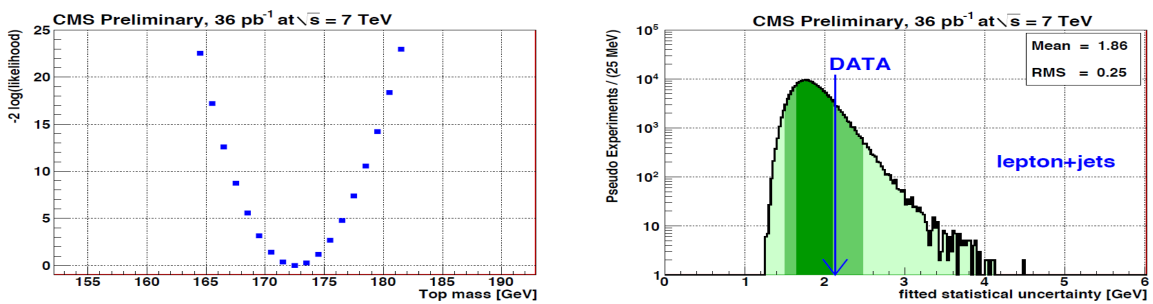
Application of FBPS Generators: MEM@NLO

In collaboration with Ciaran Williams and John Campbell
arXiv:1204.4424v1 [hep-ph]

- The Matrix Element Methods (MEM)
- Issues with the current MEM
- Scattering probabilities for “exclusive” events.
- Defining a perturbative calculable MEM
- Some examples

The Matrix Element Method

- This method was developed by experimenters
  - Mainly used in top event analysis.
  - Popular as it extracts the maximum amount of information from the event.



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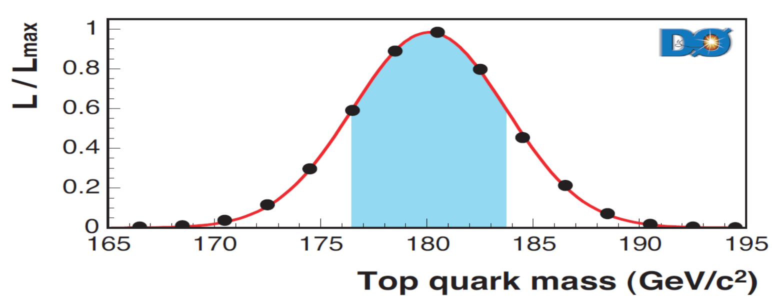


D0: Nature **429**, 638-642

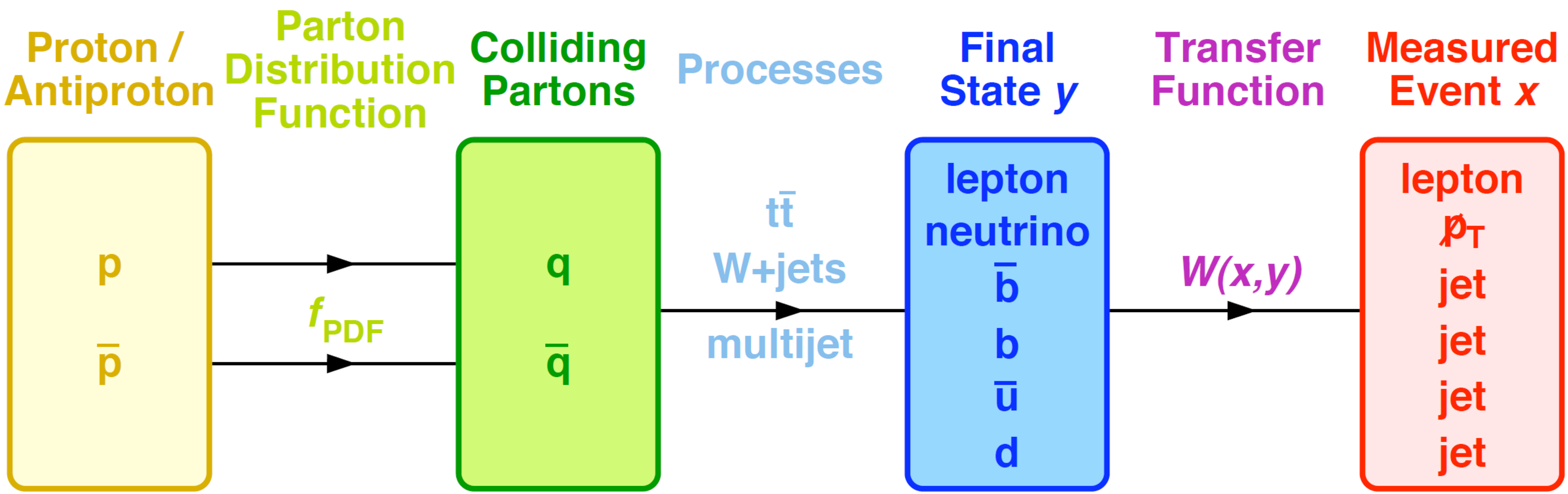
The Matrix Element Method

- Mostly only experimental in-house MEM codes
- Not much attention from theorists for the MEM
  - Only MADGRAPH has an implementation
    $\longrightarrow$ MEM for general BSM scenarios
    P. Artoisenet, V. Lemaitre,F. Maltoni, O. Mattelaer 1007.3300
    $\longrightarrow$ Including some IRS modeling
    J. Alwall, A. Freitas, O. Mattelaer 1010.2263
- The current MEM is very phenomenological, not well suited for proper theoretical treatment such as NLO.

