Two-loop calculations of the trilinear Higgs coupling in general renormalisable theories

Based on work in progress
in collaboration with Henning Bahl, Johannes Braathen and Sebastian Paßehr

Martin Gabelmann
KUTS @ DESY
Hamburg, Germany | 27 June 2024

HELMHOLTZ RESEARCH FOR GRAND CHALLENGES
DESY.

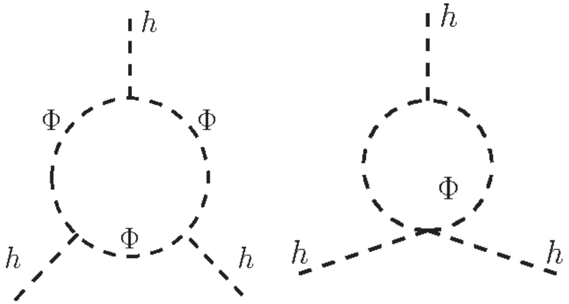
Why two loops ?

1) Capturing all types of effects entering λ_{hhh}

2) Properly interpreting experimental bounds on κ_λ

Mass-splitting effects in λ_{hhh} – how well established are they?

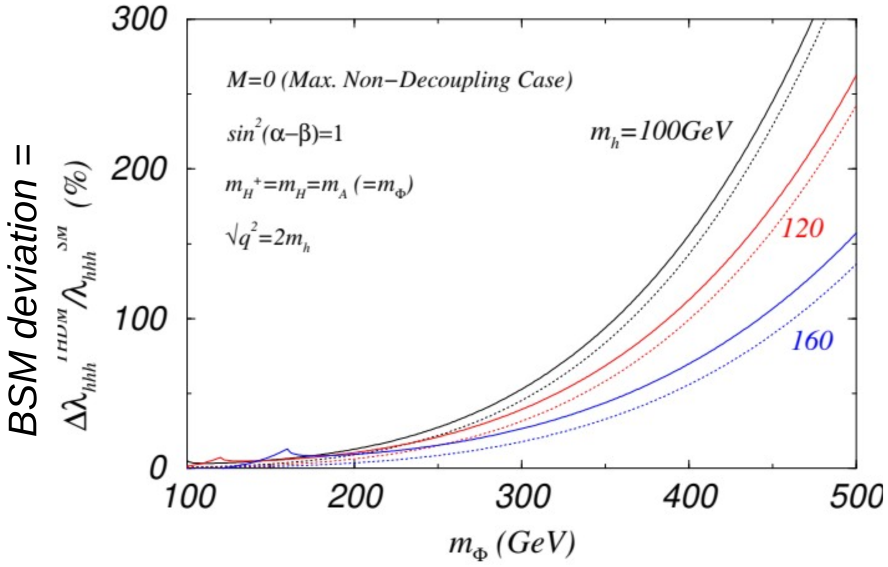
- First investigation of 1L BSM contributions to λ_{hhh} in 2HDM: [Kanemura, (Kiyoura), Okada, Senaha, Yuan ‘02, ‘04]



$$g_{hh\Phi\Phi} = -\frac{2(M^2 - m_\Phi^2)}{v^2}$$

$(\Phi \in \{H, A, H^\pm\})$

$[m_\Phi^2 = M^2 + \tilde{\lambda}_\Phi v^2]$



- **Deviations of tens/hundreds of % from SM possible,** for **large $g_{h\Phi\Phi}$ or $g_{hh\Phi\Phi}$ couplings**
- **Mass splitting effects**, now found in various models (2HDM, inert doublet model, singlet extensions, etc.)

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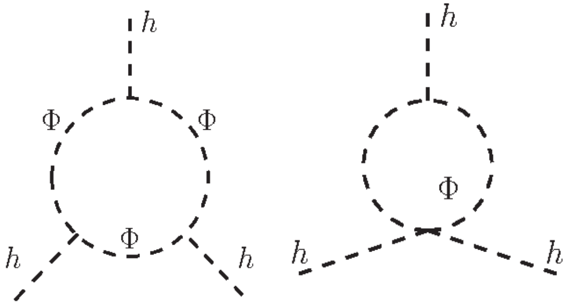

