Precision top physics at hadron colliders

Alexander Mitov

Theory Division, CERN

Loosely based on works with:

Michal Czakon '11 and (in progress)
Cacciari, Czakon, Mangano and Nason '11
George Sterman '09 and (in progress)
Beneke, Falgari, Schwinn '09

What is well established

Top-pair production is completely understood within NLO/NNLL QCD

✓ NLO QCD corrections computed long ago:

✓ Inclusive production (first numerical fits; now analytic)

Nason, Dawson, Ellis `88
Beenakker, Kuijf, van Neerven, Smith `89
Czakon, Mitov `08

✓ Differential (all numeric of course)

Nason, Dawson, Ellis `89
Beenakker, Kuijf, van Neerven, Smith `89

✓ Fully differential

Mangano, Nason, Ridolfi '92

✓ NLO EW known. Relatively small (1.5%) [more later]

Hollik, Kollar `07
Beenakker, Denner, Hollik, Mertig, Sack, Wackeroth `93

What is well established

Top-pair production is completely understood within NLO/NNLL QCD

Main features:

- ✓ Very large NLO QCD corrections ~50%
- ✓ Total theory uncertainty at ~10%
- ✓ Important for Higgs and bSM physics (M. Peskin: "*BSM Hides beneath Top*")
- ✓ Experimental improvements down to 5% (at LHC)
- ✓ Current LHC data agrees well with SM theory
- ✓ Tevatron data generally agrees too.

The notable exception: Forward-backward asymmetry from CDF.

Conclusion: "further scrutiny is needed"

Improve theory beyond NLO

1990's: the rise of soft gluon resummation

- All improvements in the last 15-20 years are based on soft gluon resummation.

- ✓ NLL resummation for top developed
  - ✓ For total inclusive
  - ✓ For differential

Bonciani, Catani, Mangano, Nason `98
Sterman, Kidonakis, Oderda `96-`98

NOTE: these are different resummations (i.e. different things are being resumed)!

- ✓ Minimal Prescription: important step in the practical implementation

Catani, Mangano, Nason, Trentadue `96

  - ✓ Still a subject of discussions

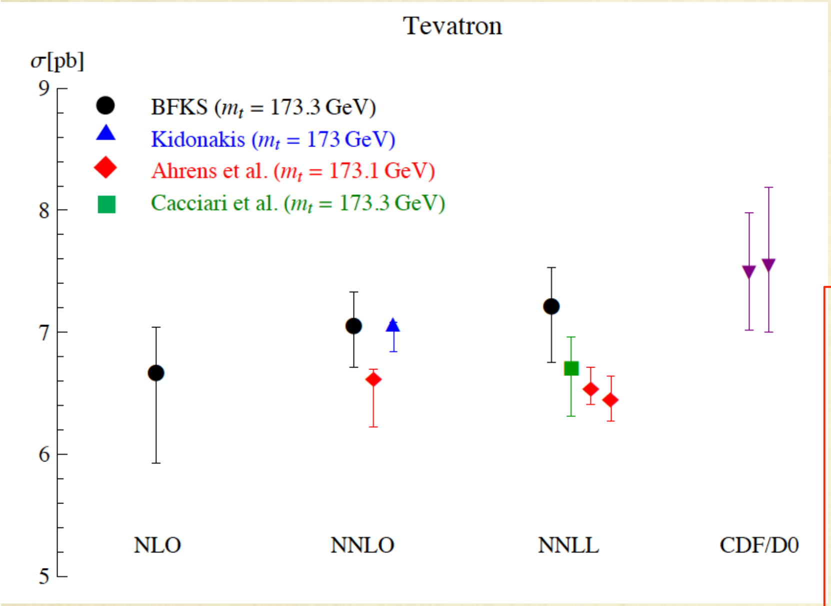
- ✓ This has recently been analyzed by a number of groups:

- ✓ Many differences among groups

Ahrens, Ferroglia, Neubert, Pecjak, Yang
Kidonakis
Aliev, Lacker, Langenfeld, Moch, Uwer, Wiedermann
Beneke, Falgari, Klein, Schwinn
Cacciari, Czakon, Mangano, Mitov, Nason

✓ Nice illustration:

Beneke, Falgari, Klein, Schwinn `11



FAQ: is it possible that some groups have better predictions that others?

Answer: Not really!

All approaches (have to) agree at leading power.

The real question is:

✓ What is the size of the sub-leading terms?

✓ Are they included in the error estimate?

