N_c^2 corrections to the Higgs mass in the MSSM

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DESY Theory

Overview

1. The calculation
2. Aspects of two-loop renormalisation
3. The numerical results
4. Summary & Outlook

1. The calculation

The pole mass of a particle

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- two-loop pole mass

$$M_h^2 = m_h^2 - \mathrm{Re}\,\hat{\Sigma}_{hh}^{(1)}(m_h^2) - \mathrm{Re}\,\hat{\Sigma}_{hh}^{(2)}(m_h^2) + \mathrm{Re}\left\{\hat{\Sigma}_{hh}^{(1)}(m_h^2)\partial\hat{\Sigma}_{hh}^{(1)}(m_h^2)\right\} - \mathrm{Re}\,\frac{\left(\hat{\Sigma}_{hH}^{(1)}(m_h^2)\right)^2}{m_H^2 - m_h^2}$$

Current status and goal

- Known contributions:
  - full one-loop $\mathcal{O}(\alpha_{\text{em}} + \alpha_t + \alpha_b)$
  - full two-loop QCD $\mathcal{O}((\alpha_{\text{em}} + \alpha_t + \alpha_b)\alpha_s)$
  - full two-loop third generation Yukawa $\mathcal{O}\big((\alpha_t + \alpha_b)^2\big)$
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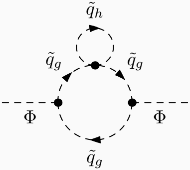
  - full two-loop third generation Yukawa $\mathcal{O}\big((\alpha_t + \alpha_b)^2\big)$
  - some three-loop parts
- We calculate $\mathcal{O}\big((\alpha_{\text{em}} + \alpha_t + \alpha_b)^2 N_c^2\big)$ contributions

The Feynman diagrams



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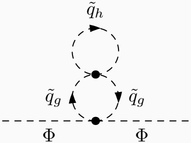

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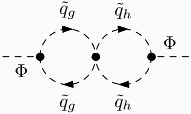

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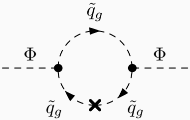

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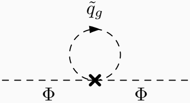

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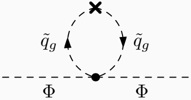

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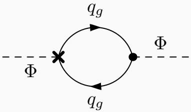

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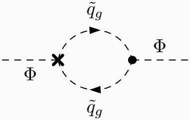

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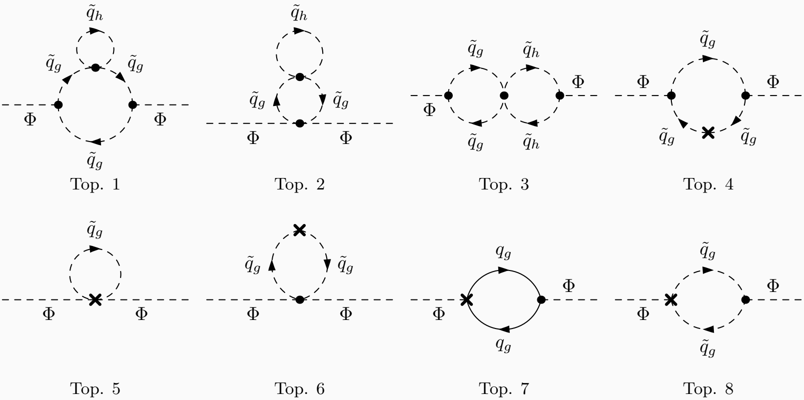

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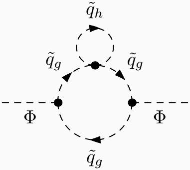

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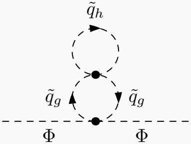

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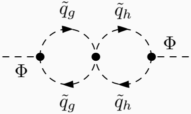

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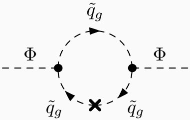

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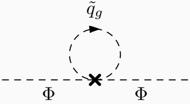

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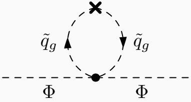

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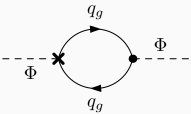

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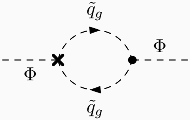

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- Third generation only: 5322 diagrams
- All generations: 40446 diagrams

Getting started

- diagrams (unrenormalised SEs) generated with FeynArts
- calculated with FormCalc, OneCalc, TwoCalc in DRED regularisation
- set $\alpha_s = 0$, $m_u = m_d = m_c = m_s = 0$, and take N_c^2 terms

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- set $\alpha_s = 0$, $m_u = m_d = m_c = m_s = 0$, and take N_c^2 terms
- renormalise SEs, e.g.:

$$\hat{\Sigma}^{(2)}_{AA} = \Sigma^{(2)}_{AA}(p^2) + \delta^{(2)} Z_{AA}(p^2 - m_A^2) - \delta^{(2)} m_A^2 \\ + \tfrac{1}{4}\left(\delta^{(1)} Z_{AG}\right)^2 p^2 - \delta^{(1)} Z_{AA} \delta^{(1)} m_A^2 - \delta^{(1)} Z_{AG} \delta^{(1)} m_{AG}^2$$

- ~ 70 one-loop CTs & 14 two-loop CTs

2. Aspects of two-loop renormalisation

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- We rederived the relation from Higgs potential before min. conditions ($T_h = T_H = T_A = 0$; $\beta_\mathrm{n} = \beta_\mathrm{c} = \beta$) are applied
- Correct version:
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- Combination:
  $\delta^{(2)} m^2_{H^\pm} + \delta^{(2)} m^2_{G^\pm} = \delta^{(2)} m^2_A + \delta^{(2)} m^2_G + \delta^{(2)} M^2_W$

The relation to all orders

- Mass term in Higgs potential for gauge eigenstates:

$$\mathcal{L}_{\text{Higgs}} \supset -\frac{1}{2}\begin{pmatrix}\phi_1 & \phi_2 & \chi_1 & \chi_2\end{pmatrix}\begin{pmatrix}\mathbf{M}^2_{\phi\phi} & \mathbf{M}^2_{\phi\chi} \\ \mathbf{M}^2_{\chi\phi} & \mathbf{M}^2_{\chi\chi}\end{pmatrix}\begin{pmatrix}\phi_1 \\ \phi_2 \\ \chi_1 \\ \chi_2\end{pmatrix}$$

