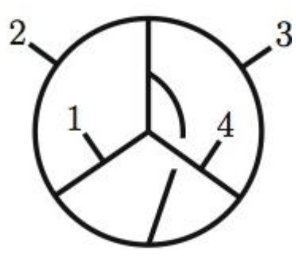
Relations Between Gravity and Gauge Theory and Implications for UV Properties

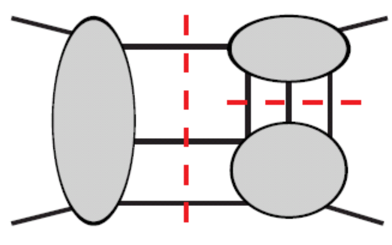
April 16, 2012

Loops and Legs in Quantum Field Theory.

Zvi Bern, UCLA

Based on papers with John Joseph Carrasco, Scott Davies, Tristan Dennen, Lance Dixon, Yu-tin Huang, Harald Ita, Henrik Johansson and Radu Roiban.





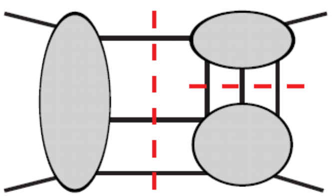
Multiloop Non-Planar Amplitudes

In this talk we will explain how we carry out high-loop-order calculations in supergravity theories, proving supergravity is much better behaved in the UV than had been anticipated.
Update on latest developments in 2012.

Nonplanar is less developed than planar but we do have powerful general purpose tools (not limited to susy):

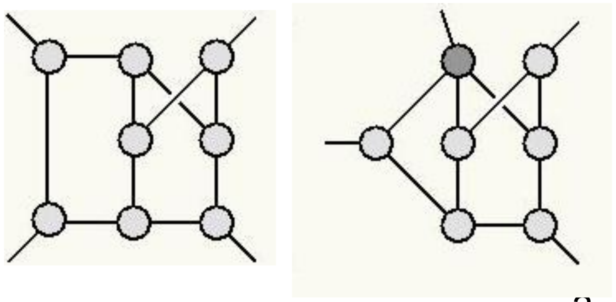
- **Unitarity method**

  ZB, Dixon, Dunbar, Kosower



- **Method of maximal cuts**

  ZB, Carrasco, Johansson , Kosower



- **Duality between color and kinematics**

  ZB, Carrasco and Johansson

Gravity vs Gauge Theory

Consider the gravity Lagrangian

$$L_{\text{gravity}} = \frac{2}{\kappa^2}\sqrt{-g}\,R$$

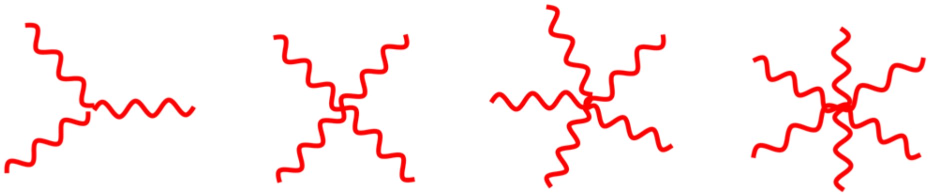
$$\kappa^2 = 32\pi G_{\text{Newton}}$$

flat metric

graviton field

$$\underbrace{g_{\mu\nu}}_{\text{metric}} = \underbrace{\eta_{\mu\nu}}_{\text{flat metric}} + \kappa \underbrace{h_{\mu\nu}}_{\text{graviton field}}$$

Infinite number of complicated interactions



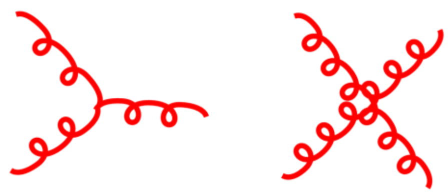
$+ \cdots$

terrible mess

Non-renormalizable

Compare to Yang-Mills Lagrangian on which QCD is based

$$L_{\text{YM}} = \frac{1}{g^2} F^2$$

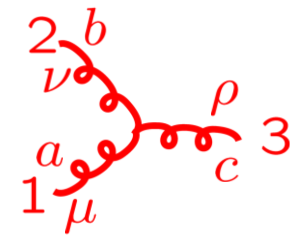


Only three and four point interactions

Gravity seems so much more complicated than gauge theory.

Three Vertices

Standard Feynman diagram approach.



Three-gluon vertex:

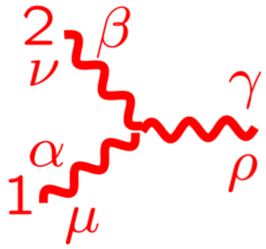
$$V_{3\,\mu\nu\sigma}^{abc} = -g f^{abc}(\eta_{\mu\nu}(k_1-k_2)_\rho+\eta_{\nu\rho}(k_1-k_2)_\mu+\eta_{\rho\mu}(k_1-k_2)_\nu)$$

$$k_i^2 = E_i^2 - \vec{k}_i^2 \neq 0$$

Three-graviton vertex:

$$G_{3\mu\alpha,\nu\beta,\sigma\gamma}(k_1, k_2, k_3) =$$

$$\begin{aligned}
&\mathrm{sym}[-\frac{1}{2}P_3(k_1\cdot k_2\eta_{\mu_1\nu_1}\eta_{\mu_2\nu_2}\eta_{\mu_3\nu_3}) - \frac{1}{2}P_6(k_{1\mu_1}k_{1\nu_2}\eta_{\mu_1\nu_1}\eta_{\mu_3\nu_3}) + \frac{1}{2}P_3(k_1\cdot k_2\eta_{\mu_1\mu_2}\eta_{\nu_1\nu_2}\eta_{\mu_3\nu_3})\\
&\quad +P_6(k_1\cdot k_2\eta_{\mu_1\nu_1}\eta_{\mu_2\mu_3}\eta_{\nu_2\nu_3}) + 2P_3(k_{1\mu_2}k_{1\nu_3}\eta_{\mu_1\nu_1}\eta_{\nu_2\mu_3}) - P_3(k_{1\nu_2}k_{2\mu_1}\eta_{\nu_1\mu_1}\eta_{\mu_3\nu_3})\\
&\quad +P_3(k_{1\mu_3}k_{2\nu_3}\eta_{\mu_1\mu_2}\eta_{\nu_1\nu_2}) + P_6(k_{1\mu_3}k_{1\nu_3}\eta_{\mu_1\mu_2}\eta_{\nu_1\nu_2}) + 2P_6(k_{1\mu_2}k_{2\nu_3}\eta_{\nu_2\mu_1}\eta_{\nu_1\mu_3})\\
&\quad +2P_3(k_{1\mu_2}k_{2\mu_1}\eta_{\nu_2\mu_3}\eta_{\nu_3\nu_1}) - 2P_3(k_1\cdot k_2\eta_{\nu_1\mu_2}\eta_{\nu_2\mu_3}\eta_{\nu_3\mu_1})]
\end{aligned}$$



About 100 terms in three vertex

Naïve conclusion: Gravity is a nasty mess.