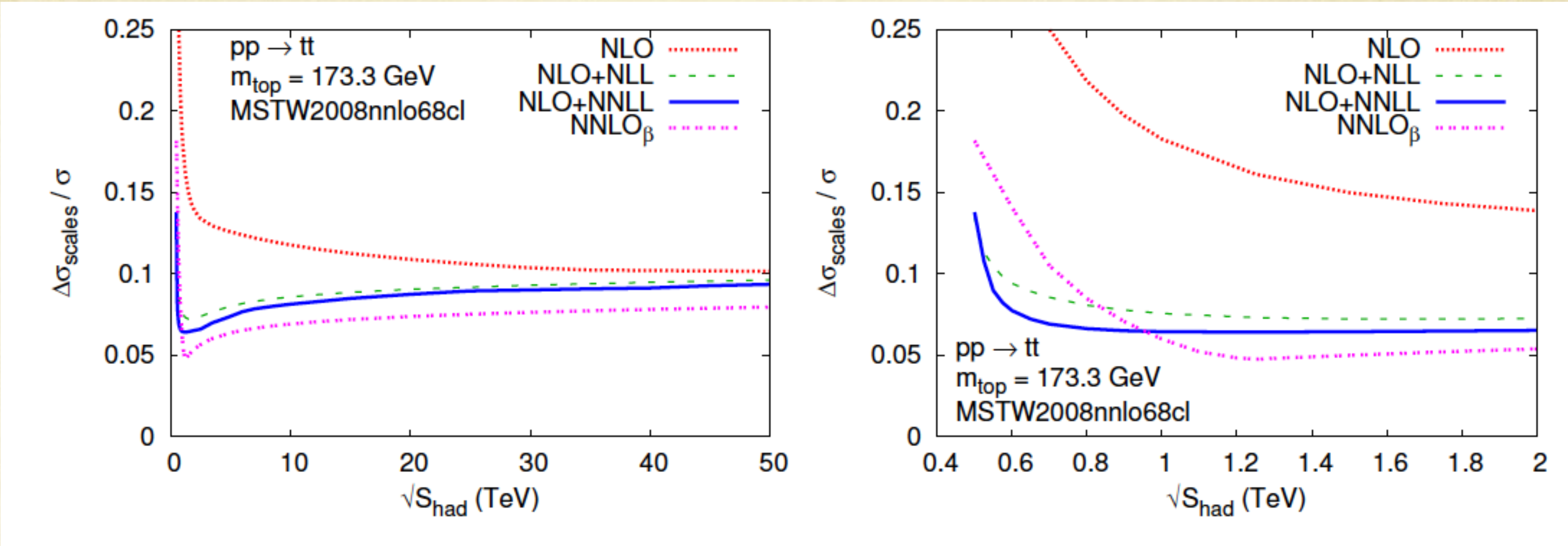
✓ Significant differences between various predictions

✓ Suggests the true current uncertainty

Recent results

Textbook example: by changing the collider energy go into (out of) the threshold region

Cacciari, Czakon, Mangano, Mitov, Nason `11



- Resummed results are better when close to threshold (as expected)
- One can quantify the question: when are we close to threshold? (below 1 TeV or so)
- Approx_NNLO is a subset of the resumed result.
  Has accidentally small scale dependence.

Open questions

✓ Reliably increase the precision of the theoretical predictions?
Possible. Need NNLO.

✓ What to make of the forward – backward asymmetry?

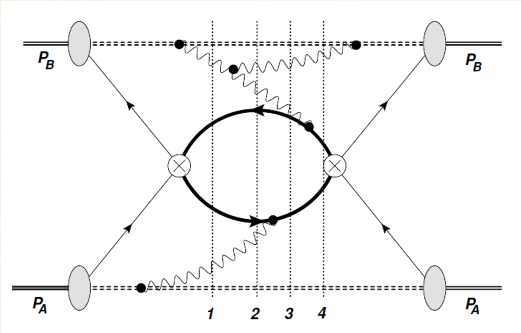
✓ Can missing theory corrections be large? Yes!
(almost universally assumed small)

✓ Is this enough to explain A_{FB}? Perhaps not!

This will only be settled once the SM predictions have been exhausted
(currently A_{FB}(tT+X) is at LO QCD!)

A_FB: non-factorizing contributions?

Motivating question: can A_{FB} be generated (or enhanced) by tT final state interactions?



Work with George Sterman, to appear

Prompted, in turn, by older work in QED

See, for example, Brodsky, Gillespie`68

- ✓ We have devised an all-order proof of the cancellations of such interactions
- ✓ The subtle point is: What is the remainder? All depends on observables' definition.
- ✓ For inclusive observables (with conventional factorization) the remainder is small.
- ✓ For observables with rapidity gaps: large corrections are possible.

