NLO QCD corrections to off-shell $\mathrm{t}\bar{\mathrm{t}}$ production at hadron colliders

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Introduction

$\mathrm{t\bar{t}}$ production @ Tevatron/LHC – physics issues

- precision measurement of m_t
  $\hookrightarrow$ cornerstone of EW precision physics / SM fit
- FB asymmetry @ Tevatron
  $\hookrightarrow$ measurement challenges SM (new physics?)
- top-spin physics @ LHC
- EW top couplings via $\mathrm{t\bar{t}} + \gamma/\mathrm{Z}/\mathrm{H}$
- $\mathrm{t\bar{t}}$ delivers background to many new-physics searches
  (large cross section with signatures of missing $\not{E}_\mathrm{T}$, jets, leptons)

$\Rightarrow$ Precision calculations necessary that

- comprise the relevant fixed-order QCD & EW corrections
- include the top-quark decays to the relevant partonic final states
  $\hookrightarrow$ full reactions $\mathrm{pp/p\bar{p}} \;\to\; \mathrm{b\bar{b}} + 2\ell 2\nu \;/\ell\nu 2\mathrm{j}/4\mathrm{j}$
- are improved by QCD resummations or matched parton showers
- are interfaced to PYTHIA/HERWIG/SHERPA for detector simulations

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Considered in this talk !

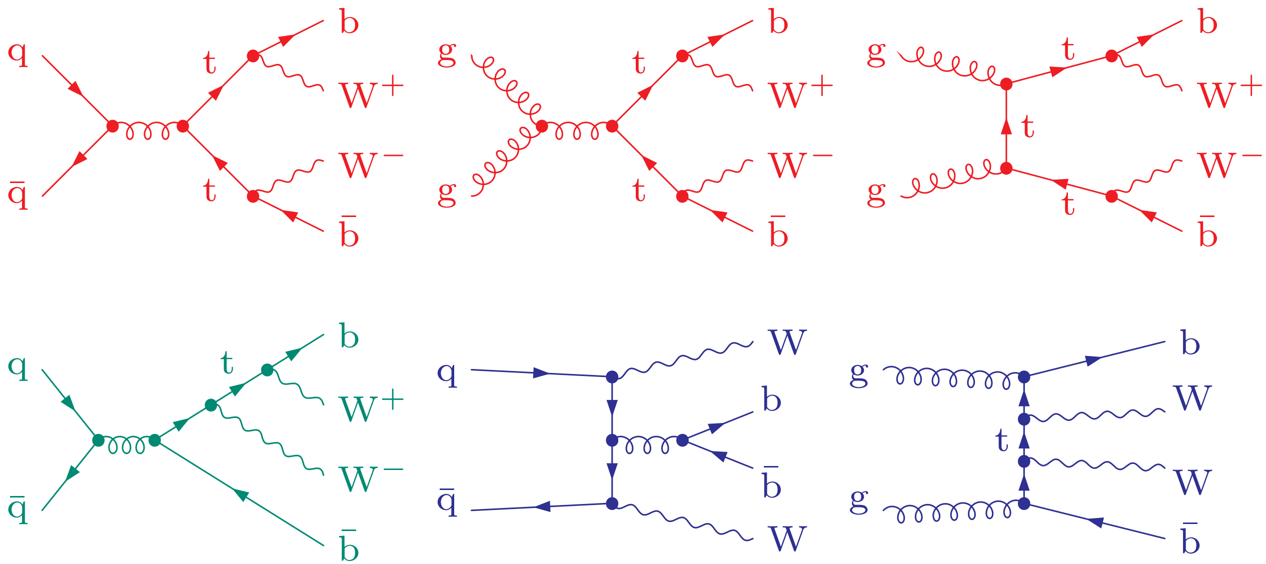
Brief history of precision calculations for hadronic $\mathrm{t\bar{t}}$ production

- NLO QCD corrections — Nason et al. ’89; Beenakker et al. ’91; Mangano et al. ’92; Frixione et al. ’95
- NLO EW corrections — Beenakker et al. ’94; S.Moretti et al. ’06; Kühn et al. ’06; Hollik et al. ’07–’11; Bernreuther et al. ’08
- QCD resummations — Laenen et al. ’92; Catani et al. ’96; Berger et al. ’96; Kidonakis et al. ’97–’01; Bonciani et al. ’98; Beneke et al. ’09–’11; Czakon et al. ’09–’11; Ahrens et al. ’10–’11; Kidonakis ’10–’11; Aliev et al. ’10; Cacciari et al. ’11
- Steps towards NNLO QCD — Czakon et al. ’07–’08; S.D. et al. ’07; Kniehl et al. ’08; Anastasiou et al. ’08; Bonciani et al. ’08–’09; Gehrmann-De Ridder et al. ’09; Czakon ’10–’11
- NLO QCD – inclusion of top decays in NWA — Bernreuther et al. ’04–’10; Melnikov et al. ’09
- NLO QCD – full $\mathrm{b\bar{b}} + 2\ell 2\nu$ final states — Denner et al. ’10; Bevilacqua et al. ’10

$\hookrightarrow$ results extended in the following (paper in preparation)

Features of the NLO calculation

Leading-order calculation



... W decays attached everywhere

- specific process: $\mathrm{pp}/\mathrm{p\bar{p}} \to \nu_{\mathrm{e}}\mathrm{e}^{+}\mu^{-}\bar{\nu}_{\mu}\mathrm{b\bar{b}}$
- # tree diagrams: $q\bar{q}$: 31 (14 for on-shell W's)
  gg: 79 (38 for on-shell W's)
- 2, 1, or 0 intermediate top-quark resonances
- 2 or 1 intermediate W-boson resonances
- 14-dim. phase space $\to$ multi-channel Monte Carlo integration

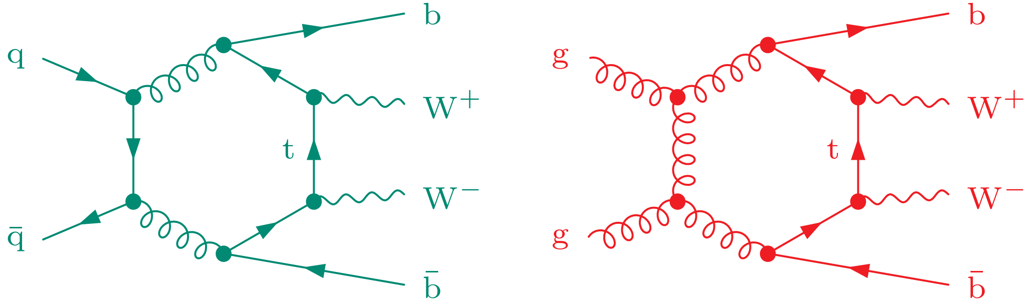
NLO – virtual corrections

- # 1-loop diagrams: (on-shell W's, fermion loops of one generation)

  $q\bar{q}$: 294 (4 hexagons, 24 pentagons, ...)

  gg: 795 (21hexagons, 96 pentagons, ...)

- most complicated representatives:
  hexagons with tensors up to rank 4 for $q\bar{q}$ and rank 5 for gg



... W decays attached everywhere

- 2, 1, 0 intermediate top-quark resonances

Treatment of intermediate resonances

Top-quark resonances

- full off-shellness kept everywhere

  $\hookrightarrow$ NLO accuracy in resonant and non-resonant regions

- complex-mass scheme Denner, S.D., Roth, Wieders '05

  $\hookrightarrow$ $\mu_\mathrm{t}^2 = m_\mathrm{t}^2 - \mathrm{i} m_\mathrm{t} \Gamma_\mathrm{t} =$ location of complex pole in propagator

- generic size of off-shell effects in σ_tot: $\sim \Gamma_\mathrm{t}/m_\mathrm{t} \lesssim 1\%$ (numerically confirmed)

W-boson resonances

- leptonic W decays included

- W off-shell effects to "top-inclusive observables" doubly suppressed:

  $\sigma_{\mathrm{b\bar{b}}2\ell 2\nu} \sim \sigma_{\mathrm{t\bar{t}}} \times (\Gamma_{\mathrm{t}\to\mathrm{b}\ell\nu}/\Gamma_\mathrm{t})^2 \quad \leftarrow$ W off-shellness cancels in top BR's

- treatment of W off-shellness:

  ◇ full off-shellness kept at LO and in real corrections (complex-mass scheme)

  ◇ virtual corrections in "double-pole approximation" (=resonance expansion)

  $\hookrightarrow$ NLO accuracy near W resonances, LO for far-off-shell W's

  But: concept applicable to electroweak corrections (otherwise proliferation of complexity)

