

Towards H+jet production at NNLO

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Introduction

Higgs search at the LHC

- ▶ Select events according to jet multiplicity
 - ▶ Different backgrounds for different multiplicity
 - ▶ Optimize signal vs. background
 - ▶ $H+0j$ and $H+1j$ of about equal importance
- ▶ Jet veto
 - ▶ Veto on central jet production to optimize Higgs signal
- ▶ Higgs boson transverse momentum distribution
 - ▶ Important input to signal predictions
 - ▶ Can observe high- p_T Higgs production from boosted jets



Higgs production at LHC

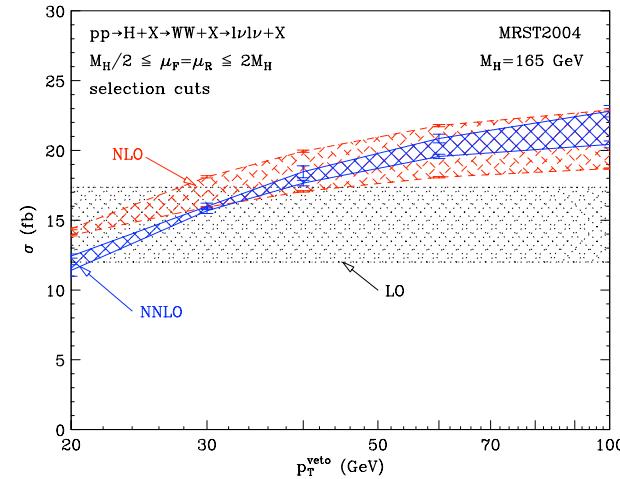
- ▶ Dominant production process: gluon fusion
 - ▶ $gg \rightarrow H$ mediated through top quark loop
 - ▶ Use effective theory with $m_t \rightarrow \infty$

$$\mathcal{L}_{\text{int}} = -\frac{\lambda}{4} H G_a^{\mu\nu} G_{a,\mu\nu}$$

- ▶ yields tree-level couplings Hgg , $Hggg$, $Hgggg$
- ▶ matching coefficients known to 3 loops
(K. Chetyrkin, B. Kniehl, M. Steinhauser)

Higgs production: QCD corrections

- ▶ Gluon fusion: $gg \rightarrow H$
 - ▶ Fixed-order QCD corrections
 - ▶ Fully exclusive NNLO corrections, including decays $H \rightarrow WW, H \rightarrow \gamma \gamma$
(C. Anastasiou, K. Melnikov, F. Petriello; S. Catani, M. Grazzini)
 - ▶ Allow arbitrary final state cuts, including jet veto
 - ▶ NNLO corrections substantial, needed for reliable prediction
 - ▶ Contains $H+j$ at NLO accuracy
 - ▶ Transverse momentum resummation
 - ▶ Known to NNLL+NLO (NNLO for inclusive cross section)
(G. Bozzi, S. Catani, G. Ferrera, M. Grazzini)