The Matrix Element Method

- For the experiments the MEM is simply:
  - Given an reconstructed event {y} (jets, leptons, photons) one calculates the LO scattering probability resulting in the observed final state.
  - The connection between the partons, bare leptons and direct photons and the final state objects is contained in the transfer function W(x,y).
  - The transfer function is determined using parton showers and detector response models

The Matrix Element Method



$$\mathcal{P}(\mathbf{x}|\Omega) = \frac{1}{\sigma_\Omega} \int dx_a dx_b \, d\mathbf{y} \sum_{ij} \frac{f_i(x_a) f_j(x_b)}{x_a x_b s} \mathcal{B}^{ij}_{\Omega}(p_a, p_b, \mathbf{y}) \, W(\mathbf{x}, \mathbf{y})$$

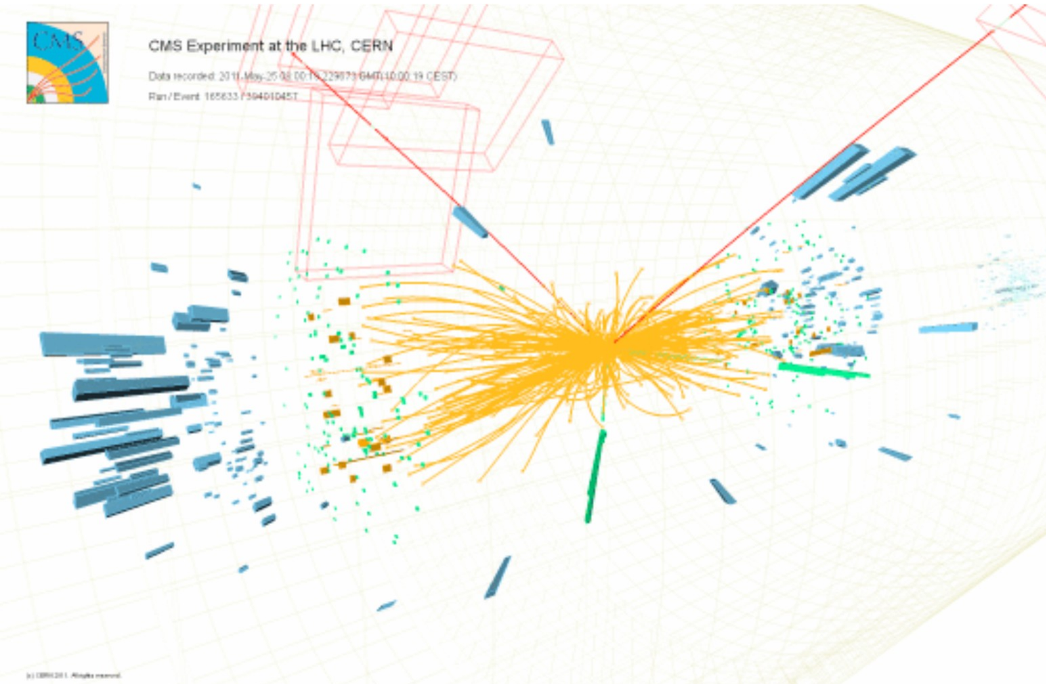
The Matrix Element Method

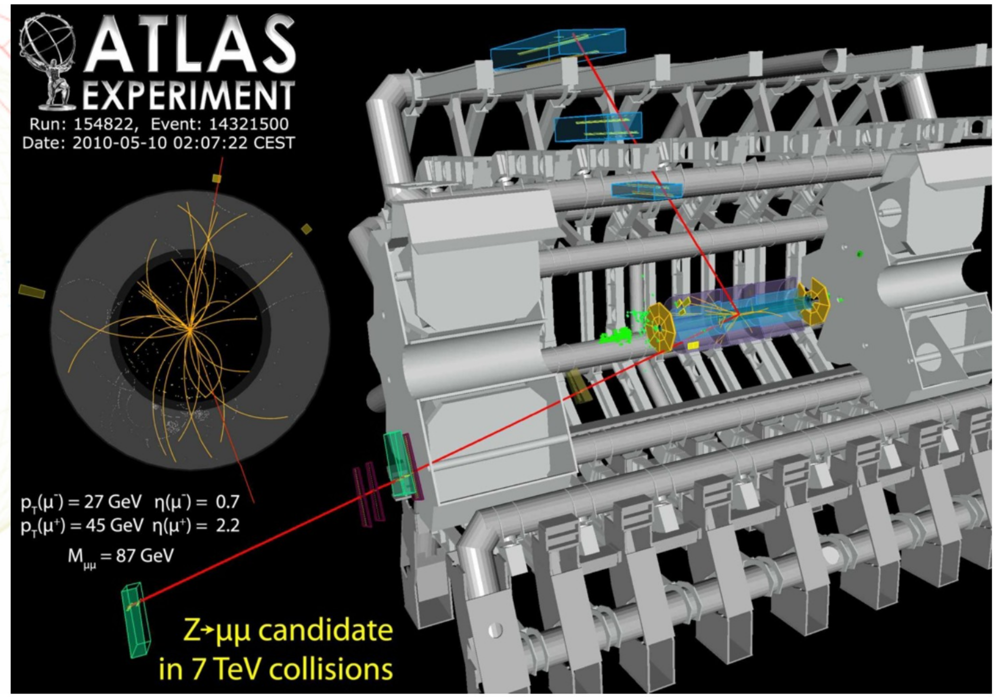
- This approach is based on the LO/LL view that a parton showers and turns into a jet.
- At NLO this is untenable, a correct view is that one should calculate the scattering probability order by order for a given final state.
- The transfer function is then nothing more than the detector mismeasurement of the particle final state. It cannot contains events physics.
- The MEM ***must*** be defined without the presence of a transfer function.