Our Feynman-diagrammatic approach for virtual 1-loop corrections

$$\mathcal{M}_{1-\mathrm{loop}} = \sum_{\mathrm{(sub)diagrams}\ \Gamma} \mathcal{M}_\Gamma$$ generated with FEYNARTS (Küblbeck et al. '90; Hahn '01)

$$\mathcal{M}_\Gamma = \sum_n \underbrace{C^{(\Gamma)}}_{\text{colour factor}} \underbrace{F_n^{(\Gamma)}}_{\uparrow} \underbrace{\hat{\mathcal{M}}_n}_{\text{spin structures like } [\bar{u}_\mathrm{b}(k_\mathrm{b})\not{\varepsilon}_{\mathrm{g}_1}(k_{\mathrm{g}_1})v_{\bar{\mathrm{b}}}(k_{\bar{\mathrm{b}}})](\varepsilon_{\mathrm{g}_2}(k_{\mathrm{g}_2})\cdot k_\mathrm{b})\ldots}$$

invariant functions containing

1-loop tensor integrals $T^{\mu\nu\rho\ldots}$

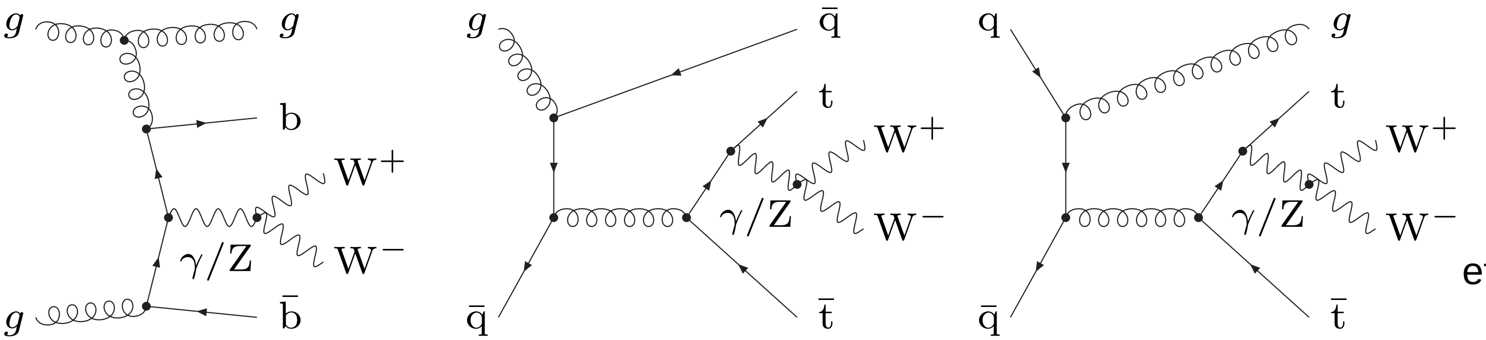
$$T^{\mu\nu\rho\ldots} = (p_k^\mu p_l^\nu p_m^\rho \ldots) T_{kl\ldots} + (g^{\mu\nu} p_m^\rho \ldots) T_{00m\ldots} + \ldots$$

$T_{kl\ldots}$ = linear combination of scalar 1-loop integrals A_0, B_0, C_0, D_0

- 5-/6-point integrals reduced to 4-point integrals Denner, S.D. '02,'05
- 4-/3-point integrals reduced à la Passarino/Veltman '79 for regular points
- specially designed methods for rescuing cases with small Gram dets. Denner, S.D. '05
- $A_0, ..., D_0$ with complex masses Denner, S.D. '10

Features:
- advantage: get all colour/spin channels in one stroke
- lengthy algebra $\rightarrow$ automation (MATHEMATICA) $\rightarrow \sim 4.5$ Mio lines of code
- two independent calculations, one using features of FORMCALC (Hahn)

Corrections due to real radiation



Salient features:

- fast evaluation of amplitudes $\rightarrow$ spinor methods / MadGraph / OpenLoops
  Stelzer, Long — Cascioli, Maierhoefer, Pozzorini '11
- multi-channel Monte Carlo integration over phase space
- soft and collinear divergences
  $\hookrightarrow$ dipole subtraction formalism
  Catani, Seymour '96; S.D. '99
  Phaf, Weinzierl '01
  Catani, S.D., Seymour, Trócsányi '02

$$\sigma^{\rm NLO} = \underbrace{\int_{m+1}\left[\mathrm{d}\sigma^{\rm real} - \mathrm{d}\sigma^{\rm sub}\right]}_{\text{finite}} + \underbrace{\int_{m}\left[\mathrm{d}\sigma^{\rm virtual} + \mathrm{d}\bar{\sigma}_1^{\rm sub}\right]}_{\text{finite}} + \int_0^1 \mathrm{d}x\, \underbrace{\int_m\left[\mathrm{d}\sigma^{\rm fact}(x) + \left(\mathrm{d}\bar{\sigma}^{\rm sub}(x)\right)_+\right]}_{\text{finite}}$$

- two alternative IR regularizations: dim. reg. / mass reg. (small $m_{\rm q}, m_{\rm b}$)

Runtime of various parts of the calculation

Typical setup:
- 2×10^7 events before applying cuts
- single 3 GHz Intel Xeon processor
- pgf77 compiler

Some statistics:

	$\sigma/\sigma_{\mathrm{LO}}$	# events (after cuts)	$(\Delta\sigma)_{\mathrm{stat}}/\sigma$	runtime	time/event
tree level	86%	5.3×10^6	0.4×10^{-3}	38 min	0.4 ms
virtual	−11%	0.26×10^6	0.6×10^{-3}	13 h	180 ms
real + dipoles	49%	10×10^6	3×10^{-3}	40 h	14 ms
total	124%		4×10^{-3}	53 h	

$\hookrightarrow$ Performance very good !

Numerical results

Setup – most relevant details

- two scale choices: $(\mu = \mu_{\rm R} = \mu_{\rm F})$

  fixed scale (FS): $\mu_0 = m_{\rm t}$

  dynamical scale (DS): $\mu_{\rm dyn} = \sqrt{\sqrt{m_{\rm t}^2 + p_{\rm T,t}^2}\sqrt{m_{\rm t}^2 + p_{\rm T,\bar{t}}^2}}$

- top-quark width: two different values with on- or off-shell $\rm W$'s

  Jezabek/Kühn '89

  $\hookrightarrow$ necessary to receive consistent (effective) branching ratios

$$\left.\begin{array}{ll} \text{W off shell:} & \Gamma_{\rm t,LO} = 1.4655\,{\rm GeV}, \quad \Gamma_{\rm t,NLO} = 1.3376\,{\rm GeV} \\ \text{W on shell:} & \Gamma_{\rm t,LO} = 1.4426\,{\rm GeV}, \quad \Gamma_{\rm t,NLO} = 1.3167\,{\rm GeV} \end{array}\right\} \text{differ by } 1.6\%$$

- $\rm W/Z$-boson widths: NLO QCD predictions everywhere

  (only leptonic $\rm W$ decays, no imbalance in BR's)

- More details:

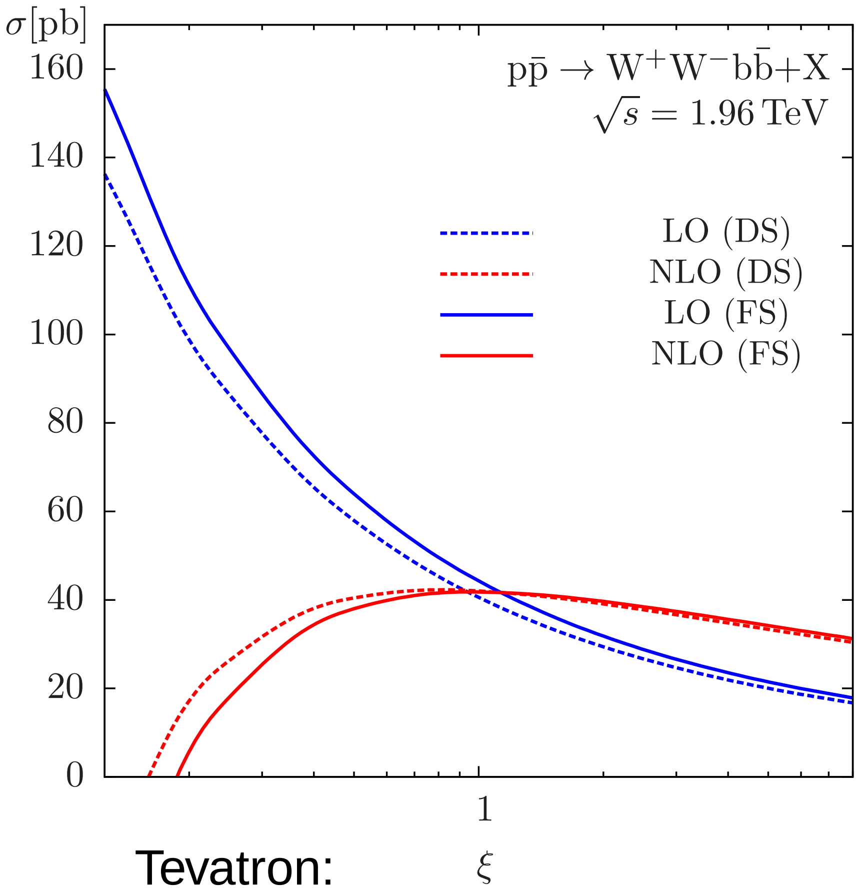
  G_μ scheme for EW couplings, $m_{\rm b} = 0$, $M_{\rm H} \to \infty$, MSTW2008(N)LO PDFs,
  $N_F = 5$, anti-$k_{\rm T}$ algorithm with $R = 0.4(0.5)$ for Tev.(LHC), cuts:
  $p_{\rm T,b} > 20(30)\,{\rm GeV}, |\eta_{\rm b}| < 2.5, p_{\rm T,miss} > 25(20)\,{\rm GeV}, p_{\rm T,l} > 20\,{\rm GeV}, |\eta_{\rm l}| < 2.5$