Definitely not a good approach.

Simplicity of Gravity Amplitudes

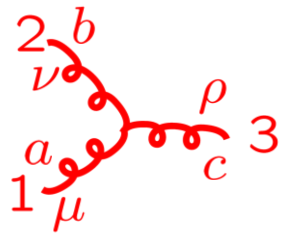
People were looking at gravity the wrong way.

On-shell viewpoint more powerful.

***On-shell* three vertices contains all information:**

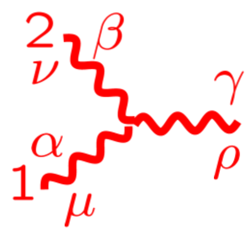
$$k_i^2 = 0$$

gauge theory:



$$-gf^{abc}(\eta_{\mu\nu}(k_1 - k_2)_\rho + \text{cyclic})$$

gravity:



$$i\kappa(\eta_{\mu\nu}(k_1 - k_2)_\rho + \text{cyclic})$$
$$\times\, (\eta_{\alpha\beta}(k_1 - k_2)_\gamma + \text{cyclic})$$

double copy of Yang-Mills vertex.

- **Using modern on-shell methods, any gravity scattering amplitude constructible solely from *on-shell* 3 vertices. BCFW recursion for trees, BDDK unitarity method for loops.**
- **Higher-point vertices irrelevant!**

Gravity vs Gauge Theory

Consider the gravity Lagrangian

$$L_{\text{gravity}} = \frac{2}{\kappa^2}\sqrt{-g}\,R \qquad g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}$$

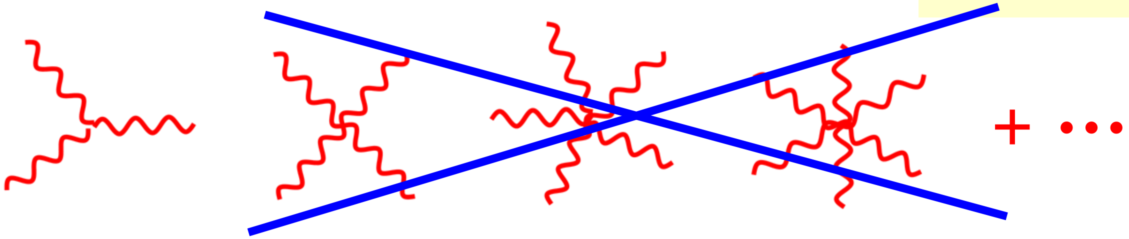
flat metric

graviton field

metric

$$\kappa^2 = 32\pi\, G_{\text{Newton}}$$

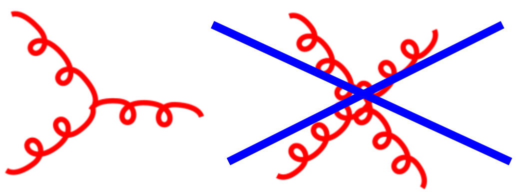
Infinite number of irrelevant interactions!



Simple relation to gauge theory

Compare to Yang-Mills Lagrangian

$$L_{\text{YM}} = \frac{1}{g^2} F^2$$



Only three-point Interactions needed

Gravity seems ~~so much~~ no more complicated than gauge theory.

Duality Between Color and Kinematics

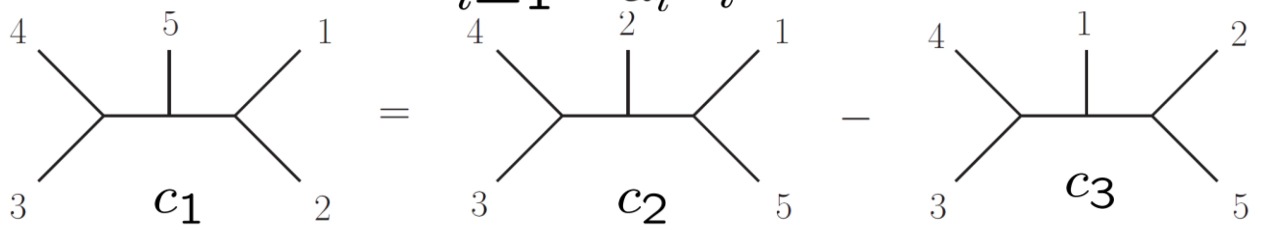
Consider five-point tree amplitude: ZB, Carrasco, Johansson (BCJ)

color factor

kinematic numerator factor

$$\mathcal{A}_5^{\text{tree}} = \sum_{i=1}^{15} \frac{c_i\, n_i}{\prod_{\alpha_i} p_i^2}$$

Feynman propagators



$$c_1 \equiv f^{a_3 a_4 b} f^{b a_5 c} f^{c a_1 a_2}, \quad c_2 \equiv f^{a_3 a_4 b} f^{b a_2 c} f^{c a_1 a_5}, \quad c_3 \equiv f^{a_3 a_4 b} f^{b a_1 c} f^{c a_2 a_5}$$

$$n_i \sim k_4 \cdot k_5\, k_2 \cdot \varepsilon_1\, \varepsilon_2 \cdot \varepsilon_3\, \varepsilon_4 \cdot \varepsilon_5 + \cdots$$

$$\boxed{c_1 - c_2 + c_3 = 0 \Leftrightarrow n_1 - n_2 + n_3 = 0}$$

Claim: At n-points we can always find a rearrangement so color and kinematics satisfy the same algebraic constraint equations.