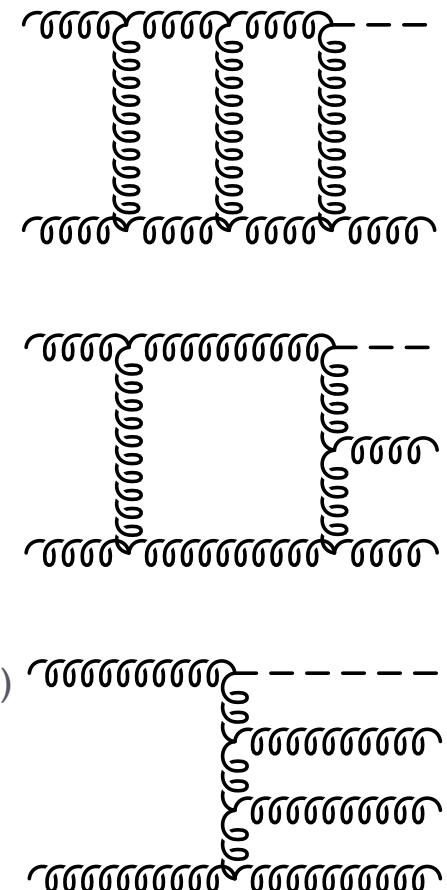
Higgs plus jets production

NLO results

- ▶ **H+1 jet** (D. de Florian, M. Grazzini, Z. Kunszt; V. Ravindran, J. Smith, W. van Neerven)
 - ▶ Jet veto cross section
 - ▶ Higgs boson p_T distribution
- ▶ **H+2 jets**
 - ▶ Vector boson fusion signal
(VBFNLO: D. Zeppenfeld, C. Oleari et al., E. Berger, J. Campbell)
 - ▶ QCD process (J. Campbell, R.K. Ellis, G. Zanderighi)

Ingredients to H+j production at NNLO

- ▶ Two-loop matrix elements: H+3 partons
(M. Jaquier, E.W.N. Glover, A. Koukoutsakis, TG)
 - ▶ Explicit infrared poles from loop integrals
- ▶ One-loop matrix elements: H+4 partons
(S. Badger, E.W.N. Glover, P. Mastrolia, C. Williams; L. Dixon, Y. Sofianatos)
 - ▶ Explicit infrared poles from loop integrals
 - ▶ Implicit infrared poles from single real radiation
- ▶ Tree-level matrix elements: H+5 partons
(V. Del Duca, A. Frizzo, F. Maltoni; S. Badger, L. Dixon, E.W.N. Glover, V.V. Khoze)
 - ▶ Implicit infrared poles from double real radiation



Infrared poles cancel in the sum

Two-loop corrections to $H \rightarrow 3$ partons

Basic helicity amplitudes

▶ $H \rightarrow ggg$

$$|\mathcal{M}_{ggg}^{+++}\rangle = \alpha \frac{1}{\sqrt{2}} \frac{M_H^4}{\langle p_1 p_2 \rangle \langle p_2 p_3 \rangle \langle p_3 p_1 \rangle}, \quad |\mathcal{M}_{ggg}^{++-}\rangle = \beta \frac{1}{\sqrt{2}} \frac{[p_1 p_2]^3}{[p_2 p_3][p_1 p_3]}$$

▶ $H \rightarrow q\bar{q}g$

$$|\mathcal{M}_{q\bar{q}g}^{-++}\rangle = \gamma \frac{1}{\sqrt{2}} \frac{[p_2 p_3]^2}{[p_1 p_2]}$$

- ▶ Leading order: $\alpha = \beta = \gamma = 1$
- ▶ Extract helicity coefficients from matrix elements using d -dimensional projectors

Two-loop corrections to $H \rightarrow 3$ partons

Feynman Diagrams

- ▶ $H \rightarrow ggg$
 - ▶ tree level: 4, one loop: 60, two loops: 1306
- ▶ $H \rightarrow q\bar{q}g$
 - ▶ tree level: 1, one loop: 15, two loops: 228
- ▶ Diagram generation: QGRAF/FORM (P. Nogueira/J. Vermaseren)
- ▶ Four-point functions with one off-shell leg
 - ▶ Up to tensor rank 5 at two loops
 - ▶ Reduction with to master integrals: REDUZE (C. Studerus)
 - ▶ Two-loop master integrals from $\gamma^* \rightarrow q\bar{q}g$ (E. Remiddi, TG)
 - ▶ Analytically continued to scattering kinematics (E. Remiddi, TG)

Two-loop corrections to $H \rightarrow 3$ partons

- ▶ Infrared structure given by Catani formula (S. Catani)

