

anyH3 and di-Higgs production with loop-corrected trilinear couplings in the 2HDM

Kateryna Radchenko Serdula

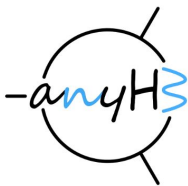
based on [arXiv: 2305.03015](#) (H. Bahl, J. Braathen, M. Gabelmann, G. Weiglein),

[arXiv: 2403.14776](#) (S. Heinemeyer, M. Mühlleitner, K. R. , G. Weiglein)

and work in progress (H. Bahl, J. Braathen, M. Gabelmann, K. R. , G. Weiglein)

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Motivation

Couplings to fermions and bosons: $m_i = \lambda_i v/2$

(λ_i are renormalizable parameters that cannot be predicted in the SM)

→ proof of the Brout-Englert-Higgs mechanism

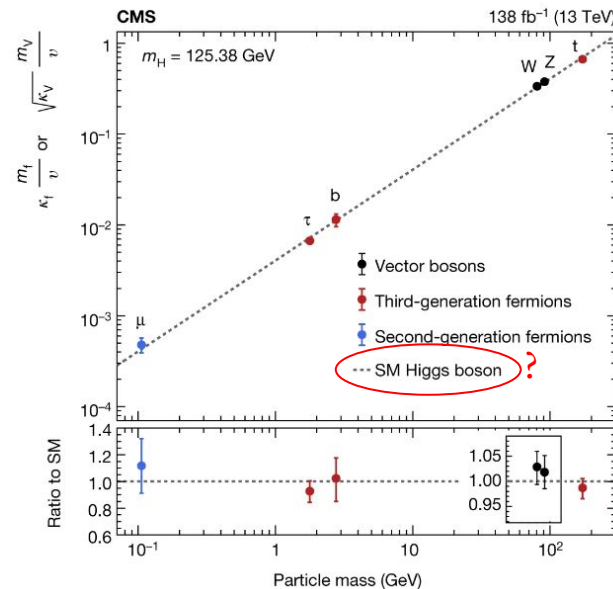
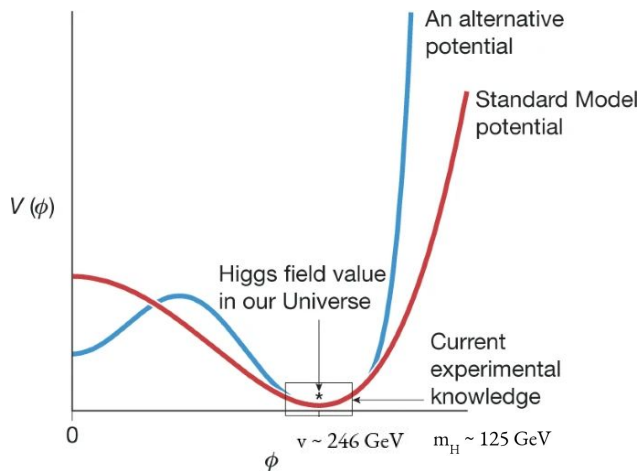


The new particle seems to be
**a Higgs boson ... but is it the
SM Higgs boson?**

Without the measurement of
the **triple Higgs coupling
(THC)** the shape of the
potential is unknown!

A potential barrier requires large deviations in the trilinears and can
be related to a **strong first order electroweak phase transition**

[Kanemura, Okada, Senaha: [arxiv: 0411354](https://arxiv.org/abs/0411354), Noble, Perelstein: [arXiv: 0711.3018](https://arxiv.org/abs/0711.3018)]



[Nature 2022]

Outline

1. Trilinear Higgs self-coupling (λ_{hhh})
 - theoretical status and experimental constraints $\rightarrow$ potential to probe BSM models
2. Generalization to **any BSM model** (anyH3)
 - methodology, user input, applications
3. Generalization to **any BSM trilinear scalar coupling** (anyH3 v2)
4. Future prospects: insertion in gluon fusion **double Higgs production**
 - developments, applications, impact on constraints, ongoing work

Higgs self coupling measurements

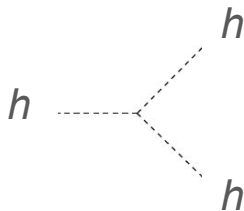
[ATLAS: [arXiv: 2211.01216](https://arxiv.org/abs/2211.01216)]

- Motivation: probe of **Higgs potential** and a **window to BSM**
- Can have **large deviations** from SM predictions in BSM while the couplings to gauge bosons and fermions are very close to the SM values (in agreement with existing constraints)
- Improving limits already impact the phenomenology

Notation:

$$\kappa_\lambda = \lambda_{hhh} / \lambda_{hhh}^{\text{SM}(0)}$$

$$\lambda_{hhh}^{\text{SM}(0)} = 3m_h/v$$



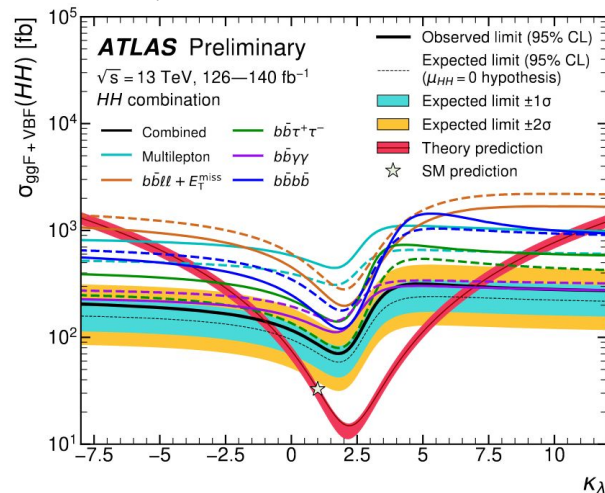
→ in the SM it is fully fixed by m_h (125 GeV) and v (246 GeV)

Experimental status:

- access through **Higgs pair production**

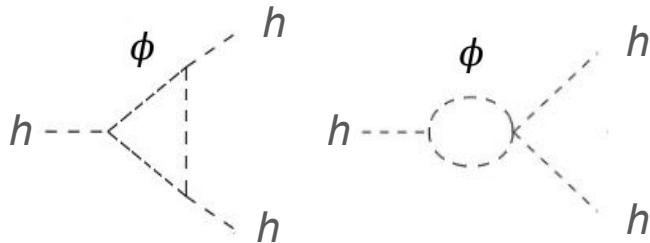
$$\mu_{hh} \leq 2.4$$

$[-1.2 < \kappa_\lambda < 7.2]$ (95% CL at LHC Run II)



Radiative corrections to the trilinear couplings

- These become extremely important in BSM theories because of the effect of heavy New Physics: ϕ



- Theoretical predictions of the 1 loop results of the trilinear already exist in many models:

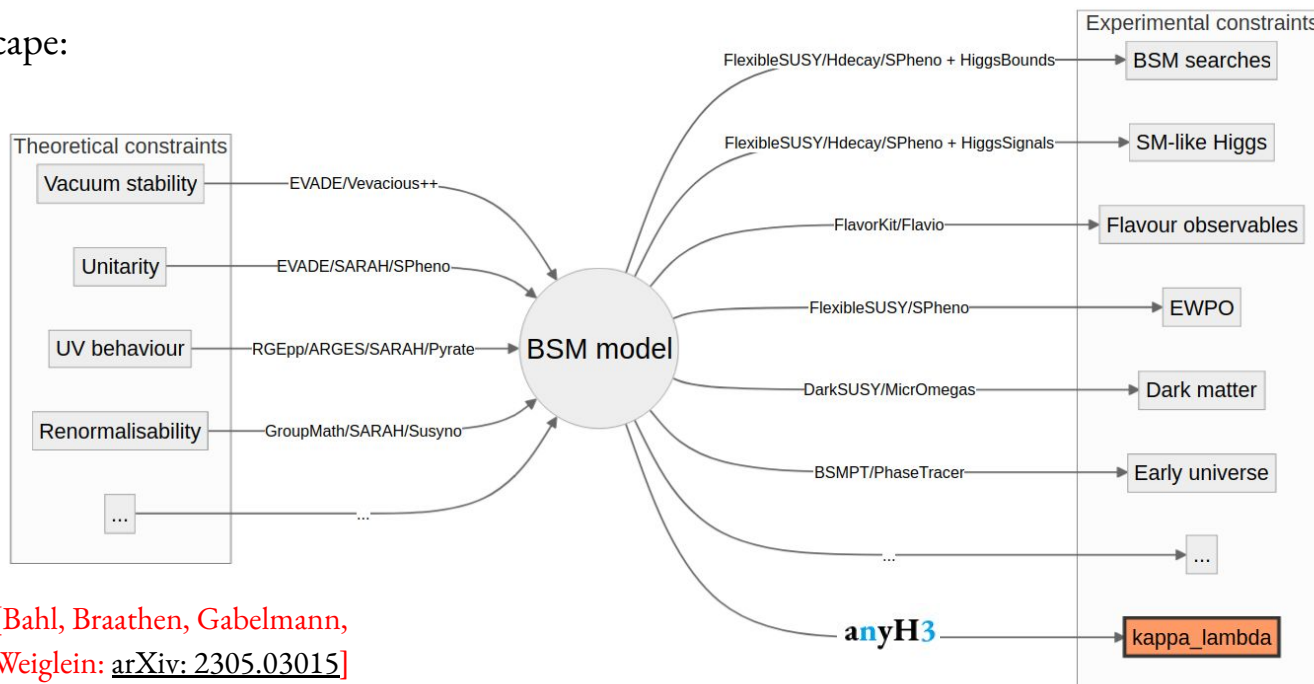
SM + singlet [Kanemura et al. '16]; **2HDMs** [Kanemura et al. '04], [Basler et al. '17]; **N2HDM (2HDM + singlet)** [Basler et al. '19]; **triplet extensions** [Aoki et al. '12], [Chiang et al. '18]; **MSSM** [Hollik, Penaranda '04]; **NMSSM** [Dao et al. '13]; **models with classical scale invariance** [Hashino, Kanemura, Orikasa '16], etc.