Percent Level Precision Physics at the Tevatron: Top-pair production in NNLO QCD qq->tt+X

P. Barnreuther, M. Czakon and A. Mitov, to appear

CERN-PH-TH/2012-092
TTK-12-13

- ✓ First ever hadron collider calculation at NNLO with more than 2 colored partons.
- ✓ First ever NNLO hadron collider calculation with massive fermions.

Structure of the cross-section

$$\sigma = \frac{\alpha_s^2}{m_t^2} \sum_{ij} \int_0^{\beta_{\max}} \mathcal{L}_{ij}(\beta)\hat{\sigma}(\beta)$$

$$\rho = \frac{4m_t^2}{s} \qquad \beta = \sqrt{1-\rho}$$

Relative velocity of tT

- ✓ The partonic cross-section computed numerically in 80 points. Then fitted.
- ✓ Many contributing partonic channels:

Computed. Dominant at Tevatron (~85%)

$$q\bar{q} \to t\bar{t}$$
$$q\bar{q} \to t\bar{t}g$$
$$q\bar{q} \to t\bar{t}gg$$
$$q\bar{q} \to t\bar{t}q'\bar{q}' , \qquad q \neq q'$$

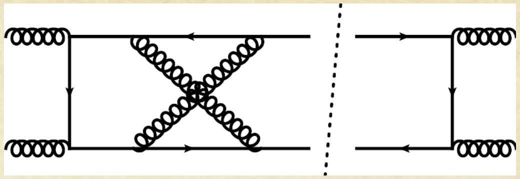
$$gg \to t\bar{t}$$
$$gg \to t\bar{t}g$$
$$gg \to t\bar{t}gg$$
$$gg \to t\bar{t}q\bar{q}$$

$$qg \to t\bar{t}q$$
$$qg \to t\bar{t}qg$$
$$qq' \to t\bar{t}qq' , \qquad q \neq \bar{q}'$$
$$q\bar{q} \to t\bar{t}q\bar{q}$$

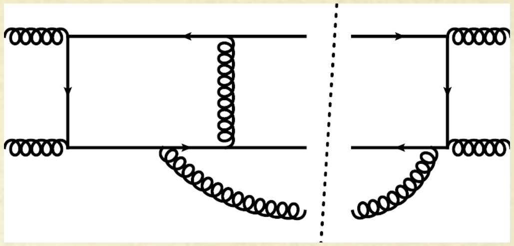
All of the same complexity. No more conceptual challenges expected (just lots of CPU)

What's needed for NNLO?

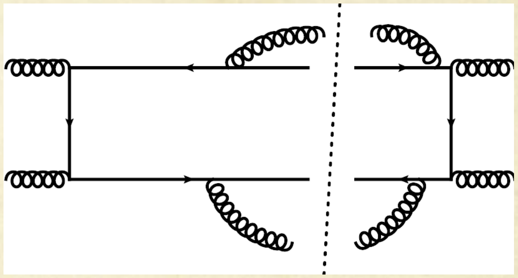
There are 3 principle contributions:



- ✓ 2-loop virtual corrections (V-V)



- ✓ 1-loop virtual with one extra parton (R-V)



- ✓ 2 extra emitted partons at tree level (R-R)

And 2 secondary contributions:

- ✓ Collinear subtraction for the initial state

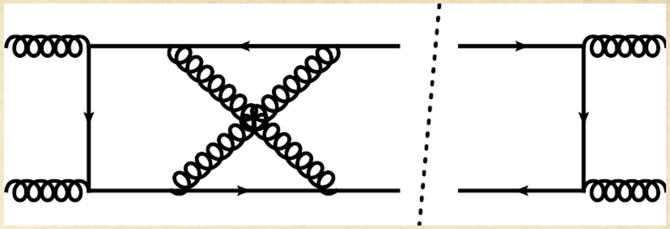
Known, in principle. Done numerically.

- ✓ One-loop squared amplitudes (analytic)

Korner, Merebashvili, Rogal `07

What’s needed for NNLO? V-V

Required are the two loop amplitudes:
qq → QQ and gg → QQ.



- ✓ Their high energy limits and their poles are known analytically. Fermionic corrections too.