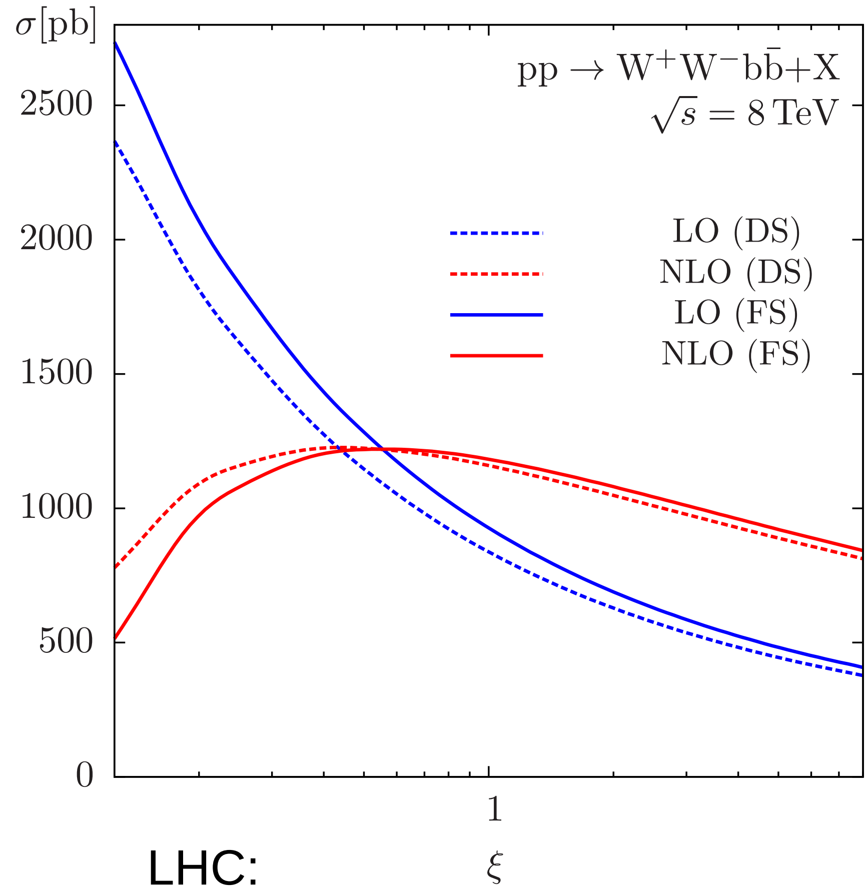
3.1 Fixed versus dynamical scale

Scale dependence of integrated cross sections

$$\mu = \mu_{\rm R} = \mu_{\rm F}, \quad \mu_{\rm FS} = \xi\, m_{\rm t}, \quad \mu_{\rm DS} = \xi\, \mu_{\rm dyn}$$

Denner, S.D., Kallweit, Pozzorini '10,'12





Tevatron:

$q\bar{q}$ dominance (s-channel)

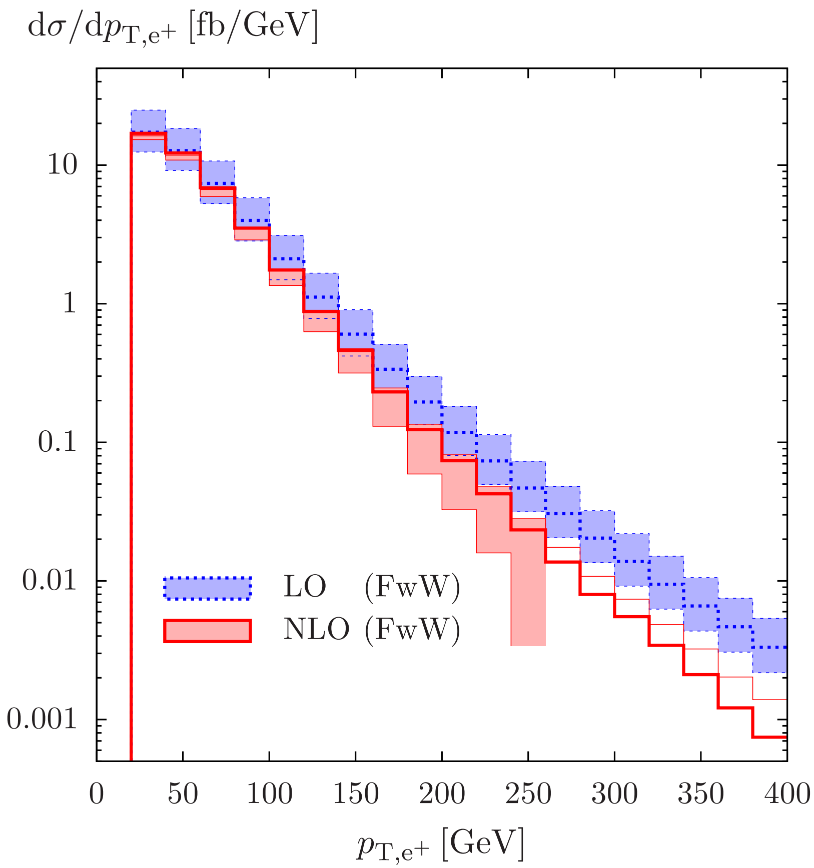
LHC:

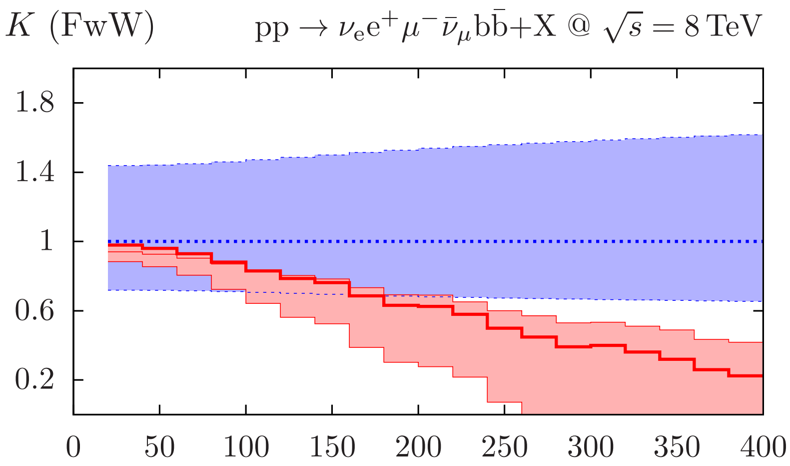
gg dominance (t-channel)

$\hookrightarrow$ smaller scales relevant

NLO QCD corrections to p_T distributions – example: leptonic p_T

Denner, S.D., Kallweit, Pozzorini '12





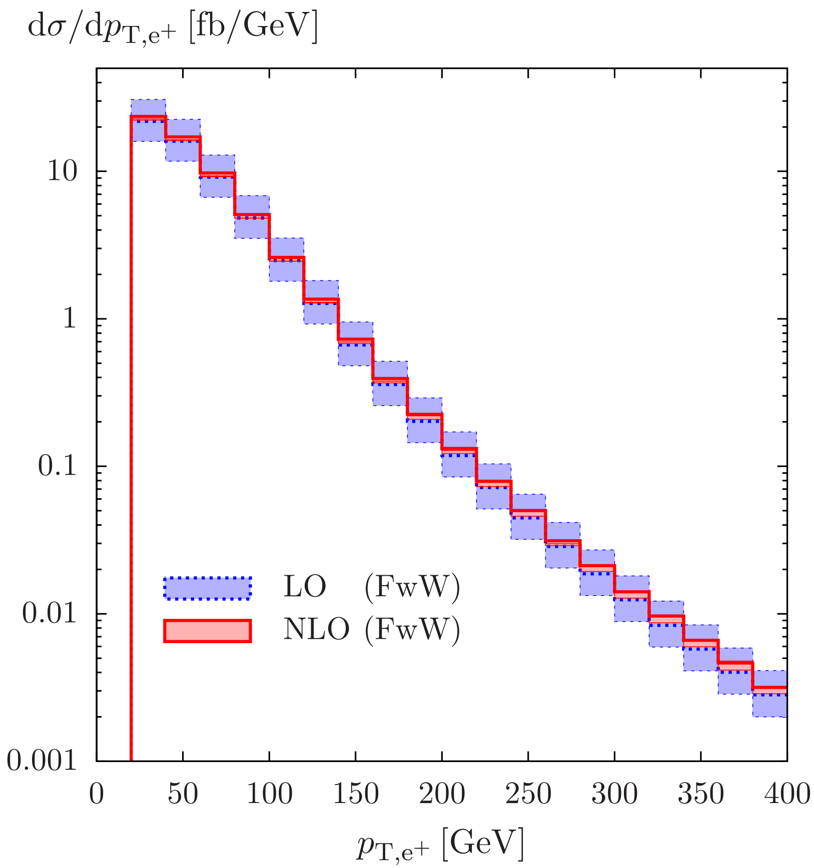
central scale: $\mu = m_\mathrm{t}/2 \quad (\xi = 1/2)$

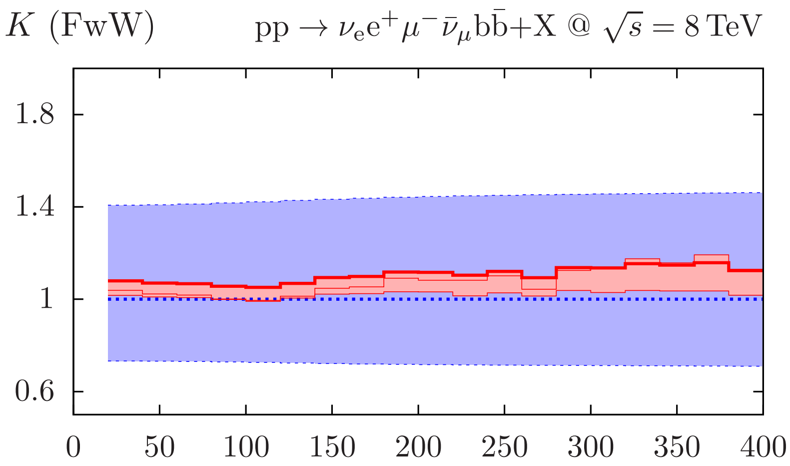
Fixed scale implies large negative corrections at high p_T

(Bands correspond to scale variations by factors of 2.)

