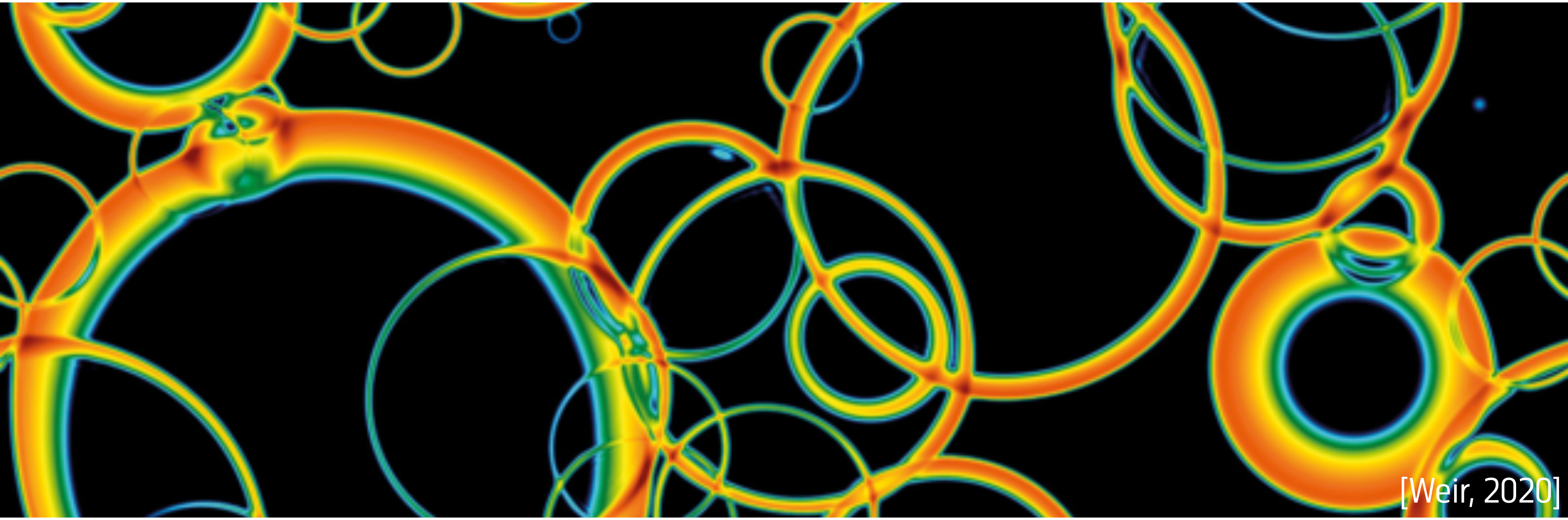
# Katharsis of Universe's Transition Studies

## Effective Potential, Thermal Phase Transitions, Dimensional Reduction

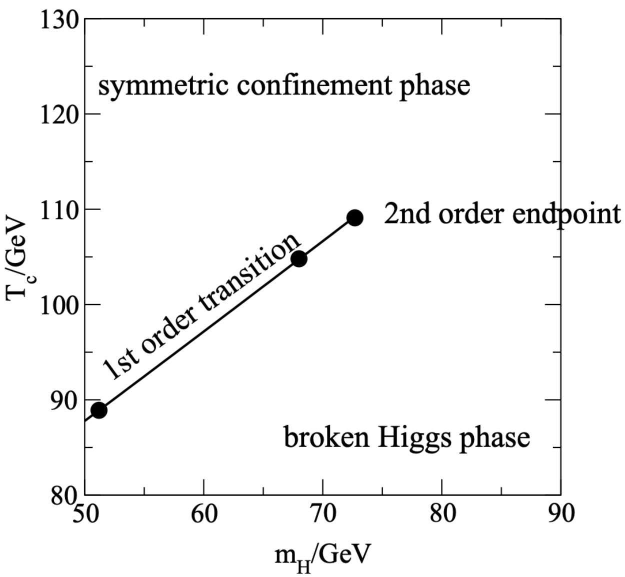



Thomas Biekötter, **Andrii Dashko**, Maximilian Löschner, Georg Weiglein

KUTS, Hamburg, 27.06.2024

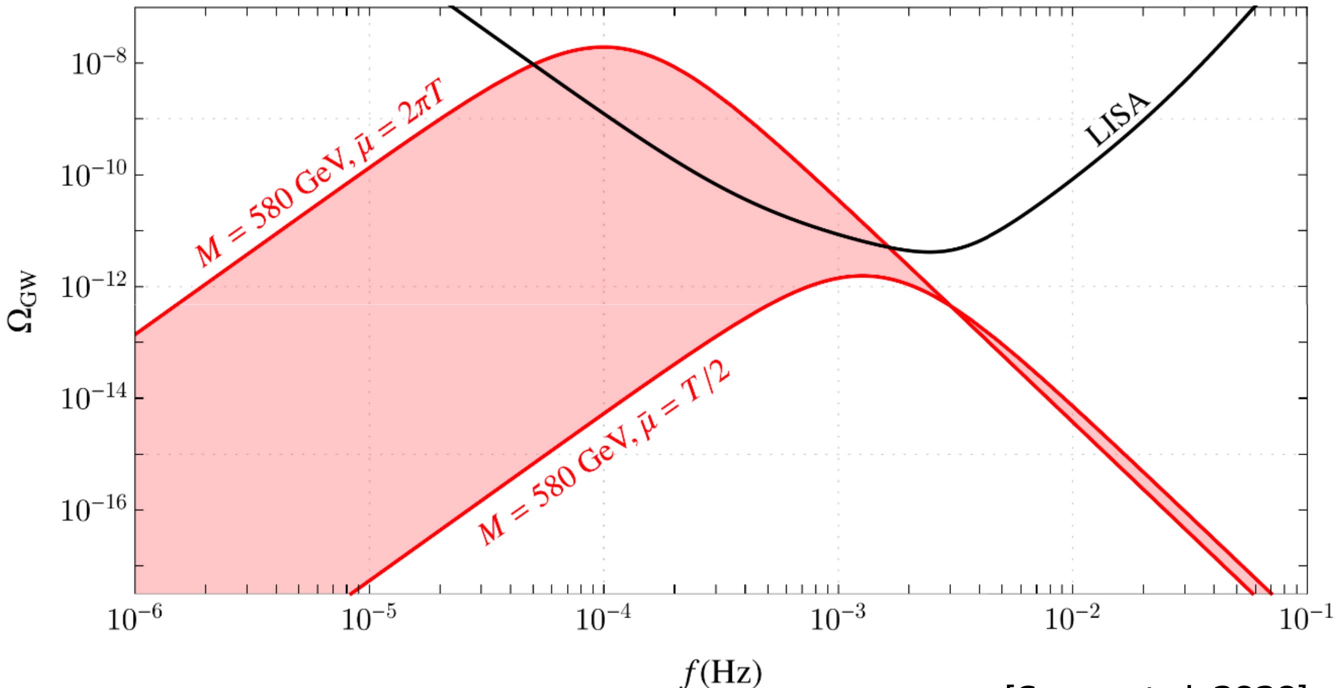
# Motivation: First Order Phase Transition in the Early Universe