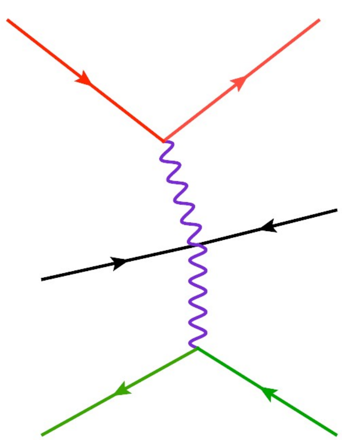
Scattering Probabilities for Exclusive Events

- Exclusive in the sense of reconstructed objects (not particles).
- The lowest order estimate has distinct kinematic feature: Jets are massless and transverse momentum in the final state is conserved (each parton is assigned to a jet).
- Beyond LO and real events have massive jets and have unclustered hadrons, i.e. no transverse momentum balance.

Scattering Probabilities for Exclusive Events

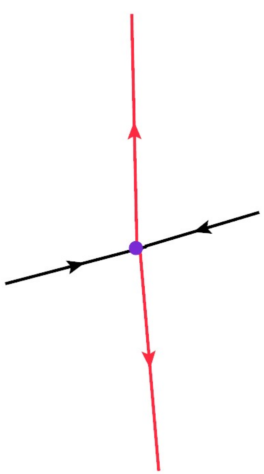






How can we relate an experimental event to our theoretical model?

Experimental events are unbalanced (for the LO final state), LO events have exact balance.



Scattering Probabilities for Exclusive Events

- To deal with jet mass: Integrate out the jet mass while keeping fixed (Pt,eta,phi) (changes initial state)
- To deal with Pt-imbalance: apply transverse boost to restore balance.
- The scattering probability is Lorentz Invariant. The colliding partons beyond LO are not along the beam axis.
- This only affects non-Lorentz invariant observables. Nothing else...

Scattering Probabilities for Exclusive Events

- For the remainder of the talk we restrict ourselves to leptonic final states.
- This means that for the first implementation we only have to deal with initial state radiation.
- This covers DY production, Higgs search using 4 lepton final states and anomalies coupling searches.

Scattering Probabilities for Exclusive Events

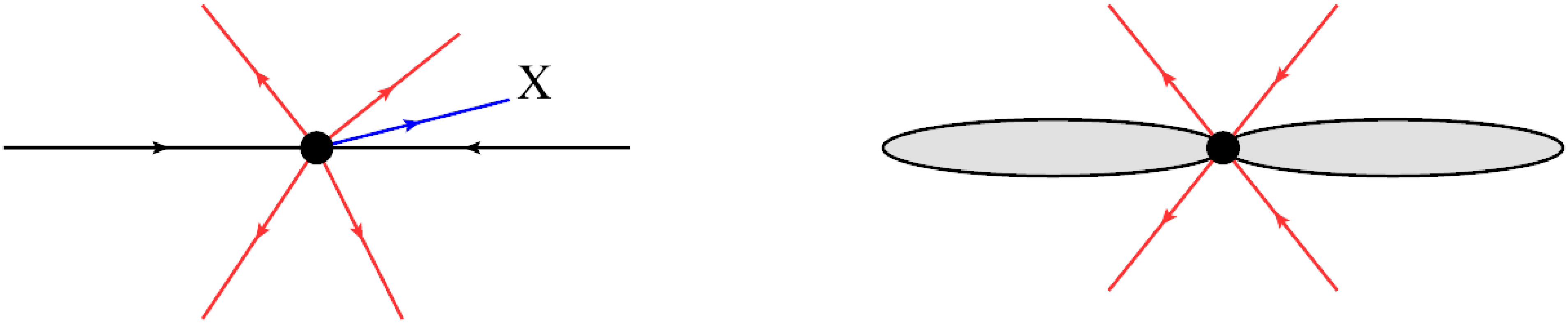


Figure 1: The generation of the Born (and virtual) phase space from a given experimental event. The left hand side depicts a collision that results in the production of a colour neutral final state (represented here by four leptons in red) that nearly balance in the transverse plane. The resulting imbalance (X, in blue) represents the remaining event which is not modeled in the Born matrix element. We apply a Lorentz transformation such that X has no components in the transverse plane, with the remaining longitudinal and energy components absorbed into the colliding partons.

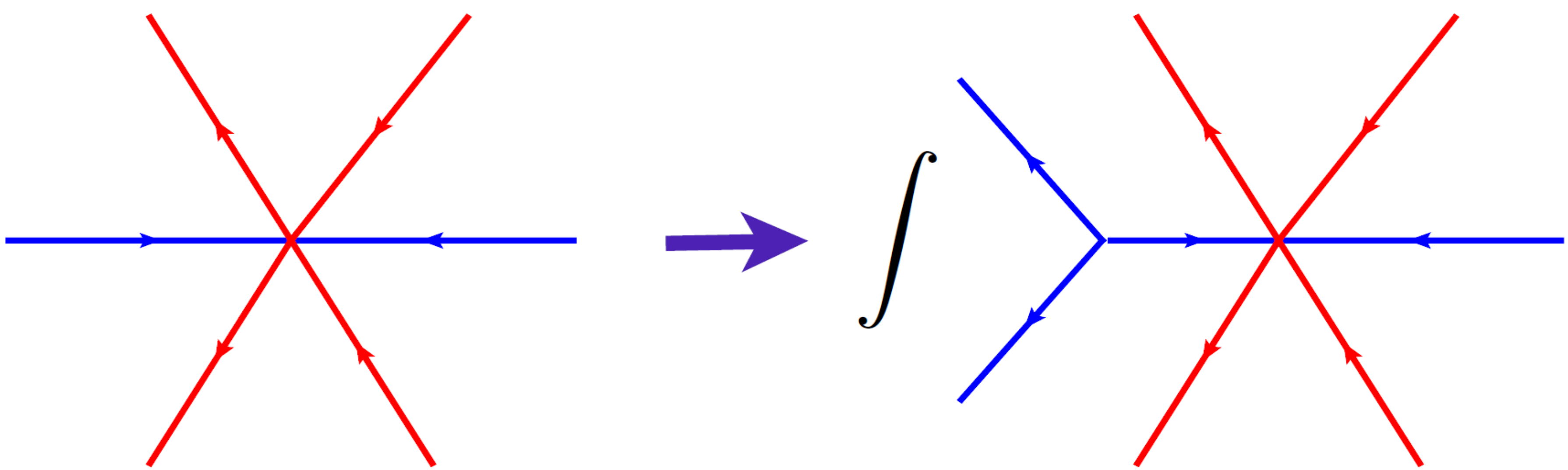
Scattering Probabilities for Exclusive Events