NLO QCD corrections to p_{T} distributions – example: leptonic p_{T}

Denner, S.D., Kallweit, Pozzorini ’12





central scale: $\mu = \mu_{\mathrm{dyn}}/2 \quad (\xi = 1/2)$

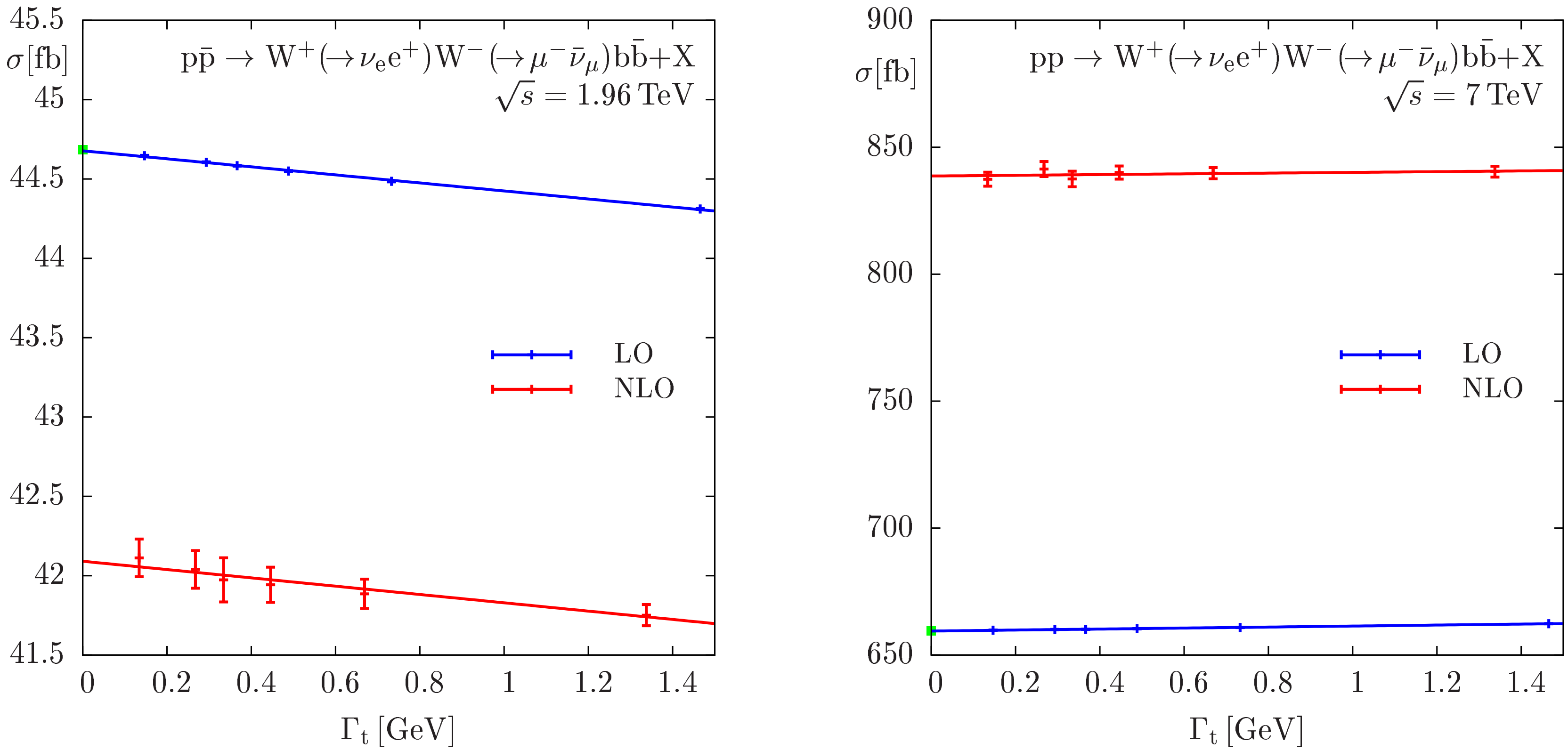
Dynamical scale leads to much flatter K factor

(Bands correspond to scale variations by factors of 2.)

3.2 Off-shell effects of the top quarks

Numerical $\Gamma_t \to 0$ limit for integrated cross sections

Denner, S.D., Kallweit, Pozzorini ’10,’12



Extrapolation: $$\sigma_{\mathrm{WWb\bar{b}}}(\Gamma_t) \times \left(\frac{\Gamma_t}{\Gamma_t^{\mathrm{fix}}}\right)^2 \xrightarrow[\Gamma_t \to 0]{} \sigma_{t\bar{t}} \times \mathrm{BR}(t \to b\ell\nu_\ell)^2 = \sigma_{t\bar{t}}^{\mathrm{NWA}}$$

- virtual and real corrections involve terms $\propto \alpha_s \ln \Gamma_t$

  $\hookrightarrow$ terms connected to IR singularities of on-shell top’s and cancel in sum

- linear dependence on Γ_t = non-trivial check

$\mathrm{t\bar{t}}$ NWA versus $\mathrm{WWb\bar{b}}$ calculation

Denner, S.D., Kallweit, Pozzorini, Schulze [arXiv:1203.6803]

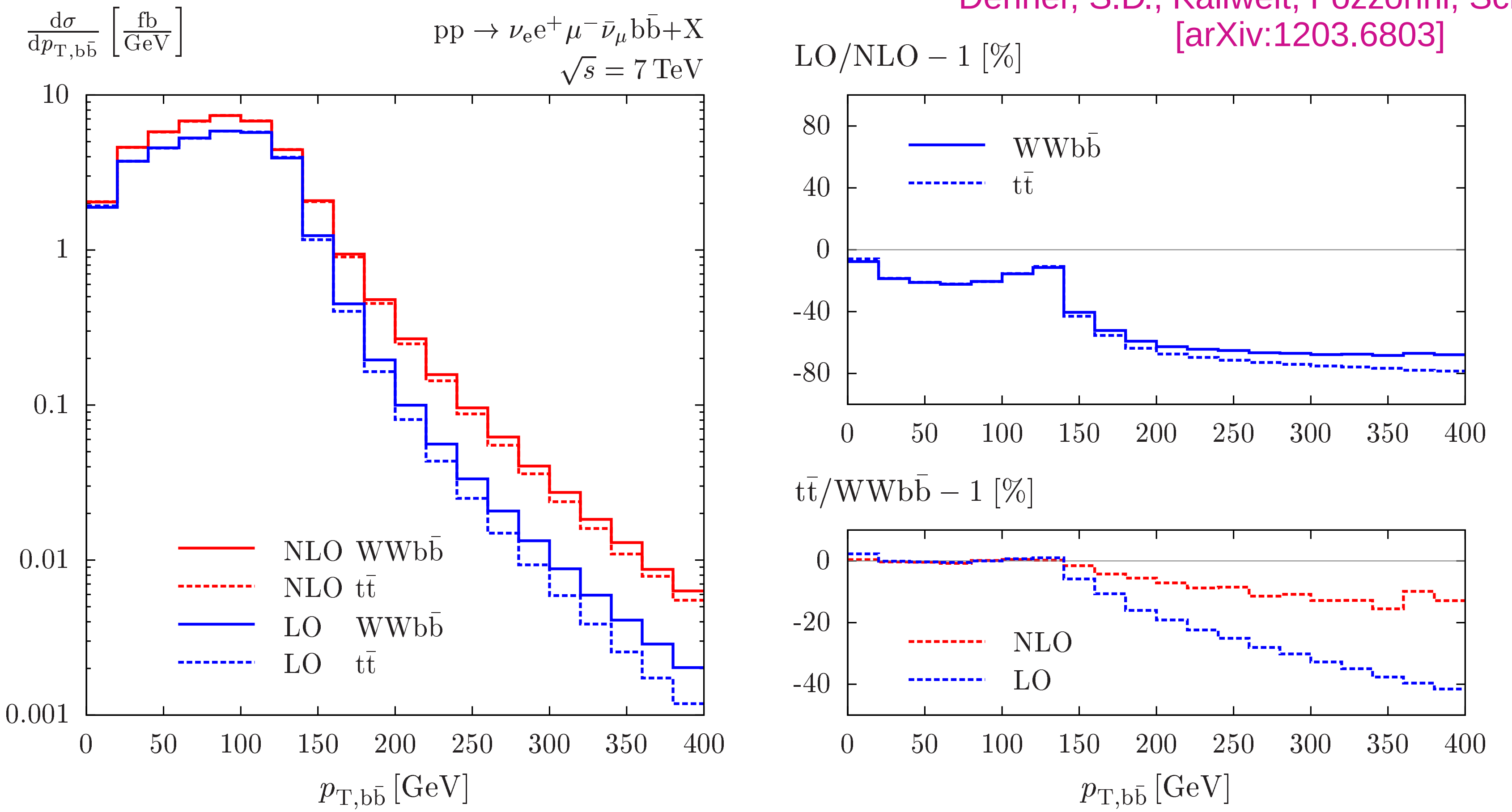
Collider	$\sqrt{s}$ [TeV]	order	$\sigma^{\mathrm{NWA}}_{\mathrm{t\bar{t}}}$ [fb]	$\sigma_{\mathrm{WWb\bar{b}}}$ [fb]	$\frac{\sigma^{\mathrm{NWA}}_{\mathrm{t\bar{t}}}}{\sigma_{\mathrm{WWb\bar{b}}}} - 1$	$\frac{\sigma^{\Gamma_{\mathrm{t}}\to 0}_{\mathrm{WWb\bar{b}}}}{\sigma_{\mathrm{WWb\bar{b}}}} - 1$
Tevatron	1.96	LO	$44.691(8)$	$44.310(3)$	$+0.86(2)\%$	$+0.8\%$
		NLO	$42.16(3)$	$41.75(5)$	$+0.98(14)\%$	$+0.9\%$
LHC	7	LO	$659.5(1)$	$662.35(4)$	$-0.43(2)\%$	-0.4%
		NLO	$837(2)$	$840(2)$	$-0.41(31)\%$	-0.2%
LHC	14	LO	$3306.3(1)$	$3334.6(2)$	$-0.85(1)\%$	$-$
		NLO	$4253(3)$	$4286(7)$	$-0.77(19)\%$	$-$