[List taken from J. Braathen]

- .. and also at 2 loops [See talk by M. Gabelmann on Friday]
- Would be useful to automate the prediction of this parameter in any BSM model $\rightarrow$ **anyH3**

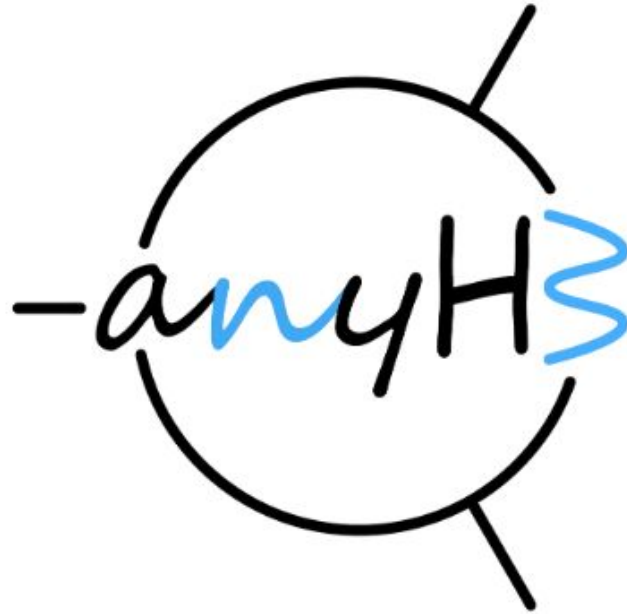
anyH3 purpose

- Close the gap regarding κ_λ predictions in the landscape of BSM tool-boxes:
- The landscape:



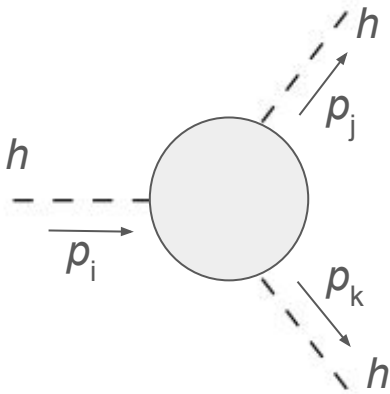
[Bahl, Braathen, Gabelmann,
Weiglein: [arXiv: 2305.03015](#)]

Generic predictions for the trilinear couplings



Trilinear Higgs couplings (generic approach)

- Trilinear couplings = renormalized 3 point functions : $\lambda_{hhh} = -\hat{\Gamma}_{hhh}(p_1^2, p_2^2, p_3^2)$

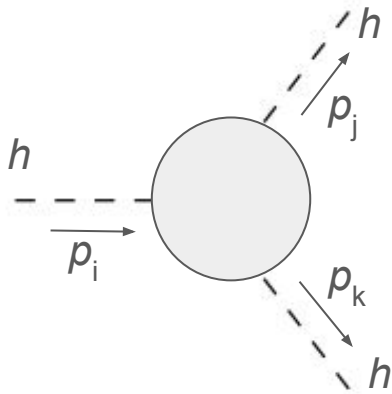


Possible topologies:

$$= \lambda_{hhh}^{(0)} + \delta_{\text{genuine}}^{(1)} \lambda_{hhh} + \delta_{\text{tadpoles}}^{(1)} \lambda_{hhh} + \delta_{\text{WFR}}^{(1)} \lambda_{hhh} + \delta_{\text{CT}}^{(1)} \lambda_{hhh}$$

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$$= \underbrace{\text{tree-level: } \lambda_{hhh}^{(0)}}_{\text{tree-level: } \lambda_{hhh}^{(0)}} + \underbrace{\text{one-particle irreducible: } \delta_{\text{genuine}}^{(1)} \lambda_{hhh}}_{\text{one-particle irreducible: } \delta_{\text{genuine}}^{(1)} \lambda_{hhh}}$$

$$+ \underbrace{\text{external leg corrections: } \delta_{\text{WFR}}^{(1)} \lambda_{hhh}}_{\text{external leg corrections: } \delta_{\text{WFR}}^{(1)} \lambda_{hhh}} + \underbrace{\text{tadpoles: } \delta_{\text{tad. WFR}}^{(1)} \lambda_{hhh} + \delta_{\text{tadpoles}}^{(1)} \lambda_{hhh}}_{\text{tadpoles: } \delta_{\text{tad. WFR}}^{(1)} \lambda_{hhh} + \delta_{\text{tadpoles}}^{(1)} \lambda_{hhh}} + \underbrace{\text{renormalisation: } \delta_{\text{CT}}^{(1)} \lambda_{hhh}}_{\text{renormalisation: } \delta_{\text{CT}}^{(1)} \lambda_{hhh}}$$

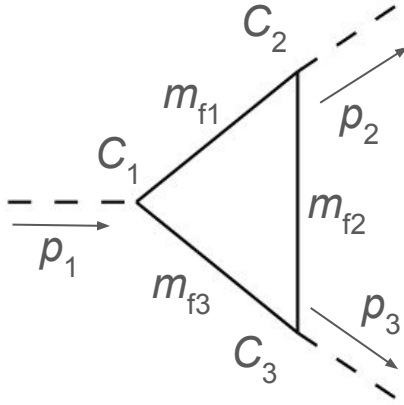
note: solid line here means generic field

Automation methodology

1. Identify all possible Feynman diagram **topologies** contributing to the three point function
2. Insert all allowed fields (**S** = scalar, **V** = vector, **F** = fermion, **U** = ghost)
3. Substitute in the general analytic expressions (FeynArts + FormCalc) $\rightarrow$ calculated from generic Lagrangian in terms of couplings (C_i), masses of the particles in the loop (m_i) and the external leg momenta (p_i)