- Mechanism to satisfy Sakharov's conditions for generation matter-antimatter asymmetry
- Naturally occurring in most of extensions of Higgs sectors
- Perfect candidate for gravitation wave detection in close future experiments (LISA, etc)



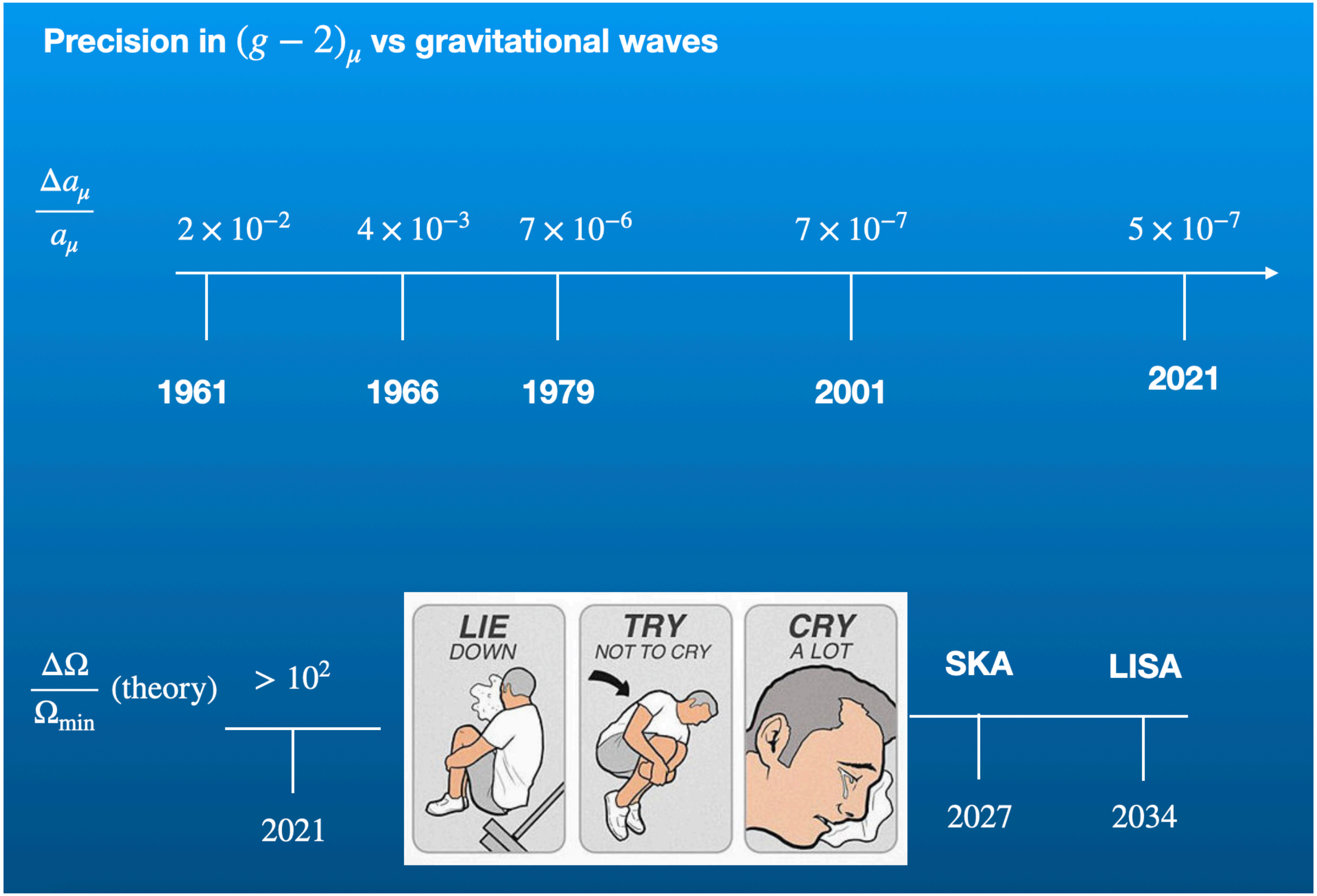

[Laine, 2000]

# Motivation: uncertainties




[Croon et al, 2020]

# Motivation: uncertainties




[White, 2024]

# Effective action/potential

$$\Gamma[\phi] = W[J] - \int d^4x\, J(x)\phi(x),$$

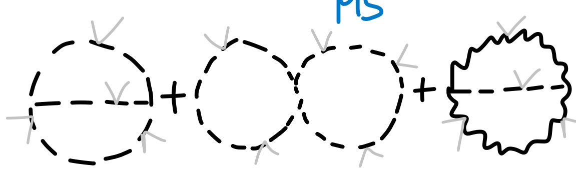
$$V_{eff}[\phi] = \frac{\Gamma[\phi]}{(vol)}$$

$$V^{eff} = V_{tree} + V_{1\text{-}loop} + V_{2\text{-}loop} + \dots$$

$$V^{T=0}_{1\text{-}loop} = -\frac{i}{2}\int \frac{d^4k}{(2\pi)^4} \ln\left(k^2 - m_i^2\right) = \sum_p \frac{n_i \cdot m_i^4}{64\pi^2}\left(\log\left(\frac{m_i^2}{\mu^2}\right) - c_i\right)$$

$\overline{MS}$

$$V^{T=0}_{2\text{-}loop} \supset$$



$+ \dots$

*full 2-loop result in arbitrary gauge is known

# Renormalization scale independence at zero temperature

$$V_{tree} = -m^2 h^2 + \lambda h^4$$

$$V_{1loop} = \frac{1}{64\pi^2} \sum_i M_i^4(h) \left[ \log\left( \frac{M_i^2(h)}{\mu^2} \right) + C \right]$$

run with $\beta \sim g^2$

explicit $\mu$-dependence $M_i^2 \sim g^2$

$$\mu \frac{d}{d\mu} \left( V_{tree} + V_{1loop} \right) = 0$$

* this holds irregardless of renormalization scheme, as beta functions and Logs in 1loop potential stay the same

# Renormalization scale dependence at non-zero temperature

- Thermal loop corrections, which are dominant during the phase transition, introduce large renormalization scale dependence.

$$V_{\text{high-}T} = -m^2 h^2 + \lambda h^4 + (\# g^2 T^2) \cdot h^2$$

$\Rightarrow$ phase transition: $m^2 \sim g^2 T^2$

$\Rightarrow$ strong: $h \sim T$

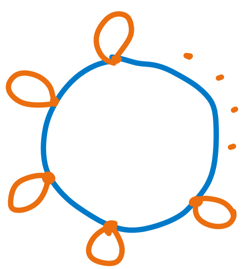
$$\mu \frac{d}{d\mu} V_{\text{high-}T} \sim g^4 T^4 \sim \mu \frac{dV_{\text{tree}}}{d\mu}$$ !