Nontrivial constraints on amplitudes in field theory and string theory

BCJ, Bjerrum-Bohr, Feng,Damgaard, Vanhove, ; Mafra, Stieberger, Schlotterer; Tye and Zhang; Feng, Huang, Jia; Chen, Du, Feng; Du, Feng, Fu; Naculich, Nastase, Schnitzer

BCJ

Gravity and Gauge Theory

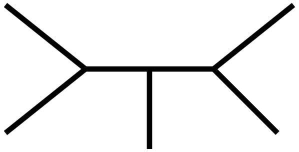
kinematic numerator — color factor

gauge theory: $$\frac{1}{g^{n-2}}\mathcal{A}_n^{\text{tree}}(1,2,3,\ldots,n) = \sum_i \frac{n_i\, c_i}{\prod_{\alpha_i} p_{\alpha_i}^2}$$ **sum over diagrams with only 3 vertices**

$$c_i \sim f^{a_1 a_2 b_1} f^{b_1 b_2 a_5} f^{b_2 a_4 a_5}$$

Assume we have:

$$c_i + c_j + c_k = 0 \quad \Leftrightarrow \quad n_i + n_j + n_k = 0$$



Then: $c_i \Rightarrow \tilde{n}_i$ **kinematic numerator of second gauge theory**

Proof: ZB, Dennen, Huang, Kiermaier

gravity:

$$\boxed{-i\left(\frac{2}{\kappa}\right)^{(n-2)} \mathcal{M}_n^{\text{tree}}(1,2,\ldots,n) = \sum_i \frac{n_i\, \tilde{n}_i}{\prod_{\alpha_i} p_{\alpha_i}^2}}$$

Gravity numerators are a double copy of gauge-theory ones.

This works for ordinary Einstein gravity and susy versions.

Cries out for a unified description of the sort given by string theory!

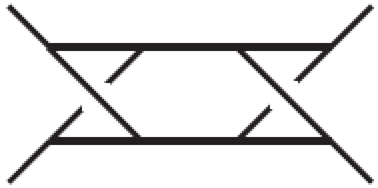
Gravity From Gauge Theory

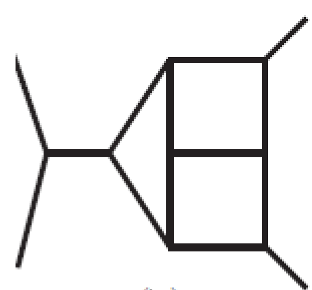
$$-i\left(\frac{2}{\kappa}\right)^{(n-2)}\mathcal{M}_n^{\text{tree}}(1,2,\ldots,n)=\sum_i\frac{n_i\,\tilde{n}_i}{\prod_{\alpha_i}p_{\alpha_i}^2}$$

	n	$\tilde{n}$
N **= 8 sugra:**	**(*N* = 4 sYM)** $\times$	**(*N* = 4 sYM)**
N **= 4 sugra:**	**(*N* = 4 sYM)** $\times$	**(*N* = 0 sYM)**

Loop-Level Conjecture

ZB, Carrasco, Johansson (2010)





$$c_i + c_j + c_k = 0$$
$$n_i + n_j + n_k = 0$$

sum is over diagrams

kinematic numerator

color factor

$$\frac{(-i)^L}{g^{n-2+2L}} \mathcal{A}_n^{\text{loop}} = \sum_j \int \prod_{l=1}^{L} \frac{d^D p_l}{(2\pi)^D} \frac{1}{S_j} \frac{n_j c_j}{\prod_{\alpha_j} p_{\alpha_j}^2}$$

gauge theory

propagators

$$\frac{(-i)^{L+1}}{(\kappa/2)^{n-2+2L}} \mathcal{M}_n^{\text{loop}} = \sum_j \int \prod_{l=1}^{L} \frac{d^D p_l}{(2\pi)^D} \frac{1}{S_j} \frac{n_j \tilde{n}_j}{\prod_{\alpha_j} p_{\alpha_j}^2}$$

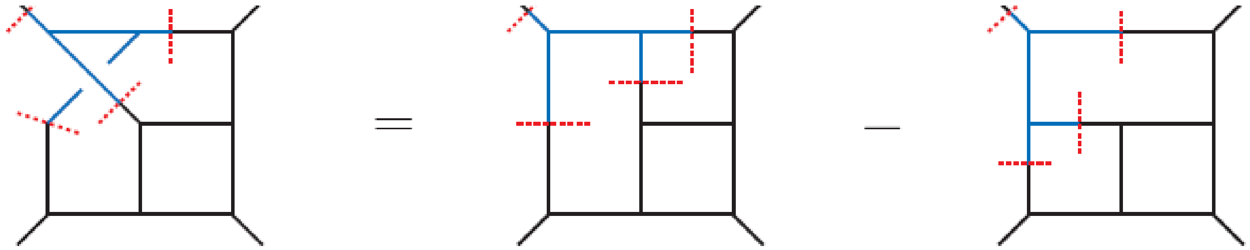
gravity

symmetry factor

Loop-level is identical to tree-level one except for symmetry factors and loop integration.
Double copy works if numerator satisfies duality.

Gravity from Gauge Theory Amplitudes

ZB, Carrasco, Johansson



$$c_k = c_i - c_j$$
$$n_k = n_i - n_j$$

If you have a set of duality satisfying numerators. To get:

gauge theory $\longrightarrow$ gravity theory

simply take

color factor $\longrightarrow$ kinematic numerator

Gravity loop integrands are free!

Generalized Gauge Invariance

BCJ
Bern, Dennen, Huang, Kiermaier
Tye and Zhang

gauge theory

$$\frac{(-i)^L}{g^{m-2+2L}}\mathcal{A}_m^{\rm loop} = \sum_j \int \frac{d^{DL}p}{(2\pi)^{DL}} \frac{1}{S_j} \frac{n_j c_j}{\prod_{\alpha_j} p^2_{\alpha_j}}$$

$$n_i \to n_i + \Delta_i \qquad \sum_j \int \frac{d^{DL}p}{(2\pi)^{DL}} \frac{1}{S_j} \frac{\Delta_j c_j}{\prod_{\alpha_j} p^2_{\alpha_j}} = 0$$

$$(c_\alpha + c_\beta + c_\gamma) f(p_i) = 0$$

Above is just a definition of generalized gauge invariance

gravity

$$\frac{(-i)^{L+1}}{(\kappa/2)^{n-2+2L}}\mathcal{M}_n^{\rm loop} = \sum_j \int \prod_{l=1}^{L} \frac{d^D p_l}{(2\pi)^D} \frac{1}{S_j} \frac{n_j \tilde{n}_j}{\prod_{\alpha_j} p^2_{\alpha_j}}$$

$$n_i \to n_i + \Delta_i \qquad \sum_j \int \frac{d^{DL}p}{(2\pi)^{DL}} \frac{1}{S_j} \frac{\Delta_j \tilde{n}_j}{\prod_{\alpha_j} p^2_{\alpha_j}} = 0$$

- **Gravity inherits generalized gauge invariance from gauge theory.**
- **Double copy works even if only one of the two copies has duality manifest.**
- **Used to find expressions for $N \geq 4$ supergravity amplitudes at 1, 2 loops.**

ZB, Boucher-Veronneau and Johansson; Boucher-Veroneau and Dixon

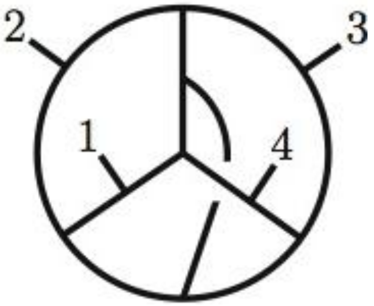
Application: UV Properties of Gravity

UV properties of gravity

Study of UV divergences in gravity has a long history.
Here, I will only skim over the history.