- good agreement between our $\sigma^{\Gamma_{\mathrm{t}}\to 0}_{\mathrm{WWb\bar{b}}}$ and $\sigma^{\mathrm{NWA}}_{\mathrm{t\bar{t}}}$ from Melnikov/Schulze
  $\hookrightarrow$ confirms $\Gamma_{\mathrm{t}} \to 0$ extrapolation
- size of top off-shell effects to integrated cross sections
  $\sim 1\% \;\sim\; \Gamma_{\mathrm{t}}/m_{\mathrm{t}}$ $\rightarrow$ corresponds to naive expectation

top off-shell effects in the $p_{\mathrm{T,b\bar{b}}}$ distribution

$\hookrightarrow$ relevant for background studies to $\mathrm{WH}(\to \mathrm{b\bar{b}})$ searches

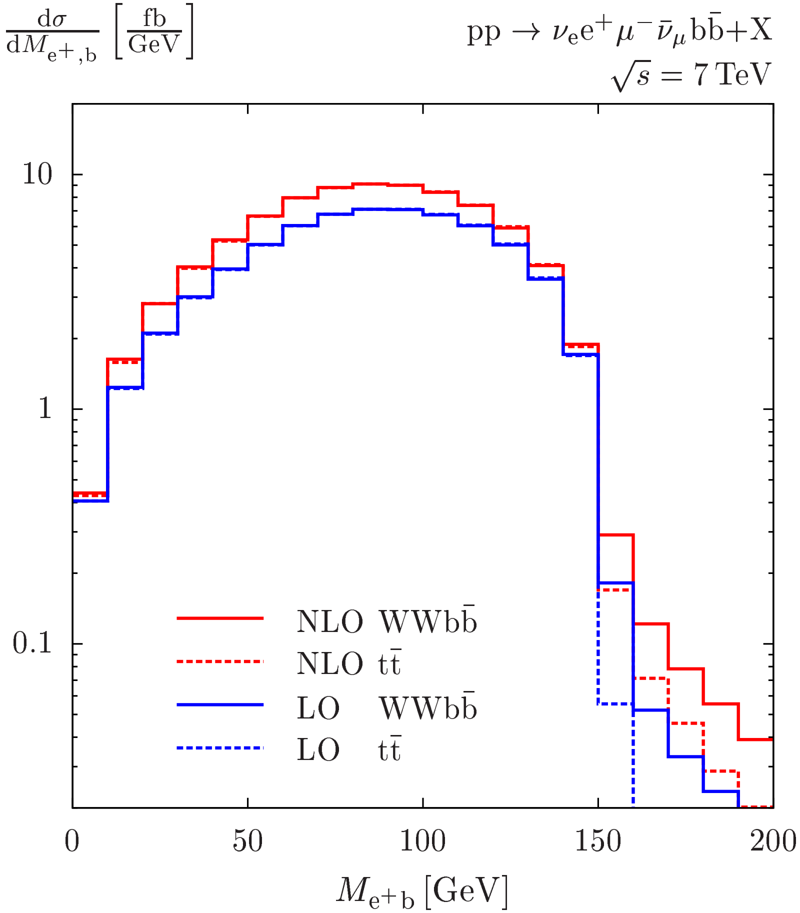
Denner, S.D., Kallweit, Pozzorini, Schulze [arXiv:1203.6803]



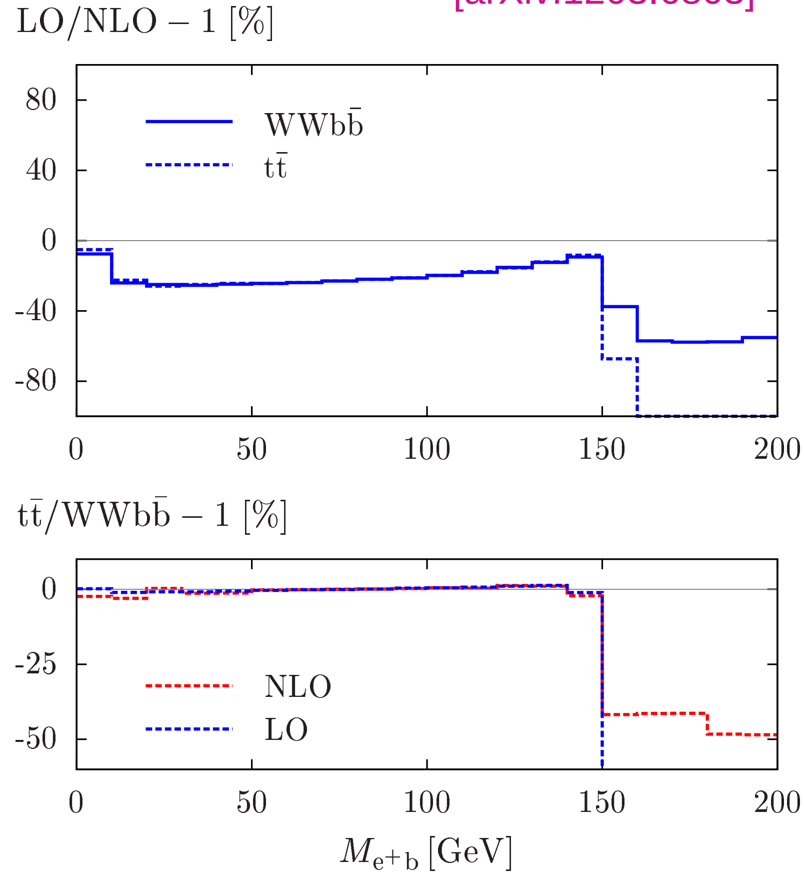
- kinematical suppression for on-shell top's in LO: $p_{\mathrm{T,b\bar{b}}} < (m_t^2 - M_W^2)/m_t \sim 135\,\mathrm{GeV}$
- cross section and K factor well described by NWA for $p_{\mathrm{T,b\bar{b}}} \lesssim 150\,\mathrm{GeV}$
- top off-shell effects reach $10-40\%$ for $p_{\mathrm{T,b\bar{b}}} \sim 200-400\,\mathrm{GeV}$

top off-shell effects in the $M_{\mathrm{e^+b}}$ distribution

$\hookrightarrow$ important for m_{t} measurement



Denner, S.D., Kallweit, Pozzorini, Schulze
[arXiv:1203.6803]



- kinematical edge for on-shell top's in LO: $M_{\mathrm{e^+b}} < \sqrt{m_{\mathrm{t}}^2 - M_{\mathrm{W}}^2} \sim 150\,\mathrm{GeV}$
- cross section and K factor well described by NWA for $M_{\mathrm{e^+b}} \lesssim 150\,\mathrm{GeV}$
- top off-shell effects reach some 10% for $M_{\mathrm{e^+b}} \gtrsim 150\,\mathrm{GeV}$

3.3 Off-shell effects of the W bosons

Preliminary consideration

Cancellation of effects in σ and Γ_t ("effective BR's")

$\hookrightarrow$ double suppression $\sim \frac{\Gamma_\mathrm{t}}{m_\mathrm{t}} \frac{\Gamma_\mathrm{W}}{M_\mathrm{W}} \times \ldots < 0.5\%$

effect hardly visible where top-quark resonances dominate

Total cross section @ LHC with CM energy $8\,\mathrm{TeV}$ Denner, S.D., Kallweit, Pozzorini '12

scale	Γ_W	$\sigma_\mathrm{LO}[\mathrm{fb}]$	$\sigma_\mathrm{NLO}[\mathrm{fb}]$
FS	narrow	$1283.3(2)^{+43.1\%}_{-27.8\%}$	$1219(3)^{-11.3\%}_{-3.0\%}$
FS	finite	$1278.1(2)^{+43.2\%}_{-27.8\%}$	$1212(3)^{-11.4\%}_{-3.0\%}$
DS	narrow	$1146.3(2)^{+41.1\%}_{-26.9\%}$	$1225(3)^{-5.2\%}_{-5.3\%}$
DS	finite	$1141.7(2)^{+41.1\%}_{-26.9\%}$	$1218(2)^{-5.3\%}_{-5.3\%}$