- We implemented the FBPS (Giele, Stavenga, Winter 1106.5045) in NLOME which is based on MCFM

$$d\Phi(p_a + p_b \to Q + p_r) = d\,\Phi(\widehat{p}_a + \widehat{p}_b \to Q) \times d\,\Phi_{\mathrm{FBPS}}(p_a, p_b, p_r) \times \theta_{\mathrm{veto}}$$

$$\theta_{\mathrm{veto}}(p_r) = \theta\left[p_T^{lab}(p_r) < p_T^{\mathrm{min}}(\mathrm{jet})\right]$$

$$d\,\Phi_{\mathrm{FBPS}}(p_a, p_b, p_r) = \frac{1}{(2\pi)^3}\left(\frac{\widehat{s}_{ab}}{s_{ab}}\right) d\,t_{ar}\, d\,t_{rb}\, d\,\phi$$



Scattering Probabilities for Exclusive Events

- It seems we did not achieve anything, but in fact lots of things change because we map the bremsstrahlung phase space onto the LO phase space.
- We can express the exclusive NLO as a K-factor to the exclusive LO (the observed final state is unaltered under the FBPS integration)
- This leads immediately to the notion of unweighted NLO:
  unweighted LO $\rightarrow$ K-factor $\rightarrow$ unweighted NLO

Scattering Probabilities for Exclusive Events

W. Giele, G. Stavenga, J. Winter
arXiv:1106.5045 [hep-ph]

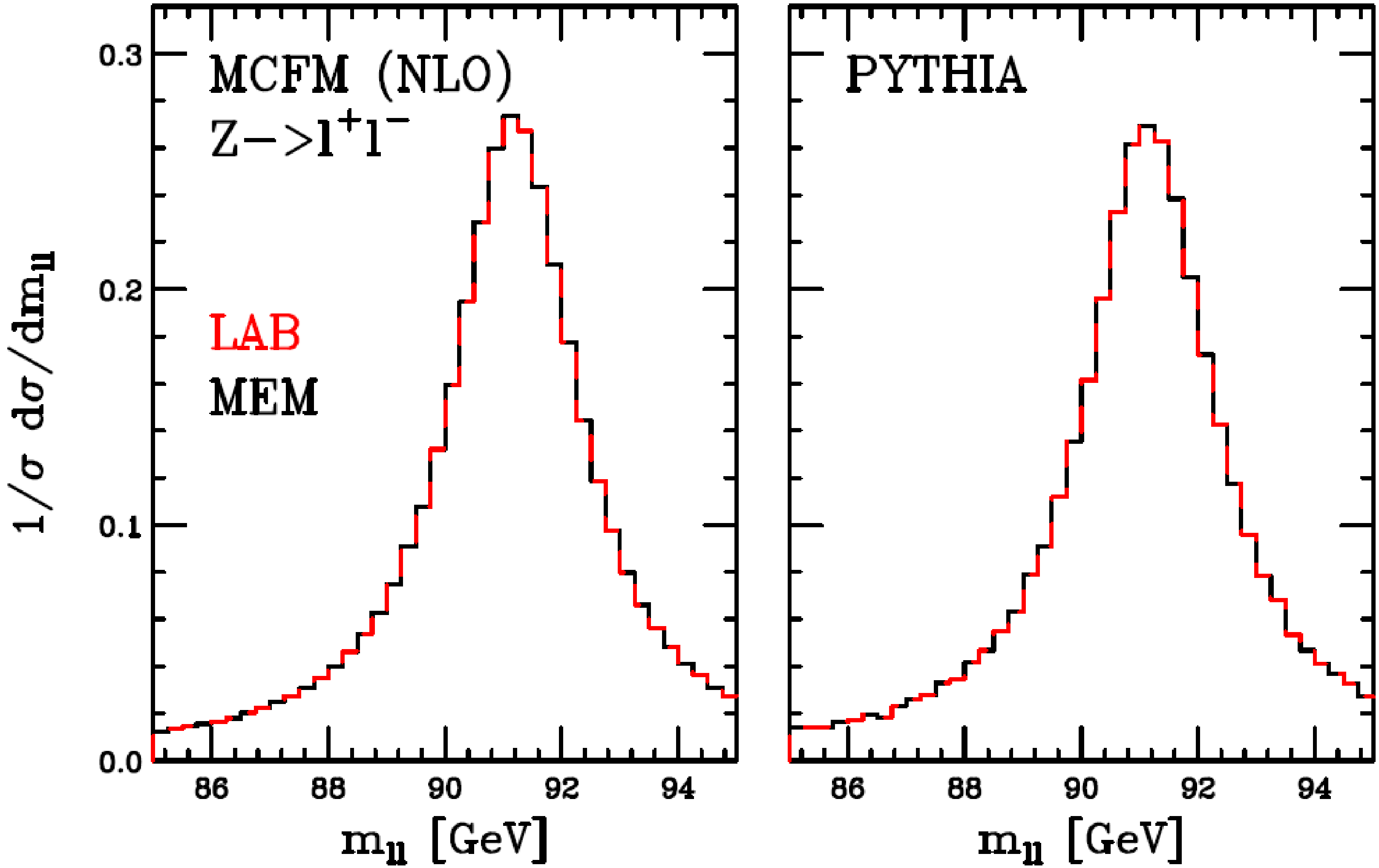
Gluons only...