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  $m^2_{H^\pm} + m^2_{G^\pm} = m^2_A + m^2_G + M^2_W$
- With ren. transformation we obtain to all orders:
  $\delta^{(n)} m^2_{H^\pm} + \delta^{(n)} m^2_{G^\pm} = \delta^{(n)} m^2_A + \delta^{(n)} m^2_G + \delta^{(n)} M^2_W$

Renormalisation at the two-loop level

- On-shell: $\delta^{(2)}T_i$, $\delta^{(2)}M_W^2$, $\delta^{(2)}M_Z^2$, $\delta^{(2)}m_A^2$
- $\overline{\mathrm{DR}}$: $\delta^{(2)}Z_{\mathcal{H}_1}$, $\delta^{(2)}Z_{\mathcal{H}_2}$

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- Two schemes: $\delta^{(2)}t_\beta$ (OS, $\overline{\mathrm{DR}}$)
- $t_\beta^{\overline{\mathrm{DR}}}$ from demanding the finiteness of $\hat{\Sigma}_{AZ}^{L,(2)}$:

$$\begin{aligned}\delta^{(2)}t_\beta^{\mathrm{DR}} = &\frac{1}{\mathrm{i}c_\beta^2 M_Z}\left[\Sigma_{AZ}^{L,(2)}\right]_{\mathrm{Div}} - \frac{\delta^{(2)}Z_{AG}^{\mathrm{DR}}}{2c_\beta^2} \\ &+ c_\beta s_\beta\left(\delta^{(1)}t_\beta^{\mathrm{DR}}\right)^2 - \tfrac{1}{2}\delta^{(1)}t_\beta^{\mathrm{DR}}\delta^{(1)}Z_{AA}^{\mathrm{DR}} \\ &- \frac{1}{2}\left(\delta^{(1)}t_\beta^{\mathrm{DR}} + \frac{\delta^{(1)}Z_{AG}^{\mathrm{DR}}}{2c_\beta^2}\right)\left[\frac{\delta^{(1)}M_Z^2}{M_Z^2} + \delta^{(1)}Z_{ZZ}\right]_{\mathrm{Div}}\end{aligned}$$

$\delta^{(2)} t_\beta^{\mathbf{OS}}$ via the decay $A \to \tau^-\tau^+$

- Starting point:

$$\hat{\Gamma}^{\text{ph}}_{A\tau\tau} = \sqrt{\hat{Z}_A}\big(\hat{\Gamma}_{A\tau\tau} + \hat{Z}_{AG}\hat{\Gamma}_{G\tau\tau} + AZ \text{ mixing}\big)$$

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- Renormalisation condition:

$$\begin{aligned} \left|\hat{\Gamma}^{\text{ph}}_{A\tau\tau}\right|^2 &= \left|\Gamma^{(0)}_{A\tau\tau} + \hat{\Gamma}^{\text{ph},(1)}_{A\tau\tau} + \hat{\Gamma}^{\text{ph},(2)}_{A\tau\tau} + \cdots\right|^2 \\ &= \left|\Gamma^{(0)}_{A\tau\tau}\right|^2\Big(1 + 2\,\text{Re}\,\tilde{\Gamma}^{\text{ph},(1)}_{A\tau\tau} + 2\,\text{Re}\,\tilde{\Gamma}^{\text{ph},(2)}_{A\tau\tau} + \left|\tilde{\Gamma}^{\text{ph},(1)}_{A\tau\tau}\right|^2 + \cdots\Big) \\ &\stackrel{!}{=} \left|\Gamma^{(0)}_{A\tau\tau}\right|^2, \end{aligned}$$

where $\hat{\Gamma}^{\text{ph},(\text{i})}_{A\tau\tau} \equiv \tilde{\Gamma}^{\text{ph},(\text{i})}_{A\tau\tau}\Gamma^{(0)}_{A\tau\tau}$.

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$$\frac{\delta^{(1)} t_\beta^{\text{OS}}}{t_\beta} = -\delta^{(1)} Z_\text{w} + \frac{1}{2}\operatorname{Re}\partial\Sigma^{(1)}_{AA}(m_A^2) + \frac{1}{t_\beta}\frac{\operatorname{Im}\Sigma^{L,(1)}_{AZ}(m_A^2)}{M_Z},$$

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- Same procedure at two-loop level. Field ren. constants do not appear anywhere

Evaluating the finite pieces

- Self-energies are products of one-loop integrals
- Expansion in $\varepsilon = \frac{4-D}{2}$: $L = L^{\text{div}}\frac{1}{\varepsilon} + L^{\text{fin}} + L^{\varepsilon}\varepsilon + \mathcal{O}\left(\varepsilon^2\right)$

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- Alternatively: Different implementation of $\overline{\text{DR}}$ CT

Cancellation of $\mathcal{O}(\varepsilon)$ terms

- Starting point: $\Sigma^{(2)} = \sum_i L_i M_i + \sum_j N_j \delta c_j$, where L_i, M_i, N_j are one-loop integrals and δc_j are one-loop CTs

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- Expand loop integrals and CTs in ε and use relations between divergent parts:

$$\Sigma^{(2)} = -\frac{L_i^{\text{div}} M_i^{\text{div}}}{\varepsilon^2} + \frac{N_j^{\text{div}} \delta c_j^{\text{fin}}}{\varepsilon} + L_i^{\text{fin}} M_i^{\text{fin}} + N_j^{\text{fin}} \delta c_j^{\text{fin}} + N_j^{\text{div}} \delta c_j^{\varepsilon} + \mathcal{O}(\varepsilon)$$

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- If a mixed scheme is implemented via a re-parameterisation from a full $\overline{\text{DR}}$ calculation, this issue is circumvented.