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$$\begin{aligned}
 &= 2\mathbf{B0}(p_3^2, m_2^2, m_3^2)(C_1^L(C_2^L C_3^R m_{f_1} + C_2^R C_3^R m_{f_2} + C_2^R C_3^L m_{f_3}) + C_1^R(C_2^R C_3^L m_{f_1} + \\
 &C_2^L C_3^L m_{f_2} + C_2^L C_3^R m_{f_3})) + m_{f_1} \mathbf{C0}(p_2^2, p_3^2, p_1^2, m_1^2, m_2^2, m_3^2)((C_1^L C_2^L C_3^R + \\
 &C_1^R C_2^R C_3^L)(p_1^2 + p_2^2 - p_3^2) + 2(C_1^L C_2^L C_3^L + C_1^R C_2^R C_3^R)m_{f_2} m_{f_3} + \\
 &2m_{f_1}(C_1^L(C_2^L C_3^R m_{f_1} + C_2^R C_3^R m_{f_2} + C_2^R C_3^L m_{f_3}) + C_1^R(C_2^R C_3^L m_{f_1} + C_2^L C_3^L m_{f_2} + \\
 &C_2^L C_3^R m_{f_3}))) + \mathbf{C1}(p_2^2, p_3^2, p_1^2, m_1^2, m_2^2, m_3^2)(2p_2^2(C_1^L C_3^R(C_2^L m_{f_1} + C_2^R m_{f_2}) + \\
 &C_1^R C_3^L(C_2^R m_{f_1} + C_2^L m_{f_2})) + (p_1^2 + p_2^2 - p_3^2)((C_1^L C_2^L C_3^R + C_1^R C_2^R C_3^L)m_{f_1} + \\
 &(C_1^L C_2^R C_3^L + C_1^R C_2^L C_3^R)m_{f_3})) + \mathbf{C2}(p_2^2, p_3^2, p_1^2, m_1^2, m_2^2, m_3^2)((p_1^2 + p_2^2 - \\
 &p_3^2)(C_1^L C_3^R(C_2^L m_{f_1} + C_2^R m_{f_2}) + C_1^R C_3^L(C_2^R m_{f_1} + C_2^L m_{f_2})) + 2p_1^2((C_1^L C_2^L C_3^R + \\
 &C_1^R C_2^R C_3^L)m_{f_1} + (C_1^L C_2^R C_3^L + C_1^R C_2^L C_3^R)m_{f_3}))
 \end{aligned}$$

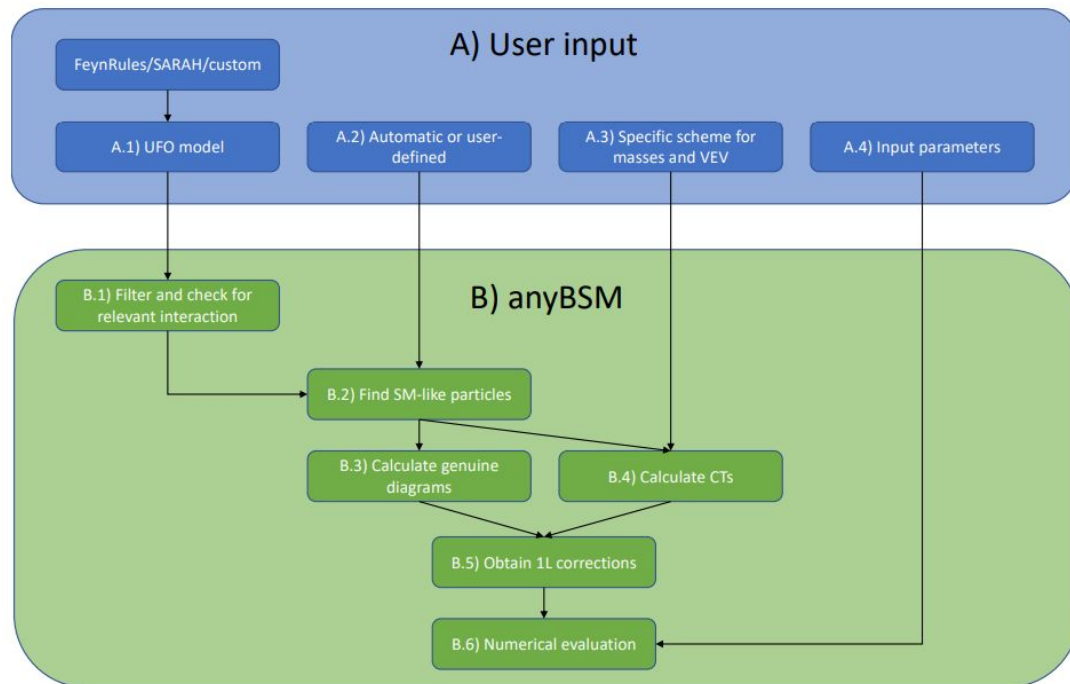
$$C_i = C_i^L P_L + C_i^R P_R, \text{ where } P_{L,R} \equiv \frac{1}{2}(1 \mp \gamma_5)$$

*loop integrals evaluated with pyCollier
(original Fortran code: COLLIER)

[Denner, Dittmaier, Hofer: [arXiv: 1604.06792](https://arxiv.org/abs/1604.06792)]

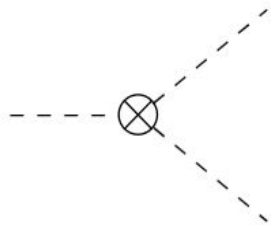
User input

- anyH3 takes as input **UFO model files** that can be generated by SARAH, FeynRules, etc.
(this sets particles, parameters, couplings, vertices, lorentz structures in the model)
- to convert to anyH3 notation one can use the inbuilt tool, which will map to the corresponding conventions automatically
- optionally, the user can provide a renormalisation scheme for the model



Renormalisation

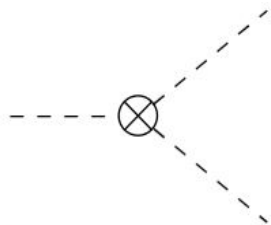
- The trilinear coupling is not an observable!
- Easily automated schemes like $\overline{\text{MS}}$ or $\overline{\text{DR}}$ can lead to artificially large corrections and other issues
[See previous talk by Andrii Dashko]
- Many options in anyH3:
 - * SM sector: fully OS or $\overline{\text{MS}}/\overline{\text{DR}}$
 - * BSM sector masses: fully OS or $\overline{\text{MS}}/\overline{\text{DR}}$, BSM sector fields: OS or $\overline{\text{MS}}/\overline{\text{DR}}$
 - * BSM sector couplings and vevs: custom or $\overline{\text{MS}}$



$$\delta_{\text{CT}}^{(1)} \lambda_{hhh} = \sum_i \frac{\partial}{\partial m_i^2} \lambda_{hhh}^{(0)} \cdot \delta_{\text{CT}}^{(1)} m_i^2 + \frac{\partial}{\partial v} \lambda_{hhh}^{(0)} \cdot \delta_{\text{CT}}^{(1)} v + \delta_{\text{custom-CT}}^{(1)} \lambda_{hhh}$$

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$$\delta_{\text{CT}}^{(1)} \lambda_{hhh} = \sum_i \frac{\partial}{\partial \boxed{m_i^2}} \lambda_{hhh}^{(0)} \cdot \delta_{\text{CT}}^{(1)} m_i^2 + \frac{\partial}{\partial v} \lambda_{hhh}^{(0)} \cdot \delta_{\text{CT}}^{(1)} v + \boxed{\delta_{\text{custom-CT}}^{(1)} \lambda_{hhh}}$$

$$\boxed{m_i} = m_h(\text{SM}), m_\phi(\text{BSM})$$

$$\boxed{\delta_{\text{custom-CT}}^{(1)} \lambda_{hhh}} = \sum_{\alpha_i} \frac{\partial}{\partial \boxed{\alpha_i}} \lambda_{hhh}^{(0)} \cdot \delta_{\text{CT}}^{(1)} \alpha_i$$

$$\boxed{\alpha_i} = \text{BSM parameters (mixing angles, indep. couplings, etc.)}$$

What does OS (on-shell) mean?

- Counterterms in this expression:
$$\delta_{\text{CT}}^{(1)} \lambda_{hhh} = \sum_i \frac{\partial}{\partial m_i^2} \lambda_{hhh}^{(0)} \cdot \boxed{\delta_{\text{CT}}^{(1)} m_i^2} + \frac{\partial}{\partial v} \lambda_{hhh}^{(0)} \cdot \boxed{\delta_{\text{CT}}^{(1)} v} + \boxed{\delta_{\text{custom-CT}}^{(1)} \lambda_{hhh}}$$

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- On shell **masses**: $\boxed{\delta_{\text{CT}}^{(1)} m_i^2} = -\text{Re} \Sigma_{ii}(p^2 = m_i^2)$
- On shell **vev**: $v^{\text{OS}} \equiv \frac{2M_W}{e} \sqrt{1 - \frac{M_W^2}{M_Z^2}} \rightarrow \frac{\boxed{\delta_{\text{CT}}^{(1)} v}}{v^{\text{OS}}} = \frac{\delta^{(1)} M_W^2}{2M_W^2} + \frac{\cos^2 \theta_w}{2 \sin^2 \theta_w} \left(\frac{\delta^{(1)} M_Z^2}{M_Z^2} - \frac{\delta^{(1)} M_W^2}{M_W^2} \right) - \frac{\delta^{(1)} e}{e}$
- On shell **tadpoles**: $\boxed{\delta_{\text{custom-CT}}^{(1)} \lambda_{hhh}} = \frac{\partial}{\partial t_h} \lambda_{hhh}^{(0)} \cdot \delta t_h^{\text{OS}}, \delta t_h^{\text{OS}} = -T_h = \text{unrenormalised scalar one point function}$
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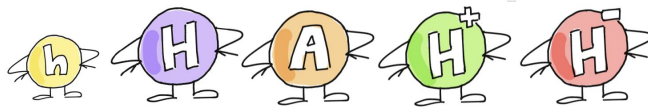
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- On shell **fields**: $\delta Z_{hh}^{\text{OS}} = \left. \frac{-\partial \Sigma_{hh}(p^2)}{\partial p^2} \right|_{p^2=m_h^2}$ included in $\delta_{\text{WFR}}^{(1)} \lambda_{hhh}$