jets	r-factor	$--++\cdots+$		$---+\cdots++$		$-+-+\cdots-+$	
		$\left\|m^{(0)}\right\|^2$	k-factor	$\left\|m^{(0)}\right\|^2$	k-factor	$\left\|m^{(0)}\right\|^2$	k-factor
2	172 ± 1	1.72216	1.15 ± 0.05	1.6×10^{-31}	$---$	0.00552438	1.09 ± 0.05
3	243 ± 2	120.638	1.13 ± 0.08	0.043632	1.18 ± 0.08	5.98249	1.10 ± 0.08
4	392 ± 3	125.234	1.30 ± 0.13	0.282847	1.17 ± 0.13	0.0498892	1.18 ± 0.13
5	366 ± 4	5941.55	0.94 ± 0.17	849.054	0.87 ± 0.17	31.5083	0.80 ± 0.17
6	529 ± 5	1202.54	1.15 ± 0.24	69.0066	1.06 ± 0.24	0.469815	0.82 ± 0.24
8	650 ± 7	26732.0	1.41 ± 0.34	1364.49	1.32 ± 0.34	1.41604	1.15 ± 0.34
10	844 ± 11	6575.23	1.49 ± 0.49	579.066	1.26 ± 0.49	6.09232×10^{-6}	0.97 ± 0.49
15	1264 ± 20	4690.02	1.39 ± 0.95	671.554	1.28 ± 0.95	4.37178×10^{-7}	1.24 ± 0.95

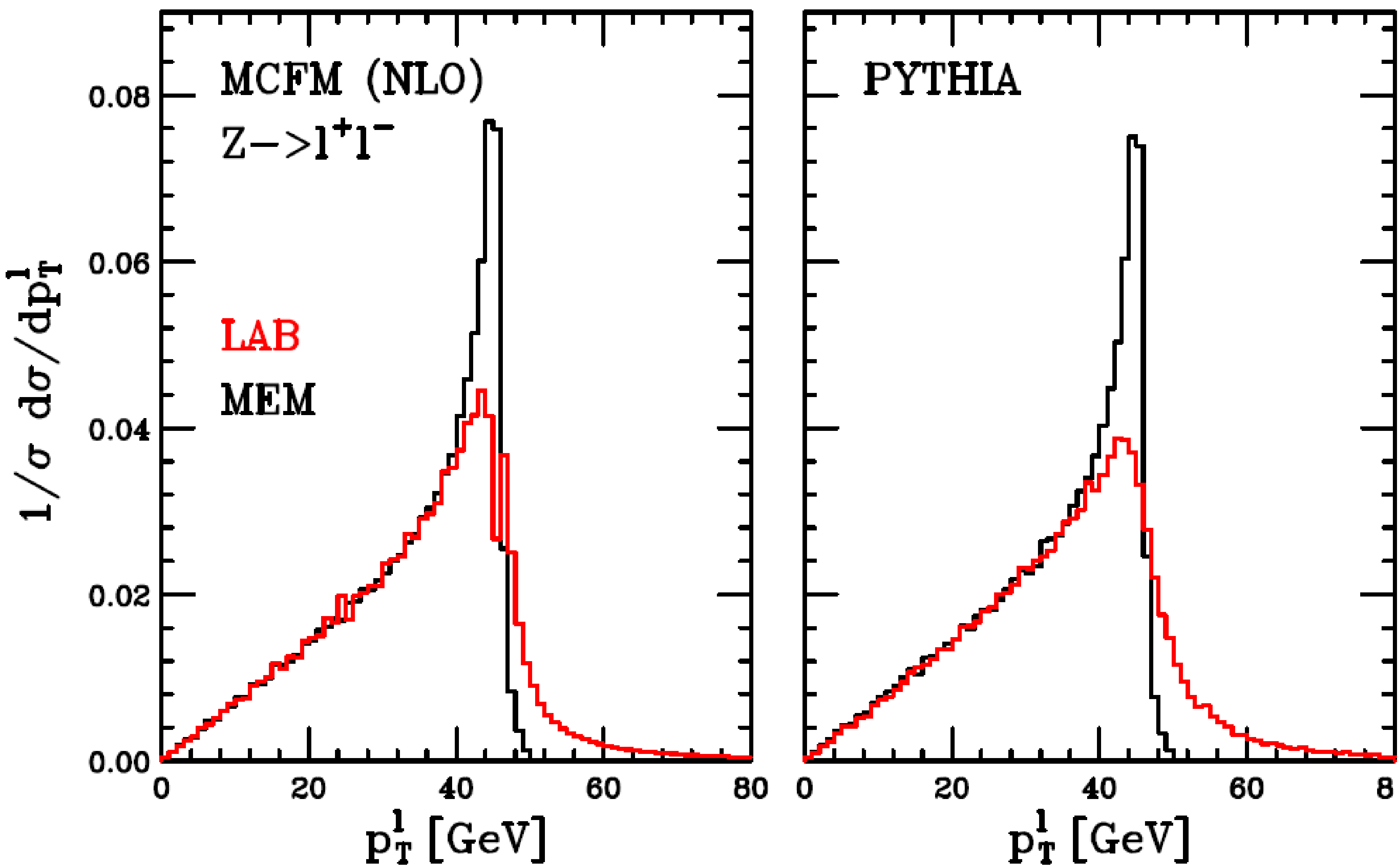
Table 1: The LO ordered amplitude squared $|m^{(0)}(J_a, J_b, J_1, \ldots, J_n)|^2$ and its corresponding $r(s_{\min})$ and ordered k-factor as defined in Eq. (33) for an exclusive n-jet event. The explicit jet momenta for the different jet multiplicities are given in Appendix B. The slicing scale $s_{\min}$ is set to $10^{-4}\times S$ and the Monte Carlo integration over the