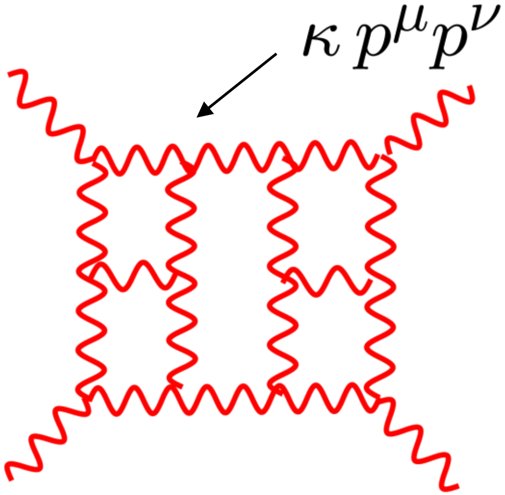
We study this issue not because we think a finite theory of gravity immediately gives a good description of Nature—long list of nontrivial issues.

We study it because the theory can only be finite due to *a fantastic* new symmetry or dynamical mechanism.



Power Counting at High Loop Orders

$$\kappa = \sqrt{32\pi G_N} \longleftarrow$$ Dimensionful coupling



Gravity: $$\int \prod_{i=1}^{L} \frac{dp_i^D}{(2\pi)^D} \frac{(\kappa\, p_j^\mu p_j^\nu)\cdots}{\text{propagators}}$$

Gauge theory: $$\int \prod_{i=1}^{L} \frac{d^D p_i}{(2\pi)^D} \frac{(g\, p_j^\nu)\cdots}{\text{propagators}}$$

Extra powers of loop momenta in numerator means integrals are badly behaved in the UV.

Non-renormalizable by power counting.

Reasons to focus on *N* = 8 supergravity:

- **With more susy expect better UV properties.**
- **High symmetry implies technical simplicity.**

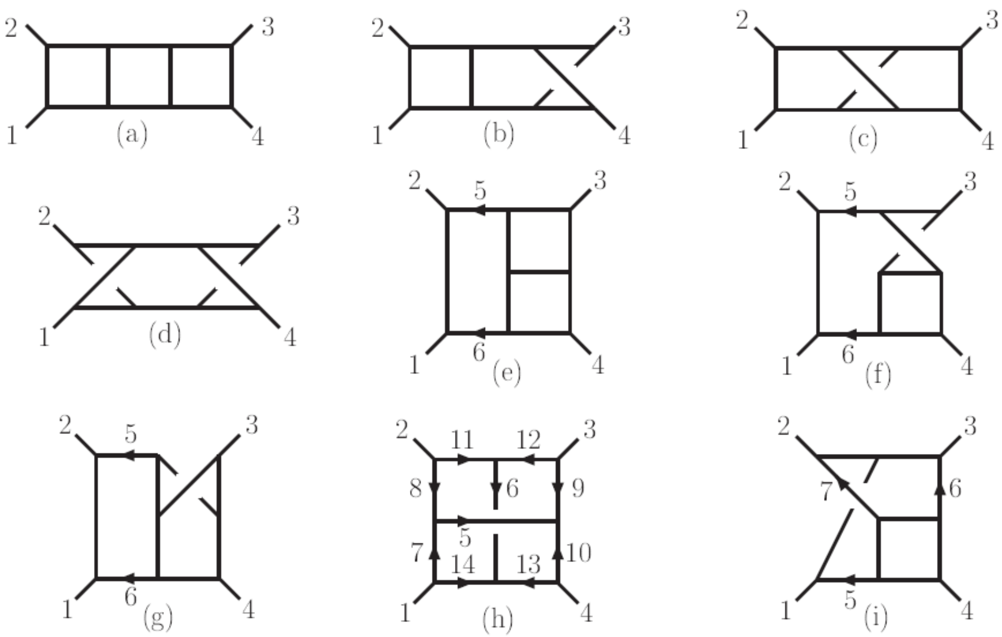
Cremmer and Julia

Three loop Finitenes of $N = 8$ Supergravity

Analysis of unitarity cuts shows highly nontrivial all-loop cancellations. ZB, Dixon and Roiban (2006); ZB, Carrasco, Forde, Ita, Johansson (2007)

To test completeness of cancellations, we decided to directly calculate potential three-loop divergence.

ZB, Carrasco, Dixon, Johansson, Kosower, Roiban (2007)



Three loops is not only ultraviolet finite it is "superfinite"— finite for $D < 6$. Very finite!!

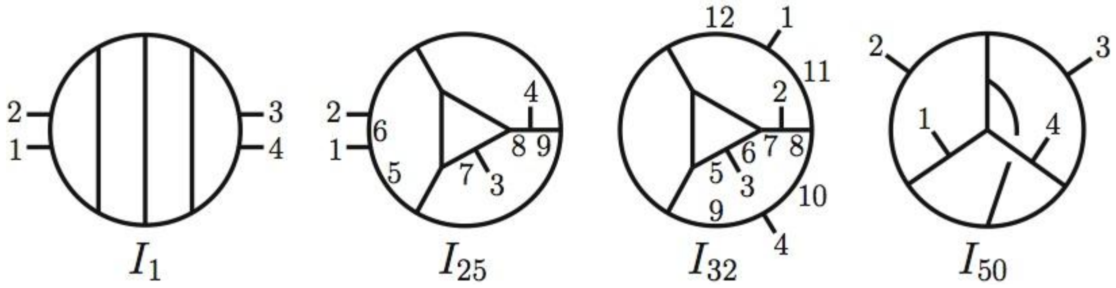
Supergravity brought back from dustbin of discarded theories.

Obtained via unitarity method

Four-Loop Amplitude Construction

ZB, Carrasco, Dixon, Johansson, Roiban

Get 50 distinct diagrams or integrals (ones with two- or three-point subdiagrams not needed).



$$M_4^{4\text{-loop}} = \left(\frac{\kappa}{2}\right)^{10} stu M_4^{\text{tree}} \sum_{S_4} \sum_{i=1}^{50} c_i I_i$$

Integral

symmetry factor

leg perms

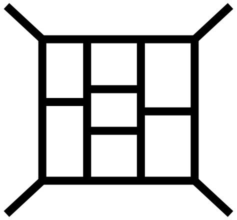
UV finite for $D < 11/2$
It's very finite!