Czakon, Mitov, Moch ’07
Czakon, Mitov, Sterman ‘09
Ferroglia, Neubert, Pecjak, Yang ’09
Bociani et al. `09-`11

- ✓ Directly used here: The qq→QQ amplitude is known numerically

Czakon `07

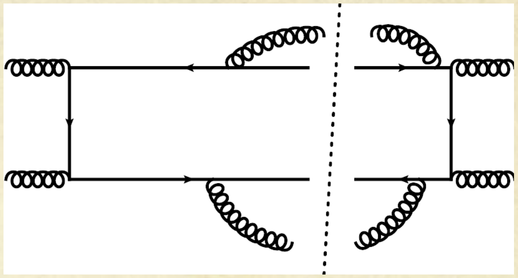
- ✓ Numerical work underway for the gg→QQ

Czakon, Bärnreuther, to appear

What’s the future here?

- ✓ Right now this is the biggest (and perhaps only) obstacle for NNLO phenomenology on a mass scale

What's needed for NNLO? R-R



- ✓ A wonderful result By M. Czakon

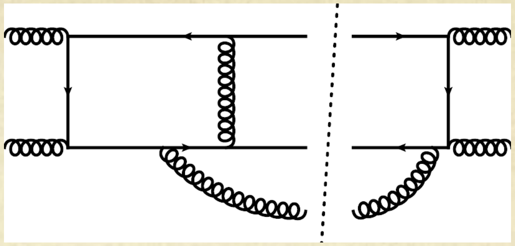
Czakon `10-11

- ✓ The method is general (also to other processes, differential kinematics, etc).

- ✓ Explicit contribution to the total cross-section given.

- ✓ Just been verified in an extremely non-trivial problem.

What's needed for NNLO? R-V



✓ Counterterms all known (i.e. all singular limits)

Bern, Del Duca, Kilgore, Schmidt '98-99
Catani, Grazzini '00
Bierenbaum, Czakon, Mitov '11

The finite piece of the one loop amplitude computed with a private code of Stefan Dittmaier.

Extremely fast code!

A great help!

Many thanks!

How is the calculation organized?

- ✓ Guiding principle: do not try to combine all cuts into a single "finite" integration
  - ✓ To have the flexibility to somehow compute each cut,
  - ✓ Everything is done numerically. And in an independent approach.

- ✓ The right subtraction scheme has existed for a long time: FKS

Frixione, Kunszt, Signer `96

- ✓ FKS can easily be extended to NNLO:

"the subtraction terms are defined by the phase space, not us"

✓ Example: VV

$$\mathcal{O}_{VV} \sim \int d^d\Phi_2 \, |M_{2\to 2}|^2(\epsilon)$$

$$|M_{2\to 2}|^2(\epsilon) \sim \sum_{i\leq 4} \frac{M_i}{\epsilon^i}$$

✓ Since the phase space integration is non-singular:

1. Expand phase-space and matrix element in Eps
2. Integrate each term separately (i.e. derive 5 results; 4 will cancel)
3. Amplitude is known numerically. But its poles are known analytically.

Ferroglia, Neubert, Pecjak, Yang '09

✓ The poles of any 2-loop amplitude (with masses too) can be predicted

Mitov, Sterman, Sung '09-'10
Ferroglia, Neubert, Pecjak, Yang '09-'10

✓ Example RR : The basic logic is very simple: Czakon `10

1. Split the phase-space into sectors (algorithmic and process independent)

$$
\begin{aligned}
1 \; = \; & \\
& \left.\begin{array}{l} +\theta_1(k_1)\theta_1(k_2) \\ +\theta_2(k_1)\theta_2(k_2) \end{array}\right\} \text{triple-collinear sector} \\
& \left.\begin{array}{l} +\theta_1(k_1)\theta_2(k_2)(1-\theta_3(k_1,k_2)) \\ +\theta_2(k_1)\theta_1(k_2)(1-\theta_3(k_1,k_2)) \end{array}\right\} \text{double-collinear sector} \\
& +(\theta_1(k_1)\theta_2(k_2)+\theta_2(k_1)\theta_1(k_2))\theta_3(k_1,k_2) \; \} \text{single-collinear sector} \, .
\end{aligned}
$$

2. Remap the phase-space integration variables in each sector (algorithmic)

3. The singularities are factored out explicitly (no counterterms needed)

$$\mathcal{O}_S = \mathcal{N} \int_0^1 d\zeta d\eta_1 d\eta_2 d\xi_1 d\xi_2 \; \theta_1(k_1)\theta_1(k_2) \; \frac{1}{\eta_1^{1-b_1\epsilon}} \frac{1}{\eta_2^{1-b_2\epsilon}} \frac{1}{\xi_1^{1-b_3\epsilon}} \frac{1}{\xi_2^{1-b_4\epsilon}} \; \int d\Phi_n(Q) \, F_J \, \mathcal{M}_S$$