$\hookrightarrow$ W off-shell effect $\sim 0.4\%$ (decreasing with looser cuts)

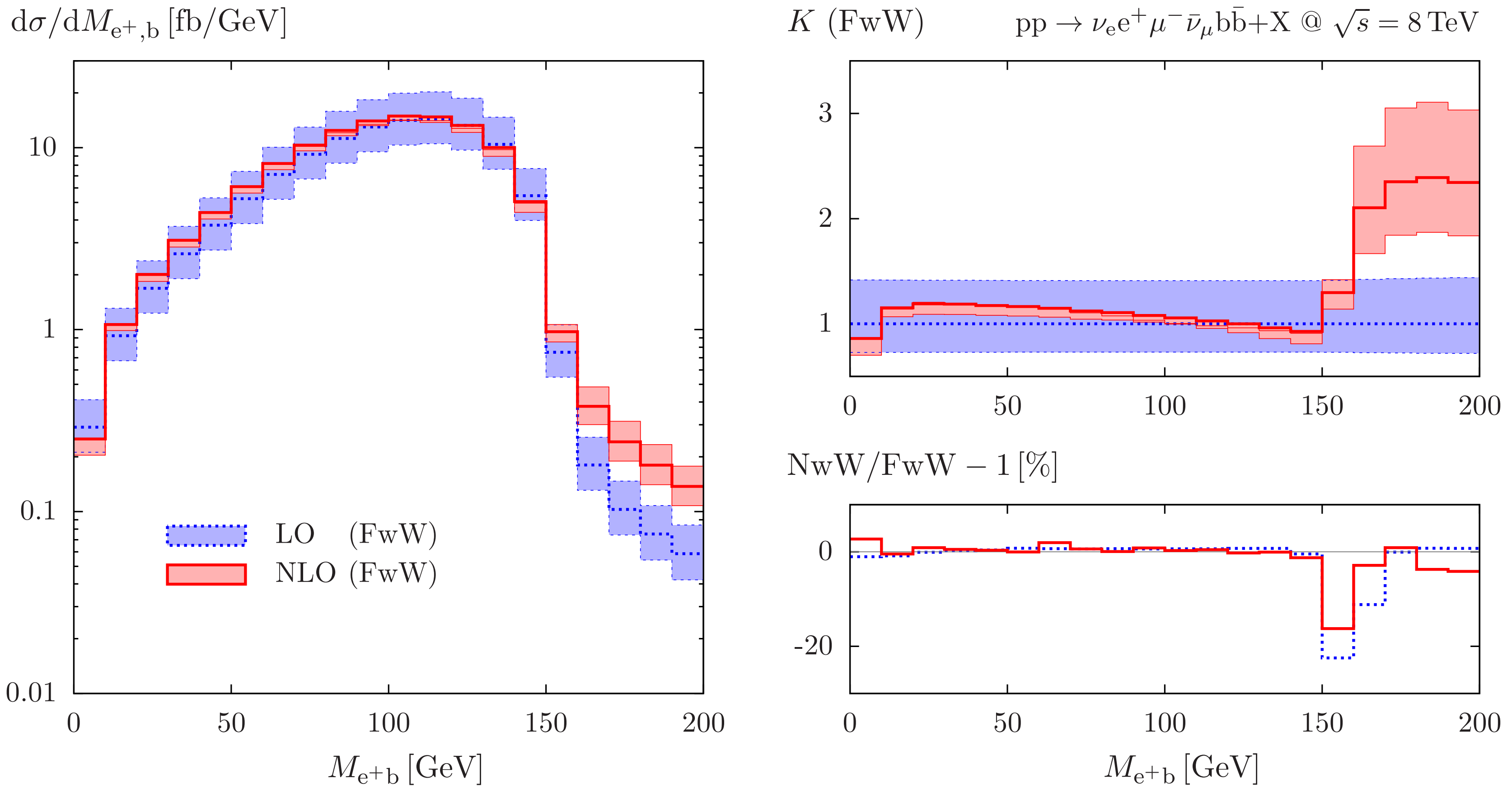
(similar at Tevatron and LHC @ 7 and $14\,\mathrm{TeV}$)

Note: $\Gamma_\mathrm{t}/m_\mathrm{t} = 0.8\%$, $\Gamma_\mathrm{W}/M_\mathrm{W} = 3\%$

$\hookrightarrow$ additional suppression beyond $\Gamma_\mathrm{W}/M_\mathrm{W}$ confirmed !

W off-shell effects in the $M_{\mathrm{e^+b}}$ distribution

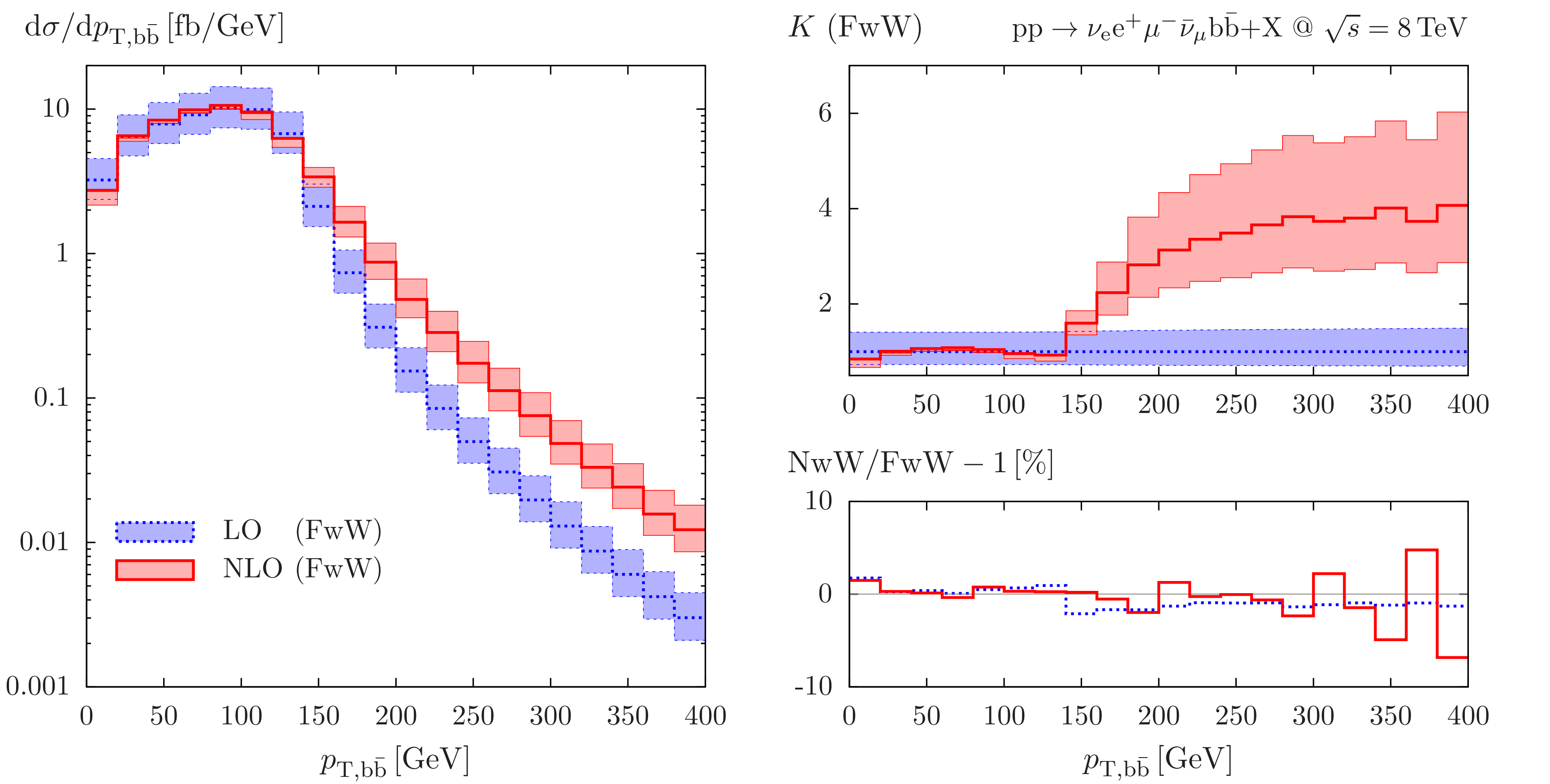
Denner, S.D., Kallweit, Pozzorini '12



W off-shell effect relevant ($\sim 10{-}20\%$) near on-shell edge at $M_{\mathrm{e^+b}} \sim 150\,\mathrm{GeV}$

W off-shell effects in the $p_{\mathrm{T},\mathrm{b\bar{b}}}$ distribution

Denner, S.D., Kallweit, Pozzorini '12



W off-shell effects reach some % for $p_{\mathrm{T},\mathrm{b\bar{b}}} > 150\,\mathrm{GeV}$.