BSM Example: 2HDM

[T. D. Lee (1973) *Physical Review*, Branco, Ferreira et al: [arXiv: 1106.0034](#)]

- **CP conserving** 2HDM with two complex doublets: $\Phi_1 = \begin{pmatrix} \phi_1^+ \\ \frac{v_1 + \rho_1 + i\eta_1}{\sqrt{2}} \end{pmatrix}, \Phi_2 = \begin{pmatrix} \phi_2^+ \\ \frac{v_2 + \rho_2 + i\eta_2}{\sqrt{2}} \end{pmatrix}$



- **Softly broken $\mathbb{Z}_2$ symmetry** ($\Phi_1 \rightarrow \Phi_1; \Phi_2 \rightarrow -\Phi_2$) entails 4 Yukawa types

- Potential:

$$V_{2\text{HDM}} = m_{11}^2(\Phi_1^\dagger \Phi_1) + m_{22}^2(\Phi_2^\dagger \Phi_2) - m_{12}^2(\Phi_1^\dagger \Phi_2 + \Phi_2^\dagger \Phi_1) + \frac{\lambda_1}{2}(\Phi_1^\dagger \Phi_1)^2 + \frac{\lambda_2}{2}(\Phi_2^\dagger \Phi_2)^2 + \lambda_3(\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \lambda_4(\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) + \frac{\lambda_5}{2}((\Phi_1^\dagger \Phi_2)^2 + (\Phi_2^\dagger \Phi_1)^2),$$

- Free parameters:

$$m_h, m_H, m_A, m_{H^\pm}, M, c_{\beta-\alpha}, t_\beta, v$$

$$\begin{aligned} \tan \beta &= v_2/v_1 \\ v^2 &= v_1^2 + v_2^2 \sim (246 \text{ GeV})^2 \end{aligned}$$

$$c_\theta \equiv \cos \theta, s_\theta \equiv \sin \theta, t_\theta \equiv \tan \theta$$

$$M^2 = \frac{m_{12}^2}{c_\beta s_\beta}$$

OS renormalisation example: THDM

- **tadpoles:**

$$\begin{array}{c} \bigcirc \\ \vdots \\ T_h \end{array} + \begin{array}{c} \times \\ \vdots \\ \delta t \end{array} = 0$$

Conditions

Counterterms

$$\hat{T}_h = \cancel{t_h^0} + T_h + \delta t \stackrel{!}{=} 0 \Rightarrow \boxed{\text{CT}}: \delta t_h^{\text{OS}} = -T_h$$

OS renormalisation example: THDM

- **tadpoles:**

$$\text{tadpole diagram} + \text{crossed tadpole} = 0$$

The first diagram is a tadpole with a solid line loop and a dashed line labeled T_h attached. The second diagram is a tadpole with a solid line loop and a dashed line labeled δt attached, with a cross over the loop.

- **masses:**

$$\text{mass diagram} + \text{crossed mass diagram} = 0$$

The first diagram is a mass correction with a solid line loop and two dashed lines labeled h_i and h_j attached, with $\Sigma h_i h_j$ below. The second diagram is a mass correction with a solid line loop and two dashed lines attached, with a cross over the loop and δm_h^2 below.

Conditions

Counterterms

$$\hat{T}_h = t_h^0 + T_h + \delta t \stackrel{!}{=} 0 \Rightarrow \boxed{\text{CT}}: \delta t_h^{\text{OS}} = -T_h$$

$$\text{Re } \hat{\Sigma}_{hh}(m_h^2) \stackrel{!}{=} 0 \Rightarrow \boxed{\text{CT}}: \delta m_h^{2\text{OS}} = \text{Re}(\Sigma_{hh}(m_h^2))$$

OS renormalisation example: THDM

- **tadpoles:**

$$\text{tadpole diagram} + \text{crossed tadpole} = 0$$

- **masses:**

$$\text{self-energy diagram} + \text{crossed self-energy} = 0$$

- **fields:**

renormalized self energies:

$$\hat{\Sigma}_{hh}(p^2) = \Sigma_{hh}(p^2) - \delta m_h^2 + (p^2 - m_h^2) \delta Z_{hh}$$

$$\hat{\Sigma}_{hH}(p^2) = \Sigma_{hH}(p^2) + (p^2 - m_h^2) \frac{\delta Z_{hH}}{2} + (p^2 - m_H^2) \frac{\delta Z_{Hh}}{2}$$

Conditions

Counterterms

$$\hat{T}_h = t_h^0 + T_h + \delta t \stackrel{!}{=} 0 \Rightarrow \text{CT}: \delta t_h^{\text{OS}} = -T_h$$

$$\text{Re } \hat{\Sigma}_{hh}(m_h^2) \stackrel{!}{=} 0 \Rightarrow \text{CT}: \delta m_h^{2\text{OS}} = \text{Re}(\Sigma_{hh}(m_h^2))$$

$$\left. \frac{\partial \hat{\Sigma}_{hh}}{\partial p^2} \right|_{p^2=m_h^2} \stackrel{!}{=} 0 \Rightarrow \text{CT}: \delta Z_{hh}^{\text{OS}} = \left. \frac{-\partial \Sigma_{hh}(p^2)}{\partial p^2} \right|_{p^2=m_h^2}$$

$$\hat{\Sigma}_{hH}(m_h^2) \stackrel{!}{=} 0 \Rightarrow \text{CT}: \delta Z_{Hh} = -2 \frac{\Sigma_{hH}(m_h^2)}{m_h^2 - m_H^2}$$

$$\hat{\Sigma}_{hH}(m_H^2) \stackrel{!}{=} 0 \Rightarrow \text{CT}: \delta Z_{hH} = 2 \frac{\Sigma_{hH}(m_H^2)}{m_h^2 - m_H^2}$$

OS renormalisation example: THDM

- **tadpoles:**

$$\text{tadpole diagram} + \text{crossed tadpole diagram} = 0$$

$T_h + \delta t = 0$

- **masses:**

$$\text{self-energy diagram} + \text{crossed self-energy diagram} = 0$$

$\Sigma_{hh} + \delta m_h^2 = 0$

- **fields:**

renormalized self energies:

$$\hat{\Sigma}_{hh}(p^2) = \Sigma_{hh}(p^2) - \delta m_h^2 + (p^2 - m_h^2) \delta Z_{hh}$$

$$\hat{\Sigma}_{hH}(p^2) = \Sigma_{hH}(p^2) + (p^2 - m_h^2) \frac{\delta Z_{hH}}{2} + (p^2 - m_H^2) \frac{\delta Z_{HH}}{2}$$

- **other parameters:** δM^2 can be defined through some physical process or another parameter (e.g. independent couplings) [See morning talk by Kei Yagyu and next talk by Alain Verduras]

Conditions

Counterterms

$$\hat{T}_h = t_h^0 + T_h + \delta t \stackrel{!}{=} 0 \Rightarrow \text{CT: } \delta t_h^{\text{OS}} = -T_h$$

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Renormalisation example: THDM user defined scheme

- **tadpoles:** $\frac{\partial V}{\partial \rho_1} = \left|_{\substack{\Phi_1 = \langle \Phi_1 \rangle, \\ \Phi_2 = \langle \Phi_2 \rangle}} \right. \stackrel{!}{=} t_1, \quad \frac{\partial V}{\partial \rho_2} = \left|_{\substack{\Phi_1 = \langle \Phi_1 \rangle, \\ \Phi_2 = \langle \Phi_2 \rangle}} \right. \stackrel{!}{=} t_2$

user defined:

$$\begin{aligned} t_1 &= \cos \alpha t_H - \sin \alpha t_h \\ t_2 &= \sin \alpha t_H + \cos \alpha t_h \end{aligned}$$

these terms need to be kept in the UFO file so that all parameters depend on t_1, t_2 , including the trilinears