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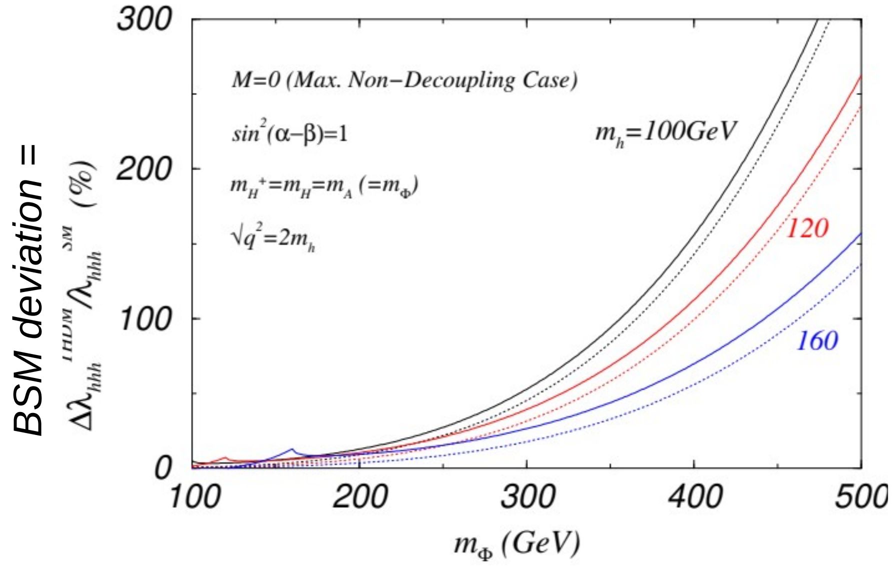

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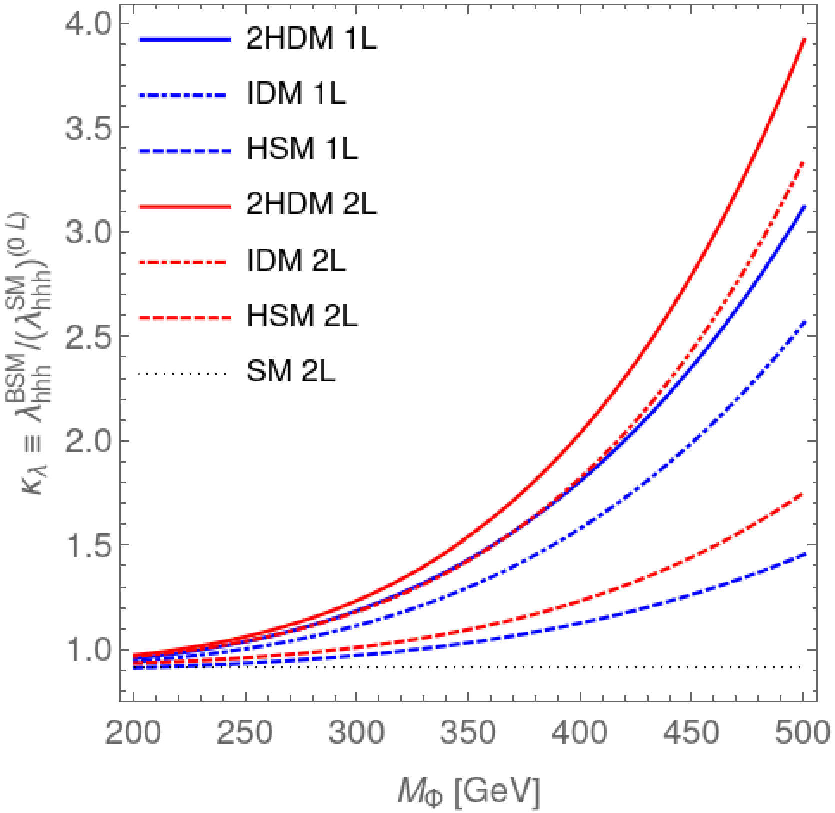
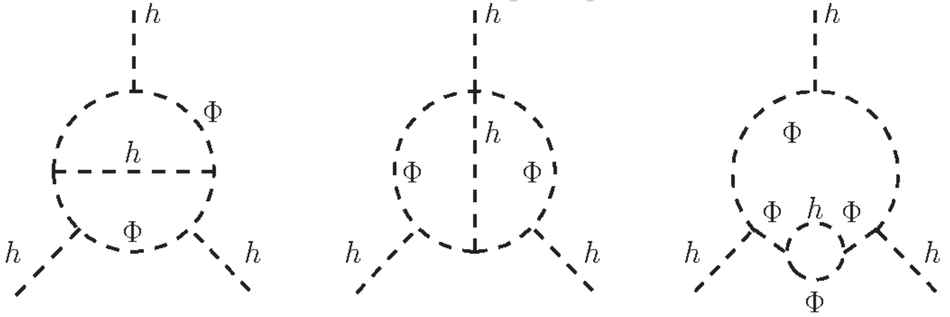
- **Mass splitting effects**, now found in various models (2HDM, inert doublet model, singlet extensions, etc.)