4. Apply the usual identities:

$$\frac{1}{\lambda^{1-b\epsilon}} = \frac{1}{b}\frac{\delta(\lambda)}{\epsilon} + \sum_{n=0}^{\infty} \frac{(b\epsilon)^n}{n!} \left[\frac{\ln^n(\lambda)}{\lambda}\right]_+$$

$$\int_0^1 d\lambda \left[\frac{\ln^n(\lambda)}{\lambda}\right]_+ f(\lambda) = \int_0^1 \frac{\ln^n(\lambda)}{\lambda}(f(\lambda) - f(0))$$

All is driven by phase-space

Effective counterterm.
Known from singular limits.

✓ Example RV : Similar to RR.

1. Less singular regions (one soft and/or one collinear)
2. Remap the phase-space integration variables in each sector (algorithmic)
3. The singularities are factored out explicitly (no counterterms needed)
4. Apply the usual identities:

$$\frac{1}{\lambda^{1-b\epsilon}} = \frac{1}{b}\frac{\delta(\lambda)}{\epsilon} + \sum_{n=0}^{\infty}\frac{(b\epsilon)^n}{n!}\left[\frac{\ln^n(\lambda)}{\lambda}\right]_+$$

$$\int_0^1 d\lambda \left[\frac{\ln^n(\lambda)}{\lambda}\right]_+ f(\lambda) = \int_0^1 \frac{\ln^n(\lambda)}{\lambda}(f(\lambda) - f(0))$$

Effective counterterm.
Known from singular limits.

5. The matrix elements are now divergent – no problem: expand and integrate
6. Counterterms more complicated, but known analytically.

Results @ parton level

Partonic cross-section through NNLO:

$$\sigma_{ij}\left(\beta,\frac{\mu^2}{m^2}\right) = \frac{\alpha_S^2}{m^2}\Big\{\sigma_{ij}^{(0)} + \alpha_S\left[\sigma_{ij}^{(1)} + L\,\sigma_{ij}^{(1,1)}\right] + \alpha_S^2\left[\sigma_{ij}^{(2)} + L\,\sigma_{ij}^{(2,1)} + L^2\sigma_{ij}^{(2,2)}\right] + \mathcal{O}(\alpha_S^3)\Big\},$$

The NNLO term:

$$\sigma_{q\bar q}^{(2)}(\beta) = F_0(\beta) + F_1(\beta)N_L + F_2(\beta)N_L^2$$

Numeric

Analytic

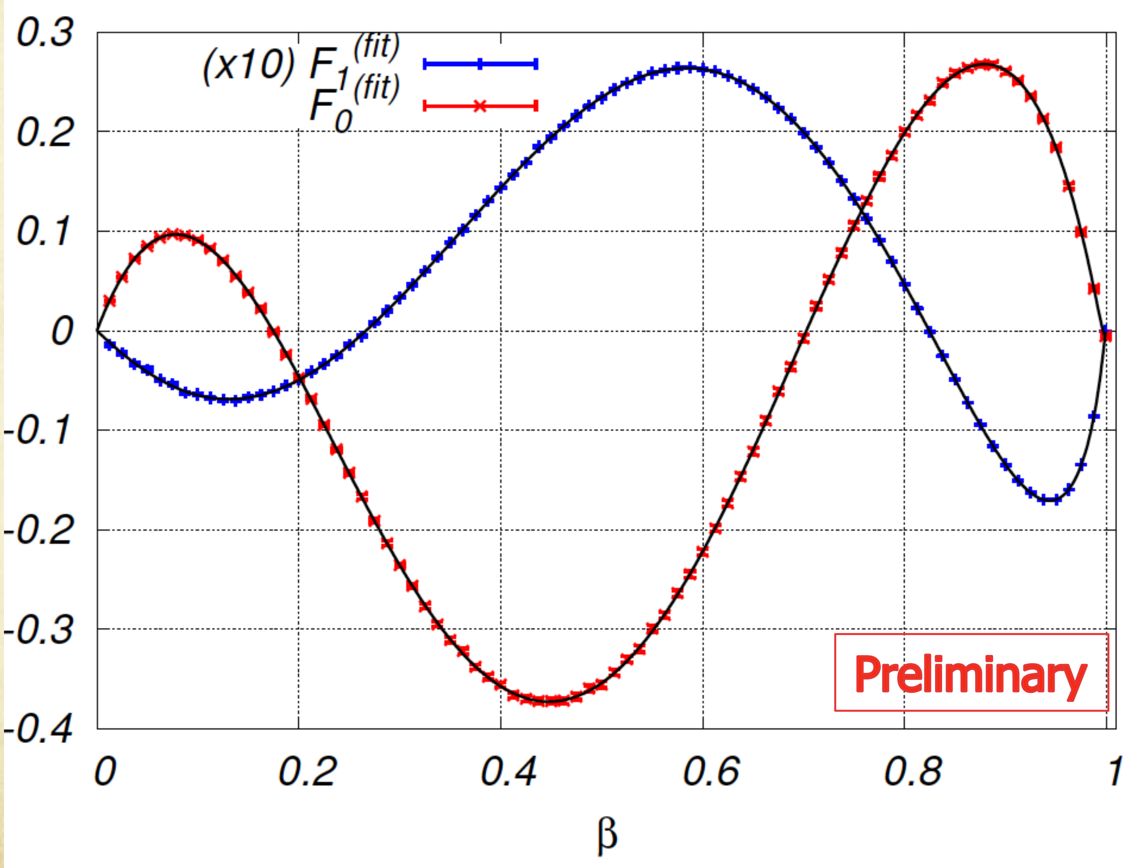
$$F_i \equiv F_i^{(\beta)} + F_i^{(\mathrm{fit})}, i = 0,1$$

