Updates of FlexibleEFTHiggs with applications to the MSSM and NMSSM

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1. Introduction

- experimental Higgs mass: $M_h^{\mathrm{exp}} = 125.25 \pm 0.11\ \mathrm{GeV}$ [PDG,2024]
- Why NMSSM?
  - simple perturbation of MSSM with interesting Higgs phenomenology
  - solution of μ and little hierarchy problem
  - no fine tuning of x_t necessary
- Why `FlexibleEFTHiggs`?
  - hybrid approach = combination of fixed-order and EFT approach in Higgs mass calculation
  - structurally simple matching conditions $\Rightarrow$ easily automatizable
  - implemented in `FlexibleSUSY` for MSSM and NMSSM

1. Introduction

- NMSSM Higgs sector:
  - superpotential:

$$W \supset -\lambda \hat{S}\left(\hat{H}_d \cdot \hat{H}_u\right) + \frac{\kappa}{3}\hat{S}^3$$

  $\Rightarrow$ generation of effective μ-parameter: $\mu_{\text{eff}} = \lambda v_s$

  - soft SUSY-breaking Lagrangian:

$$\mathcal{L}_{\text{soft}} \supset -M_{H_d}^2|H_d|^2 - M_{H_u}^2|H_u|^2 - M_S^2|S|^2 - \left(\lambda A_\lambda S\left(H_d \cdot H_u\right) - \frac{\kappa}{3}A_\kappa S^3 + \text{h.c.}\right)$$

  - assumption: **CP-conserving** Higgs sector $\Rightarrow$ λ, κ real!
  - expansion:

$$H_d = \begin{pmatrix} v_d + \frac{1}{\sqrt{2}}\left(\phi_d^0 - i\chi_d^0\right) \\ -\phi_d^- \end{pmatrix} \qquad H_u = \begin{pmatrix} \phi_u^+ \\ v_u + \frac{1}{\sqrt{2}}\left(\phi_u^0 + i\chi_u^0\right) \end{pmatrix} \qquad S = v_s + \frac{1}{\sqrt{2}}(s + ia)$$

  $\Rightarrow$ mass mixing: $\phi_u^0, \phi_d^0, s \;\rightarrow\; h_1, h_2, h_3 \qquad \chi_u^0, \chi_d^0, a \;\rightarrow\; a_1, a_2, G^0 \qquad \phi_u^\pm, \phi_d^\pm \;\Rightarrow\; H^\pm, G^\pm$

  $\Rightarrow$ case $v \rightarrow 0$: $\phi_u^0 \sim h_1 \mathrel{\hat{=}} h \qquad \phi_d^0 \sim h_2 \qquad s \sim h_3$

1. Introduction

- NMSSM Higgs sector:
  - tree-level Higgs mass:
    - upper limit on lightest CP-even Higgs (for $v \ll m_A$) :

$$m_h^2 < M_Z^2 \cos^2 2\beta + \lambda^2 v^2 \sin^2 2\beta$$

    - in approximation $v \ll m_s$:

$$m_h^2 = M_Z^2 \cos^2 2\beta + \lambda^2 v^2 \sin^2 2\beta - \frac{\lambda^3 \left(2\mu_{\text{eff}} - (A_\lambda + 2\frac{\kappa}{\lambda}\mu_{\text{eff}}) \sin 2\beta\right)^2}{\kappa\mu_{\text{eff}}(A_\kappa + 4\frac{\kappa}{\lambda}\mu_{\text{eff}})}$$

1. Introduction

- recent status of NMSSM Higgs mass calculation:
  - **FO calculations**:

    $\rightarrow$ full/partial N^2LO calculations [0907.4682] [1411.4665] [1601.08100] [2106.06990]
  - **EFT calculations**:

    $\rightarrow$ NNLL calculations with (partial) NMSSM-specific 2-loop matching corrections [1703.03267] [2206.04618]
  - **hybrid calculations**:

    $\rightarrow$ LL + NNLO calculation implemented with `SARAH/SPheno` [1411.0675]

    $\rightarrow$ NLL + NLO calculation with `FlexibleEFTHiggs` method, implemented in `FlexibleSUSY` [1609.00371] [1710.03760]

2. FlexibleEFTHiggs Higgs Mass Calculation

- FlexibleEFTHiggs method = hybrid approach:
  - calculation structure:

Q

$Q_{\text{match}} \sim M_S$

(1) Matching, determine $\hat{\lambda}$:

$$\left(M_h^{\text{SUSY}}\right)^2 = \left(M_h^{\text{SM}}\right)^2 \;\Rightarrow\; \hat{\lambda}(Q_{\text{match}}) = \frac{1}{\hat{v}^2}\left[\left(M_h^{\text{SUSY}}\right)^2 - \Delta s_h^{\text{SM}}\left(M_h^{\text{SUSY}}\right)\right]$$

$\Rightarrow$ include power-suppressed contributions of $\mathcal{O}(v^2/M_S^2)$!

(2) RG running $\Rightarrow$ resummation of large logs!

Q_{low}

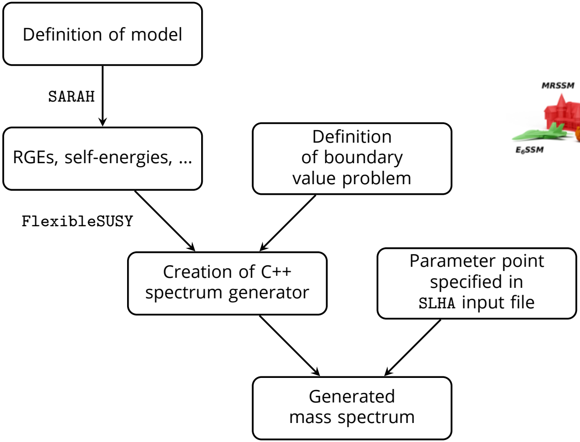
(3) Calculate M_h in SM: $\left(M_h^{\text{SM}}\right)^2 = \hat{\lambda}\hat{v}^2 + \Delta s_h^{\text{SM}}$

2. FlexibleEFTHiggs Higgs Mass Calculation

- FlexibleEFTHiggs method = hybrid approach:
  - advantages:
    - structurally simple, easily applicable to any BSM model
    - easy to automate
  - disadvantages:
    - difficult to transfer to other EFTs than SM
    - careful treatment of parameter matching for 2-loop accuracy needed
  - differences between EFT- and full-model parametrization

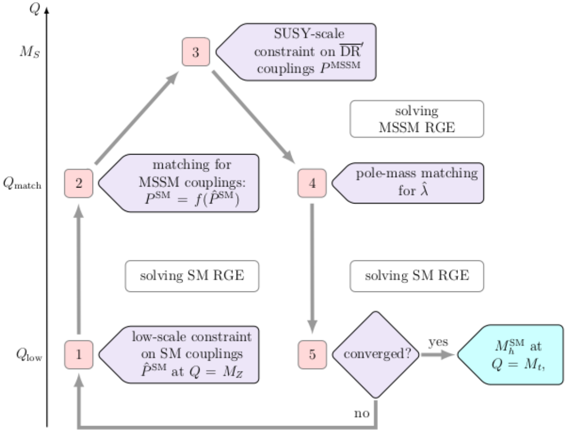
2. FlexibleEFTHiggs Higgs Mass Calculation

- FlexibleSUSY = spectrum generator generator



2. FlexibleEFTHiggs Higgs Mass Calculation

- FlexibleEFTHiggs implementation in `FlexibleSUSY`:
  - two-scale solver:



2. FlexibleEFTHiggs Higgs Mass Calculation

- current status on FlexibleEFTHiggs implementation in `FlexibleSUSY`:
  - MSSM:
    - so far: NLL + NLO FEFT-calculation implemented
    - **new:** N^{3}LL + N^{3}LO FEFT-calculation added