- Large effects **confirmed at 2L** in [Braathen, Kanemura ‘19] → **leading 2L corrections involving BSM scalars (H,A,H±) and top quark**, computed in effective potential approximation

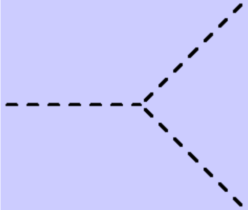
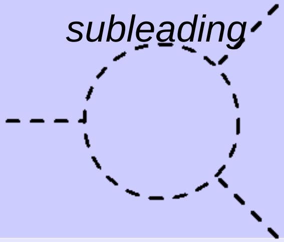
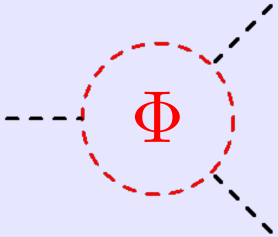
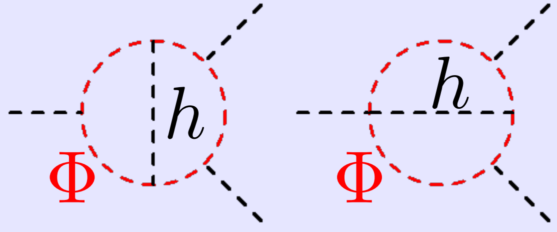
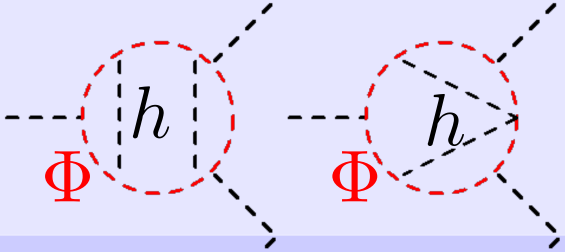
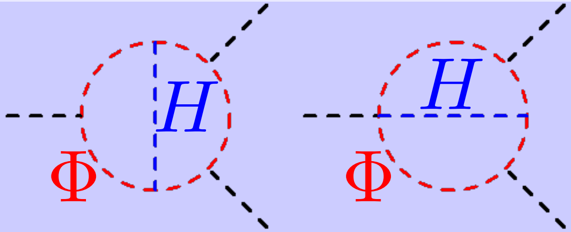
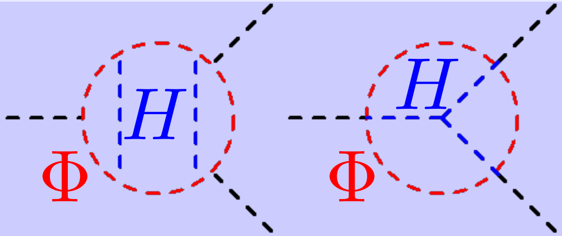
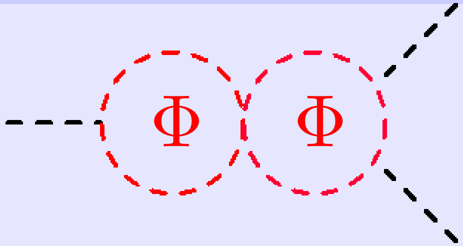
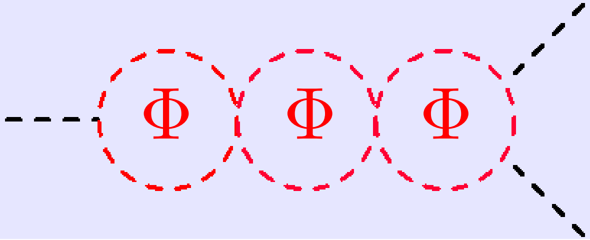