(uncancelled $\mu$ dependence)

# Daisy resummation

- The presence of hierarchy between hard ( $\sim T$ ) and soft ( $\sim gT$ ) scales requires resummation of the hard modes, which messes up with the loop order

$$V_{\text{daisy}} \sim -\frac{T}{12\pi} \sum_{\text{soft}} \left( M_i^2 + \Sigma_i(T) \right)^{3/2}$$



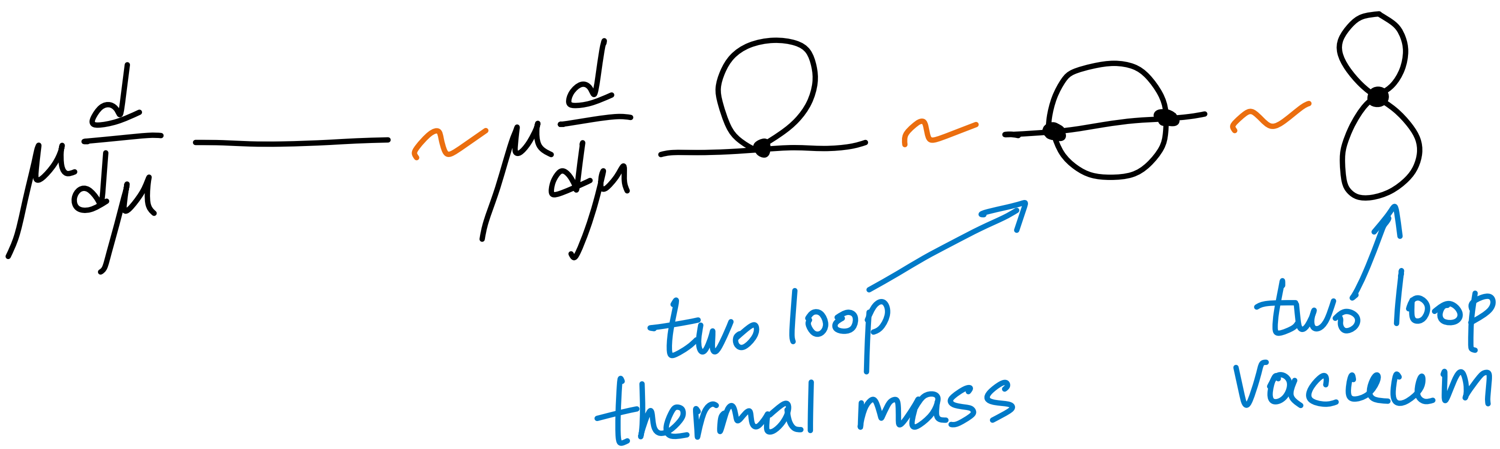
$$\#N \sim m^3 T \left( \frac{gT}{m} \right)^{2N} \sim g^3$$

$$V_{\text{thermal}} = V_{\text{tree}} + V_{\text{1-loop}}^{T=0} + V_{\text{1-loop}}^{T\neq 0} + V_{\text{daisy}}$$

# Renormalization scale independence at non-zero temperature

- To restore renormalisation scale invariance, we need to add higher loop thermal contributions

$$V_{thermal} = \underbrace{V_{tree} + (\# g^2 T^2) h^2}_{\sim g^2} - \underbrace{\frac{T}{12\pi}\left(M_i^2(h) - \Sigma_i(T)\right)^{3/2}}_{\sim g^3} + \underbrace{(\dots)}_{c \sim g^4}$$



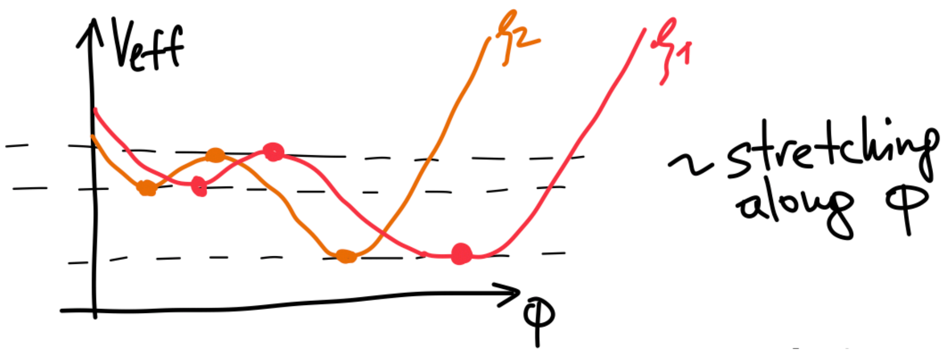
# Gauge dependency

- The effective action itself is an intrinsically gauge dependent quantity, as it's defined for the non-zero source term, and the system has a non-dynamical background change.

- But, it's gauge dependent according to Nielsen identity:

$$\frac{\partial V_{eff}}{\partial \xi} = C_i(\varphi_i, \xi) \frac{\partial V_{eff}}{\partial \varphi_i}$$

- $V_{eff}$ is gauge invariant at stationary point (extremums)






[Nielsen, 1975]

# Gauge independency

- Gauge invariant results can be obtained by systematic $\hbar$-expansion

$$V_{eff}(\varphi) = V_{tree}(\varphi) + \hbar V_{1loop}(\varphi) + \hbar^2 V_{2loop}(\varphi) + \dots$$

$$\varphi_{min} = \varphi_0 + \hbar \varphi_1 + \hbar^2 \varphi_2 + \dots \quad \text{with} \quad \left.\frac{\partial V_{eff}}{\partial \varphi}\right|_{\varphi_{min}} = 0$$

$$\downarrow$$

$$V_{eff}(\varphi_{min}) = V_0(\varphi_0) + \hbar V_1(\varphi_0) + \hbar^2 \left[ V_2(\varphi_0) + \frac{1}{2}\left(\frac{\partial V_1}{\partial \varphi}\right)^2 \left(\frac{\partial^2 V_0}{\partial \varphi^2}\right)^{-1} \right] + \dots$$



[Patel, et.al., 2011]

# Gauge dependency (again)

- Issue with DAISY resummation - not following strict loop expansion

$$V_{daisy} \sim \hbar \sum_{soft} \left[ \left( M_i^2 + \underbrace{\Sigma_i(T)}_{\sim \hbar} \right)^{3/2} - \left( M_i^2 \right)^{3/2} \right]$$

- There's a way to get gauge-independent DAISYies

Evaluate $\sum_{soft} \left[ \left( M_i^2 + \Sigma_i(T) \right)^{3/2} \right]$ at $\phi_0^{high\text{-}T}$ $\begin{pmatrix} \text{minimum of} \\ V_{tree} + V_{high\text{-}T} \end{pmatrix}$

$\sum_{soft} \left( M_i^2 \right)^{3/2}$ at $\phi_0$

# High-T EFT

## Separating logarithms

- Large $\mu$ - dependence & need for resummations indicate large separation of scales

- Relevant scales:

  - hard mode $\sim \pi T$
  - soft $\sim gT \sim m_D$
  - ultrasoft $\sim g^2 T$

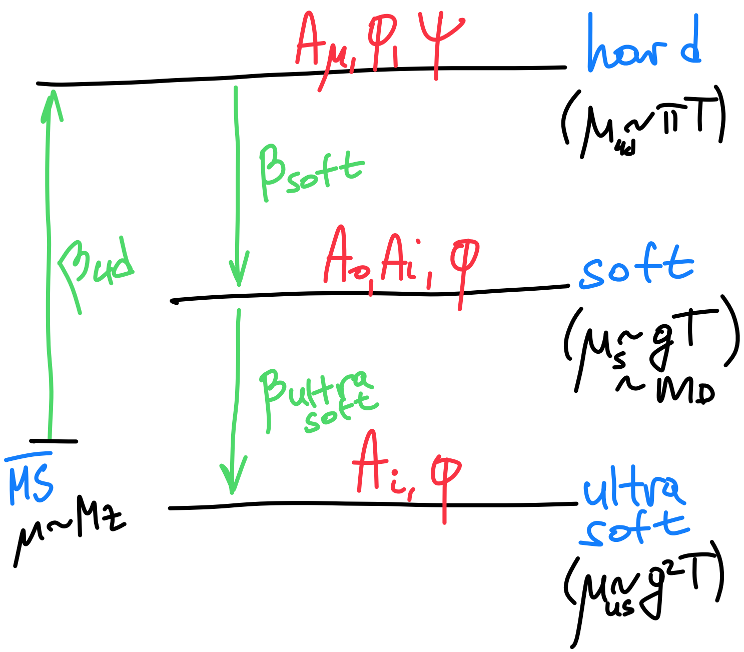
- Basically, we have too many logarithms:

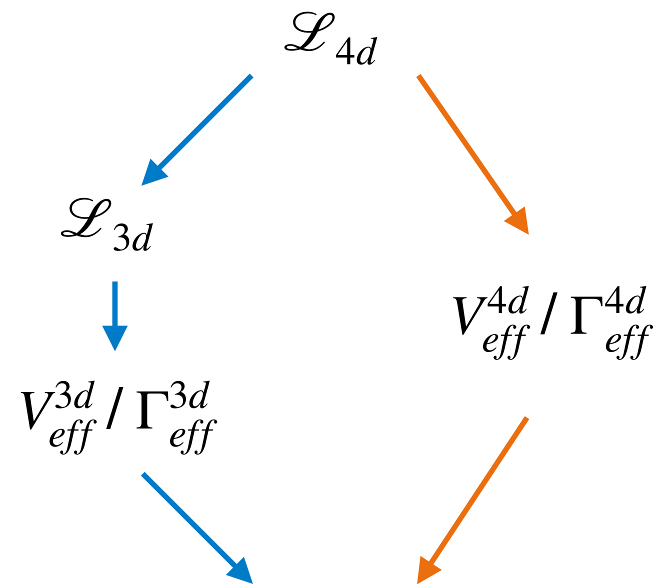
$$\log\left(\frac{\pi T}{\mu}\right), \log\left(\frac{gT}{\mu}\right), \log\left(\frac{g^2 T}{\mu}\right)$$



[Kajantie et al, 1994]

# High-T EFT




Physical observables $(T_c, T_n, \Omega_{GW}, \dots)$

High-T EFT lives in 3D:

- Simpler integrals
- Superrenormalizable theory

Automated matching tool:
**DRalgo** [Ekstedt, et.al. 2022]

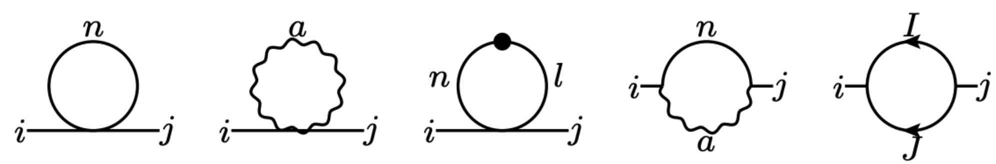
Temperature is integrated out

- Use $T = 0$ QFT framework
- Resummations are already included
- Gauge invariance is straightforward

# Dimensional reduction

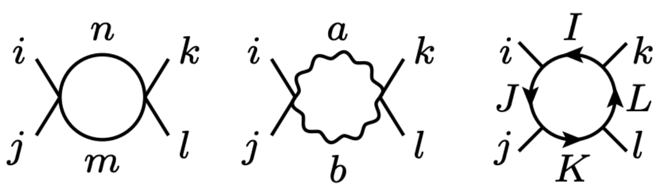
- Mass parameters

$$\int \mathrm{d}^4x \langle R_i R_j \rangle_{4\mathrm{d}} = T^{-1} \int \mathrm{d}^3x \langle R_i R_j \rangle_{4\mathrm{d}} = \int \mathrm{d}^3x \langle R_i R_j \rangle_{3\mathrm{d}}$$




- Quartics

$$\int \mathrm{d}^4x \langle R_i R_j R_k R_l \rangle_{4\mathrm{d}} = T^{-1} \int \mathrm{d}^3x \langle R_i R_j R_k R_l \rangle_{4\mathrm{d}} = \int \mathrm{d}^3x \langle R_i R_j R_k R_l \rangle_{3\mathrm{d}}$$



# DRalgo

[Ekstedt, Schicho, Tenkanen, 2022]

Automated matching (hard and soft):
- 2 loop mass parameters matching
- 2 loop Debye mass calculation
- 1 loop couplings matching

$$\bar{\mu}_3^2 = \underbrace{\boxed{\overset{\text{tree-level}}{\mu^2} + \overset{\text{1-loop}}{\#g^2T^2}}}_{\mathcal{O}(g^2)} + \underbrace{\boxed{\overset{\text{1-loop}}{\#g^2\mu^2} + \overset{\text{2-loop}}{\#g^4T^2}}}_{\mathcal{O}(g^4)} + \mathcal{O}(g^6)$$

$$+ \underbrace{\boxed{\overset{\text{1-loop}}{\#g^2 m_{\text{D}}}}}_{\mathcal{O}(g^3)} + \underbrace{\boxed{\overset{\text{2-loop}}{\#g^4}}}_{\mathcal{O}(g^4)} + \mathcal{O}(g^5) \; ,$$

Automated effective potential calculation
- 2 loop effective potential in soft/ultra-soft theory (*3loop is coming)

Automated calculation of beta function and anomalous dimensions
- In 4D parent theory
- In 3D EFT