Our implementation

- Example: Calculation with two CTs, δc_1 at one-loop level in OS scheme, δc_2 at two-loop level in $\overline{\mathrm{DR}}$ scheme:

$$\hat{\Sigma}^{(1)} = \Sigma^{(1)} + \delta^{(1)} c_2,$$
$$\hat{\Sigma}^{(2)} = \Sigma^{(2)} + \delta^{(2)} c_2$$

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- Our implementation:

$$\delta^{(1)} c_2^{\overline{\text{DR}}} = -\frac{\Sigma^{(1),\text{div}}(c_1^{\text{OS}}, c_2^{\overline{\text{DR}}})}{\varepsilon},$$
$$\delta^{(2)} c_2^{\overline{\text{DR}}} = -\frac{\Sigma^{(2),\text{ddiv}}}{\varepsilon^2} - \frac{\Sigma^{(2),\text{div}}}{\varepsilon}$$
$$= \frac{L_i^{\text{div}} M_i^{\text{div}}}{\varepsilon^2} - \frac{N_1^{\text{div}} \delta c_1^{\text{fin}}}{\varepsilon}$$

Alternative implementation

- Alternative (suggested by P. Slavich):

$$\begin{aligned}\delta^{(1)} c_2^{\overline{\mathrm{DR}},PS} &= -\frac{\Sigma^{(1)}(c_1^{\overline{\mathrm{DR}}}, c_2^{\overline{\mathrm{DR}}})}{\varepsilon} \\ &= \delta^{(1)} c_2^{\overline{\mathrm{DR}}} - \frac{N_1^{\mathrm{div}} \delta c_1^{\mathrm{fin}}}{\varepsilon} - N_1^{\mathrm{div}} \delta c_1^{\varepsilon}, \\ \delta^{(2)} c_2^{\overline{\mathrm{DR}},PS} &= \frac{L_i^{\mathrm{div}} M_i^{\mathrm{div}}}{\varepsilon^2}\end{aligned}$$

- This holds since

$$c_1^{\overline{\mathrm{DR}}} = c_1^{\mathrm{OS}} + \delta c_1^{\mathrm{fin}} + \varepsilon \delta c_1^{\varepsilon}.$$

and

$$N_1^{\mathrm{div}} = \frac{\mathrm{d}\Sigma^{(1),\mathrm{div}}}{\mathrm{d}c_1}.$$

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- $\delta c_2^{\overline{\text{DR}},PS} = \delta c_2^{\overline{\text{DR}}} - N_1^{\text{div}} \delta c_1^{\varepsilon}$

3. The numerical results

Setup

- 5 different $\mathcal{CP}$-conserving scenarios

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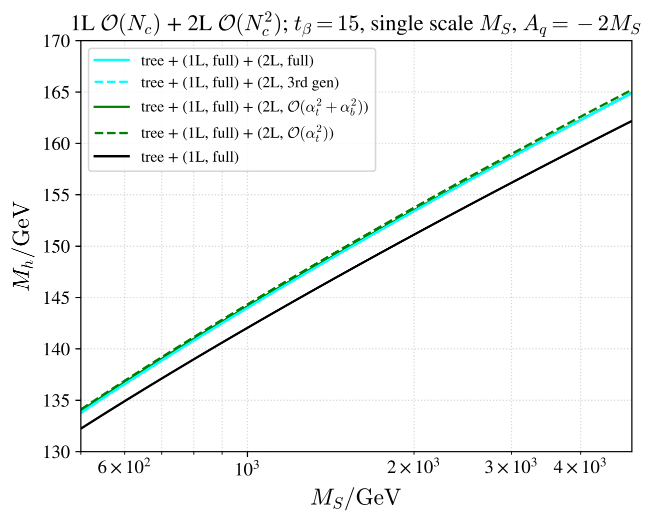
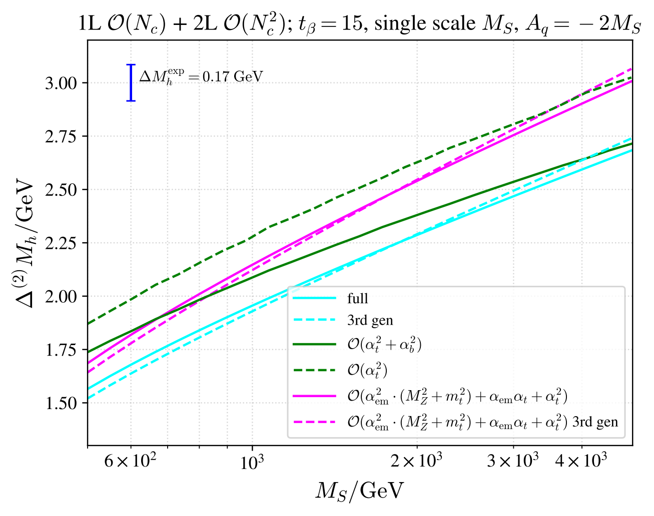
- Comparison with $M_h^{\text{exp}} = 125.25 \pm 0.17\,\text{GeV}$
- Size of new contributions: $\mathcal{O}\big((\alpha_{\text{em}} + \alpha_t + \alpha_b)^2 N_c^2\big)$ vs known $\mathcal{O}\big((\alpha_t + \alpha_b)^2 N_c^2\big)$

Scenario 2



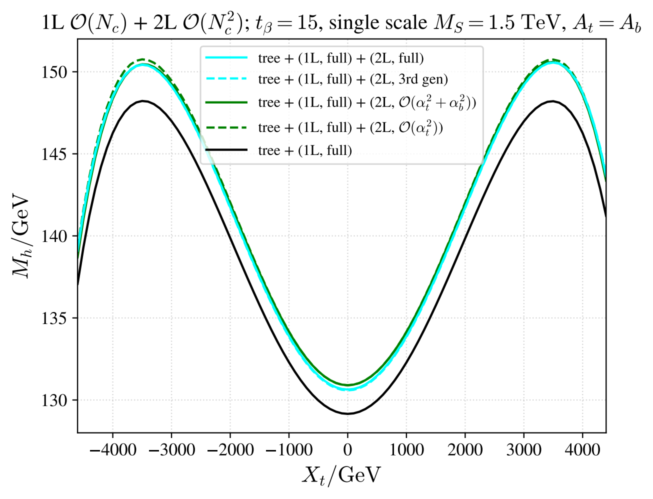
Two-loop QCD corrections lower prediction by up to 10 GeV