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user defined:

- **masses and fields:** automatic
- **mixing angles** (through off diagonal self energies):

$$\begin{pmatrix} H_0 \\ h_0 \end{pmatrix} = \begin{pmatrix} 1 + \frac{1}{2}\delta Z_{HH} & \frac{1}{2}Z_{Hh} + \delta\alpha \\ \frac{1}{2}Z_{hH} - \delta\alpha & 1 + \frac{1}{2}\delta Z_{HH} \end{pmatrix} \begin{pmatrix} H \\ h \end{pmatrix}$$

$$\begin{aligned} \delta\alpha &= \frac{\Sigma_{hH}(m_h^2) + \Sigma_{hH}(m_H^2)}{2(m_H^2 - m_h^2)} \\ \delta\beta &= -\frac{\Sigma_{G^{(0)}A}(0) + \Sigma_{G^{(0)}A}(m_A^2)}{2m_A^2} \end{aligned}$$

→ this is a version of an OS scheme, see also:

[Krause et al. [1605.04853](#)

Denner et al. [1808.03466](#)

Kanemura et al. [1705.05399](#)]

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- **other parameters:**

user defined:

$$\begin{aligned} \delta M &= \frac{M}{32\pi^2 v^2} \left\{ -3 (2M_W^2 + M_Z^2) + 6 \left(\frac{m_u^2 + m_c^2 + m_t^2}{t_\beta^2} + (m_d^2 + m_s^2 + m_b^2) t_\beta^2 \right) \right. \\ &\quad \left. + 2t_\beta^2 (m_e^2 + m_\mu^2 + m_\tau^2) - \frac{s_{2\alpha}}{s_{2\beta}} (M_h^2 - M_H^2) + 4M^2 - 2M_{H^\pm}^2 - M_A^2 \right\} \log \left(\frac{4Q^2}{(M + m_t)^2} \right) \end{aligned}$$

Renormalisation example: THDM user defined scheme

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- other parameters:

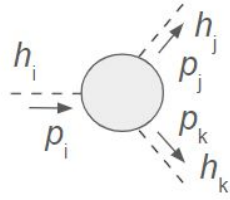
user defined:

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Applications of anyH3

- Compute loop corrected trilinears in a user defined model (14 BSM models already implemented)
- Track the momentum dependence of the coupling
- Use the result for theoretical predictions in the double Higgs (hh) production cross section
- Estimate higher order uncertainties
- Phenomenology and powerful exclusion potential of the parameter space of BSM models → easily incorporated in large parameter scans

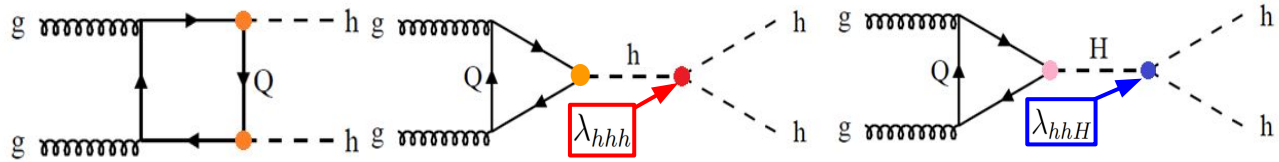
Generalization to BSM trilinear Higgs couplings



anyH3 v2: [Bahl, Braathen, Gabelmann, KR, Weiglein: TBP]

- An **additional argument** can be set in the calculation of the three point functions which is the **fields** involved in the external legs
- Relevant for resonant Higgs pair production processes

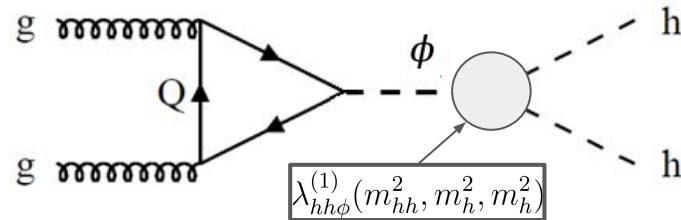
Example: 2HDM
gggh production



We include corrections to this process by means of effective trilinear Higgs couplings assuming that the largest contribution comes from this type of diagrams and others can be neglected (eg. double box diagram):

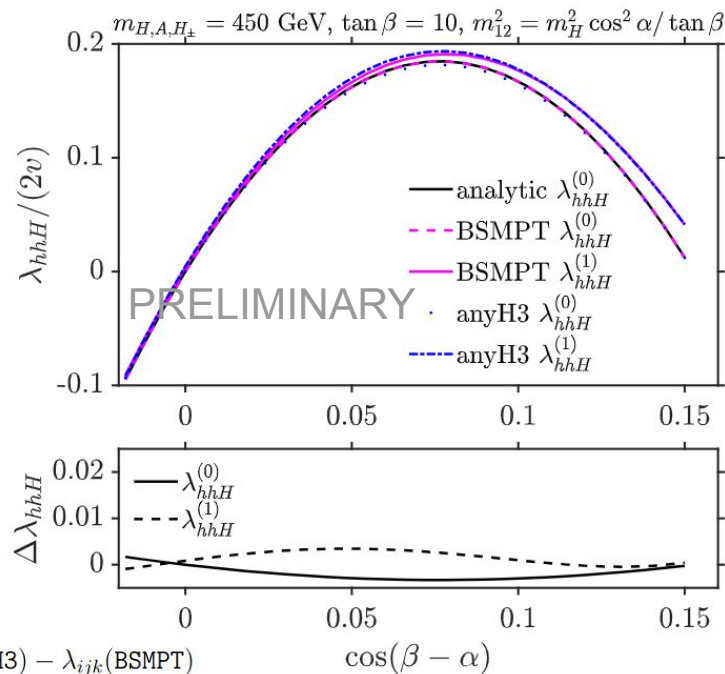
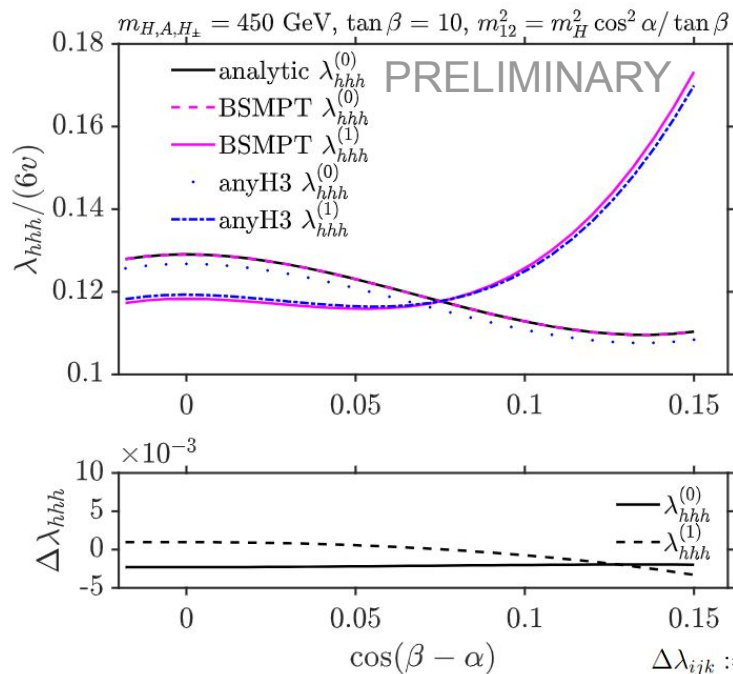
- Is this reasonable? -> modifications of λ_{hhh} are the leading source of deviations of non resonant hh production cross section

[Bahl, Braathen, Weiglein : [arXiv: 2202.03453](https://arxiv.org/abs/2202.03453)]



Comparison to other tools

THDM II



Agreement of the corrections to the trilinear couplings λ_{hhh} and λ_{hhH} with BSMPT

Note: BSMPT uses V_{eff} + different renormalization schemes + different vevs!