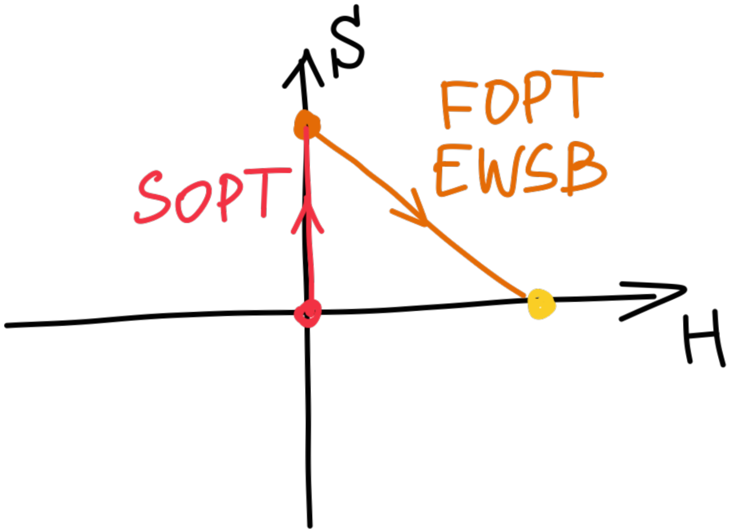
$$V_{\text{eff}}^{\text{3d}} = \underbrace{V_{\text{tree}}^{\text{3d}}}_{\mathcal{O}(g^2)} + \underbrace{V_{\text{1-loop}}^{\text{3d}}}_{\mathcal{O}(g^3)} + \underbrace{V_{\text{2-loop}}^{\text{3d}}}_{\mathcal{O}(g^4)} + \mathcal{O}(g^5)$$

Matching of higher order scalar* operators
- Dimension 6 and 8 operators

# Benchmark model: cxSM

$$\mathscr{L}_{cxSM} \supset \partial^\mu H^\dagger \partial^\mu H + \mu_h^2 H^\dagger H + \lambda_h (H^\dagger H)^2$$

$$+\frac{1}{2}|\partial^\mu S|^2 + \frac{1}{2} b_2 |S|^2 + d_2 |S|^4 + \frac{1}{2}\delta_2 |S|^2 H^\dagger H$$




$$m_{H_2} = m_A = 62.5 \text{ GeV},$$
$$d_2 = 0.5$$

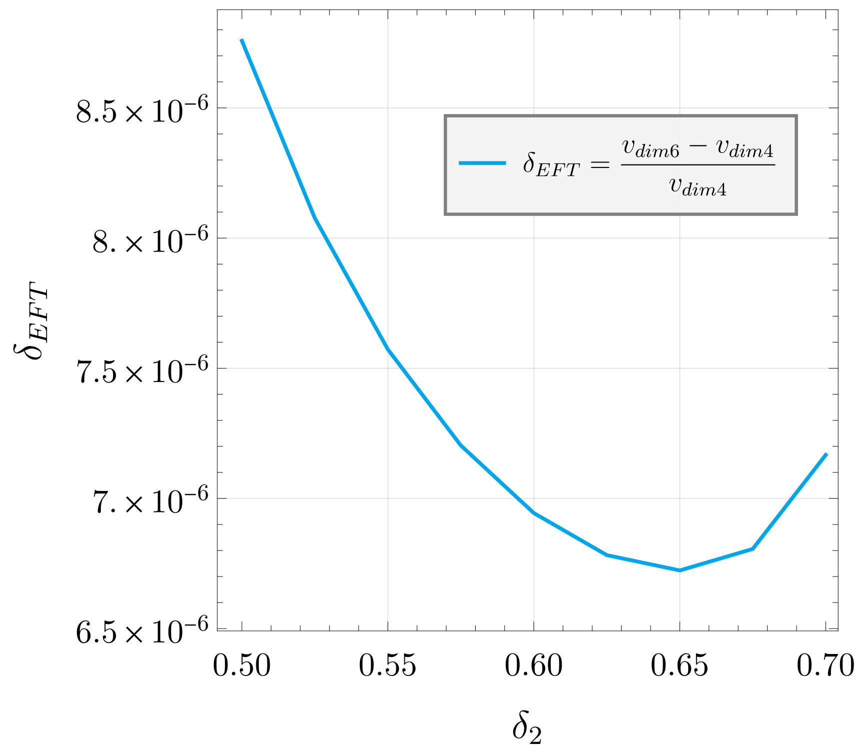
# Results: EFT validity

## Higher dimensional operators impact

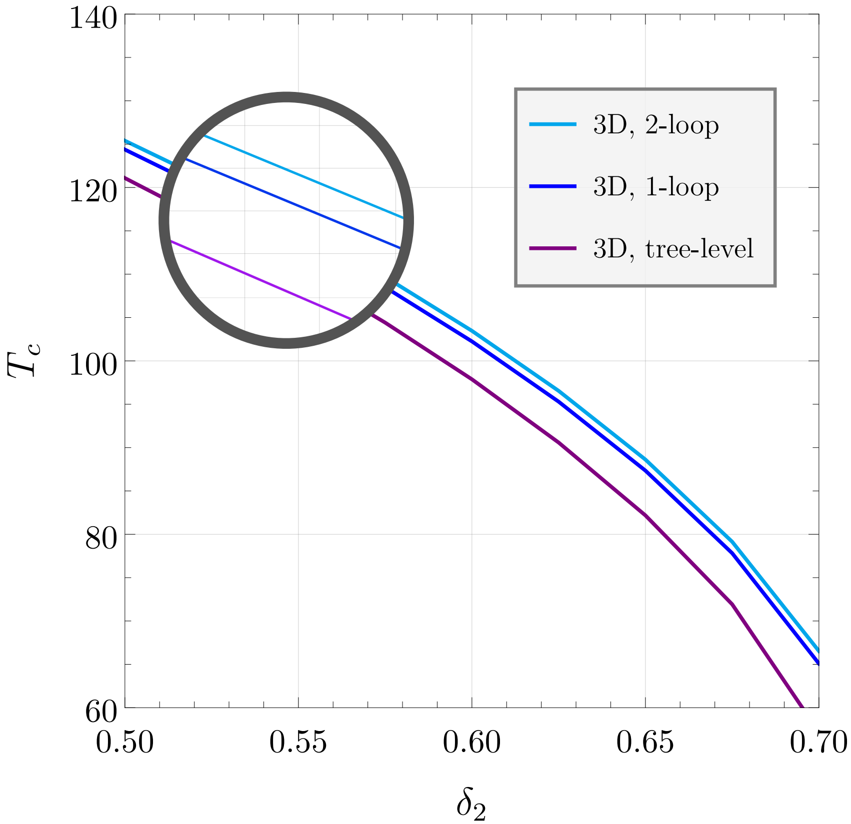
$\mathscr{L}_{4d}$ (Full theory, renormalizable*)