Scenario 2



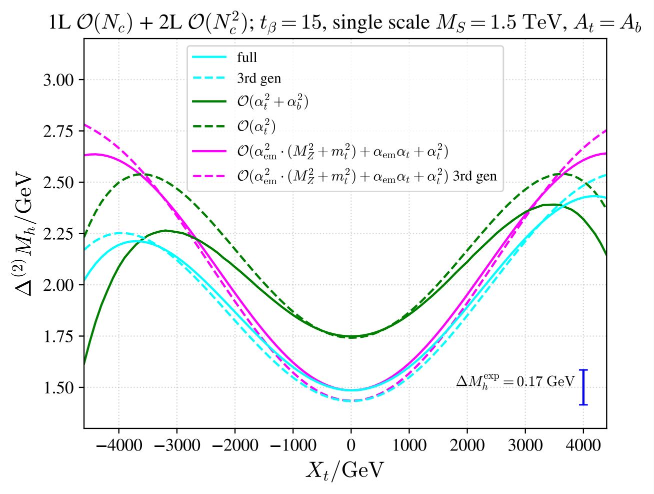
For low M_S, new corrections are comparable to ΔM_h^{exp}

Scenario 3



Two-loop QCD corrections lower prediction by up to 8 GeV

Scenario 3



For low $|X_t|$, new corrections larger than $\Delta M_h^{\rm exp}$

4. Summary & Outlook

Summary

- Successfully calculated the complete $\mathcal{O}\big((\alpha_{\text{em}} + \alpha_t + \alpha_b)^2 N_c^2\big)$ contributions to MSSM Higgs boson masses

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- Results in arXiv:2311.01889 [H. Bahl, D. Meuser, G. Weiglein]

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Outlook

- With our work, n-loop $\mathcal{O}(N_c^n)$ terms calculable
- Extension to $\mathcal{CP}$-violating scenarios possible (t_β^{OS} via decay $H^+ \to \tau^+\nu_\tau$)
- Results can be included in NMSSM Higgs boson mass prediction

5. Backup slides

The pole mass of a particle

$$m_{h/H}^2 = \frac{1}{2}\left(m_A^2 + M_Z^2 \mp \sqrt{(m_A^2 + M_Z^2)^2 - 4m_A^2 M_Z^2 \cos^2 2\beta}\right),$$
$$m_{H^\pm}^2 = m_A^2 + M_W^2,$$
$$t_\beta = \frac{v_2}{v_1}$$

- tree-level propagator:

$$\mathrm{i}\Delta_{hh}^{(0)}(p^2) = \frac{\mathrm{i}}{p^2 - m_h^2}$$

- dressed loop-corrected propagator:

$$\mathrm{i}\Delta_{hh}(p^2) = \frac{\mathrm{i}}{p^2 - m_h^2 + \hat{\Sigma}_{hh}(p^2)}$$

Particle mixing

- ansatz: 1PI two-point vertex functions as a matrix

$$\hat{\Gamma}_{hH}(p^2) = \begin{pmatrix} p^2 - m_h^2 + \hat{\Sigma}_{hh}(p^2) & \hat{\Sigma}_{hH}(p^2) \\ \hat{\Sigma}_{hH}(p^2) & p^2 - m_H^2 + \hat{\Sigma}_{HH}(p^2) \end{pmatrix}$$

- propagator matrix

$$\Delta_{hH}(p^2) = \left[\hat{\Gamma}_{hH}(p^2)\right]^{-1}$$

- diagonal entries are dressed propagators, e.g.

$$\mathrm{i}\Delta_{hh}(p^2) = \frac{\mathrm{i}}{p^2 - m_h^2 + \underbrace{\hat{\Sigma}_{hh}(p^2) - \frac{\left[\hat{\Sigma}_{hH}(p^2)\right]^2}{p^2 - m_H^2 + \hat{\Sigma}_{HH}(p^2)}}_{\hat{\Sigma}^{\mathrm{eff}}_{hh}(p^2)}}$$

Unstable particles

- external particle in self-energy unstable
  $\Rightarrow$ self-energies get imaginary part
  $\Rightarrow$ pole of propagator is complex quantity

$$\mathcal{M}_h^2 = M_h^2 - \mathrm{i} M_h \Gamma_h$$

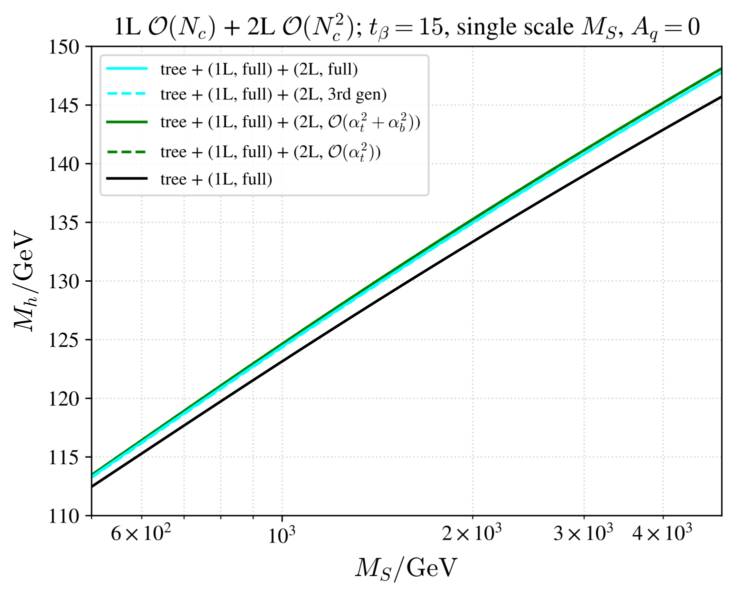
  with physical mass M_h and decay width Γ_h
- two-loop pole mass

$$\begin{aligned} M_h^2 = m_h^2 &- \mathrm{Re}\,\hat{\Sigma}_{hh}^{(1)}(m_h^2) - \mathrm{Re}\,\hat{\Sigma}_{hh}^{(2)}(m_h^2) \\ &+ \mathrm{Re}\left\{\hat{\Sigma}_{hh}^{(1)}(m_h^2)\partial\hat{\Sigma}_{hh}^{(1)}(m_h^2)\right\} - \mathrm{Re}\,\frac{\left(\hat{\Sigma}_{hH}^{(1)}(m_h^2)\right)^2}{m_H^2 - m_h^2} \end{aligned}$$