Generic conclusion on W off-shellness:

- $< 0.5\%$ where top resonances dominate
- LO inclusion generally sufficient

3.4 Comparison to other work

Input tuned to Bevilacqua et al. [HELAC-NLO] arXiv:1012.4230 [hep-ph]

$\hookrightarrow$ comparison of integrated cross sections:

collider	energy	$\sigma_{\mathrm{LO}}[\mathrm{fb}]$	$\sigma_{\mathrm{LO}}[\mathrm{fb}]$	$\sigma_{\mathrm{NLO}}[\mathrm{fb}]$	$\sigma_{\mathrm{NLO}}[\mathrm{fb}]$
		HELAC-NLO	our result	HELAC-NLO	our result
Tevatron	1.96 TeV	34.922(14)	34.921(5)	35.705(47)	35.850(45)
LHC	7 TeV	550.54(18)	550.29(7)	808.46(98)	806.0(1.0)
LHC	10 TeV	1394.72(75)	1394.6(2)	1993.3(2.5)	1984.6(3.4)
		$\Delta \sim 1\sigma < 0.1\%$		$\Delta \sim 2\sigma \sim 0.3{-}0.4\%$	

Good agreement

Loop results for single phase-space points: (m_{t} kept real)

MadLoop and GoSam results mutually agree

$\hookrightarrow$ Hirschi et al. arXiv:1103.0621 [hep-ph] and Cullen et al. arXiv:1111.2034 [hep-ph]

But: no contact yet made to HELAC-NLO and our work

Conclusions

NLO QCD predictions for $\mathrm{pp}/\mathrm{p\bar{p}} \to \mathrm{WWb\bar{b}} \to \nu_\mathrm{e}\mathrm{e}^+\mu^-\bar{\nu}_\mu\mathrm{b\bar{b}}$

- two calculations available and in agreement (DDKP and HELAC-NLO)
- top-quark off-shell effects
  - ◇ $\sim 1\%$ as long as top resonances dominate (e.g. σ_tot)
  - ◇ can rise to effects $> 10\%$ for off-shell top's (e.g. in $M_{\mathrm{e^+b}}$, $p_{\mathrm{T,b\bar{b}}}$ distributions)
    $\hookrightarrow$ relevance for m_t determination, some background studies, etc.
- W-boson off-shell effects
  - ◇ suppressed ($< 0.5\%$) as long as top resonances dominate
  - ◇ LO treatment should be sufficient also in off-shell tails
  - ◇ to be included in Γ_t as well

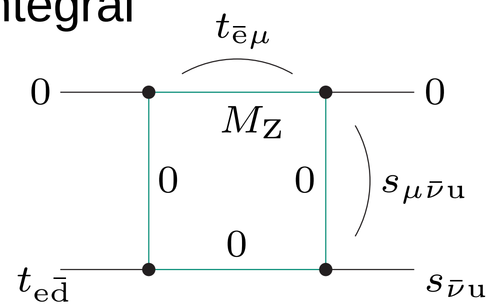
Outlook / possible use of the new results

- further correction $\sim 1\%$ to state-of-the-art prediction of $\sigma_{\mathrm{t\bar{t}}}$
- implementation into multi-purpose MC & parton-shower matching
  $\hookrightarrow$ application in m_t determinations
- further improvement by EW corrections

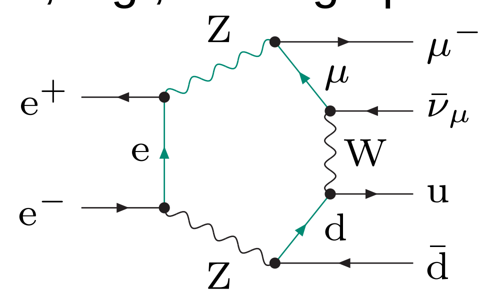
Backup slides

A typical (older) example with small Gram determinant:

Box integral

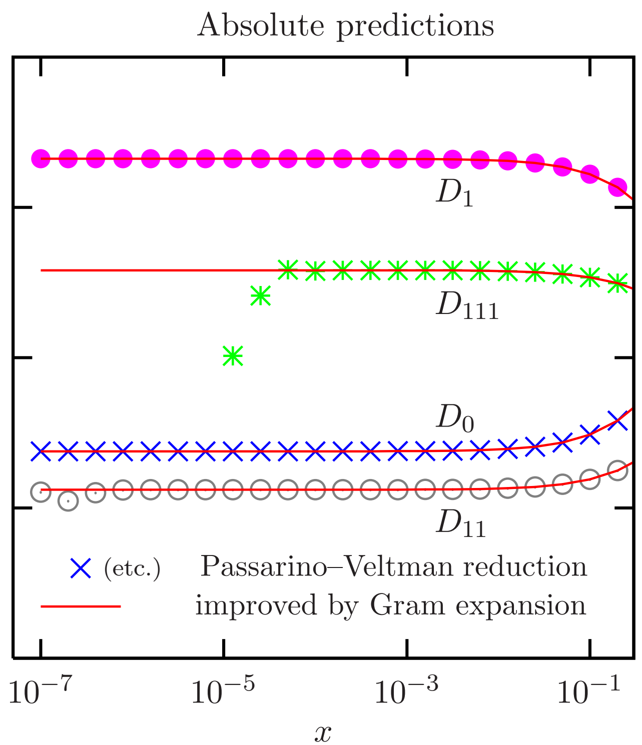


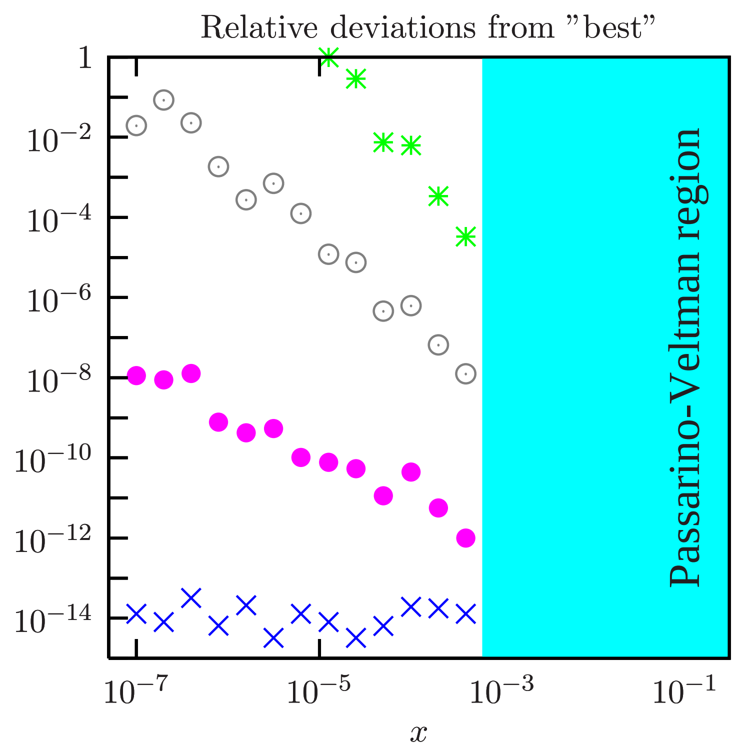
appears, e.g., in subgraph of diagram



Gram det.: $\det(\mathrm{Gram}) \to 0$ if $t_{e\bar{d}} \to t_{\mathrm{crit}} \equiv \dfrac{s_{\mu\bar{\nu}u}(s_{\mu\bar{\nu}u} - s_{\bar{\nu}u} + t_{\bar{e}\mu})}{s_{\mu\bar{\nu}u} - s_{\bar{\nu}u}}$

Numerical comparison: maximal tensor rank $= 6$ (similar to $\mathrm{ee} \to 4f$ application)





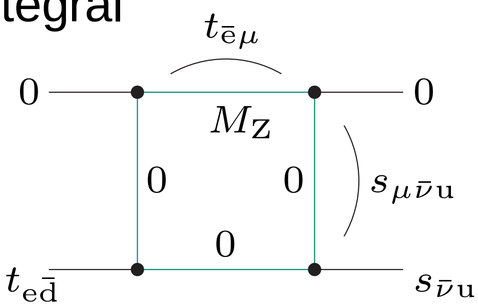
$$x \equiv \frac{t_{e\bar{d}}}{t_{\mathrm{crit}}} - 1$$

$$\begin{aligned} s_{\mu\bar{\nu}u} &= +2\times10^4\,\mathrm{GeV}^2 \\ s_{\bar{\nu}u} &= +1\times10^4\,\mathrm{GeV}^2 \\ t_{\bar{e}\mu} &= -4\times10^4\,\mathrm{GeV}^2 \\ t_{\mathrm{crit}} &= -6\times10^4\,\mathrm{GeV}^2 \end{aligned}$$

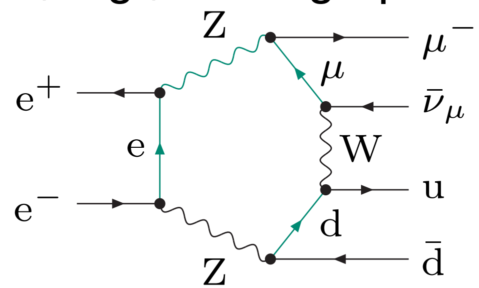
PV reduction breaks down,
but Gram exp. stable
for $\det(\mathrm{Gram}) \to 0$!