    $\rightarrow$ model name: `NUHMSSMNoFVHimalayaEFTHiggs`
  - NMSSM:
    - so far: NLL + NLO FEFT-calculation implemented
    - **new:** N^{2}LL + N^{2}LO FEFT-calculation added

    $\rightarrow$ model name: `NMSSMEFTHiggs`
  - **soon:** usage of full model parametrization $\rightarrow x_t$-resummation! [2003.04639]

    $\rightarrow$ new solver: shooting solver
  - publicly available under `github.com/FlexibleSUSY/FlexibleSUSY`

2. FlexibleEFTHiggs Higgs Mass Calculation

- NMSSM Higgs mass calculation:
  - renormalization scheme: $\overline{\mathrm{DR}}$ ($\overline{\mathrm{MS}}$ in SM)
  - 2-loop $\hat{\lambda}$-matching:

$$\Delta\hat{\lambda} = \Delta\hat{\lambda}^{1l}_{\mathrm{MSSM}} + \Delta\hat{\lambda}^{1l}_{\lambda,\kappa} + \Delta\hat{\lambda}^{2l}_{\mathrm{MSSM}}$$

    $\Delta\hat{\lambda}^{1l}_{\mathrm{MSSM}} \,\hat{=}$ full MSSM-like 1-loop contributions

    $\Delta\hat{\lambda}^{1l}_{\lambda,\kappa} \,\hat{=}$ full NMSSM-specific 1-loop contributions $\rightarrow\ \mathcal{O}\Big(\lambda^4 + \lambda^2\kappa^2 + \kappa^4\Big)$

    $\Delta\hat{\lambda}^{2l}_{\mathrm{MSSM}} \,\hat{=}$ full MSSM-like 2-loop contributions ($g_{1,2} = 0$) $\rightarrow\ \mathcal{O}\Big(g_3^2(y_t^2 + y_b^2)^2 + (y_t^2 + y_b^2 + y_\tau^2)^3\Big)$

  - 3-loop β-functions

    $\Rightarrow$ resummation of MSSM-like NNLL

    $\Rightarrow$ resummation of NMSSM-specific NLL

3. Theory Uncertainty Estimation

- high-scale uncertainty ΔM_h^{HS} from variation of Q_{match}:

$$\Delta M_h^{\text{HS}} = \max_{Q\in[M_S/2,2M_S]} \Big| M_h(Q_{\text{match}} = M_S) - M_h(Q_{\text{match}} = Q) \Big|$$

  - reliable method in MSSM $\to$ also reliable for estimation of NMSSM-specific terms?

- low-scale uncertainty ΔM_h^{LS} determined as maximum from two methods:

$$\Delta M_h^{\text{LS}} = \max\{\Delta M_h^{Q_{\text{low}}}, \Delta M_h^{\text{PLO}}\}$$

  - variation of Q_{low}:

$$\Delta M_h^{Q_{\text{low}}} = \max_{Q\in[M_t/2,2M_t]} \Big| M_h(Q_{\text{low}} = M_t) - M_h(Q_{\text{low}} = Q) \Big|$$

  - variation of loop-order of QCD-corrections in relation between M_t and $\hat{y}_t$:

$$\Delta M_h^{\text{PLO}} = \Big| M_h^{\hat{y}_t,2l} - M_h^{\hat{y}_t,3l} \Big|$$

- total uncertainty:

$$\Delta M_h = \Delta M_h^{\text{HS}} + \Delta M_h^{\text{LS}}$$

3. Theory Uncertainty Estimation

- coverage of λ-dependent contributions by estimation:
  - λ-dependent contributions covered by M_h calculation:

$L = \ln \frac{Q_\text{match}}{Q_\text{low}}$

$\kappa_L = \frac{1}{16\pi^2}$

	$\mathcal{O}(L^0)$	$\mathcal{O}(L^1)$	$\mathcal{O}(L^2)$	$\mathcal{O}(L^3)$	…
$\mathcal{O}(\kappa_L^0)$	$\propto \lambda^{\leq 2}$	–	–	–	…
$\mathcal{O}(\kappa_L^1)$	$\propto \lambda^{\leq 4}$	$\propto \lambda^{\leq 4}$	–	–	…
$\mathcal{O}(\kappa_L^2)$	$\propto \lambda^{\leq 6}$	$\propto \lambda^{\leq 6}$	$\propto \lambda^{\leq 6}$	–	…
$\mathcal{O}(\kappa_L^3)$	$\propto \lambda^{\leq 8}$	$\propto \lambda^{\leq 8}$	$\propto \lambda^{\leq 8}$	$\propto \lambda^{\leq 8}$	…
…	…	…	…	…	…

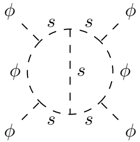
incomplete fully covered

  - effect of matching scale variation:

$$\frac{\partial M_h}{\partial(\ln Q_\text{match})} \supset \propto \kappa_L^2 \lambda^{\leq 6} + \propto \kappa_L^3 \lambda^{\leq 8} + \mathcal{O}(\kappa_L^4)$$

3. Theory Uncertainty Estimation

- coverage of κ-dependent contributions by estimation:
  - $\hat{\lambda}_{\text{tree}}$ regarded to be of $\mathcal{O}(\kappa^0)$
  - $v \to 0$ limit assumed
  - κ-dependent contributions covered by M_h calculation:



$\supset \propto \kappa^{\leq 8}$

with $\phi \in \{\phi_u^0, \phi_d^0\}$

	$\mathcal{O}(L^0)$	$\mathcal{O}(L^1)$	$\mathcal{O}(L^2)$	$\mathcal{O}(L^3)$	...
$\mathcal{O}(\kappa_L^0)$	$\propto \kappa^0$	—	—	—	...
$\mathcal{O}(\kappa_L^1)$	$\propto \kappa^{\leq 4}$	$\propto \kappa^0$	—	—	...
$\mathcal{O}(\kappa_L^2)$	$\propto \kappa^{\leq 8}$	$\propto \kappa^{\leq 4}$	$\propto \kappa^0$	—	...
$\mathcal{O}(\kappa_L^3)$	$\propto \kappa^{\leq 12}$	$\propto \kappa^{\leq 8}$	$\propto \kappa^{\leq 4}$	$\propto \kappa^{\leq 0}$	...
...	...	...	...	...	...

incomplete fully covered

$\Rightarrow$ different appearance of λ and κ in perturbation series of M_h^2!

3. Theory Uncertainty Estimation

- coverage of κ-dependent contributions by estimation:
  - effect of matching scale variation:

$$\frac{\partial M_h}{\partial(\ln Q_{\text{match}})} \supset \propto \kappa_L^2 \kappa^{\leq 6} + \kappa_L^3 \Big(\propto \kappa^{\leq 6} L + \propto \kappa^{\leq 8} \Big) + \mathcal{O}(\kappa_L^4)$$

    $\Rightarrow$ terms of highest order in κ not simulated, e.g. $\mathcal{O}(\kappa_L^2 \kappa^8)$ and $\mathcal{O}(\kappa_L^3 \kappa^8 L)$!

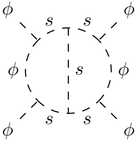
  - impact of not simulated terms:

    $\rightarrow$ rough estimation by comparison of simulated and not simulated contributions

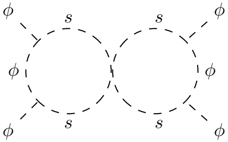
3. Theory Uncertainty Estimation

- coverage of κ-dependent contributions by estimation:
  - impact of not simulated terms $\rightarrow$ rough estimation:

$\rightarrow$ at $\mathcal{O}(\kappa_L^2 L^0)$:



$$\rightarrow \Delta\hat{\lambda} \sim \underbrace{\kappa_L^2 \left(\frac{\mu_{\text{eff}}}{M_S}\right)^6 \frac{\kappa^8}{\lambda^2}}_{\Delta_8} + \text{terms} \propto \kappa^{\leq 7}$$



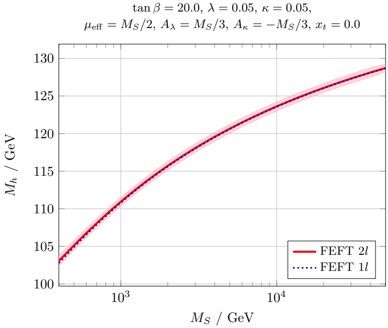
$$\rightarrow \Delta\hat{\lambda} \sim \underbrace{\kappa_L^2 \left(\frac{\mu_{\text{eff}}}{M_S}\right)^4 \kappa^6}_{\Delta_6} + \text{terms} \propto \kappa^{\leq 5}$$

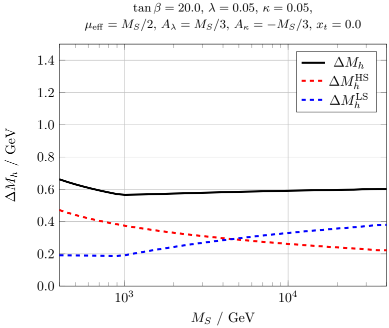
$$\Rightarrow \frac{\Delta_8}{\Delta_6} = \left(\frac{\mu_{\text{eff}}}{M_S}\right)^2 \frac{\kappa^2}{\lambda^2} \quad \Rightarrow \text{large impact if } \lambda \ll \kappa$$

$\Rightarrow$ missing κ-dependent terms reliably estimated if $\lambda \sim \kappa$

4. Results

- M_h and ΔM_h dependence on M_S:

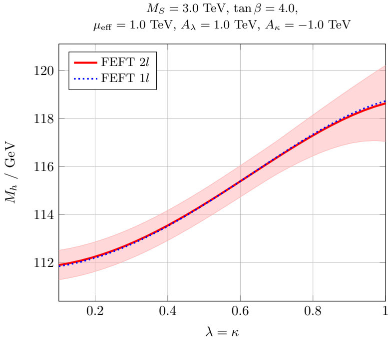


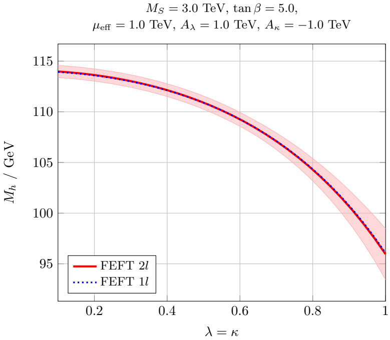


  - for $0.3\ \text{TeV} < M_S < 100\ \text{TeV}$: uncertainty almost constant with $\Delta M_h \approx 0.6\ \text{GeV}$

4. Results

- $M_h(\lambda = \kappa)$ dependence:

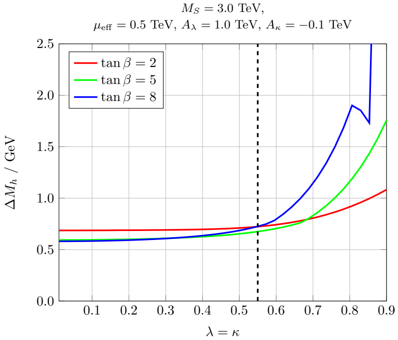


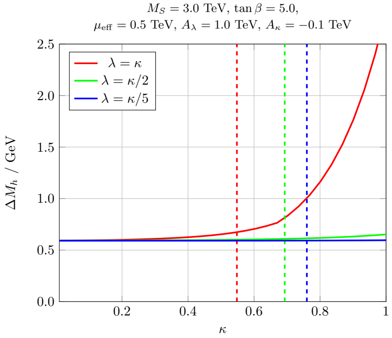


  - behaviour dominated by tree-level dependence

4. Results

- ΔM_h dependence on λ and κ :