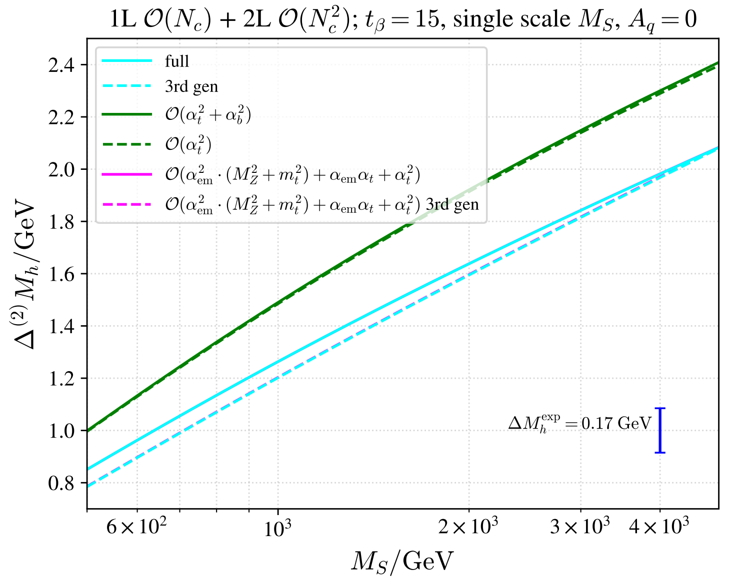
Parameter values

$$
\begin{aligned}
\alpha_{\rm em}^{-1} &= 137.035999084, & \alpha_s &= 0,\\
\Delta\alpha_{\rm lep}(M_Z^2) &= 0.031497687, & \Delta\alpha^{(5)}_{\rm had}(M_Z^2) &= 0.02766,\\
M_Z &= 91.1876\,\text{GeV}, & M_W &= 80.379\,\text{GeV},\\
m_t = M_t &= 172.76\,\text{GeV}, & m_b &= 4.18\,\text{GeV},\\
t_\beta = t_\beta^{\rm OS} &= 15, & m_A &= M_A = M_S,\\
M^2_{\tilde{q}_g} &= M_S^2, & M^2_{\tilde{u}_g} &= M_S^2,\\
M^2_{\tilde{d}_g} &= M_S^2, & \mu &= M_S,\\
A_t &= A_q, & A_b &= A_q.
\end{aligned}
$$

Scenario 1


1L $\mathcal{O}(N_c)$ + 2L $\mathcal{O}(N_c^2)$; $t_\beta = 15$, single scale M_S, $A_q = 0$
tree + (1L, full) + (2L, full)
tree + (1L, full) + (2L, 3rd gen)
tree + (1L, full) + (2L, $\mathcal{O}(\alpha_t^2 + \alpha_b^2)$)
tree + (1L, full) + (2L, $\mathcal{O}(\alpha_t^2)$)
tree + (1L, full)
M_h/GeV
M_S/GeV
150
145
140
135
130
125
120
115
110
6×10^2
10^3
2×10^3
3×10^3
4×10^3

Scenario 1


1L $\mathcal{O}(N_c)$ + 2L $\mathcal{O}(N_c^2)$; $t_\beta = 15$, single scale M_S, $A_q = 0$
full
3rd gen
$\mathcal{O}(\alpha_t^2 + \alpha_b^2)$
$\mathcal{O}(\alpha_t^2)$
$\mathcal{O}(\alpha_{\text{em}}^2 \cdot (M_Z^2 + m_t^2) + \alpha_{\text{em}}\alpha_t + \alpha_t^2)$
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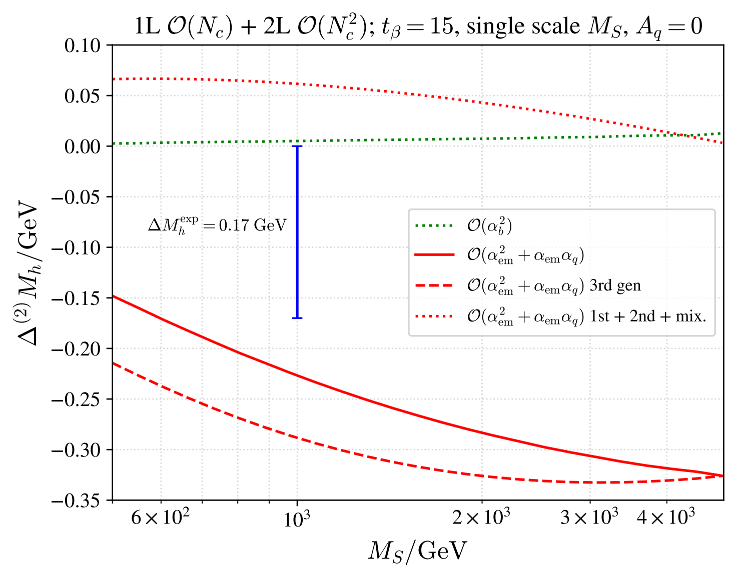
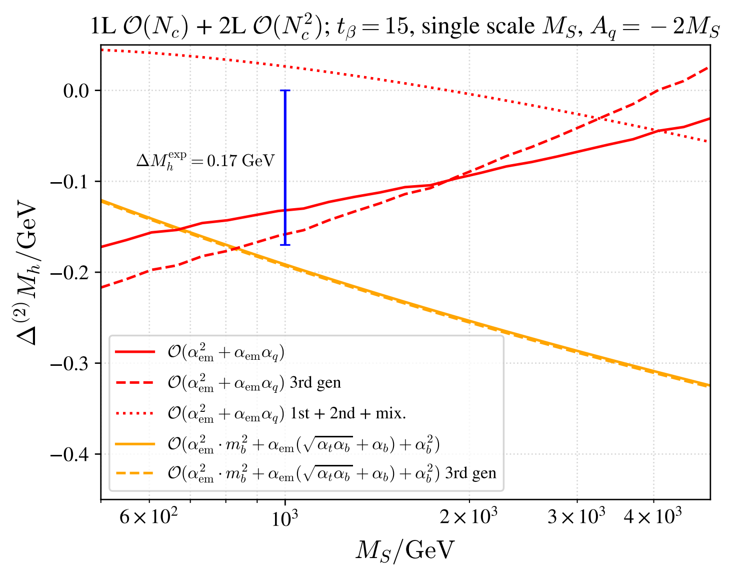
$\Delta M_h^{\text{exp}} = 0.17$ GeV
$\Delta^{(2)} M_h/\text{GeV}$
M_S/GeV
2.4
2.2
2.0
1.8
1.6
1.4
1.2
1.0
0.8
6×10^2
10^3
2×10^3
3×10^3
4×10^3