Scattering Probabilities for Exclusive Events

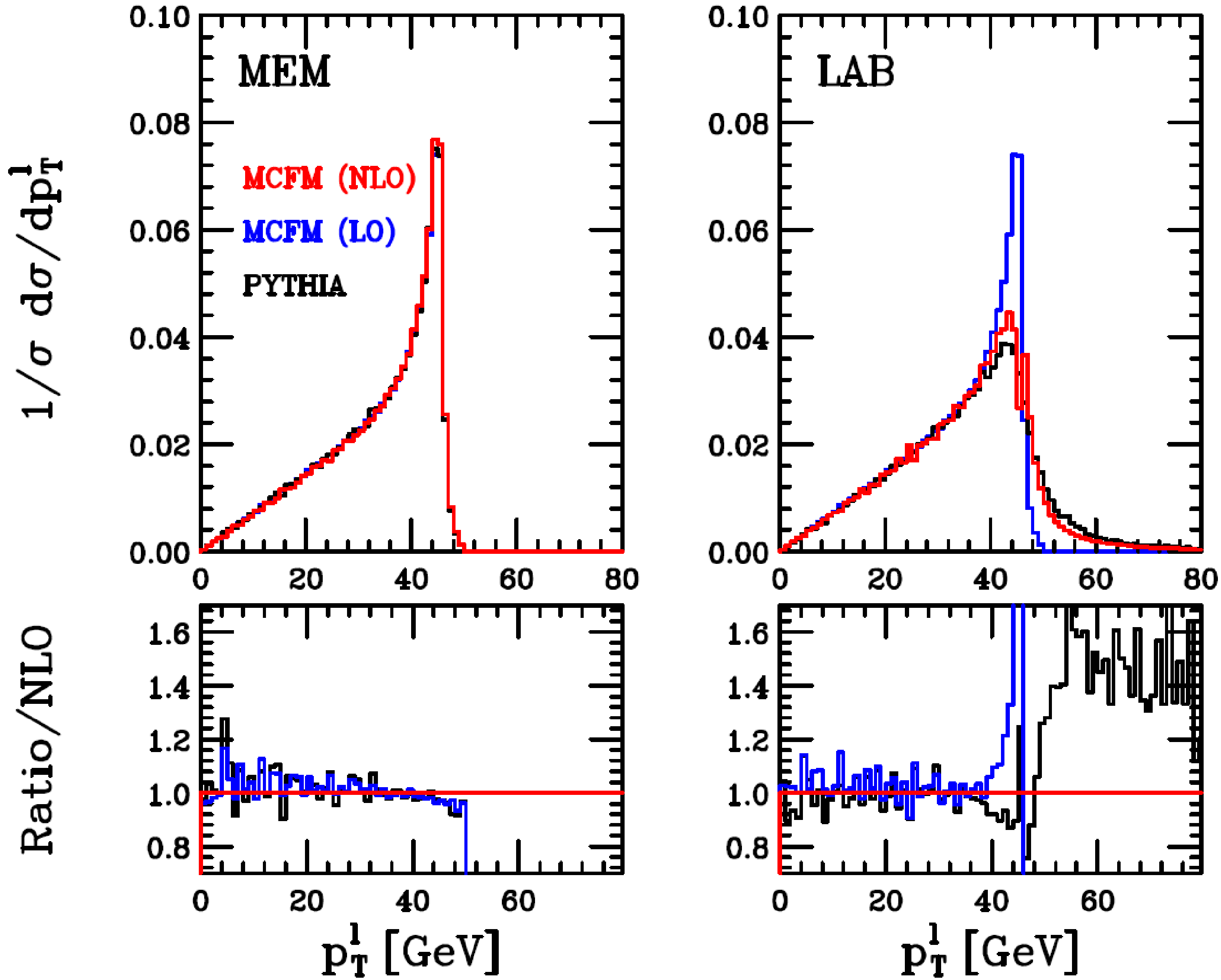
- The phenomenology can be dramatically affected



Scattering Probabilities for Exclusive Events



Scattering Probabilities for Exclusive Events



MEM@NLO

- Now that the exclusive fully differential cross sections are defined it is easy to define the MEM at LO and NLO (without the need of a transfer functions):
- At LO:

$$\mathcal{L}_{ij}(s_{ab}, x_l, x_u) = \int dx_a dx_b \, \frac{f_i(x_a) f_j(x_b)}{x_a x_b s} \, \delta(x_a x_b s - s_{ab})$$

$$\mathcal{P}(\mathbf{x}|\Omega) = \frac{1}{\sigma_\Omega^{LO}} B_\Omega(\mathbf{x}) \qquad B_\Omega(\mathbf{x}) = \mathcal{L}_{ij}(s_{ab}, x_l, x_u) \, \mathcal{B}_\Omega^{ij}(p_a, p_b, \mathbf{x})$$