Originally took more than a year.
Today with the double copy we can reproduce it in a couple of days.

Recent Status of Divergences (2011)

For $N = 8$ sugra in $D = 4$:

Consensus that in $N = 8$ supergravity trouble starts at 5 loops and by 7 loops we have valid UV counterterm in $D = 4$ under all known symmetries (suggesting divergences) .

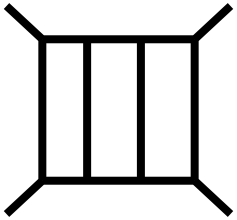


Bossard, Howe, Stelle; Elvang, Freedman, Kiermaier; Green, Russo, Vanhove ; Green and Bjornsson ; Bossard , Hillmann and Nicolai; Ramond and Kallosh; Broedel and Dixon; Elvang and Kiermaier; Beisert, Elvang, Freedman, Kiermaier, Morales, Stieberger

For $N = 4$ sugra in $D = 4$:

Very similar story except here the valid R^4 counterterm is at 3 loops .

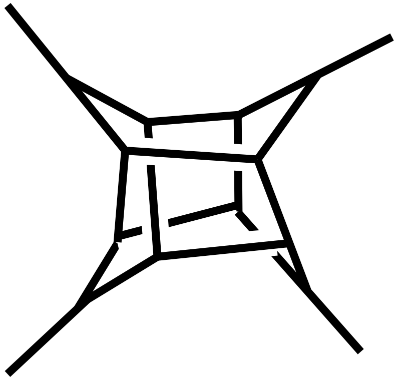
Bossard, Howe, Stelle and Vanhove



To nearly everyone, it seemed supergravity was dead once again.

Status of $N = 8$ Sugra 5 Loop Calculation

ZB, Carrasco, Dixon, Johannson, Roiban



~500 such diagrams with ~100s terms each

We have corresponding $N = 4$ sYM amplitude complete (but not in BCJ format).

Stay tuned. We are going to be able to do the corresponding $N = 8$ sugra calculation.

Place your bets on D^8R^4 counterterm:

- At 5 loops in $D = 24/5$ does $N = 8$ supergravity diverge?
- At 7 loops in $D = 4$ does $N = 8$ supergravity diverge?

5 loops

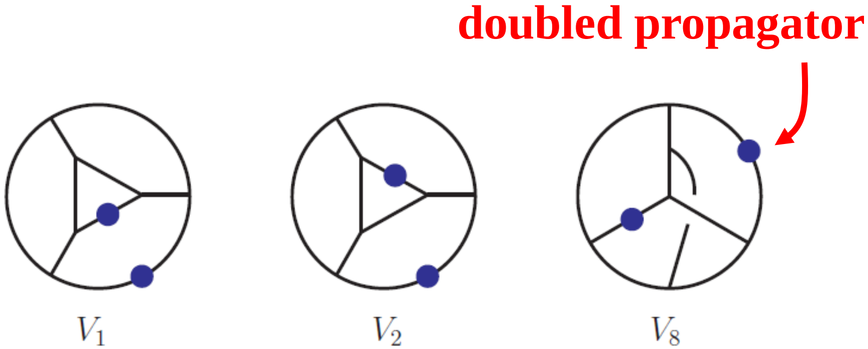
Kelly Stelle:
British wine
"It will diverge"

Zvi Bern:
California wine
"It won't diverge"

New Four-Loop Surprise

ZB, Carrasco, Dixon, Johansson, Roiban (2012)

Critical dimension D =11/2.
Express UV divergences
in terms of vacuum like integrals.



gauge theory

$$\mathcal{A}_4^{(4)}(1,2,3,4)\Big|^{SU(N_c)}_{\text{pole}} = -6\, g^{10}\, \mathcal{K}\, N_c^2 \Big(N_c^2\, V_1 + 12\, \boxed{(V_1 + 2\, V_2 + V_8)}\Big)$$
$$\times \Big(s\,(\text{Tr}_{1324} + \text{Tr}_{1423}) + t\,(\text{Tr}_{1243} + \text{Tr}_{1342}) + u\,(\text{Tr}_{1234} + \text{Tr}_{1432})\Big)$$

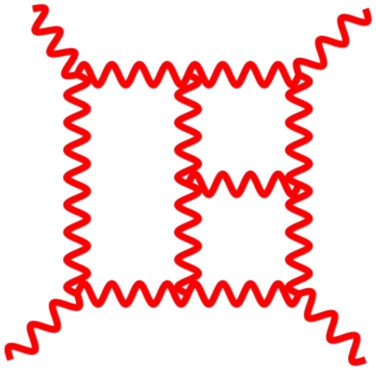
same divergence

gravity

$$\mathcal{M}_4^{(4)}\Big|_{\text{pole}} = -\frac{23}{8}\left(\frac{\kappa}{2}\right)^{10} stu(s^2+t^2+u^2)^2\, M_4^{\text{tree}}\, \boxed{(V_1 + 2V_2 + V_8)}$$

- **Gravity UV divergence is directly proportional to subleading color single-trace divergence of N = 4 super-Yang-Mills theory.**
- **Same happens at 1-3 loops.**

$N = 4$ Supergravity



Consensus had it that a valid R^4 counterterm exists for this theory in $D = 4$. Analogous to 7 loop counterterm of $N = 8$.

Bossard, Howe, Stelle; Bossard, Howe, Stelle, Vanhove

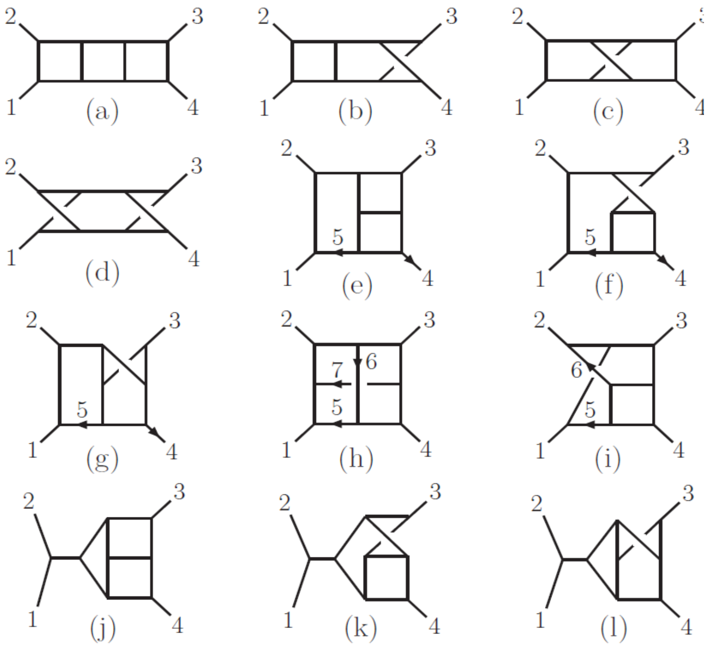
$N = 4$ sugra at 3 loops ideal test case to study.