[Basler, Biermann,
Mühlleitner, Müller, Santos,
Viana: [arXiv: 2404.19037](https://arxiv.org/abs/2404.19037)]

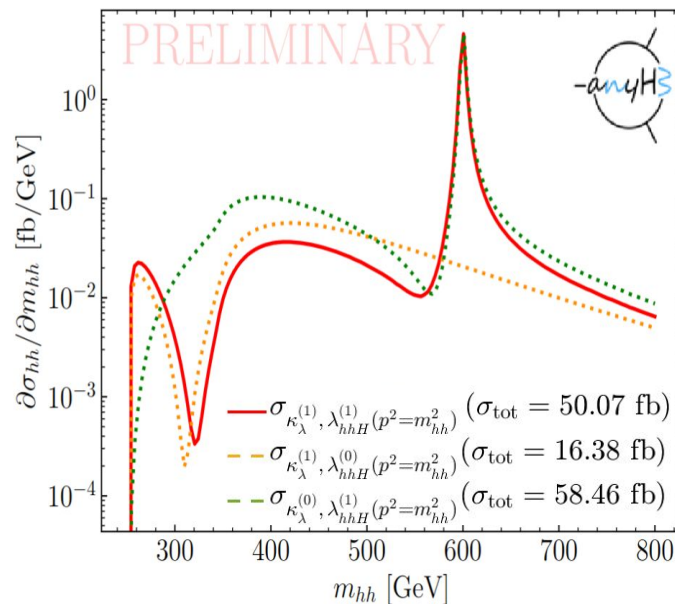
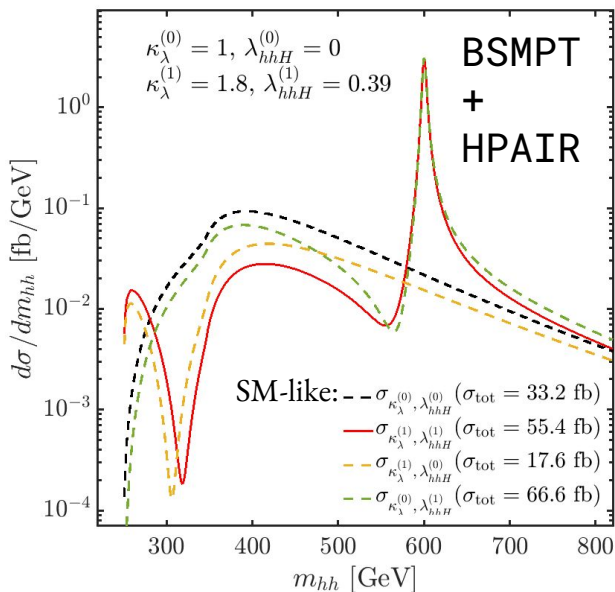
Example applications

THDM II

- Impact of loop corrections to the trilinear is very sizable in Higgs pair production searches!
- Existing analysis in 2HDM can be easily reproduced in any model with anyBSM

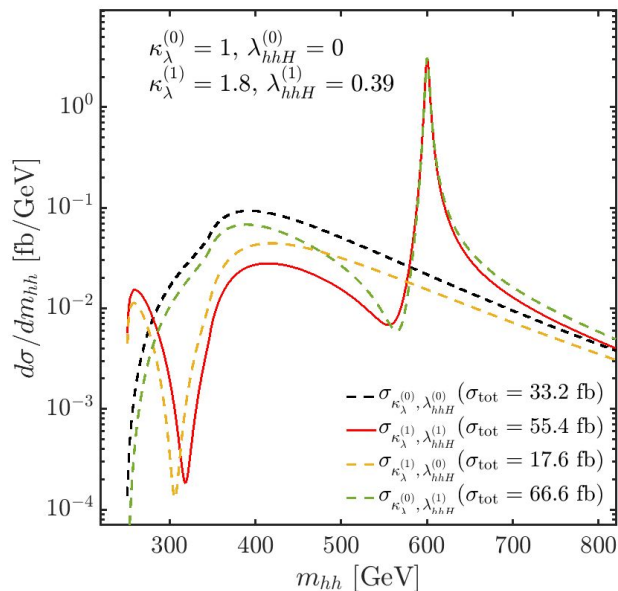
[Heinemeyer, Mühlleitner, K.R., Weiglein: [arXiv: 2403.14776](#)] [Bahl, Braathen, Gabelmann, KR, Weiglein: TBP]

HPAIR:
 [Plehn, Spira, Zerwas
 : [arXiv: 9603205](#)]
 [Dawson, Dittmaier,
 Spira:
[arXiv:9805244](#)]
 [Abouabid, Arhrib,
 Azevedo, El Falaki,
 Ferreira,
 Mühlleitner, Santos:
[arXiv: 2112.12515](#)]

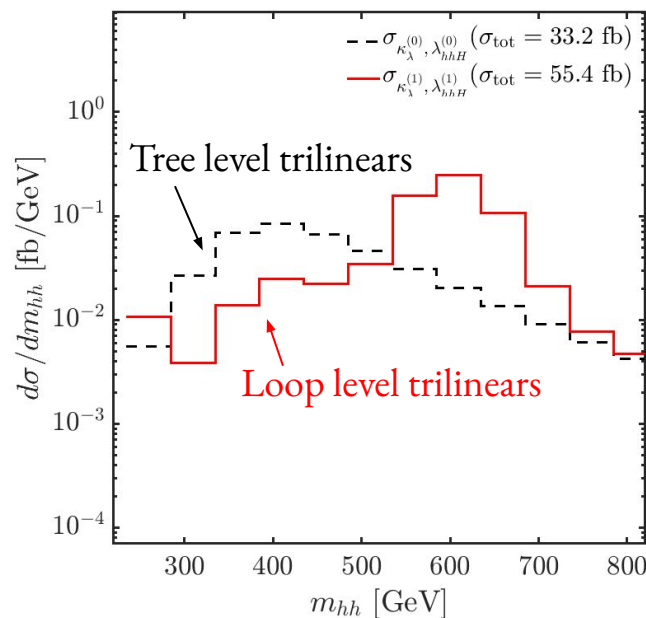
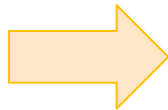


Difference in the experimental results

- Differential cross section measurements are affected by the finite resolution of particle detectors $\rightarrow$ observed spectrum is “**smeared**” $\rightarrow$ we mimic this effect by introducing *ad hoc* **Gaussian uncertainties** in m_{hh}
- Experimental data is gathered in **bins**

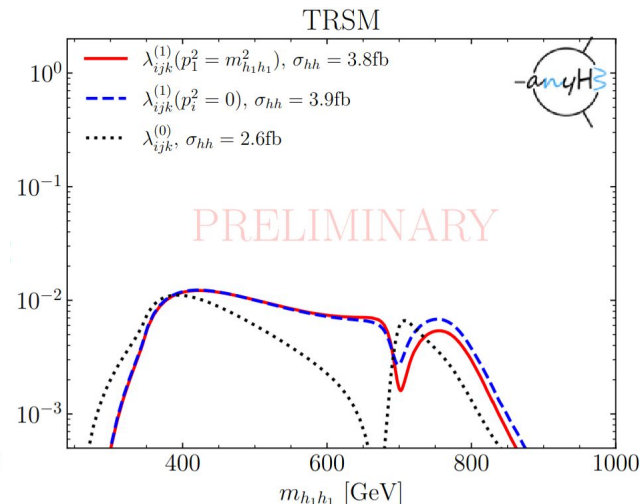
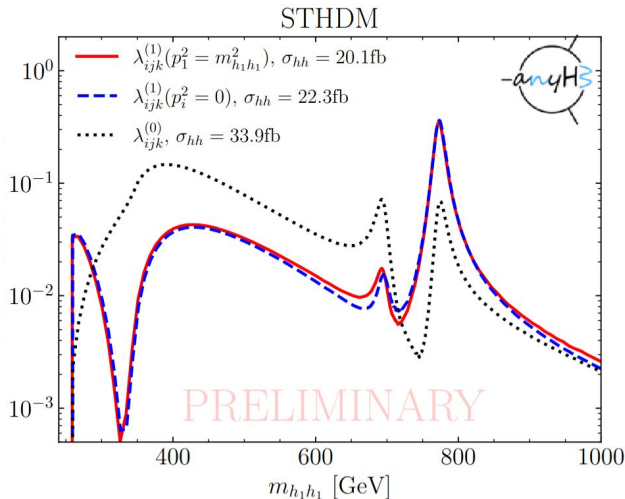
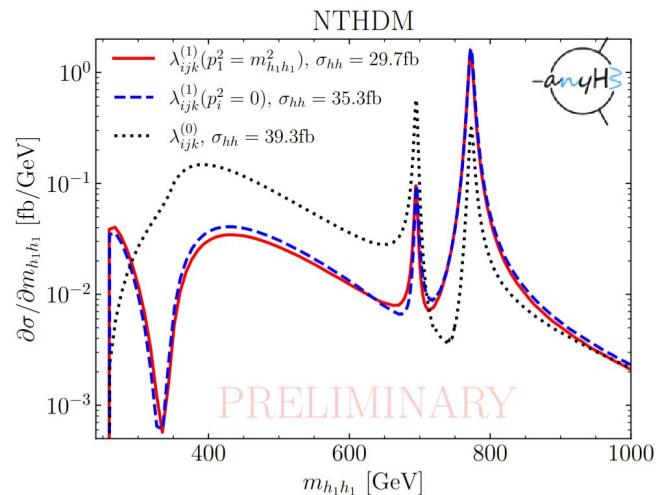