Scalar contributions to λ_{hhh} in aligned 2HDM

$g_{hh\Phi\Phi} = -\frac{2(M^2 - m_\Phi^2)}{v^2}$

BSM scalars:
$\Phi \in \{H, A, H^\pm\}$
$m_\Phi^2 = M^2 + \tilde{\lambda}_\Phi v^2$

Coupling/Order	0L	1L	2L	3L
g_{hhhh}			*subleading*	*subleading*
$g_{(h)h\Phi\Phi}$	-			
$g_{(h)H\Phi\Phi'}$ [$g_{(h)G\Phi\Phi'}$ case similar]	-	-		
$g_{\Phi\Phi\Phi'\Phi'}$ [2 BSM scalars of species Φ, 2 of species Φ']	-	-		

[NB: 1 h can be replaced by a VEV!]

→ *no further type of coupling entering after 2L*

→ *for each class of diagrams, perturbative convergence can be checked!*

e.g. in [Bahl, Braathen, Weiglein PRL '22]

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1) Capturing all types of effects entering λ_{hhh}

2) Properly interpreting experimental bounds on κ_{λ}

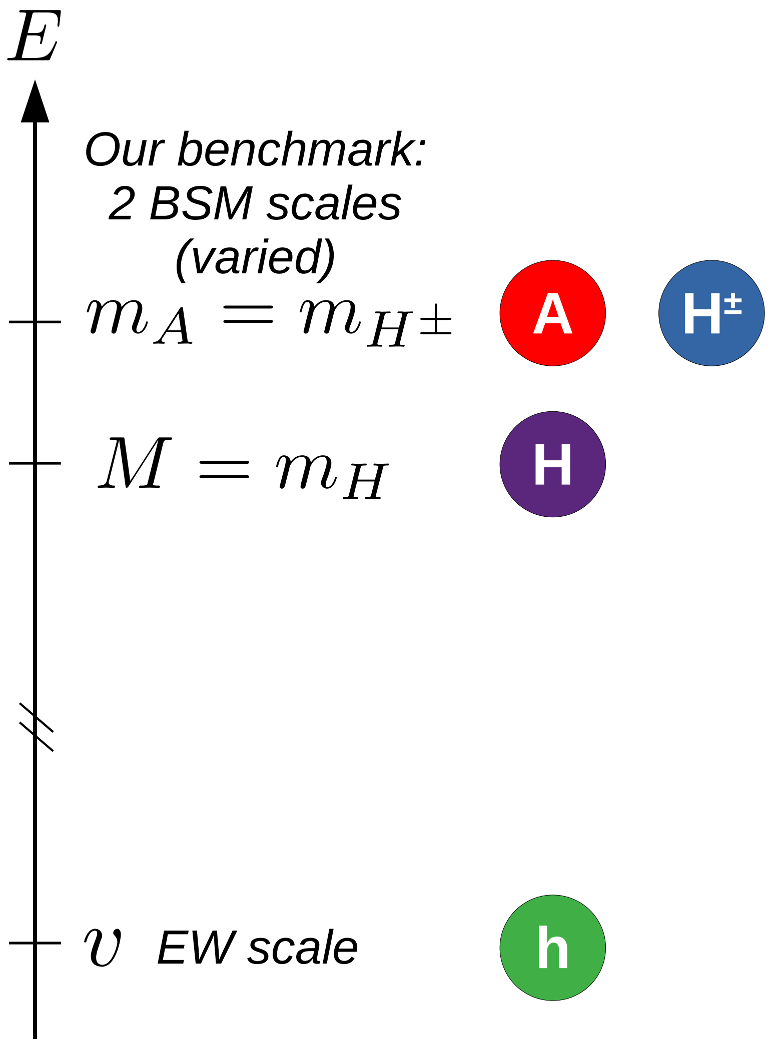
A benchmark scenario in the aligned 2HDM

[Bahl, Braathen, Weiglein '22]

- **Two-Higgs-Doublet Model (2HDM)**:
  Here: CP conservation assumed, softly-boken Z_2 symmetry to avoid FCNCs, Yukawa couplings of type I

- Mass eigenstates:
  - 2 CP-even Higgs bosons
    h (125-GeV Higgs), H
  - CP-odd Higgs boson A
  - Charged Higgs bosons $H^{\pm}$
  - M: new mass term in 2HDM,