ZB, Davies, Dennen, Huang (2012)

Three-Loop Construction

ZB, Davies, Dennen, Huang

N = 4 sugra : (N = 4 sYM) x (N = 0 YM)



- For N = 4 sYM copy use known representation where duality between color and kinematics holds.
- For N = 0 YM we use Feynman diagram representation.

Integral $I^{(x)}$	$\mathcal{N}=4$ Super-Yang-Mills ($\sqrt{\mathcal{N}=8}$ supergravity) numerator
(a)–(d)	s^2
(e)–(g)	$\left(s\,(-\tau_{35}+\tau_{45}+t)-t\,(\tau_{25}+\tau_{45})+u\,(\tau_{25}+\tau_{35})-s^2\right)/3$
(h)	$\Big(s\,(2\tau_{15}-\tau_{16}+2\tau_{26}-\tau_{27}+2\tau_{35}+\tau_{36}+\tau_{37}-u)$ $+t\,(\tau_{16}+\tau_{26}-\tau_{37}+2\tau_{36}-2\tau_{15}-2\tau_{27}-2\tau_{35}-3\tau_{17})+s^2\Big)/3$
(i)	$\Big(s\,(-\tau_{25}-\tau_{26}-\tau_{35}+\tau_{36}+\tau_{45}+2t)$ $+t\,(\tau_{26}+\tau_{35}+2\tau_{36}+2\tau_{45}+3\tau_{46})+u\,\tau_{25}+s^2\Big)/3$
(j)-(l)	$s(t-u)/3$

Form of the N = 4 sYM integrand from BCJ

Extraction of UV Divergences

- **Series expand in small external momenta (or large loop momenta) and use mass regulator for IR divergences.**
- **Get unwanted IR regulator dependence in UV divergences**

Problem solved nearly 30 years ago. Marcus, Sagnotti (1984)

Recursively subtract all subdivergences.

regulator dependent

reparametrize subintegral

$$\mathcal{S}\left[\int \prod_{i=1}^{L} dp_i \, I\right] \equiv \mathrm{Div}\left[\int \prod_{i=1}^{L} dp_i \, I\right] - \sum_{l=1}^{L-1} \sum_{\substack{l-\text{loop} \\ \text{subloops}}} \mathrm{Div}\left[\int \prod_{j=l+1}^{L} dp'_j \, \mathcal{S}\left[\int \prod_{i=1}^{l} dp'_i \, I\right]\right]$$

Regulator Independent

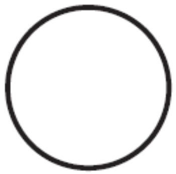
Nice consistency check: all log(*m*) terms must cancel

Extracting UV divergence in the presence of UV subdivergences and (unphysical) IR divergences is a well understood problem.

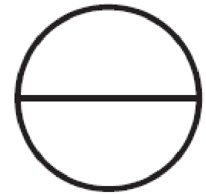
Integral Basis

Expanding in small external momenta and using FIRE we obtain a basis of integrals:

A.V. Smirnov

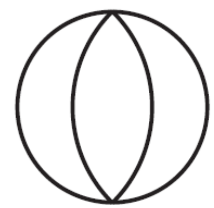


$$(m^2)^{1-\epsilon}\left(\frac{1}{\epsilon}+1+\left(1+\frac{1}{2}\zeta_2\right)\epsilon\right)$$



$$(m^2)^{1-2\epsilon}\left(\frac{3}{2\epsilon^2}+\frac{9}{2\epsilon}+\frac{21}{2}+\frac{3}{2}\zeta_2-2c\right)$$

$$(m^2)^{1-3\epsilon}\left(\frac{1}{\epsilon^3}+\frac{17}{3\epsilon^2}+\left(\frac{67}{3}+\frac{3}{2}\zeta_2-4c\right)\frac{1}{\epsilon}\right)$$



$$(m^2)^{2-3\epsilon}\left(\frac{2}{\epsilon^3}+\frac{23}{3\epsilon^2}+\left(\frac{35}{2}+3\zeta_2\right)\frac{1}{\epsilon}\right)$$

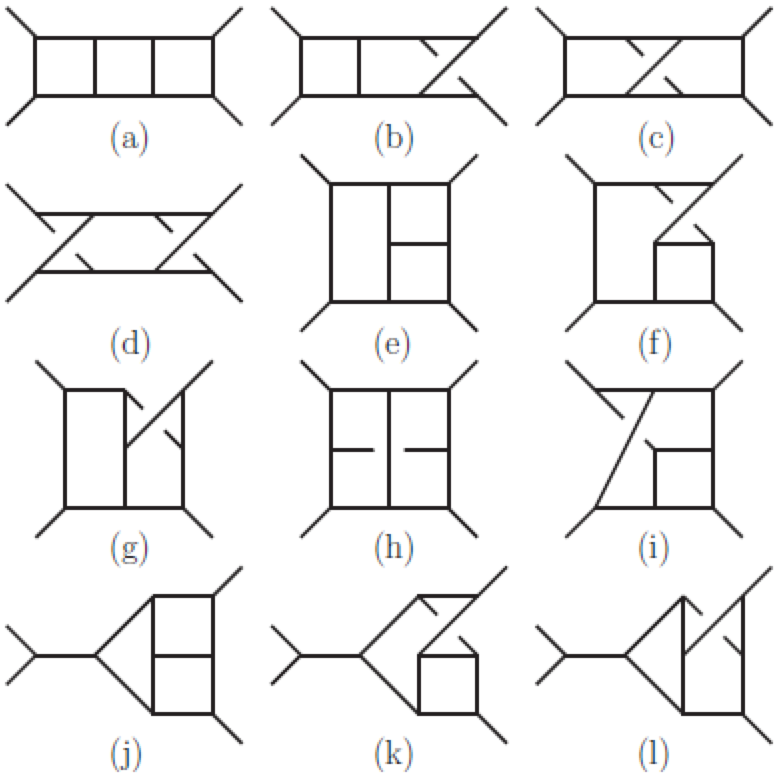
$$(m^2)^{-3\epsilon}\left(\frac{2\zeta_3}{\epsilon}\right)$$

$$c=\sqrt{3}\mathrm{Im}\left(\mathrm{Li}_2(e^{i\pi/3})\right)$$

Use Mellin-Barnes resummation of residues method of Davydychev and Kalmykov on all but last integral. Last one doable by staring at paper from Grozin (easy because no subdivergences).