- ▶ one loop:

$$\Omega^{(1)} = I_{\Omega}^{(1)}(\epsilon) \Omega^{(0)} + \Omega^{(1),finite}$$

- ▶ two loops:

$$\begin{aligned} \Omega^{(2)} = & \left(-\frac{1}{2} I_{\Omega}^{(1)}(\epsilon) I_{\Omega}^{(1)}(\epsilon) - \frac{\beta_0}{\epsilon} I_{\Omega}^{(1)}(\epsilon) + e^{-\epsilon\gamma} \frac{\Gamma(1-2\epsilon)}{\Gamma(1-\epsilon)} \left(\frac{\beta_0}{\epsilon} + K \right) I_{\Omega}^{(1)}(2\epsilon) + H_{\Omega}^{(2)}(\epsilon) \right) \Omega^{(0)} \\ & + I_{\Omega}^{(1)}(\epsilon) \Omega^{(1)} + \Omega^{(2),finite} \end{aligned}$$

- ▶ With infrared singularity operators

$$I_{ggg}^{(1)}(\epsilon) = -\frac{e^{\epsilon\gamma}}{2\Gamma(1-\epsilon)} \left[N \left(\frac{1}{\epsilon^2} + \frac{\beta_0}{N\epsilon} \right) (S_{12} + S_{13} + S_{23}) \right]$$

$$I_{q\bar{q}g}^{(1)}(\epsilon) = -\frac{e^{\epsilon\gamma}}{2\Gamma(1-\epsilon)} \left[N \left(\frac{1}{\epsilon^2} + \frac{3}{4\epsilon} + \frac{\beta_0}{2N\epsilon} \right) (S_{13} + S_{23}) - \frac{1}{N} \left(\frac{1}{\epsilon^2} + \frac{3}{2\epsilon} \right) S_{12} \right]$$

Two-loop corrections to $H \rightarrow 3$ partons

- ▶ **Finite parts** (M. Jaquier, E.W.N. Glover, A. Koukoutsakis, TG)
 - ▶ about 50k of HPLs and 2dHPL, 22 pages
- ▶ **Further simplifications using calculus of symbols**
(A. Goncharov, M. Spradlin, A. Volovich, C. Vergu)
 - ▶ **Leading transcendentality in $H \rightarrow ggg$ from N=4 SYM**
 - ▶ is compact and helicity-independent (A. Brandhuber, G. Travaglini, G. Yang)
 - ▶ **Full $H \rightarrow ggg$ amplitudes fit on two pages, e.g. $N_F N$** (C. Duhr)

$$\begin{aligned} \overline{D}_\alpha^{(2)} = & -\zeta_3 + \frac{i\pi}{4} - \frac{1}{6} (x_1 x_2 + x_3 x_2 + x_1 x_3) + \frac{67}{48} + \frac{P_5(x_1, x_2, x_3)}{72 x_1^2 x_2^2 x_3^2} \pi^2 \\ & + \frac{1}{12} \sum_{i=1}^3 \left[\frac{P_6(x_i, x_{i-1}, x_{i+1})}{x_{i-1}^2 x_{i+1}^2} \text{Li}_2(1-x_i) + \frac{P_7(x_i, x_{i-1}, x_{i+1})}{x_i^2} \ln x_{i-1} \ln x_{i+1} \right. \\ & \quad \left. + \frac{P_8(x_i, x_{i-1}, x_{i+1})}{2 x_{i-1} x_{i+1}} \ln x_i \right] \end{aligned}$$