- Scenario with **alignment**: couplings of h are SM-like at tree level



A benchmark scenario in the aligned 2HDM

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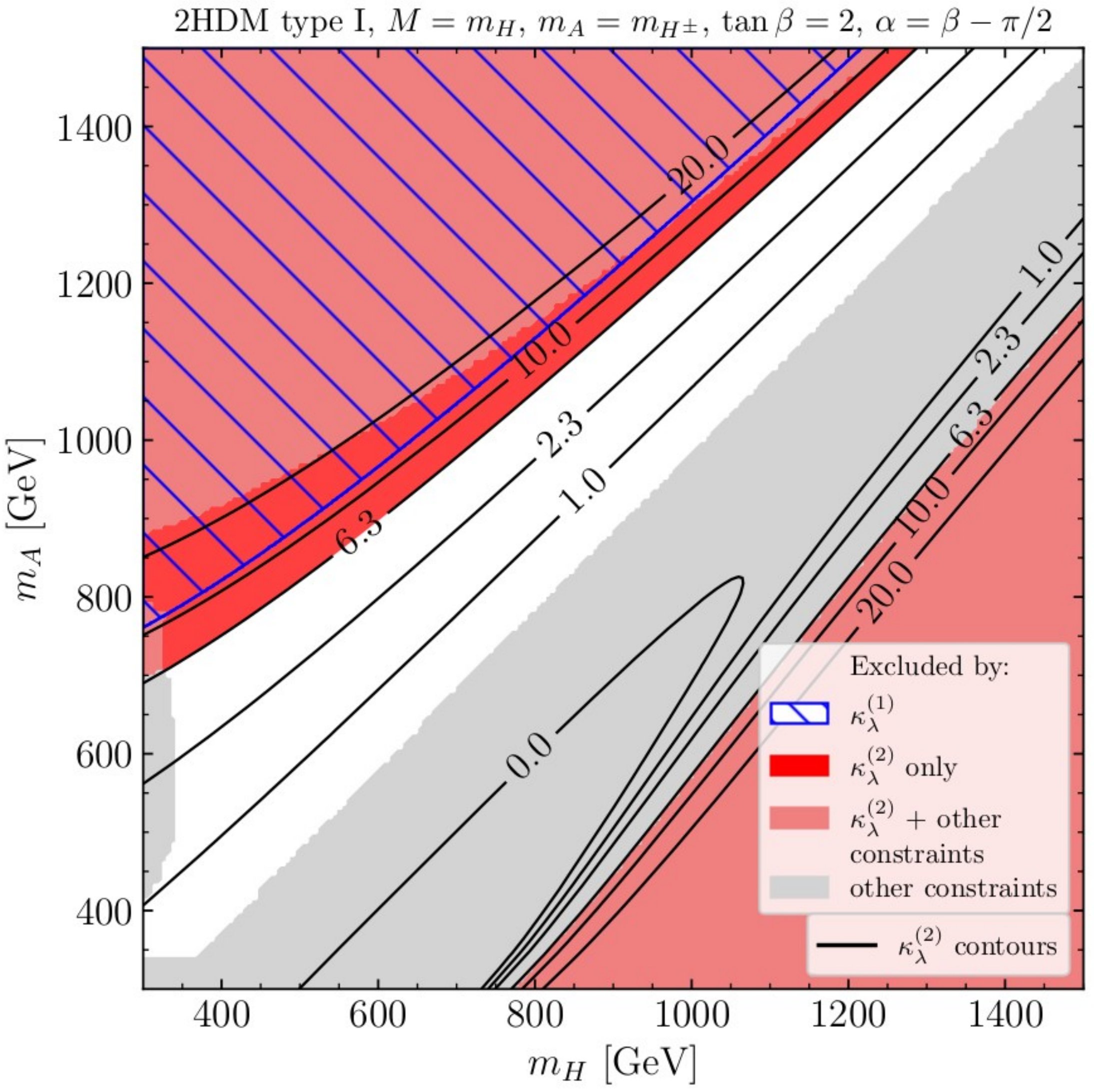
Results shown for aligned 2HDM of type-I, similar for other types (*available in backup*)

We take $m_A = m_{H\pm}$, $M = m_H$, $\tan\beta = 2$



- ***Grey area:*** area excluded by other constraints, in particular BSM Higgs searches, boundedness-from-below (BFB), perturbative unitarity
- ***Light red area:*** area excluded both by other constraints (BFB, perturbative unitarity) and by $\kappa_\lambda^{(2)} > 6.3$ [in region where $\kappa_\lambda^{(2)} < -0.4$ the calculation isn't reliable]
- ***Dark red area:*** new area that is **excluded ONLY by $\kappa_\lambda^{(2)} > 6.3$**. Would otherwise not be excluded!
- ***Blue hatches:*** area excluded by $\kappa_\lambda^{(1)} > 6.3$ → impact of including 2L corrections is significant!