The $N = 4$ Supergravity UV Cancellation

ZB, Davies, Dennen, Huang



Graph	$(\text{divergence})/(\langle 12\rangle^2[34]^2 st A^{\text{tree}}(\frac{\kappa}{2})^8)$
(a)-(d)	0
(e)	$\frac{263}{768}\frac{1}{\epsilon^3}+\frac{205}{27648}\frac{1}{\epsilon^2}+\left(-\frac{5551}{768}\zeta_3+\frac{326317}{110592}\right)\frac{1}{\epsilon}$
(f)	$-\frac{175}{2304}\frac{1}{\epsilon^3}-\frac{1}{4}\frac{1}{\epsilon^2}+\left(\frac{593}{288}\zeta_3-\frac{217571}{165888}\right)\frac{1}{\epsilon}$
(g)	$-\frac{11}{36}\frac{1}{\epsilon^3}+\frac{2057}{6912}\frac{1}{\epsilon^2}+\left(\frac{10769}{2304}\zeta_3-\frac{226201}{165888}\right)\frac{1}{\epsilon}$
(h)	$-\frac{3}{32}\frac{1}{\epsilon^3}-\frac{41}{1536}\frac{1}{\epsilon^2}+\left(\frac{3227}{2304}\zeta_3-\frac{3329}{18432}\right)\frac{1}{\epsilon}$
(i)	$\frac{17}{128}\frac{1}{\epsilon^3}-\frac{29}{1024}\frac{1}{\epsilon^2}+\left(-\frac{2087}{2304}\zeta_3-\frac{10495}{110592}\right)\frac{1}{\epsilon}$
(j)	$-\frac{15}{32}\frac{1}{\epsilon^3}+\frac{9}{64}\frac{1}{\epsilon^2}+\left(\frac{101}{12}\zeta_3-\frac{3227}{1152}\right)\frac{1}{\epsilon}$
(k)	$\frac{5}{64}\frac{1}{\epsilon^3}+\frac{89}{1152}\frac{1}{\epsilon^2}+\left(-\frac{377}{144}\zeta_3+\frac{287}{432}\right)\frac{1}{\epsilon}$
(l)	$\frac{25}{64}\frac{1}{\epsilon^3}-\frac{251}{1152}\frac{1}{\epsilon^2}+\left(-\frac{835}{144}\zeta_3+\frac{7385}{3456}\right)\frac{1}{\epsilon}$

Spinor helicity used to clean up

Sum over diagrams is gauge invariant

All UV divergences completely cancel!

Three-loop N = 4 supergravity is UV finite contrary to expectations.

Explanations?

Key Question: **Is there an ordinary symmetry explanation for this unexpected UV finiteness? Or is something extraordinary happening?**

- **Further explicit calculations needed to help settle the UV properties as well as various bets.**
- **We are currently trying to see if new information can guide a proof of UV finiteness of $N = 8$ supergravity.**

Some recent papers:

- **Non-renormalization understanding from heterotic string.** Tourkine and Vanhove (2012)
- **Quantum corrected duality current nonconservation.** Kallosh (2012)

Summary

- A new duality conjectured between color and kinematics. Locks down the multiloop structure of integrands of amplitudes.
- When duality between color and kinematics manifest, gravity integrands follow *immediately* from gauge-theory ones.
- Double copy gives us a powerful way to explore the UV properties of gravity. Appears to be a key property of quantum gravity.
- $\mathcal{N} = 4$ sugra has no three-loop four-point divergence, contrary to expectations from symmetry considerations.
- Power counting using known symmetries and their known consequences can be misleading. Concrete example.

Duality between color and kinematics offers a powerful new way to explore gauge and gravity theories at high-loop orders. Can expect many more new results in the coming years.

Extra Slides

One diagram to rule them all

ZB, Carrasco, Johansson (2010)