MEM@NLO

- At NLO:

$$\mathcal{P}(\mathbf{x}|\Omega) = \frac{1}{\sigma_\Omega^{NLO}}\Big(V_\Omega(\mathbf{x}) + R_\Omega(\mathbf{x})\Big)$$

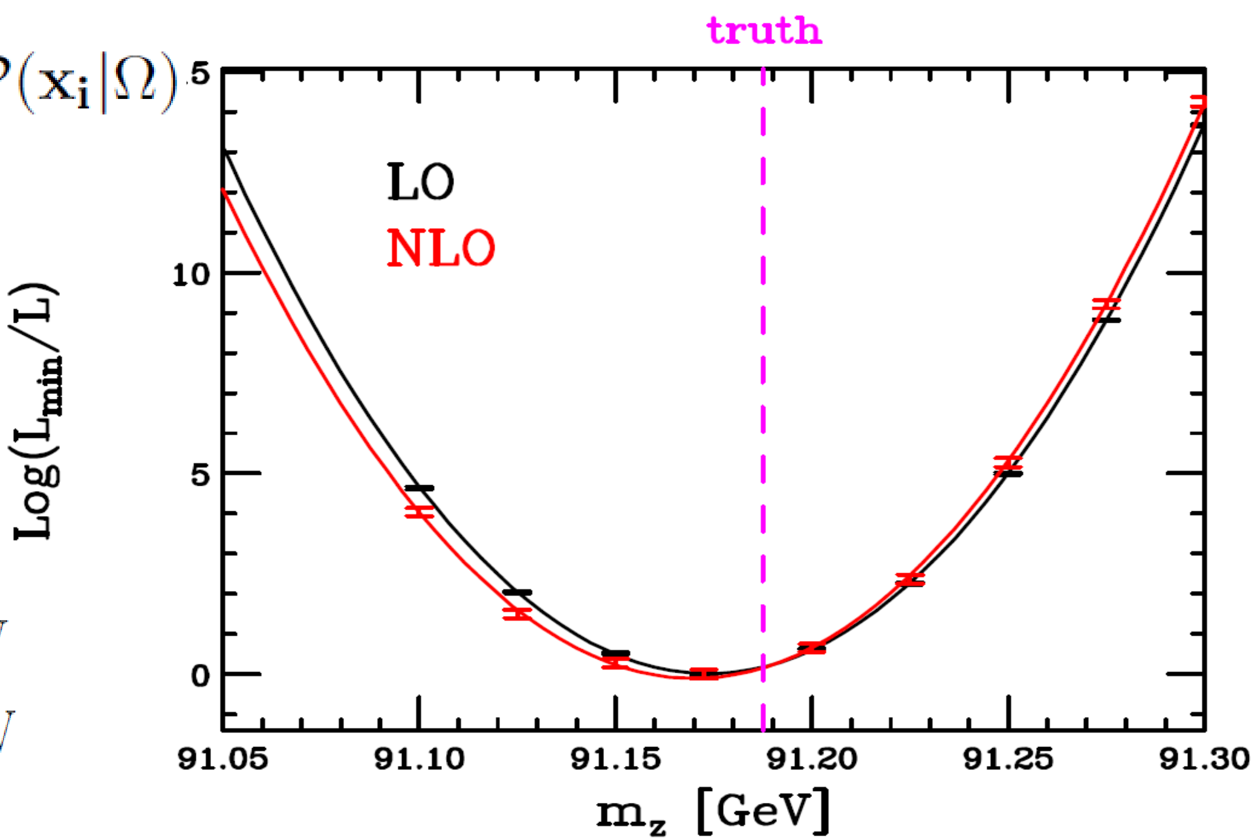
$$V_\Omega(\mathbf{x}) = \mathcal{L}_{ij}(s_{ab}, x_l, x_u)\Big(\mathcal{B}_\Omega^{ij}(p_a, p_b, \mathbf{x}) + \mathcal{V}_\Omega^{ij}(p_a, p_b, \mathbf{x})\Big)$$
$$+ \sum_{m=0}^{2}\int dz\Big(\mathcal{D}_m(z, \mathbf{x}) \otimes \mathcal{L}_m(z, s_{ab}, x_l, x_u)\Big)_{ij}\mathcal{B}_\Omega^{ij}(p_a, p_b, \mathbf{x})$$

$$R_\Omega(\mathbf{x}) = \int d\,\Phi_{\mathrm{FBPS}}(p_a, p_b, p_r)\Big(\mathcal{L}_{ij}(s_{ab}, x_l, x_u)\mathcal{R}_\Omega^{ij}(p_a, p_b, \mathbf{x}, p_r)$$
$$-\sum_m \mathcal{L}_{ij}(s_{ab}, x_l^m, x_u^m)D^m(p_a, p_b, p_r)\mathcal{B}_\Omega^{ij}(\hat{p}_a, \hat{p}_b, \mathbf{x})\Big).$$

Example: MEM for DY

- "Data" is 5000 DY Pythia events at 7 TeV (~0.1 fb) passing the cuts $p_T^\ell > 15\ \mathrm{GeV}, \quad |\eta_\ell| < 2.5, \quad 80\ \mathrm{GeV} < m_{\ell^+\ell^-} < 100\ \mathrm{GeV}$

$$\mathcal{L}(\{\mathbf{x}\}|\Omega) = f(N) \prod_{i=1}^{N} \mathcal{P}(\mathbf{x_i}|\Omega)$$



LO: $m_Z = 91.170 \pm 0.025$ GeV

NLO: $m_Z = 91.174 \pm 0.025$ GeV

Figure 4: Log-Likelihoods obtained by a MEM analysis at LO (black) and NLO (red) for the measurement of m_Z at the LHC using Pythia data. Errors represent MC integration uncertainty.