NNLO Infrared Subtraction

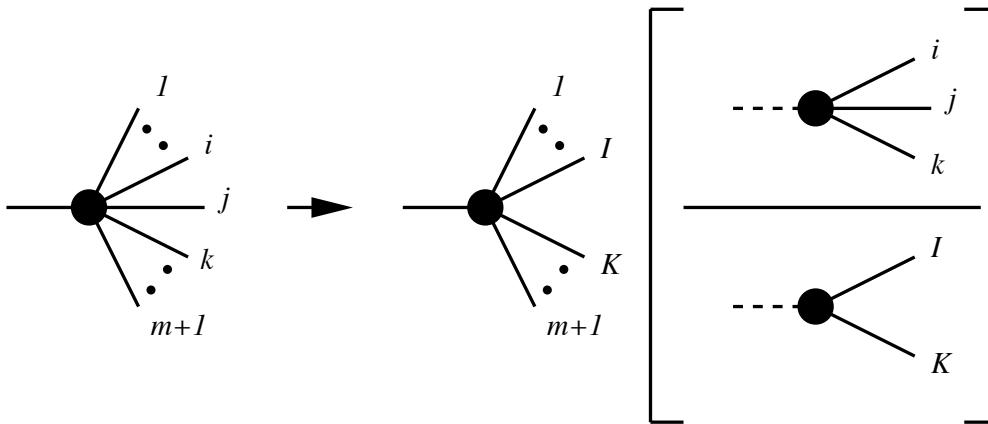
Structure of NNLO cross section

$$\begin{aligned} d\sigma_{NNLO} = & \int_{d\Phi_{m+2}} (d\sigma_{NNLO}^R - d\sigma_{NNLO}^S) \\ & + \int_{d\Phi_{m+1}} (d\sigma_{NNLO}^{V,1} - d\sigma_{NNLO}^{VS,1}) + \int_{d\Phi_{m+1}} d\sigma_{NNLO}^{MF,1} \\ & + \int_{d\Phi_m} d\sigma_{NNLO}^{V,2} + \int_{d\Phi_{m+2}} d\sigma_{NNLO}^S + \int_{d\Phi_{m+1}} d\sigma_{NNLO}^{VS,1} + \int_{d\Phi_m} d\sigma_{NNLO}^{MF,2} \end{aligned}$$

- ▶ Real and virtual contributions: $d\sigma_{NNLO}^R, d\sigma_{NNLO}^{V,1}, d\sigma_{NNLO}^{V,2}$
- ▶ Subtraction term for double real radiation: $d\sigma_{NNLO}^S$
- ▶ Subtraction term for one-loop single real radiation: $d\sigma_{NNLO}^{VS,1}$
- ▶ Mass factorization terms: $d\sigma_{NNLO}^{MF,1}, d\sigma_{NNLO}^{MF,2}$
- ▶ Each line finite and free of poles
→ numerical implementation

Antenna subtraction

- ▶ Subtraction terms constructed from antenna functions
 - ▶ Antenna function contains all emission between two partons



- ▶ Phase space factorization

$$d\Phi_{m+1}(p_1, \dots, p_{m+1}; q) = d\Phi_m(p_1, \dots, \tilde{p}_I, \tilde{p}_K, \dots, p_{m+1}; q) \cdot d\Phi_{X_{ijk}}(p_i, p_j, p_k; \tilde{p}_I + \tilde{p}_K)$$