Scenario 1

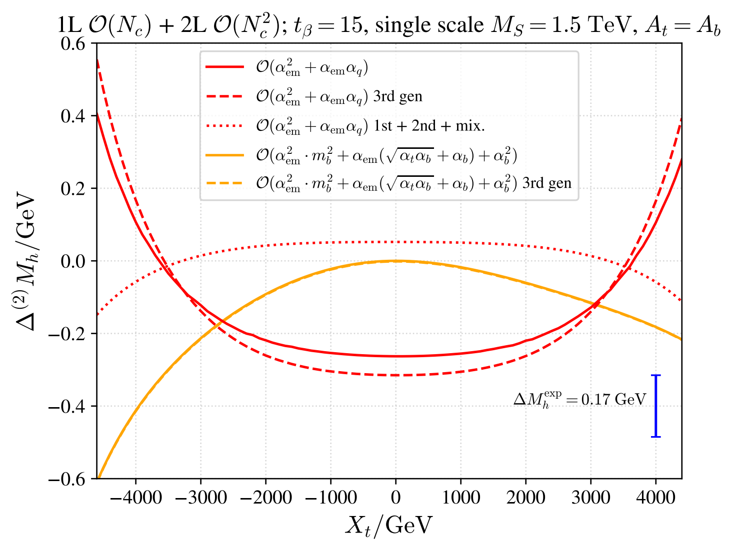


1L $\mathcal{O}(N_c)$ + 2L $\mathcal{O}(N_c^2)$; $t_\beta = 15$, single scale M_S, $A_q = 0$
0.10
0.05
0.00
−0.05
−0.10
−0.15
−0.20
−0.25
−0.30
−0.35
$\Delta^{(2)} M_h/\mathrm{GeV}$
$\Delta M_h^{\mathrm{exp}} = 0.17$ GeV
$\mathcal{O}(\alpha_b^2)$
$\mathcal{O}(\alpha_{\mathrm{em}}^2 + \alpha_{\mathrm{em}}\alpha_q)$
$\mathcal{O}(\alpha_{\mathrm{em}}^2 + \alpha_{\mathrm{em}}\alpha_q)$ 3rd gen
$\mathcal{O}(\alpha_{\mathrm{em}}^2 + \alpha_{\mathrm{em}}\alpha_q)$ 1st + 2nd + mix.
6×10^2
10^3
2×10^3
3×10^3
4×10^3
M_S/GeV

Scenario 2

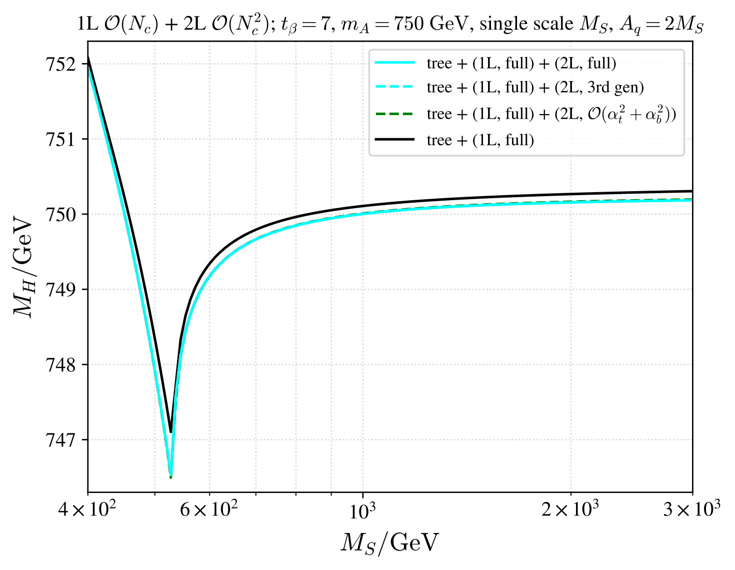

1L $\mathcal{O}(N_c)$ + 2L $\mathcal{O}(N_c^2)$; $t_\beta = 15$, single scale M_S, $A_q = -2M_S$
0.0
−0.1
−0.2
−0.3
−0.4
$\Delta^{(2)} M_h/\text{GeV}$
6×10^2
10^3
2×10^3
3×10^3
4×10^3
M_S/GeV
$\Delta M_h^{\text{exp}} = 0.17$ GeV
$\mathcal{O}(\alpha_{\text{em}}^2 + \alpha_{\text{em}}\alpha_q)$
$\mathcal{O}(\alpha_{\text{em}}^2 + \alpha_{\text{em}}\alpha_q)$ 3rd gen
$\mathcal{O}(\alpha_{\text{em}}^2 + \alpha_{\text{em}}\alpha_q)$ 1st + 2nd + mix.
$\mathcal{O}(\alpha_{\text{em}}^2 \cdot m_b^2 + \alpha_{\text{em}}(\sqrt{\alpha_t\alpha_b} + \alpha_b) + \alpha_b^2)$
$\mathcal{O}(\alpha_{\text{em}}^2 \cdot m_b^2 + \alpha_{\text{em}}(\sqrt{\alpha_t\alpha_b} + \alpha_b) + \alpha_b^2)$ 3rd gen