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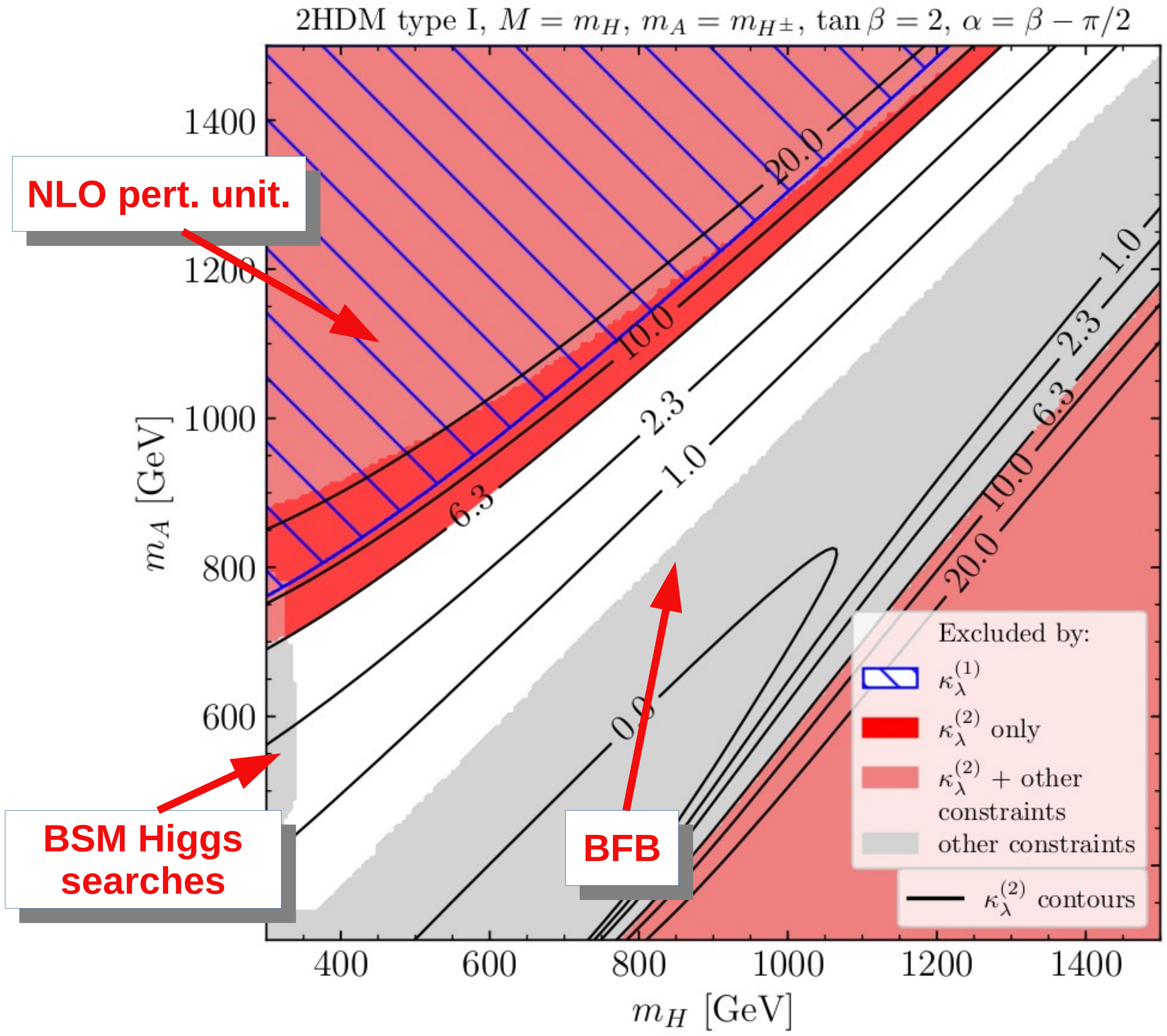