- ▶ Integrated subtraction term

$$\mathcal{X}_{ijk} = \int d\Phi_{X_{ijk}} X_{ijk}$$

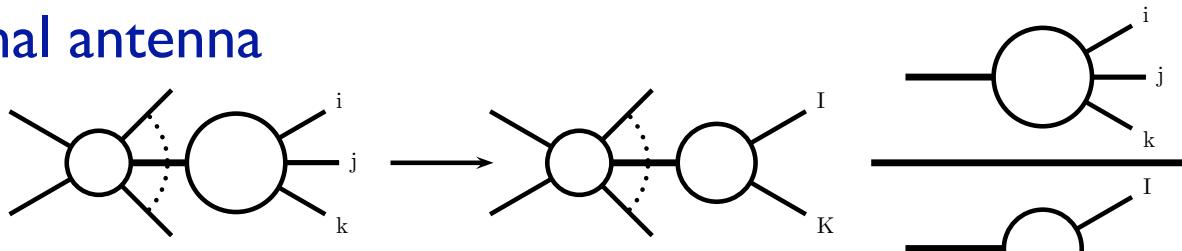
Antenna functions

- ▶ Colour-ordered pair of hard partons (radiators)
 - ▶ Hard quark-antiquark pair: A,B,C
 - ▶ Hard quark-gluon pair: D,E
 - ▶ Hard gluon-gluon pair: F,G,H
- ▶ NLO (D. Kosower; J. Campbell, M. Cullen, E.W.N. Glover)
 - ▶ Three-parton antenna: one unresolved parton X_3^0
- ▶ NNLO (A. Gehrmann-De Ridder, E.W.N. Glover, TG)
 - ▶ Four-parton antenna: two unresolved partons X_4^0
 - ▶ Three-parton antenna at one loop: X_3^1
 - ▶ Products of NLO antenna functions: $X_3^0 \otimes X_3^0$
 - ▶ Soft antenna function S

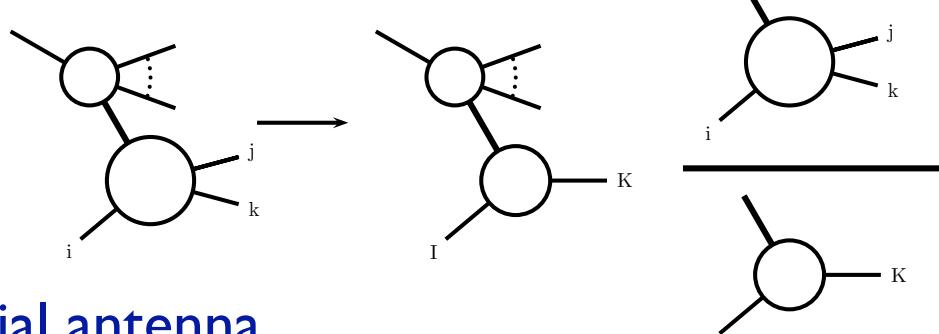
Antenna subtraction: incoming hadrons

- ▶ Three antenna types (NLO:A. Daleo, D. Maitre, TG)

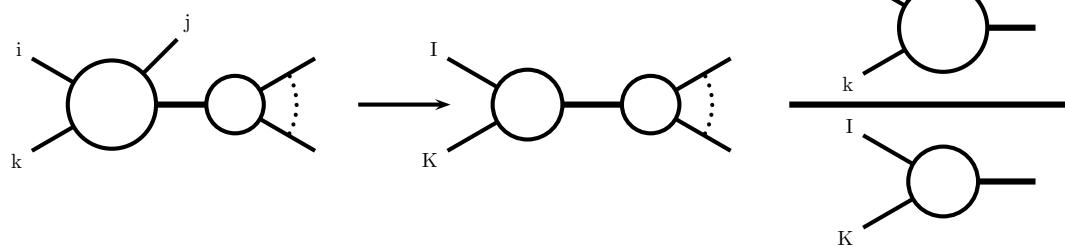
- ▶ Final-final antenna



- ▶ Initial-final antenna



- ▶ Initial-initial antenna



Final-final antenna functions

- ▶ Were derived in the calculation of NNLO corrections to $e^+e^- \rightarrow 3j$ (A. Gehrmann-De Ridder, E.W.N. Glover, G. Heinrich, TG)
- ▶ Unintegrated antennae from physical matrix elements
 - ▶ quark-antiquark antennae from $\gamma^* \rightarrow q\bar{q} + X$
 - ▶ quark-gluon antennae from $\tilde{\chi} \rightarrow \tilde{g}g + X$
 - ▶ gluon-gluon antennae from $H \rightarrow gg + X$
- ▶ Integrated antennae: inclusive $l \rightarrow 3$ and $l \rightarrow 4$ phase space integrals
- ▶ Initial-final and initial-initial antenna functions obtained by crossing

