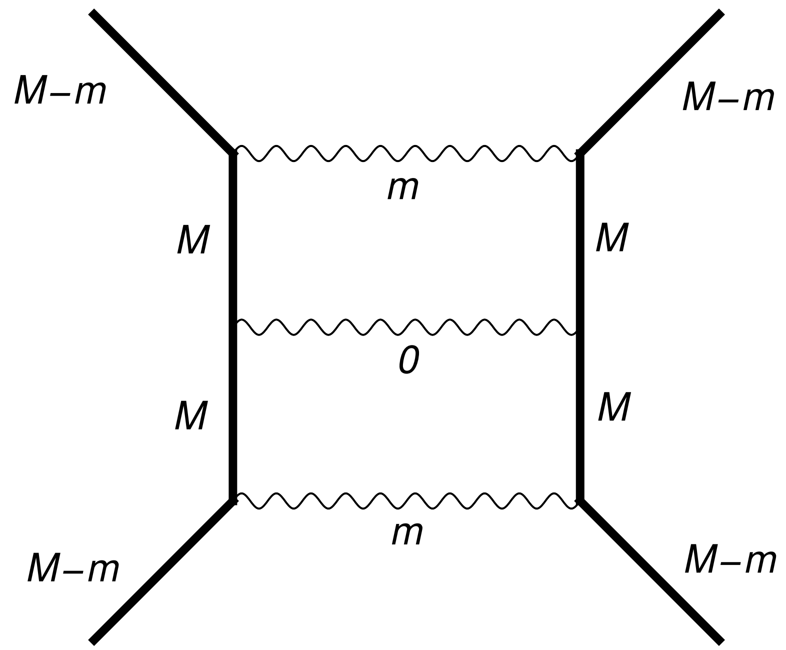
$$u = \frac{m_1 m_3}{s+(m_1-m_3)^2}\,, \qquad v = \frac{m_2 m_4}{t+(m_2-m_4)^2}$$

- relates two different physical pictures

JMH, Naculich, Spradlin, Schuster



$$(a):\quad u=\frac{m^2}{s}\,,\ v=\frac{m^2}{t}$$



$$(b):\quad u=\frac{m^2}{s}\,,\ v=\frac{M^2}{t}$$

- $u \ll 1:$ Regge limit

soft IR divergences, ``Bhabha-scattering”

$$\boxed{M(u,v) \sim e^{\log u\, \Gamma_{\rm cusp}(\lambda,\phi)}}$$

Summary and discussion

$\Gamma_{\text{cusp}}(\phi, \theta, \lambda, N)$ is interesting physical quantity

- exact result for small angles
- full three-loop result
- new limit that selects ladders; Schrödinger problem
- relation to Regge limit of massive amplitudes

- QCD result at three loops?
- systematics of $\xi = (\cos\theta - \cos\phi)/\sin\phi$ expansion
- TBA equations from integrability

Correa, Maldacena, Sever; Drukker

Known circular Wilson loops

- circular Wilson loop known

$$\langle W_{circle} \rangle = \frac{1}{N} L^1_{N-1}(-\frac{\lambda}{4N})\, e^{\frac{\lambda}{8N}} \,, \qquad \lambda = g^2_{YM} N$$

L: modified Laguerre polynomial

Ericksson, Semenoff, Zarembo

Drukker, Gross

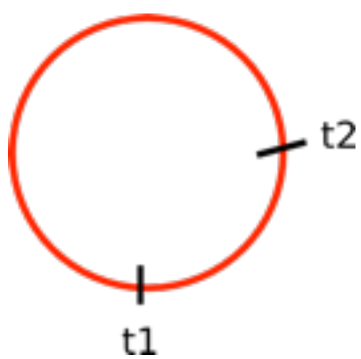
Pestun

Drukker, Giombi, Ricci, Trancanelli

- loop with non-trivial scalar profile also known

  simple replacement (!) $\lambda \to \lambda(1 - \cos^2\theta)$

- expansion in θ determines $\gamma(\lambda, N)$



$$\langle\langle \Phi(t_1)\Phi(t_2)\rangle\rangle_{circle} = \frac{\gamma(\lambda, N)}{2(1-\cos(t_1 - t_2))}$$

Final result: $$B = \frac{1}{2\pi^2}\lambda\partial_\lambda \log\langle W_{circle}\rangle$$