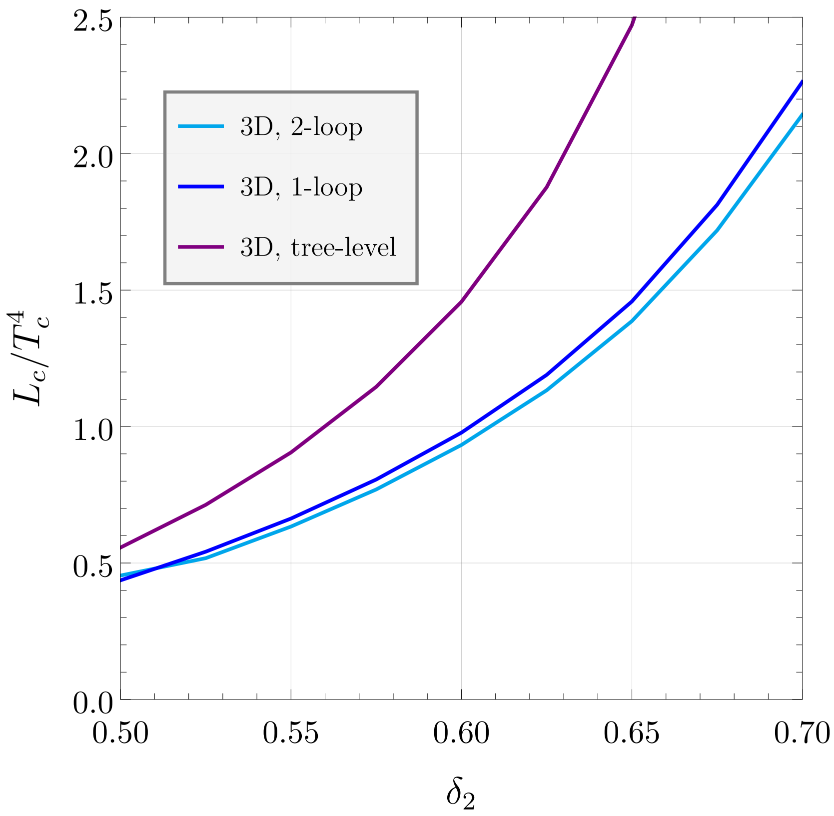
↓

$\mathscr{L}_{3d}$ (EFT, non-renormalizable)