Integrated NNLO antenna functions

- ▶ Analytical integration over unresolved part of phase space only
 - ▶ phase space integrals reduced to masters (C.Anastasiou, K. Melnikov)
 - ▶ Final-final: $q \rightarrow k_1 + k_2 + k_3 (+k_4)$, one scale: q^2
 - ▶ $1 \rightarrow 4$ tree level (4 master integrals)
 - ▶ $1 \rightarrow 3$ one loop (3 master integrals)
 - ▶ Initial-final: $q + p_1 \rightarrow k_1 + k_2 (+k_3)$, two scales: q^2, x
(A. Daleo, A. Gehrmann-De Ridder, G. Luisini, TG)
 - ▶ $2 \rightarrow 3$ tree level (9 master integrals)
 - ▶ $2 \rightarrow 2$ one loop (6 master integrals)
 - ▶ Initial-initial: $p_1 + p_2 \rightarrow q + k_1 (+k_2)$, three scales: q^2, x_1, x_2
 - ▶ $2 \rightarrow 3$ tree level (22 master integrals)
 - ▶ $2 \rightarrow 2$ one loop (5 master integrals)

Initial-initial antenna functions

- ▶ One-loop antenna functions X_3^{\perp}

- ▶ Kinematics

$$p_1 + p_2 \rightarrow q + k_j; \quad q^2 > 0$$

- ▶ Phase space factorization

$$\begin{aligned} d\Phi_{m+1}(k_1, \dots, k_{m+1}; p_1, p_2) = & d\Phi_m(\tilde{k}_1, \dots, \tilde{k}_i, \tilde{k}_k, \dots, \tilde{k}_{m+1}; x_1 p_1, x_2 p_2) \\ & \delta(x_1 - \hat{x}_1) \delta(x_2 - \hat{x}_2) [dk_j] dx_1 dx_2 \end{aligned}$$

$$\hat{x}_1 = \left(\frac{s_{12} - s_{j2}}{s_{12}} \frac{s_{12} - s_{1j} - s_{j2}}{s_{12} - s_{1j}} \right)^{\frac{1}{2}} \quad \hat{x}_2 = \left(\frac{s_{12} - s_{1j}}{s_{12}} \frac{s_{12} - s_{1j} - s_{j2}}{s_{12} - s_{j2}} \right)^{\frac{1}{2}}$$

- ▶ Phase space integral overconstrained, expand in distributions
(P.F. Monni, TG)

Initial-initial antenna functions

- ▶ are crossings of final-final antennae: four-parton case

quark-antiquark antennae

$$A_4^0 \quad A_4^0(\hat{q}, \hat{g}, g, \bar{q}), \quad A_4^0(\hat{q}, g, \hat{g}, \bar{q}), \quad A_4^0(\hat{q}, g, g, \hat{\bar{q}}), \quad A_4^0(q, \hat{g}, \hat{g}, \bar{q})$$

$$\tilde{A}_4^0 \quad \tilde{A}_4^0(\hat{q}, \hat{g}, g, \bar{q}), \quad \tilde{A}_4^0(\hat{q}, g, g, \hat{\bar{q}}), \quad \tilde{A}_4^0(q, \hat{g}, \hat{g}, \bar{q})$$

$$B_4^0 \quad B_4^0(\hat{q}, \hat{q}', \bar{q}', \bar{q}), \quad B_4^0(\hat{q}, q', \bar{q}', \hat{\bar{q}}), \quad B_4^0(q, \hat{q}', \hat{\bar{q}}', \bar{q})^*$$

$$C_4^0 \quad C_4^0(\hat{q}, \hat{\bar{q}}, q, \bar{q}), \quad C_4^0(\hat{q}, \bar{q}, \hat{q}, \bar{q}), \quad C_4^0(q, \hat{\bar{q}}, \hat{q}, \bar{q})^*, \quad C_4^0(q, \bar{q}, \hat{q}, \hat{\bar{q}})^*$$

quark-gluon antennae

$$D_4^0 \quad D_4^0(\hat{q}, \hat{g}, g, g), \quad D_4^0(\hat{q}, g, \hat{g}, g), \quad D_4^0(q, \hat{g}, \hat{g}, g), \quad D_4^0(q, \hat{g}, g, \hat{g})$$