$\mathcal{N} = 4$ super-Yang-Mills integrand

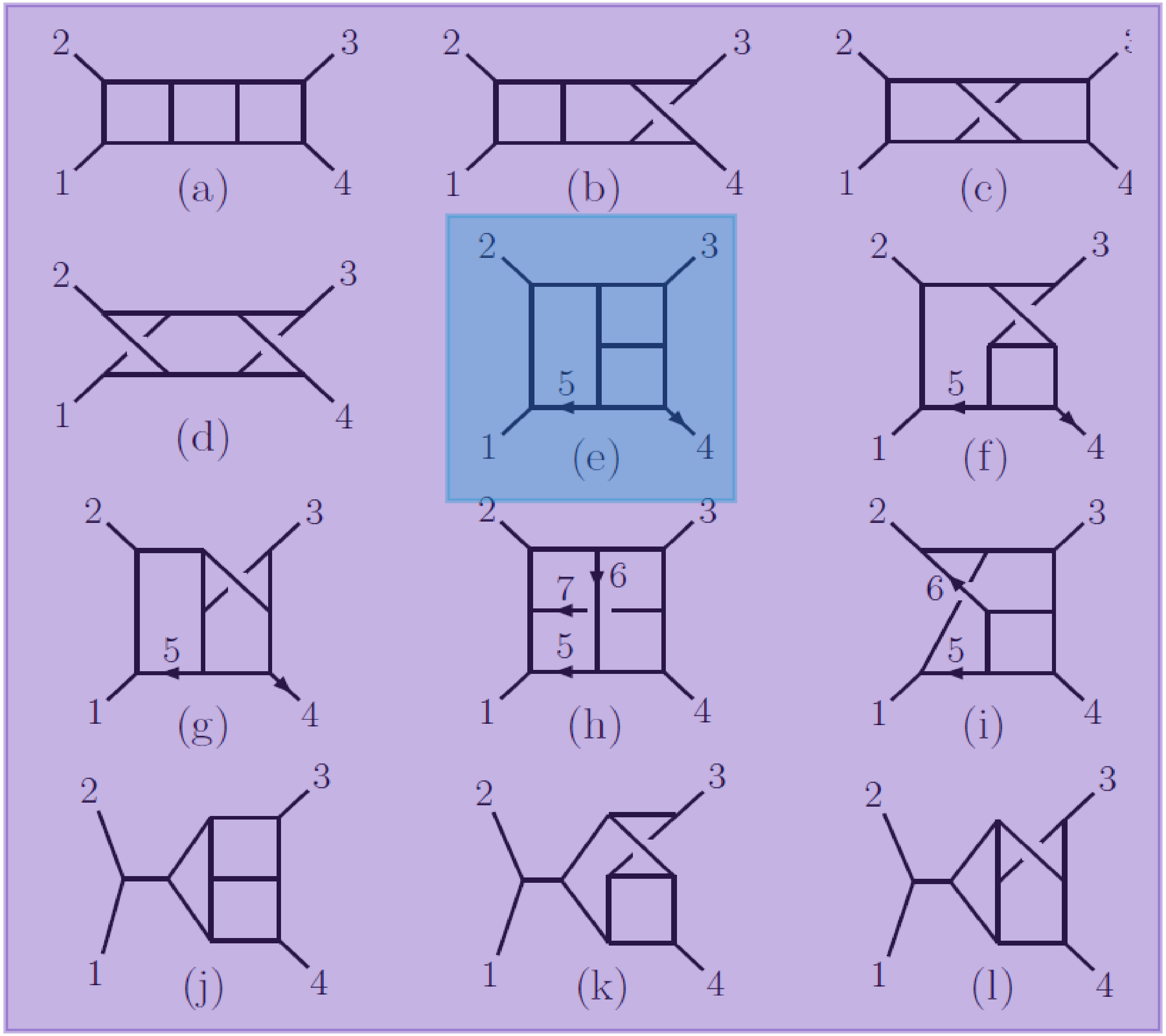


Diagram (e) is the master diagram.

Determine the master integrand in proper form and duality gives all others.

$\mathcal{N} = 8$ sugra given by double copy.

One diagram to rule them all

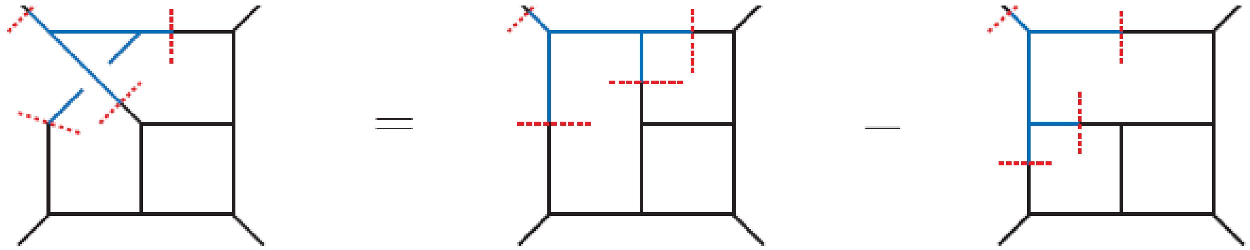
$$
\begin{aligned}
N^{(a)} &= N^{(b)}(k_1,k_2,k_3,l_5,l_6,l_7)\,,\\
N^{(b)} &= N^{(d)}(k_1,k_2,k_3,l_5,l_6,l_7)\,,\\
N^{(c)} &= N^{(a)}(k_1,k_2,k_3,l_5,l_6,l_7)\,,\\
N^{(d)} &= N^{(h)}(k_3,k_1,k_2,l_7,l_6,k_{1,3}-l_5+l_6-l_7)+N^{(h)}(k_3,k_2,k_1,l_7,l_6,k_{2,3}+l_5-l_7)\,,\\
N^{(f)} &= \boxed{N^{(e)}(k_1,k_2,k_3,l_5,l_6,l_7)\,,}\\
N^{(g)} &= \boxed{N^{(e)}(k_1,k_2,k_3,l_5,l_6,l_7)\,,}\\
N^{(h)} &= -N^{(g)}(k_1,k_2,k_3,l_5,l_6,k_{1,2}-l_5-l_7)-N^{(i)}(k_4,k_3,k_2,l_6-l_5,l_5-l_6+l_7-k_{1,2},l_6)\,,\\
N^{(i)} &= N^{(e)}(k_1,k_2,k_3,l_5,l_7,l_6)-N^{(e)}(k_3,k_2,k_1,-k_4-l_5-l_6,-l_6-l_7,l_6)\,,\\
N^{(j)} &= N^{(e)}(k_1,k_2,k_3,l_5,l_6,l_7)-N^{(e)}(k_2,k_1,k_3,l_5,l_6,l_7)\,,\\
N^{(k)} &= N^{(f)}(k_1,k_2,k_3,l_5,l_6,l_7)-N^{(f)}(k_2,k_1,k_3,l_5,l_6,l_7)\,,\\
N^{(l)} &= N^{(g)}(k_1,k_2,k_3,l_5,l_6,l_7)-N^{(g)}(k_2,k_1,k_3,l_5,l_6,l_7)\,,
\end{aligned}
$$

triangle subdiagrams vanish in *N* = 4 sYM

All numerators solved in terms of numerator (e)

Nonplanar from Planar

BCJ



$$c_k = c_i - c_j$$
$$n_k = n_i - n_j$$

Planar determines nonplanar

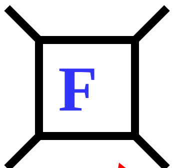
- **We can carry advances from planar sector to the nonplanar sector.**
- **Yangian symmetry and integrability clearly has echo in nonplanar sector because it is built directly from planar.**
- **Only at level of the integrands, so far, but bodes well for the future.**

Multiloop $N = 4$ supergravity

Does it work? Test at 1, 2 loops

All pure supergravities finite at 1,2 loops

One-loop: keep only box Feynman diagrams

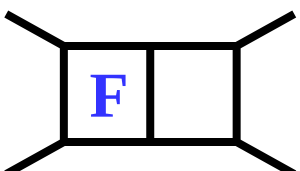
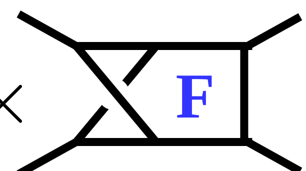
$st A_4^{\text{tree}} \times$  + perms $= \dfrac{0}{\epsilon} +$ finite

N = 4 sYM box numerator

N = 0 Feynman diagram, including ghosts

Becomes gauge invariant after permutation sum.

Two-loop: keep only double box Feynman diagrams

$s^2 t A_4^{\text{tree}} \times$  $+ s^2 t A_4^{\text{tree}} \times$  + perms $= \dfrac{0}{\epsilon}$ + finite

Feynman diagram including ghosts

Get correct results. Who would have imagined it is this simple to get gravity results?