Example:Higgs Exclusion

- We generate 250 unweighted NLO PP $\rightarrow$ ZZ $\rightarrow$ 4l events satisfying the cuts

$$p_T^{\ell_1,\ell_2} > 20 \text{ GeV} , \quad p_T^{\ell_3,\ell_4} > 5 \text{ GeV} , \quad |\eta_\ell| < 2.0 ,$$

$$15 \text{ GeV} < m_{\ell\bar{\ell}} < 115 \text{ GeV} , \quad 75 \text{ GeV} < m_{\ell'\bar{\ell}'} < 115 \text{ GeV}$$

- We look for a Higgs boson at a given mass in the sample.

the following 95% confidence exclusion range at LO,

$$\text{LO MEM exclusion:} \qquad 120 \text{ GeV} < m_H < 380 \text{ GeV} ,$$

while the NLO MEM provides the more stringent exclusion limit,

$$\text{NLO MEM exclusion:} \qquad 100 \text{ GeV} < m_H < 430 \text{ GeV} .$$

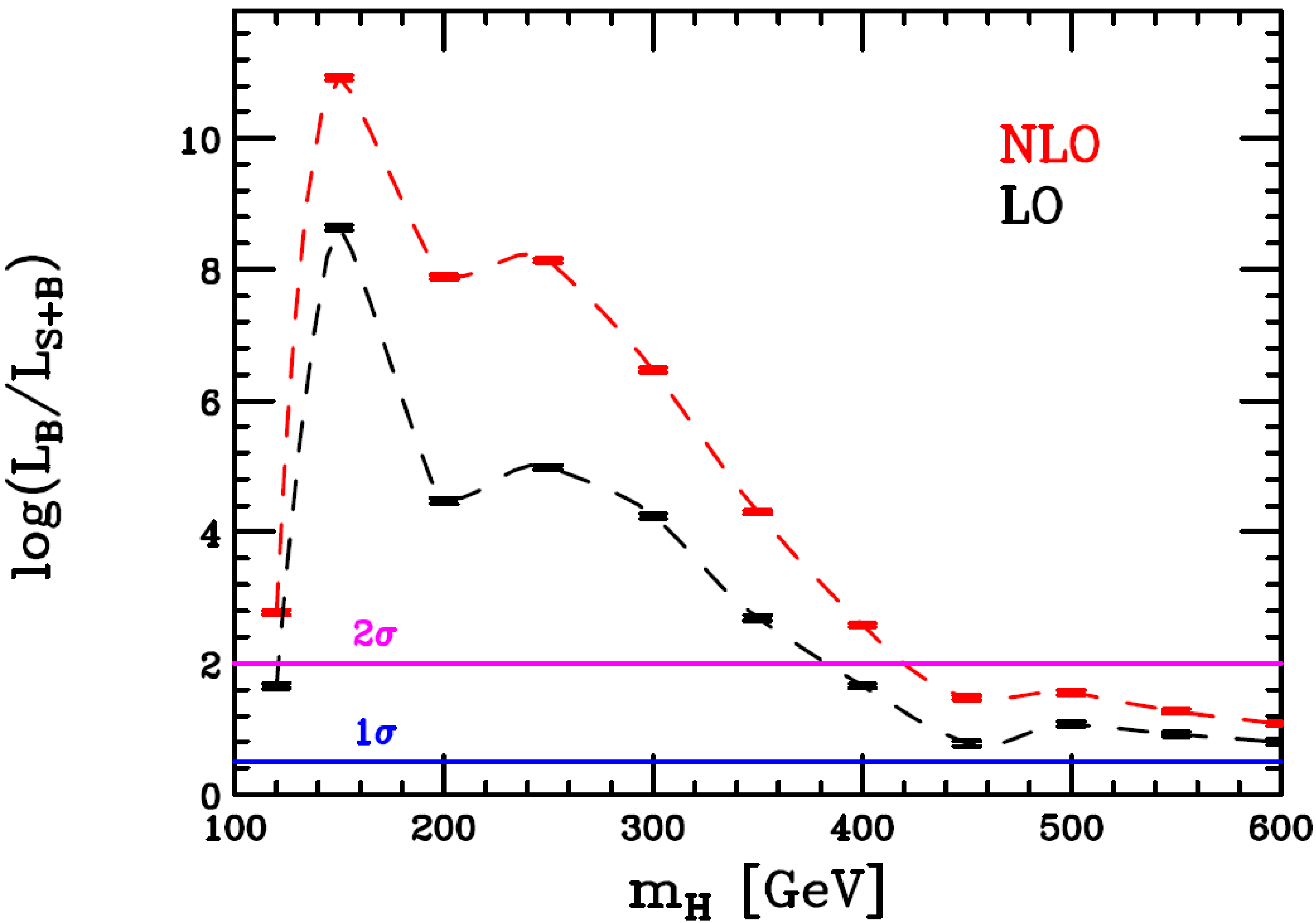


Figure 6: The log-likelihood difference for background only and signal plus background, for a Higgs boson search in the channel, $H \to ZZ^\star \to 4$ leptons. Positive values of the difference indicate that the background-only hypothesis is more likely than the signal plus background one. The blue and magenta lines represent the 1- and 2-σ limits respectively.

Conclusions

- We developed a method to define the MEM at higher order in QCD. This makes the method theoretically well defined
- To do this we had to define the NLO corrections to fully exclusive events. The result is a method where NLO correction to the fully differential event is a K-factor to the LO distribution.
- We made the first step towards phenomenological applications
- More to follow...