The known threshold approximation

Beneke et al `09

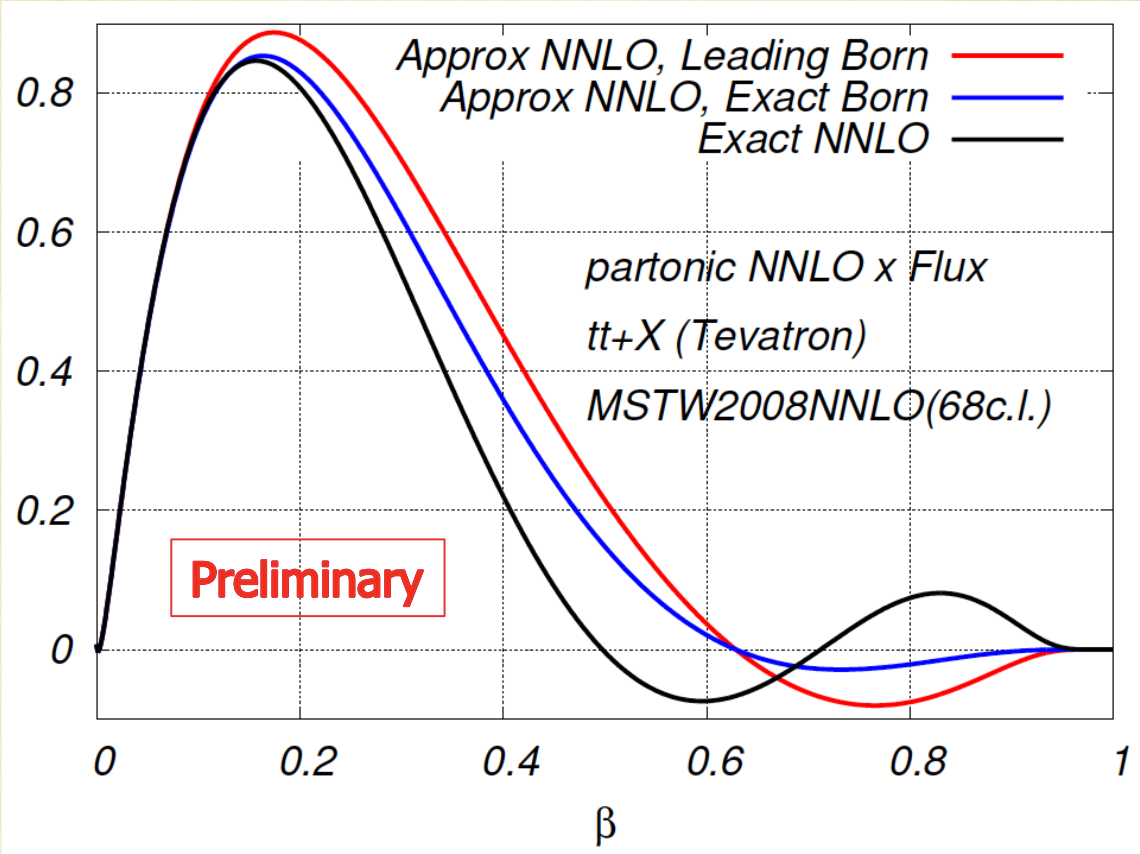
Notable features:

- ✓ Small numerical errors
- ✓ Agrees with limits



What happens once we add the flux?

$$\sigma = \frac{\alpha_s^2}{m_t^2} \sum_{ij} \int_0^{\beta_{\max}} \mathcal{L}_{ij}(\beta)\hat{\sigma}(\beta)$$



- ✓ Approximate NNLO is a an OK approximation at parton level
- ✓ There are non-trivial cancellations; the integrated numbers are closer to the exact ones than one might anticipate
- ✓ The power corrections to the Leading Born term have important effect

Here are the numbers for the Tevatron:

- ✓ Independent F/R scales
- ✓ MSTW2008NNLO
- ✓ mt=173.3

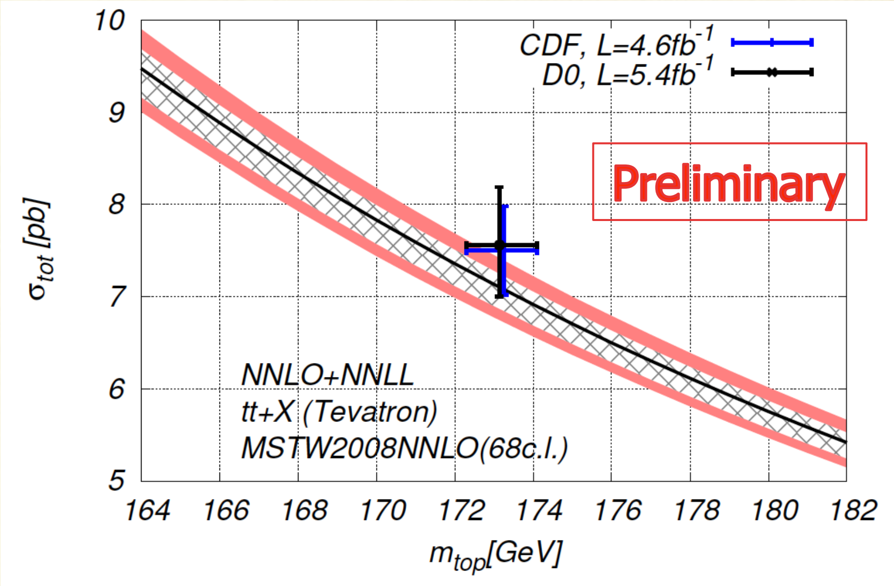
Scale variation

NNLO: Preliminary

sigma_tot = 7.005 + 2.9 % - 4.4 % [pb].

NNLO+NNLL: Preliminary

sigma_tot = 7.067 + 2.0 % - 3.3 % [pb].