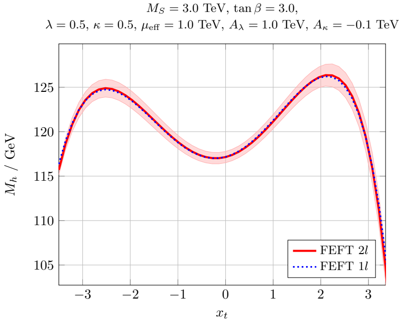


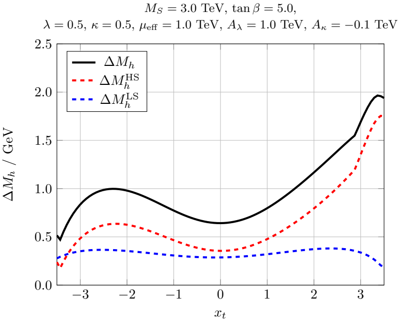


  - dashed lines indicate perturbativity limit: $\lambda^2 + \kappa^2 \leq 0.6$
  - below perturbativity limit nearly constant $\Delta M_h \sim 0.7$ GeV

4. Results

- M_h and ΔM_h dependence on x_t:





  - most relevant parameter for value of estimated uncertainty

    $\Rightarrow$ uncertainties strongly MSSM-dominated!
  - ΔM_h minimized for $x_t = 0$ (and also for $x_t \approx -3.5$)

4. Results

- benchmark points:
  - parameter choice:

BMP	$\tan\beta$	λ	κ	μ_{eff} / GeV	A_λ / GeV	A_κ / GeV	x_t
S1	20	0.05	0.05	800	1000	−500	1.78
S2	20	0.05	0.05	300	−250	−950	2.05
S4	7	0.30	0.30	800	5000	−100	2.05
S5	7	0.30	0.30	300	1200	−10	2.58
S7	3	0.60	0.30	750	2000	−1000	1.52
S8	3	0.55	0.35	300	700	−600	1.76
S10	3	1.50	1.50	800	250	−1000	0.10

  with $M_S = 3.0\,\text{GeV}$ and $A_b = A_\tau = 0$

  - all points chosen such that H_1 is SM-like

4. Results

- benchmark points:
  - calculated Higgs mass spectrum:

BMP	M_{H_1} / GeV	M_{H_2} / GeV	M_{H_3} / GeV	M_{A_1} / GeV	M_{A_1} / GeV	ΔM_h / GeV
S1	125.21	1469.62	5345.93	1095.26	5345.96	0.97
S2	125.19	274.42	432.08	431.78	924.37	0.68
S4	125.21	1573.98	5744.67	476.89	5745.13	1.09
S5	125.28	590.78	1794.14	136.81	1793.33	1.31
S7	125.27	451.87	2421.84	1070.46	2420.74	0.84
S8	125.23	169.35	935.24	593.00	931.76	0.72
S10	125.29	1159.56	1612.55	1358.08	1624.90	14.75

5. Summary and Outlook

- `FlexibleEFTHiggs` calculation, implemented in `FlexibleSUSY`:
  - MSSM: N^{3}LO + N^{3}LL level
  - NMSSM: N^{2}LO + N^{2}LL level
  - publicly available, soon also with full-model parametrization
- matching scale variation as reliable estimation of missing λ-dependent contributions, in case $\lambda \sim \kappa$ and $v \ll M_S$ also for κ-dependent ones
- ΔM_h dominated by missing MSSM contributions (λ and κ perturbative up to Planck scale)
- typical order of uncertainty: $\Delta M_h \sim 0.7\,\mathrm{GeV}$ (for $x_t \approx 0$)
- further steps:
  - more reliable estimation of missing κ-dependent terms
  - detailed comparison with other calculations
  - implementation of MSSM-like 3-loop corrections in model `NMSSMEFTHiggs`?