Scenario 3



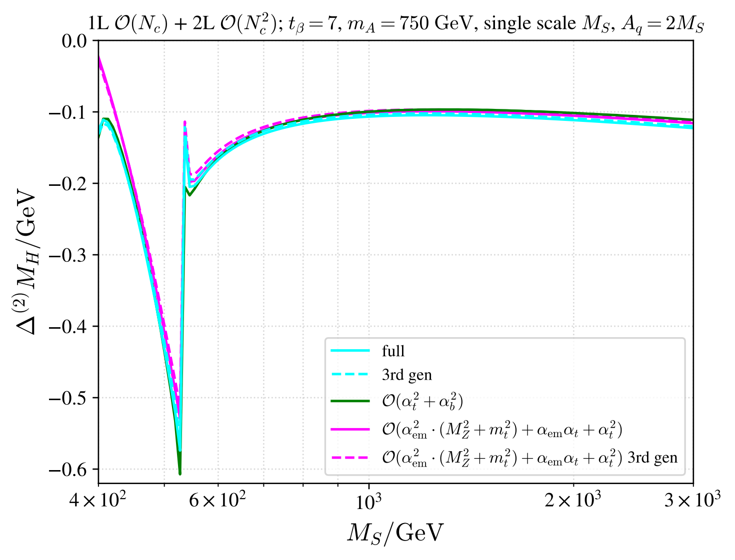
1L $\mathcal{O}(N_c)$ + 2L $\mathcal{O}(N_c^2)$; $t_\beta = 15$, single scale $M_S = 1.5$ TeV, $A_t = A_b$
$\mathcal{O}(\alpha_{\text{em}}^2 + \alpha_{\text{em}}\alpha_q)$
$\mathcal{O}(\alpha_{\text{em}}^2 + \alpha_{\text{em}}\alpha_q)$ 3rd gen
$\mathcal{O}(\alpha_{\text{em}}^2 + \alpha_{\text{em}}\alpha_q)$ 1st + 2nd + mix.
$\mathcal{O}(\alpha_{\text{em}}^2 \cdot m_b^2 + \alpha_{\text{em}}(\sqrt{\alpha_t\alpha_b} + \alpha_b) + \alpha_b^2)$
$\mathcal{O}(\alpha_{\text{em}}^2 \cdot m_b^2 + \alpha_{\text{em}}(\sqrt{\alpha_t\alpha_b} + \alpha_b) + \alpha_b^2)$ 3rd gen
$\Delta M_h^{\text{exp}} = 0.17$ GeV
$\Delta^{(2)} M_h/\text{GeV}$
0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6
−4000
−3000
−2000
−1000
0
1000
2000
3000
4000
X_t/GeV

Scenario 4



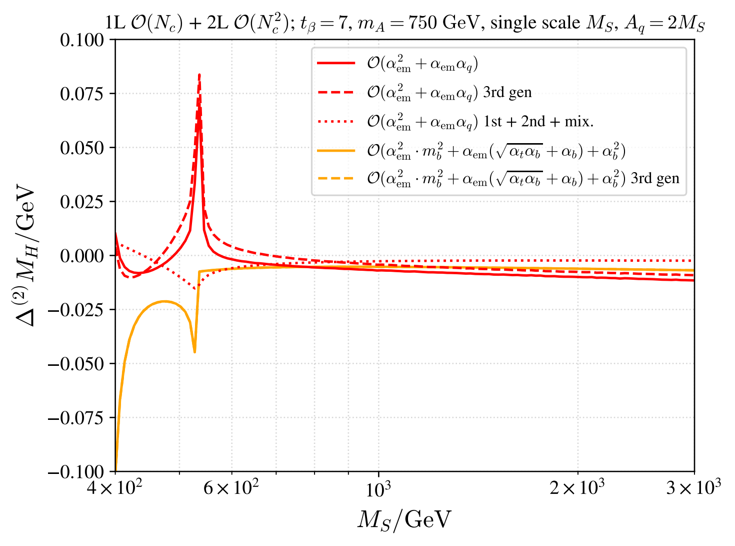
1L $\mathcal{O}(N_c)$ + 2L $\mathcal{O}(N_c^2)$; $t_\beta = 7$, $m_A = 750$ GeV, single scale M_S, $A_q = 2M_S$
tree + (1L, full) + (2L, full)
tree + (1L, full) + (2L, 3rd gen)
tree + (1L, full) + (2L, $\mathcal{O}(\alpha_t^2 + \alpha_b^2)$)
tree + (1L, full)
M_H/GeV
752
751
750
749
748
747
4×10^2
6×10^2
10^3
2×10^3
3×10^3
M_S/GeV

Scenario 4



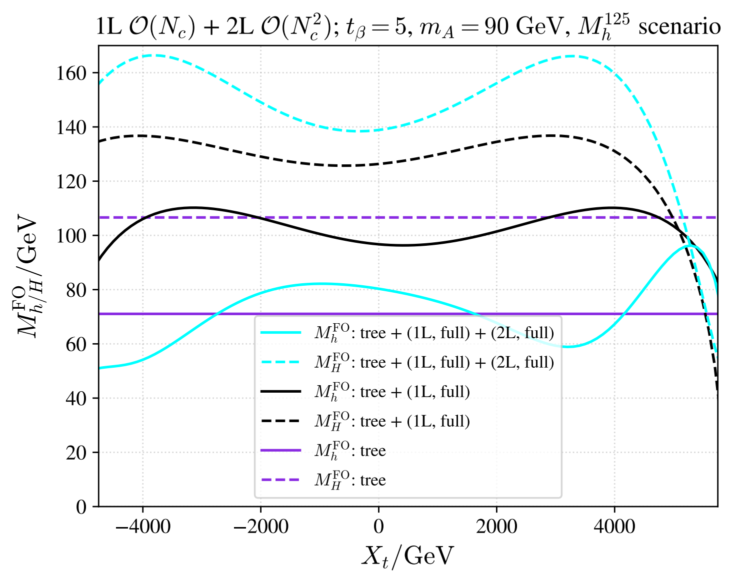
1L $\mathcal{O}(N_c)$ + 2L $\mathcal{O}(N_c^2)$; $t_\beta = 7$, $m_A = 750$ GeV, single scale M_S, $A_q = 2M_S$
0.0
−0.1
−0.2
−0.3
−0.4
−0.5
−0.6
$\Delta^{(2)} M_H/\text{GeV}$
4×10^2
6×10^2
10^3
2×10^3
3×10^3
M_S/GeV
full
3rd gen
$\mathcal{O}(\alpha_t^2 + \alpha_b^2)$
$\mathcal{O}(\alpha_{\text{em}}^2 \cdot (M_Z^2 + m_t^2) + \alpha_{\text{em}}\alpha_t + \alpha_t^2)$
$\mathcal{O}(\alpha_{\text{em}}^2 \cdot (M_Z^2 + m_t^2) + \alpha_{\text{em}}\alpha_t + \alpha_t^2)$ 3rd gen