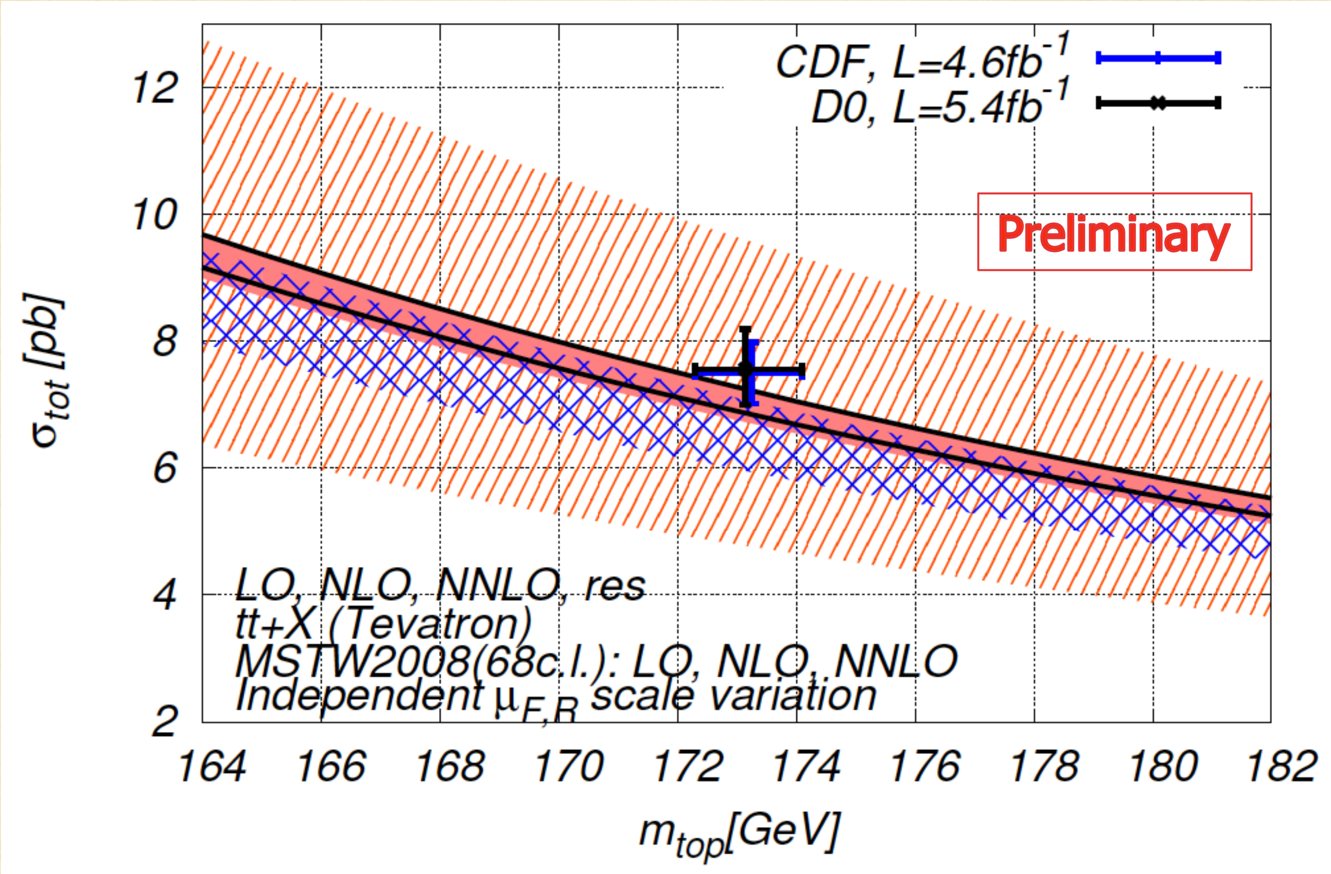
- ✓ Two loop hard matching coefficient extracted and included
- ✓ Very week dependence on unknown parameters (sub 1%): gg NNLO, A, etc.
- ✓ 50% scales reduction compared to the NLO+NNLL analysis of

Cacciari, Czakon, Mangano, Mitov, Nason ‘11

6.72 + 3.6% - 6.1%

Good perturbative convergence:

✓ Independent F/R scales
✓ mt=173.3



✓ Good overlap of various orders (LO, NLO, etc).
✓ Suggests our (restricted) independent scale variation is good

- ✓ Where to from here?
  - ✓ Compute the remaining partonic reactions
  - ✓ Compute the forward-backward asymmetry
  - ✓ Compute differential distributions
  - ✓ Add top decay
  - ✓ Compare to experiment ☺
- ✓ Compute many more processes

Facing the future, the stumbling block seems to be the availability of 2-loop amplitudes

- ✓ Our work is a strong motivation for new developments in this direction

Implementation and numbers

✓ We have also prepared the tools for top physics:

Top++ : a C++ program for the calculation of the total cross-section:

✓ Includes:

Czakon, Mitov `11

- ➢ fixed order (NLO and NNLO_approx at present)
- ➢ and resummation (full NNLL already there)

✓ It is meant to incorporate the full NNLO once available (to appear)

✓ Very user friendly.

✓ Developments:

- ✓ ver. 1.x: Approx NNLO +NNLL (Released)
- ✓ ver. 2.x: NNLO(qqbar) + NNLL. Complete Tevatron pheno. (To appear)
- ✓ ver. 3.x: Full NNLO + NNLL

Summary and Conclusions

- Long (~15 years) and turbulent chapter in top physics is closing
  - It saw uses of soft gluon resummation to a number of approximations at NNLO
  - It was theoretically very fruitful: engine for theoretical developments
- We have derived the full NNLO result qq → tt (numeric – very good precision)
  - at Tevatron it cuts scale uncertainty in half compared to NLO+NNLL
  - Words like approx_NNLO, etc., belong in the past.
- Methods are very general and applicable to differential distributions
- Applications for dijets and W+jet, H+jet, etc @ NNLO
- Only restriction – availability of two-loop amplitudes and computing speed
- We are on the verge of the NNLO revolution (NLO wish-list already exhausted ☺)