Realistic distribution:



Results in other models

- Results in many models can be obtained for the first time with an OS scheme for the couplings



Conclusions

- We presented a novel tool for calculation of the **trilinear couplings** in **arbitrary renormalizable QFTs**
 - including full **1 loop** calculation
 - including **momentum** dependence
 - flexible choices of **renormalisation** schemes → predefined or by user
- Convenient features: UFO model inputs , analytical results (Python, Mathematica), fast numerical results (with caching): SM($\sim 0.2s$); MSSM($\sim 0.5s$)
- **anyBSM** framework, currently under development but good agreement with existent codes
 - generalisation of the above to any scalar trilinear coupling
 - large impact on Higgs pair production
 - leading to interesting phenomenology within reach of HL-LHC

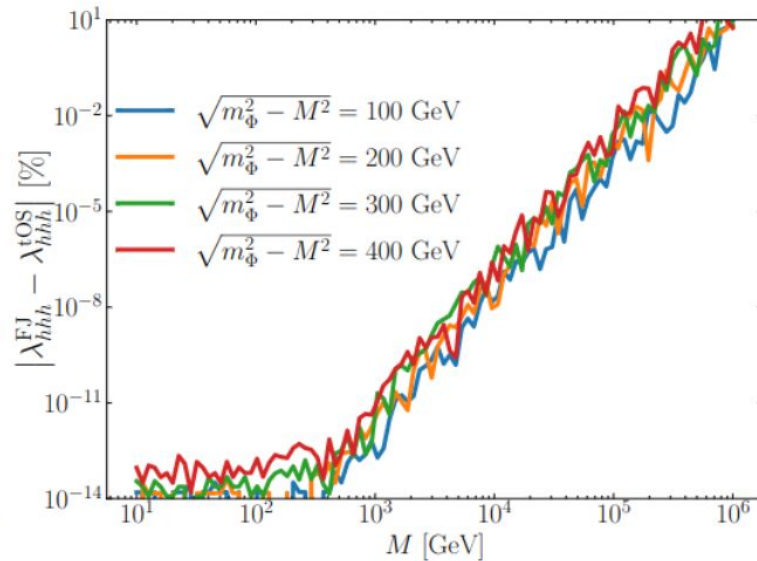
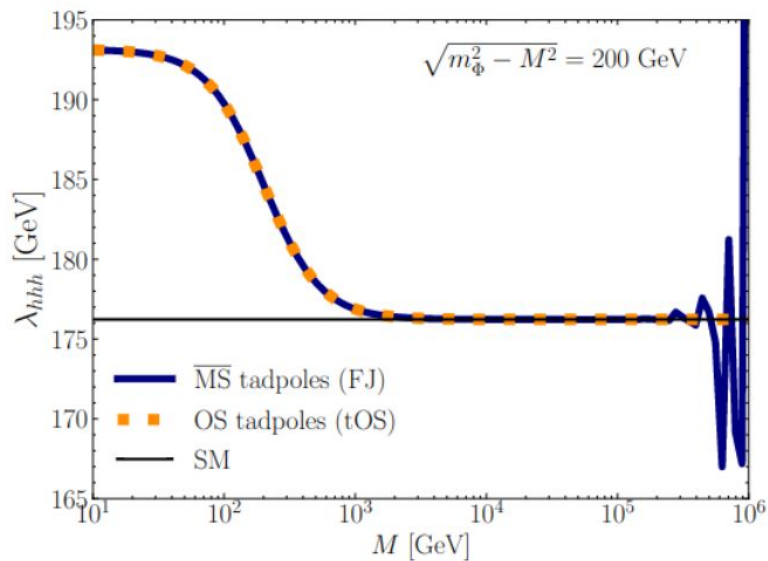
Thank you for your attention!

Backup

OS vs FJ tadpole treatment

[Slide by M. Gabelmann]

THDM type-II, $s_{\beta-\alpha} = 1$, $t_\beta = 2$, $m_{h_2} = m_A = m_{H^\pm} = m_\Phi$



Backup

A cross-check: the decoupling limit

[Slide by J. Braathen]

- Consider the decoupling limit in several BSM models

$$M_{\text{BSM}}^2 = \mathcal{M}^2 + \tilde{\lambda} v^2$$

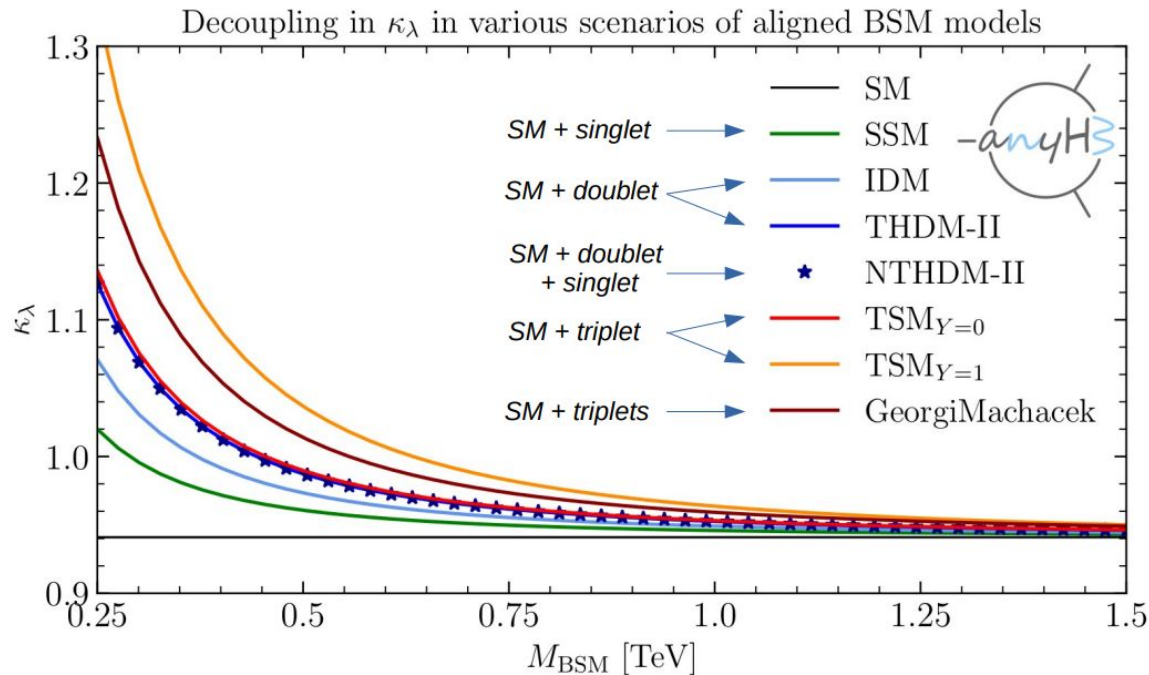
$\mathcal{M}$: BSM mass scale

$\tilde{\lambda}$: Quartic couplings

- Increase BSM mass scale

$$\mathcal{M} \rightarrow \infty$$

- BSM corrections to should vanish (c.f. decoupling theorem [Appelquist, Carrazone '75])