Scenario 4



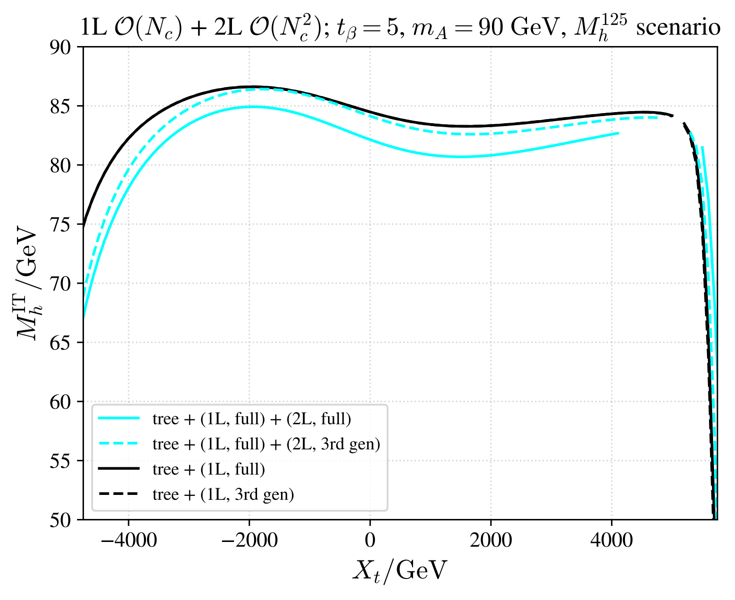
1L $\mathcal{O}(N_c)$ + 2L $\mathcal{O}(N_c^2)$; $t_\beta = 7$, $m_A = 750$ GeV, single scale M_S, $A_q = 2M_S$
$\mathcal{O}(\alpha_{\text{em}}^2 + \alpha_{\text{em}}\alpha_q)$
$\mathcal{O}(\alpha_{\text{em}}^2 + \alpha_{\text{em}}\alpha_q)$ 3rd gen
$\mathcal{O}(\alpha_{\text{em}}^2 + \alpha_{\text{em}}\alpha_q)$ 1st + 2nd + mix.
$\mathcal{O}(\alpha_{\text{em}}^2 \cdot m_b^2 + \alpha_{\text{em}}(\sqrt{\alpha_t\alpha_b} + \alpha_b) + \alpha_b^2)$
$\mathcal{O}(\alpha_{\text{em}}^2 \cdot m_b^2 + \alpha_{\text{em}}(\sqrt{\alpha_t\alpha_b} + \alpha_b) + \alpha_b^2)$ 3rd gen
$\Delta^{(2)} M_H/\text{GeV}$
0.100
0.075
0.050
0.025
0.000
−0.025
−0.050
−0.075
−0.100
4×10^2
6×10^2
10^3
2×10^3
3×10^3
M_S/GeV

Scenario 5

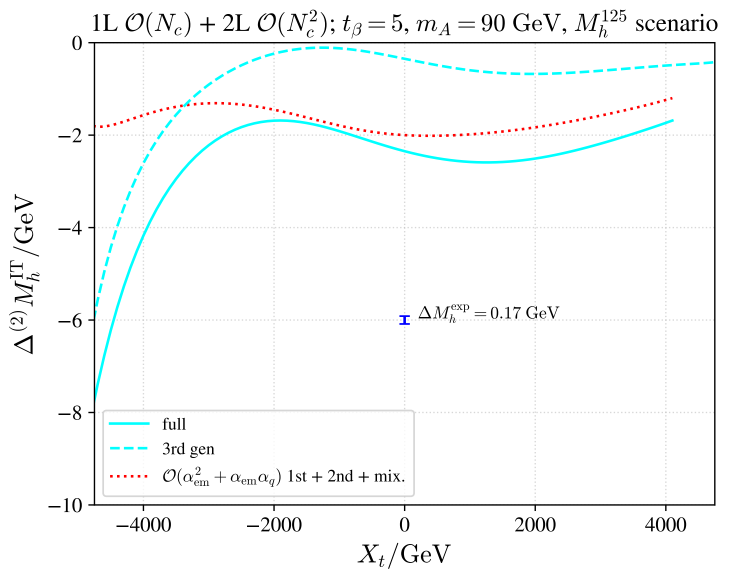


1L $\mathcal{O}(N_c)$ + 2L $\mathcal{O}(N_c^2)$; $t_\beta = 5$, $m_A = 90$ GeV, M_h^{125} scenario
$M_{h/H}^{\mathrm{FO}}$/GeV
X_t/GeV
160
140
120
100
80
60
40
20
0
−4000
−2000
0
2000
4000
M_h^{FO}: tree + (1L, full) + (2L, full)
M_H^{FO}: tree + (1L, full) + (2L, full)
M_h^{FO}: tree + (1L, full)
M_H^{FO}: tree + (1L, full)
M_h^{FO}: tree
M_H^{FO}: tree

Scenario 5


1L $\mathcal{O}(N_c)$ + 2L $\mathcal{O}(N_c^2)$; $t_\beta = 5$, $m_A = 90$ GeV, M_h^{125} scenario
M_h^{IT}/GeV
X_t/GeV
90
85
80
75
70
65
60
55
50
−4000
−2000
0
2000
4000
tree + (1L, full) + (2L, full)
tree + (1L, full) + (2L, 3rd gen)
tree + (1L, full)
tree + (1L, 3rd gen)

Scenario 5



1L $\mathcal{O}(N_c)$ + 2L $\mathcal{O}(N_c^2)$; $t_\beta = 5$, $m_A = 90$ GeV, M_h^{125} scenario
0
−2
−4
−6
−8
−10
$\Delta^{(2)} M_h^{\rm IT}$/GeV
$\Delta M_h^{\rm exp} = 0.17$ GeV
full
3rd gen
$\mathcal{O}(\alpha_{\rm em}^2 + \alpha_{\rm em}\alpha_q)$ 1st + 2nd + mix.
−4000
−2000
0
2000
4000
X_t/GeV