$$E_4^0 \quad E_4^0(\hat{q}, \hat{q}', \bar{q}', g), \quad E_4^0(\hat{q}, q', \bar{q}', \hat{g}), \quad E_4^0(q, \hat{q}', \hat{\bar{q}}', g), \quad E_4^0(q, \hat{q}', \bar{q}', \hat{g}),$$

$$\tilde{E}_4^0 \quad \tilde{E}_4^0(\hat{q}, \hat{q}', \bar{q}', g), \quad \tilde{E}_4^0(\hat{q}, q', \bar{q}', \hat{g}), \quad \tilde{E}_4^0(q, \hat{q}', \hat{\bar{q}}', g), \quad \tilde{E}_4^0(q, \hat{q}', \bar{q}', \hat{g})$$

gluon-gluon antennae

$$F_4^0 \quad F_4^0(\hat{g}, \hat{g}, g, g), \quad F_4^0(\hat{g}, g, \hat{g}, g)$$

$$G_4^0 \quad G_4^0(\hat{g}, \hat{q}, \bar{q}, g), \quad G_4^0(\hat{g}, q, \hat{\bar{q}}, g), \quad G_4^0(\hat{g}, q, \bar{q}, \hat{g}), \quad G_4^0(g, \hat{q}, \hat{\bar{q}}, g)$$

$$\tilde{G}_4^0 \quad \tilde{G}_4^0(\hat{g}, \hat{q}, \bar{q}, g), \quad \tilde{G}_4^0(\hat{g}, q, \bar{q}, \hat{g}), \quad \tilde{G}_4^0(g, \hat{q}, \hat{\bar{q}}, g)$$

$$H_4^0 \quad H_4^0(\hat{q}, \hat{\bar{q}}, q', \bar{q}'), \quad H_4^0(\hat{q}, \bar{q}, \hat{q}', \bar{q}')$$

Initial-initial antenna functions

▶ Tree-level antenna functions X_4^0

▶ Kinematics

$$p_1 + p_2 \rightarrow q + k_j + k_k$$

▶ Phase space factorization

$$d\Phi_{m+2}(k_1, \dots, k_{m+2}; p_1, p_2) = d\Phi_m(\tilde{k}_1, \dots, \tilde{k}_i, \tilde{k}_l, \dots, \tilde{k}_{m+1}; x_1 p_1, x_2 p_2)$$

$$\delta(x_1 - \hat{x}_1) \delta(x_2 - \hat{x}_2) [dk_j] [dk_k] dx_1 dx_2$$

$$\hat{x}_1 = \left(\frac{s_{12} - s_{j2} - s_{k2}}{s_{12}} \frac{s_{12} - s_{1j} - s_{1k} - s_{j2} - s_{k2} + s_{jk}}{s_{12} - s_{1j} - s_{1k}} \right)^{\frac{1}{2}}$$

$$\hat{x}_2 = \left(\frac{s_{12} - s_{1j} - s_{1k}}{s_{12}} \frac{s_{12} - s_{1j} - s_{1k} - s_{j2} - s_{k2} + s_{jk}}{s_{12} - s_{j2} - s_{k2}} \right)^{\frac{1}{2}}$$

- ▶ Integration: three-particle phase space integrals with fixed q^2, x_1, x_2
- ▶ Similar to NNLO coefficients for Drell-Yan rapidity distribution
(C.Anastasiou, L.Dixon, K.Melnikov, F.Petriello)