A typical (older) example with small Gram determinant:

Box integral

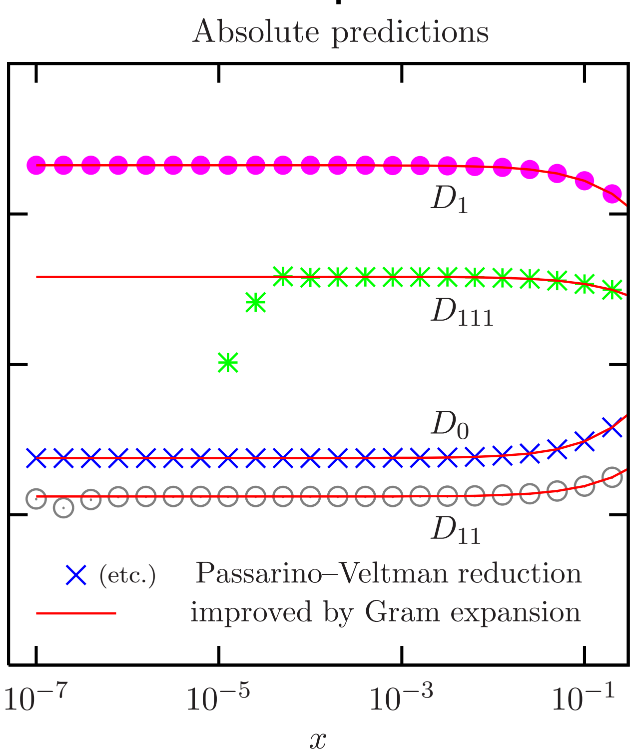


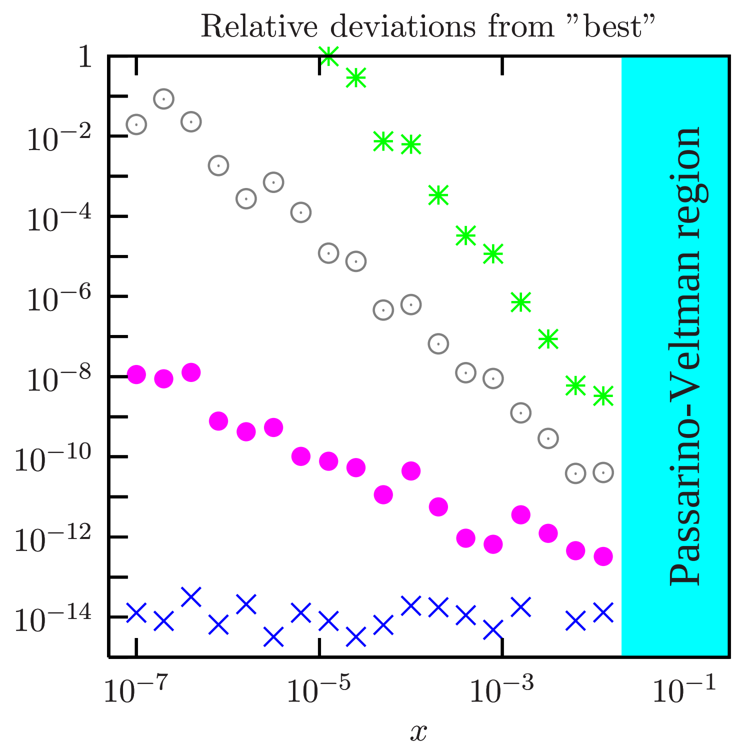
appears, e.g., in subgraph of diagram



Gram det.: $\det(\mathrm{Gram}) \to 0$ if $t_{e\bar d} \to t_{\mathrm{crit}} \equiv \dfrac{s_{\mu\bar\nu u}(s_{\mu\bar\nu u} - s_{\bar\nu u} + t_{\bar e\mu})}{s_{\mu\bar\nu u} - s_{\bar\nu u}}$

Numerical comparison: maximal tensor rank $= 12$





$x \equiv \dfrac{t_{e\bar d}}{t_{\mathrm{crit}}} - 1$

$$
\begin{aligned}
s_{\mu\bar\nu u} &= +2\times10^4\ \mathrm{GeV}^2 \\
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t_{\bar e\mu} &= -4\times10^4\ \mathrm{GeV}^2 \\
t_{\mathrm{crit}} &= -6\times10^4\ \mathrm{GeV}^2
\end{aligned}
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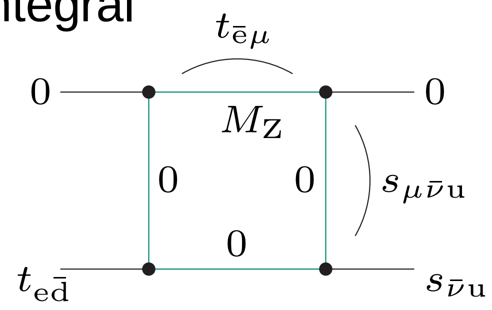

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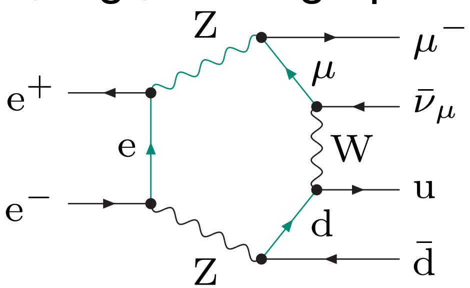

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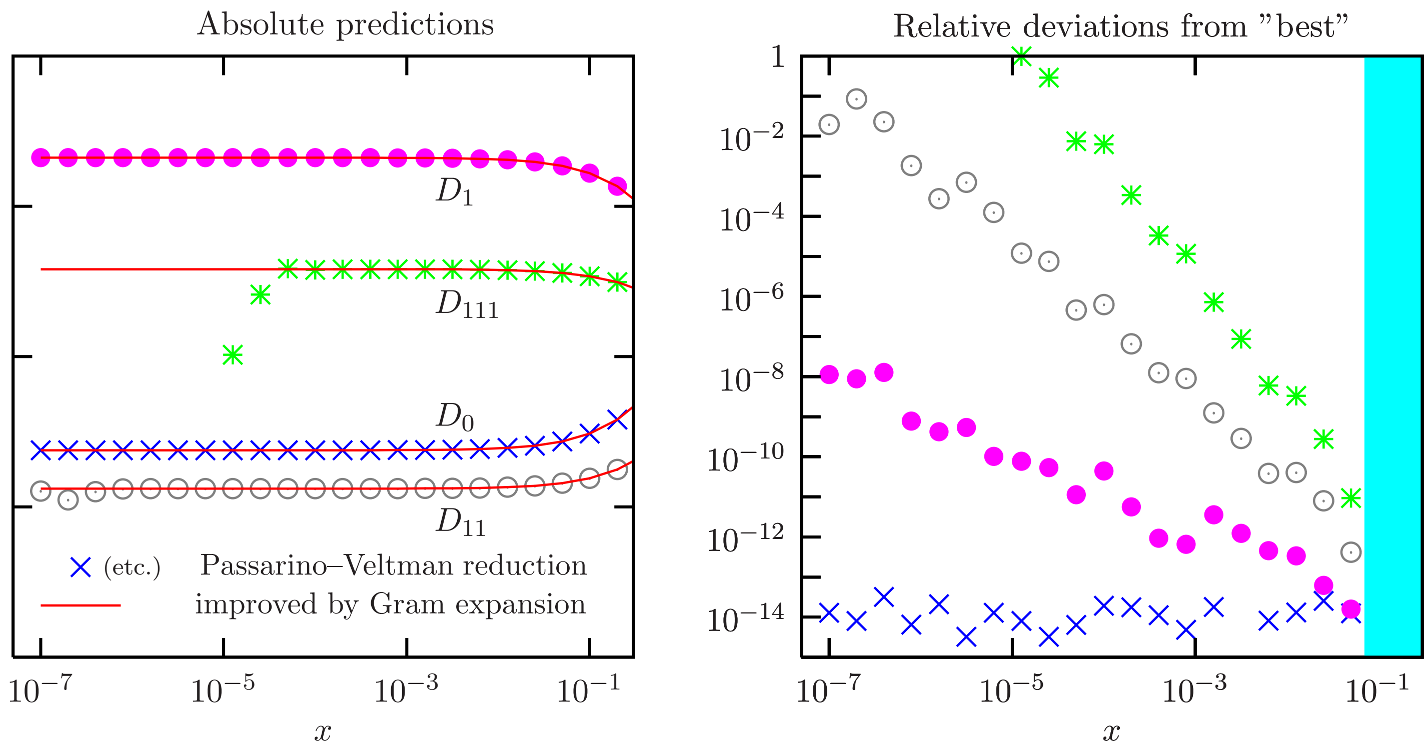
Numerical comparison: maximal tensor rank $= 25$



$x \equiv \dfrac{t_{e\bar{d}}}{t_{\mathrm{crit}}} - 1$

$$\begin{aligned} s_{\mu\bar{\nu}u} &= +2\times10^4\,\mathrm{GeV}^2 \\ s_{\bar{\nu}u} &= +1\times10^4\,\mathrm{GeV}^2 \\ t_{\bar{e}\mu} &= -4\times10^4\,\mathrm{GeV}^2 \\ t_{\mathrm{crit}} &= -6\times10^4\,\mathrm{GeV}^2 \end{aligned}$$

PV reduction breaks down,
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for $\det(\mathrm{Gram}) \to 0$!