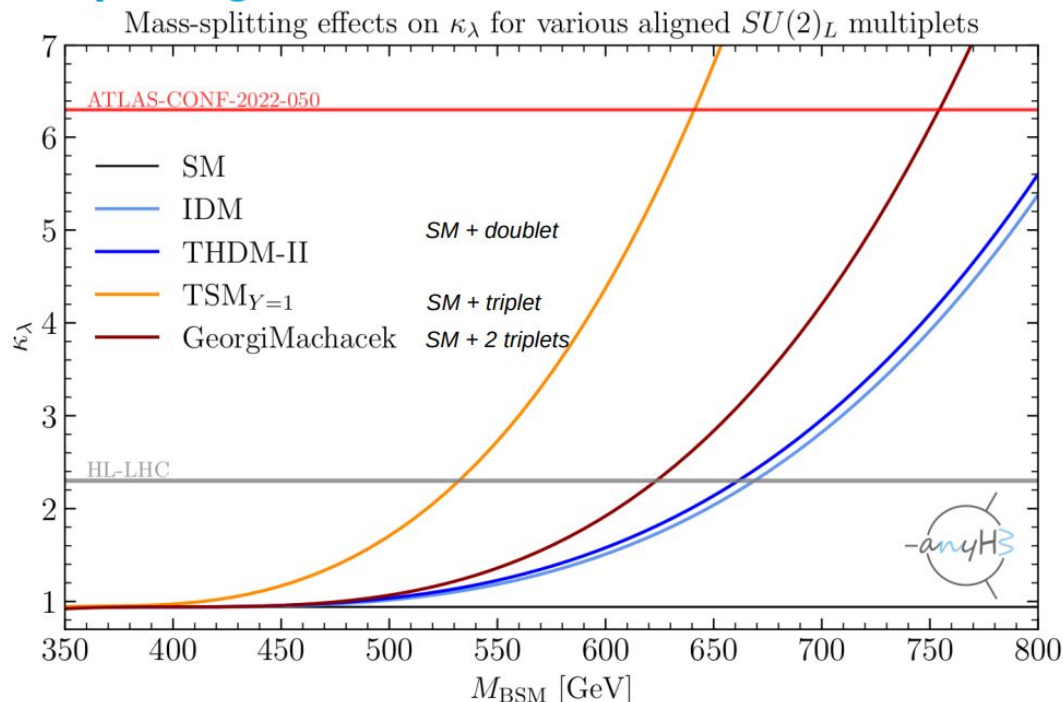
Backup

New results I: mass-splitting effects in various BSM models

- Consider the non-decoupling limit in several BSM models

$$M_{\text{BSM}}^2 = \mathcal{M}^2 + \tilde{\lambda} v^2$$

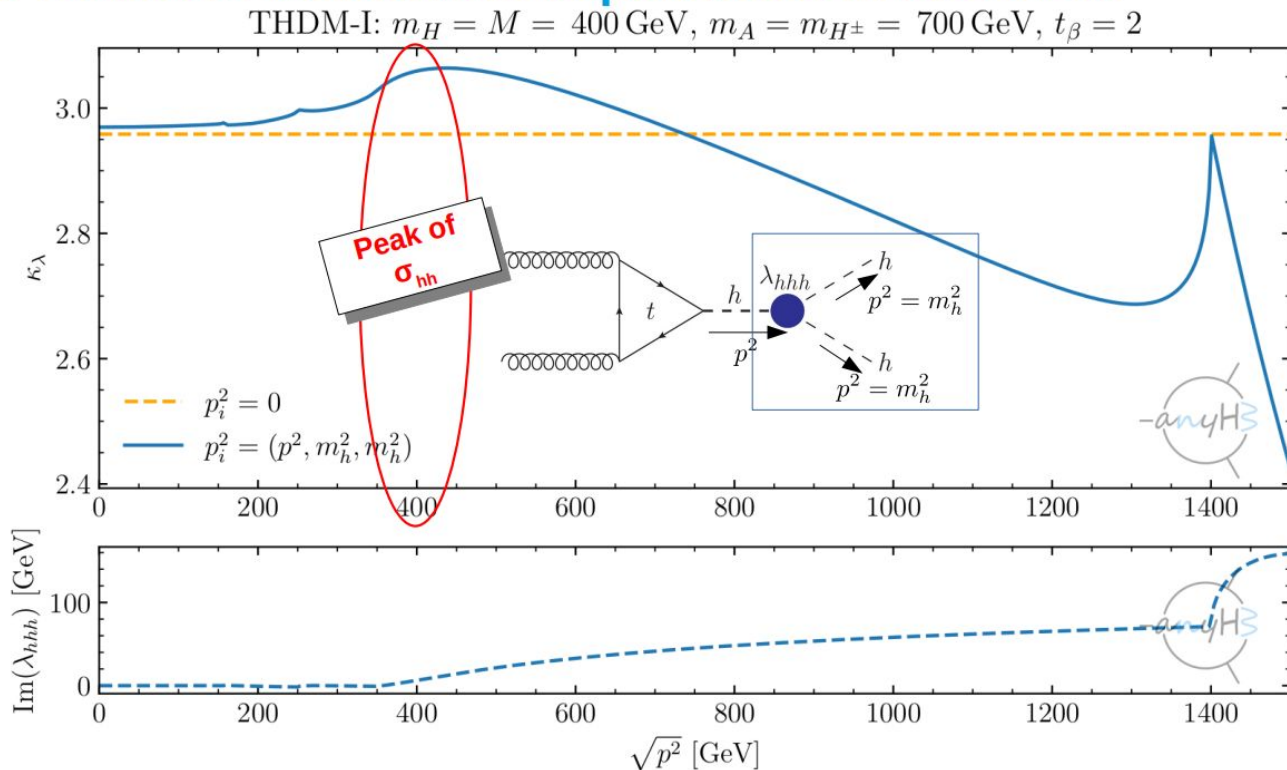
- Increase M_{BSM} , keeping $\mathcal{M}$ fixed
 - large mass splittings
 - **large BSM effects!**
- Perturbative unitarity checked with anyPerturbativeUnitarity
- Constraints on BSM parameter space!**



Here: scenarios with lightest BSM scalar mass + BSM mass param.
at 400 GeV; other BSM scalar masses = M_{BSM}

Backup

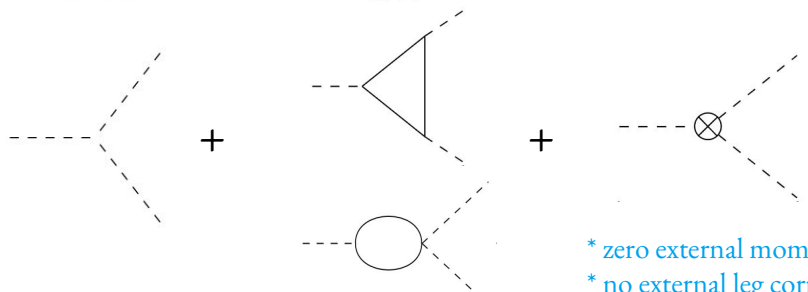
New results II: momentum dependence in the 2HDM



Effective potential approach

[Coleman, Weinberg: (1973) Physical Review]

$$V_{\text{eff}} = V_{\text{tree}} + V_{\text{CW}} + V_{\text{CT}}$$

$$\lambda_{hhh}^{\text{eff}} = \left. \frac{\partial^3 V_{\text{eff}}}{\partial h^3} \right|_{h=0} =$$


* zero external momentum
* no external leg corrections

Treatment of external leg corrections

$$\begin{aligned}\delta^{(1)}\lambda_{hhh}^{\text{WFR}} &= \sum_i \left(\frac{1}{2} \Sigma'_{hh}(p_i^2) \lambda_{hhh}^{(0)} + \sum_{j, h_j \neq h} \frac{\Sigma_{hh_j}(p_i^2)}{p_i^2 - m_{h_j}^2} \lambda_{h_j hh}^{(0)} \right) \\ &\equiv \sum_i \left(\frac{1}{2} \delta^{(1)} Z_h(p_i^2) \lambda_{hhh}^{(0)} + \sum_{j, h_j \neq h} \delta^{(1)} Z_{hh_j}(p_i^2) \lambda_{h_j hh}^{(0)} \right),\end{aligned}$$

$$\begin{aligned}\delta^{(1)}\lambda_{112}^{\text{WFR}}(p_1^2, p_2^2, p_3^2) &= \frac{1}{2} \delta^{(1)} Z_{11}(p_1^2) \lambda_{112}^{(0)} + \frac{1}{2} \delta^{(1)} Z_{11}(p_2^2) \lambda_{112}^{(0)} + \frac{1}{2} \delta^{(1)} Z_{22}(p_3^2) \lambda_{112}^{(0)} \\ &\quad + \delta^{(1)} Z_{12}(p_1^2) \lambda_{122}^{(0)} + \delta^{(1)} Z_{12}(p_2^2) \lambda_{122}^{(0)} + \delta^{(1)} Z_{21}(p_3^2) \lambda_{111}^{(0)},\end{aligned}$$