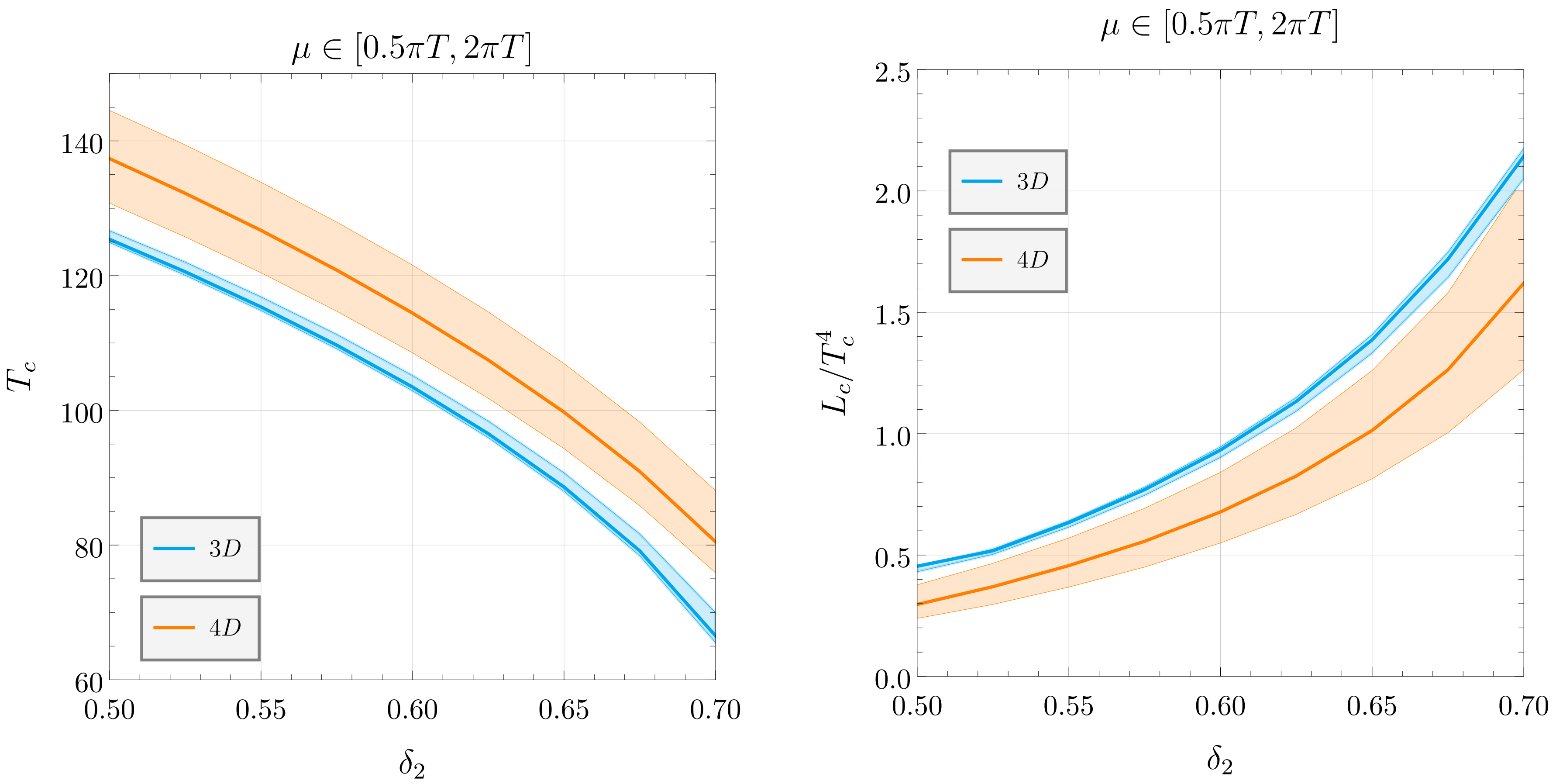


# Results: Loop convergence in EFT approach

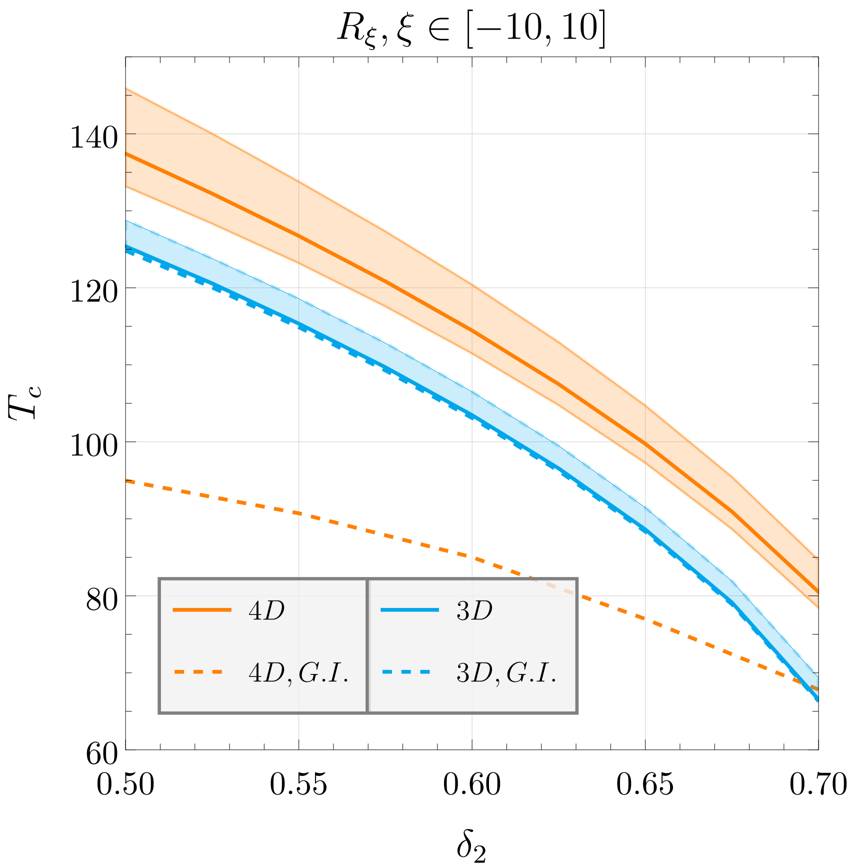


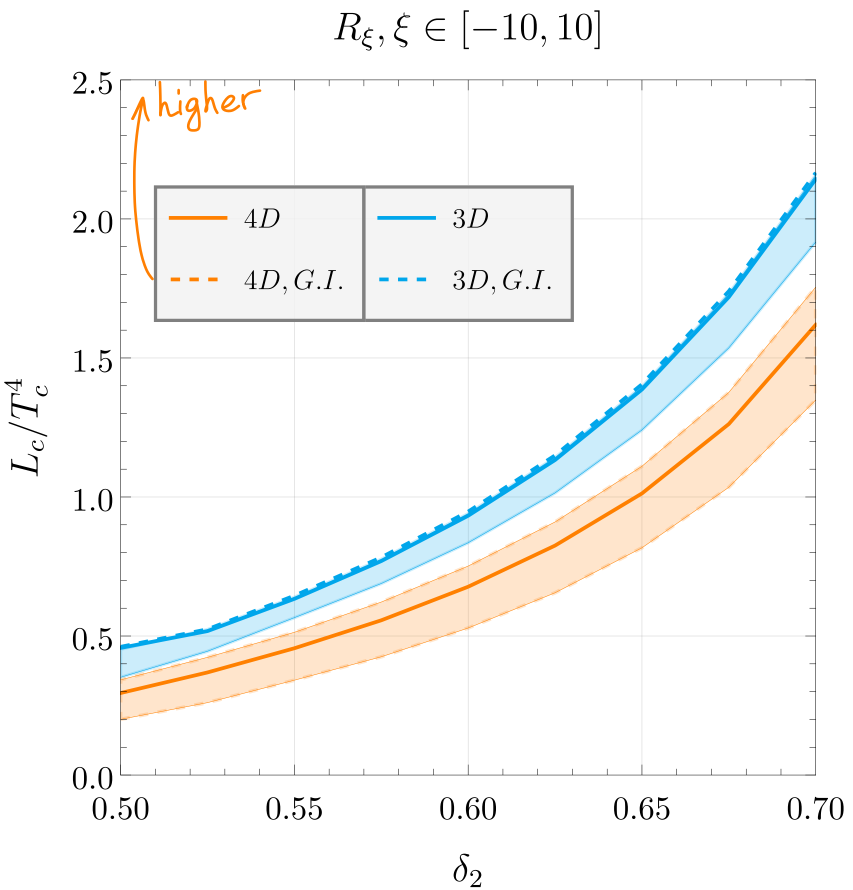



# Equilibrium Thermodynamics: renormalization scale dependence



# Equilibrium Thermodynamics: gauge (in)variance



# Lagrange parameters determination

## Another possible source of uncertainties

Physical inputs $(M_h, M_W, M_Z, G_F, \dots)$

$V_{eff}^{4d}$

n-loop $\overline{MS}$ relations

$$M_h = m_h + \Pi_h(p^2 = M_h^2)$$

$OS$-like renormalization

$$\partial_h V_{tree} = \partial_h V_{eff} + \partial_h V_{c.t.}$$

$$\partial_h^2 V_{tree} = \partial_h^2 V_{eff} + \partial_h^2 V_{c.t.}$$

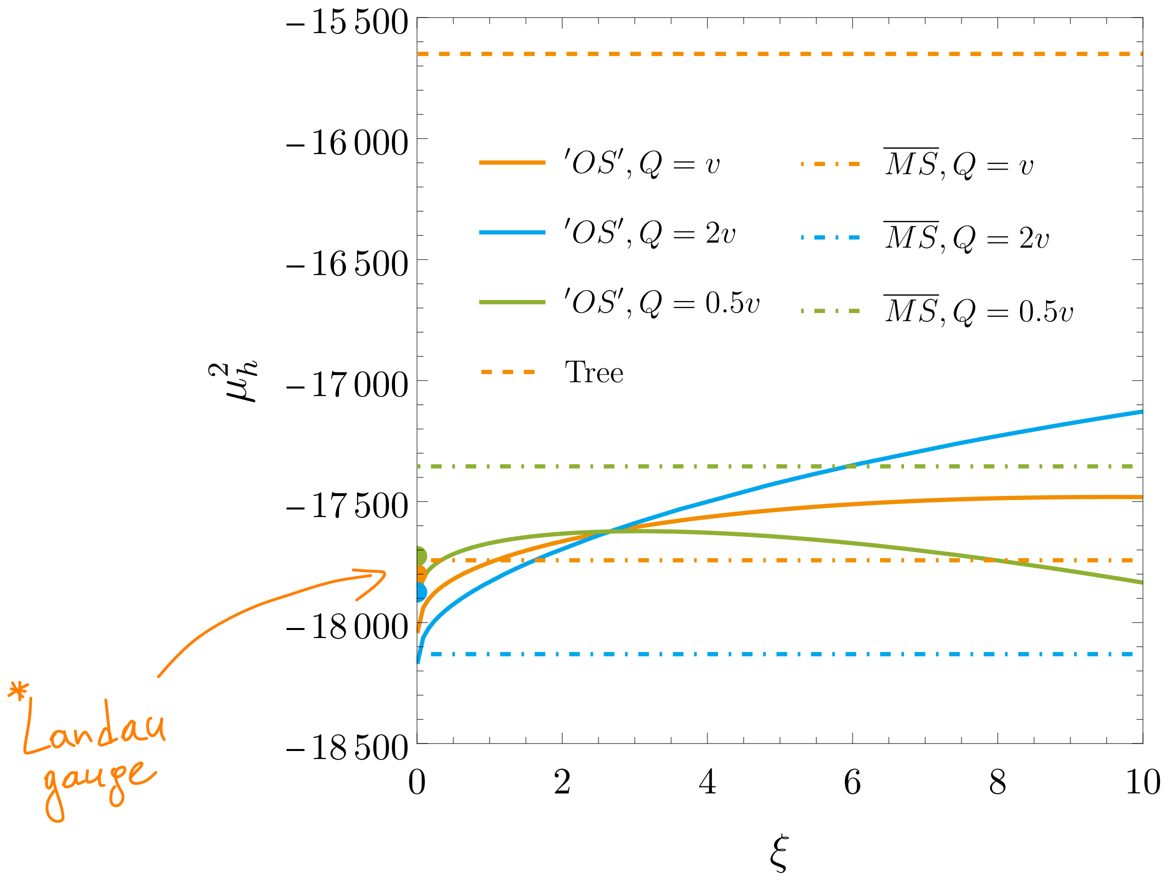
$$(\sim \Pi_h(p^2 = 0))$$

Lagrange parameters $(\mu, \lambda, g, \dots)$

- Missing momentum contribution
- Gauge dependent
- Taking derivatives must be handled carefully*
- Goldtone catastrophe in Landau gauge
- Only scalar potential couplings are renormalised