Initial-initial antenna functions

- ▶ Integration of tree-level antenna functions X_4^0
 - ▶ Express phase space integrals as masters with x_1, x_2 fixed
 - ▶ Distinguish
 - ▶ Hard region $x_1, x_2 \neq 1$: transcendentality 2
 - ▶ Collinear regions $x_1=1, x_2 \neq 1$ or $x_1 \neq 1, x_2=1$: transcendentality 3
 - ▶ Soft region: $x_1=x_2=1$: transcendentality 4
 - ▶ Determine master integrals from differential equations in x_1, x_2
 - ▶ Antenna functions with secondary fermion pair: only 12 masters
(R. Boughezal, A. Gehrmann-De Ridder, M. Ritzmann)
 - ▶ Full set of antennae now completed: contains 22 masters
(A. Gehrmann-De Ridder, M. Ritzmann, TG)
 - ▶ Observe uniform soft poles in all three kinematical situations
(S. Catani, M. Grazzini)
 - ▶ Non-abelian antennae: $3/4/e^4$, abelian antennae: $1/e^4$

Integrated antenna functions

Three-parton tree-level and one loop

	Final-final	Initial-final	Initial-initial
$A_3^0, D_3^0, E_3^0, F_3^0, G_3^0$	[1]	[2]	[2]
$\hat{A}_3^!, \tilde{A}_3^!, \hat{\bar{A}}_3^!, D_3^!,$ $\hat{D}_3^!, E_3^!, \tilde{E}_3^!, \hat{\bar{E}}_3^!, F_3^!,$ $F_3^!, G_3^!, \bar{G}_3^!, \tilde{G}_3^!$	[1]	[3]	[4]
S	[5]	[6]	[6]

[1] A. Gehrmann-De Ridder, E.W.N. Glover, TG

[2] A. Daleo, D. Maitre, TG

[3] A. Daleo, A. Gehrmann-De Ridder, G. Luisoni, TG

[4] P.F. Monni, TG

[5] S. Weinzierl; A. Gehrmann-De Ridder, E.W.N. Glover, G. Heinrich, TG

[6] A. Gehrmann-De Ridder, J. Pires, E.W.N. Glover

Integrated antenna functions

Four-parton tree-level

X_4^0	Final-final	Initial-final	Initial-initial
A, \tilde{A}	[1]	[2]	[4]
B, E, \tilde{E} , H	[1]	[2]	[3]
C	[1]	[2]	[4]
D	[1]	[2]	[4]
F	[1]	[2]	[4]
G, \tilde{G}	[1]	[2]	[4]

[1] A. Gehrmann-De Ridder, E.W.N. Glover, TG

[2] A. Daleo, A. Gehrmann-De Ridder, G. Luisoni, TG

[3] R. Boughezal, A. Gehrmann-De Ridder, M. Ritzmann

[4] A. Gehrmann-De Ridder, M. Ritzmann, TG

Integrated antenna functions

- ▶ All-gluon contribution to $H+j$ production
 - ▶ Initial-final antenna functions
 - ▶ F_3^1, F_4^0
 - ▶ Initial-initial antenna functions
 - ▶ F_3^1, F_4^0 (colour adjacent), F_4^0 (colour non-adjacent)
- ▶ All-gluon contribution to $2j$ production (talk of J. Pires)
(A. Gehrmann-De Ridder, E.W.N. Glover, J. Pires, TG)
 - ▶ Final-final and initial-final antenna functions
 - ▶ F_3^1, F_4^0
 - ▶ Initial-initial antenna functions
 - ▶ F_3^1, F_4^0 (colour adjacent), F_4^0 (colour non-adjacent)
 - ▶ Observe analytical cancellation of all infrared poles in the colour-adjacent and colour non-adjacent $2 \rightarrow 2$ processes for $2j$

Conclusions

- ▶ Higgs-plus-jet important for Higgs studies and searches
 - ▶ Samples divided according to jet multiplicity
 - ▶ Boosted Higgs studies
 - ▶ Jet veto
- ▶ Computed two-loop corrections to $H \rightarrow 3$ partons
- ▶ Calculation of NNLO corrections: antenna subtraction
 - ▶ Completed integration of NNLO antenna functions for hadron-hadron collisions: method now fully established
 - ▶ Prove analytical cancellation of poles in the all-gluon 2j production at NNLO
- ▶ Implementation in progress