# Lagrange parameters determination

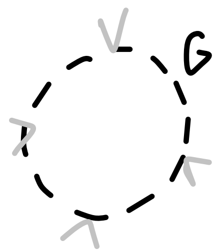
*'OS' vs $\overline{MS}$*



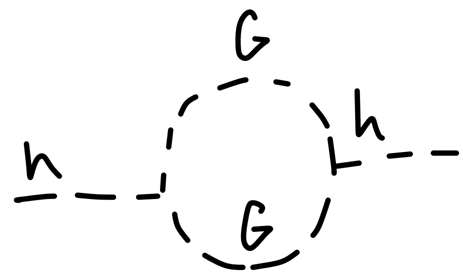
# Landau gauge: Goldstone catastrophe

- Issue arrises, as Goldstone bosons are massless at tree-level minimum in Landau gauge

$\frac{d^2}{dh^2}$




$\sim \log(m_G)$




$\sim B_0(p^2, m_G^2, m_G^2)$

$B_0(0, 0, 0) = 0$

- Smooth limit $\xi \to 0$ in $R_\xi$?

# Background vs standard $R_\xi$ gauges.

$$\mathscr{L}_{g.f.} = -\frac{1}{2\xi}(\partial^\mu A_\mu^a + i\xi g_a t_{ik}^a \tilde{\phi}_i \phi_k)^2$$

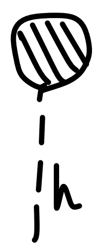
$\tilde{\phi}_i = h_i$ - background field

(!wrong derivatives*!)

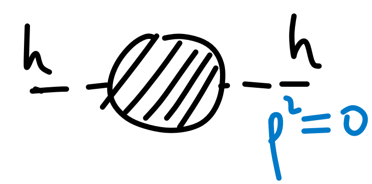
$\tilde{\phi}_i = h_{i,min} = v$ - minimal field configuration

(complicated mixings)

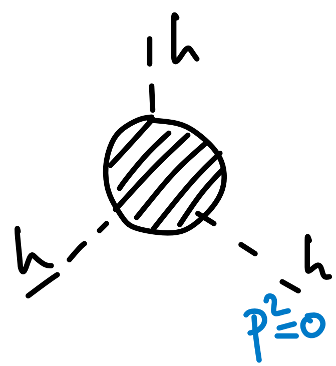
$$\left.\frac{dV_{eff}}{dh}\right|_{h=v} =$$



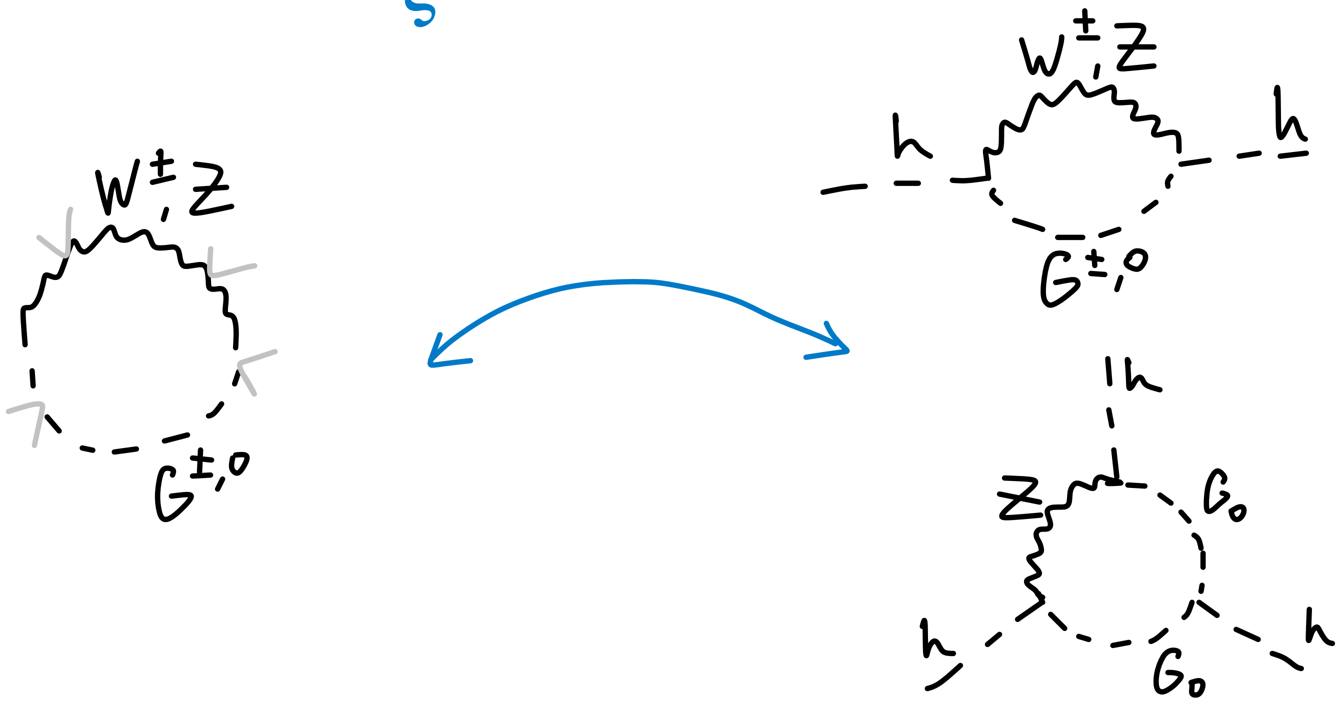

$$\left.\frac{d^2V_{eff}}{dh^2}\right|_{h=v} =$$



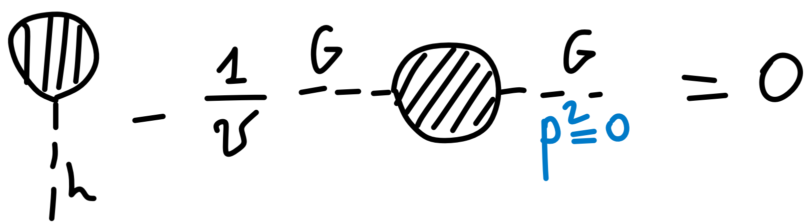

$$\left.\frac{d^3V_{eff}}{dh^3}\right|_{h=v} =$$



# Background $R_\xi$ gauges: missing parts




- Only with the correct mixing, we get the relation right:



# Conclusions.

- Thermally driven phase transitions introduce new scaling relations, which require modification of the usual perturbation theory
- Large scale separations and related resummations can be rigorously taken into account with the help of EFT techniques (dimensional reduction).
- Derivatives of the effective potential derivatives have to be taken carefully, including corresponding mixings in $R_\xi$.
- To achieve similar to equilibrium thermodynamics renormalization scale independence for non-equilibrium quantities, the bounce action has to be calculated at higher orders.