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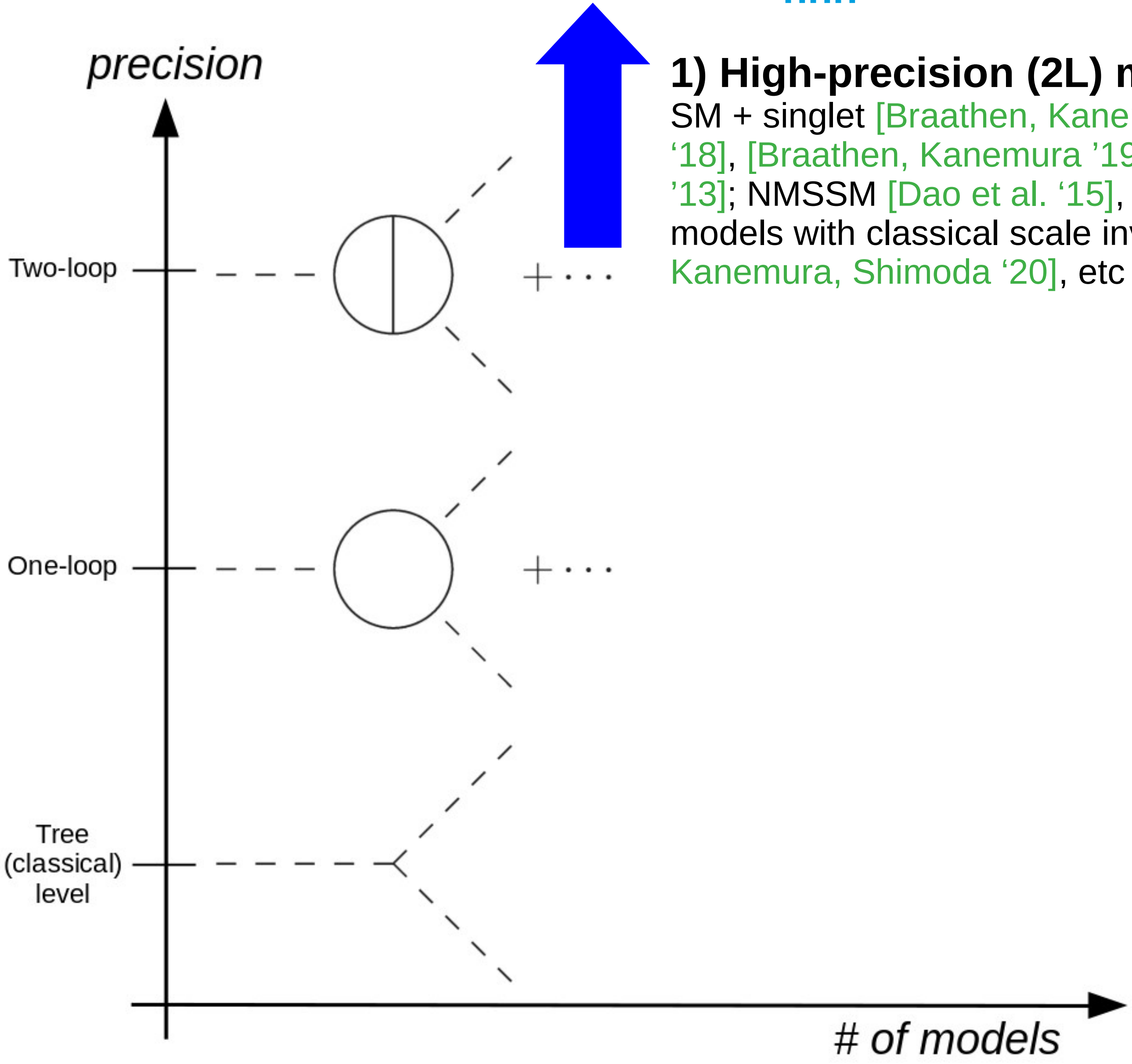
Precise predictions for λ_{hhh} in arbitrary BSM theories



1) High-precision (2L) model-specific results

SM + singlet [Braathen, Kanemura ’19]; 2HDMs [Senaha ‘18], [Braathen, Kanemura ’19]; MSSM [Brucherseifer et al. ’13]; NMSSM [Dao et al. ‘15], [Borschensky, MG et al ‘22] ; models with classical scale invariance [Braathen, Kanemura, Shimoda ‘20], etc

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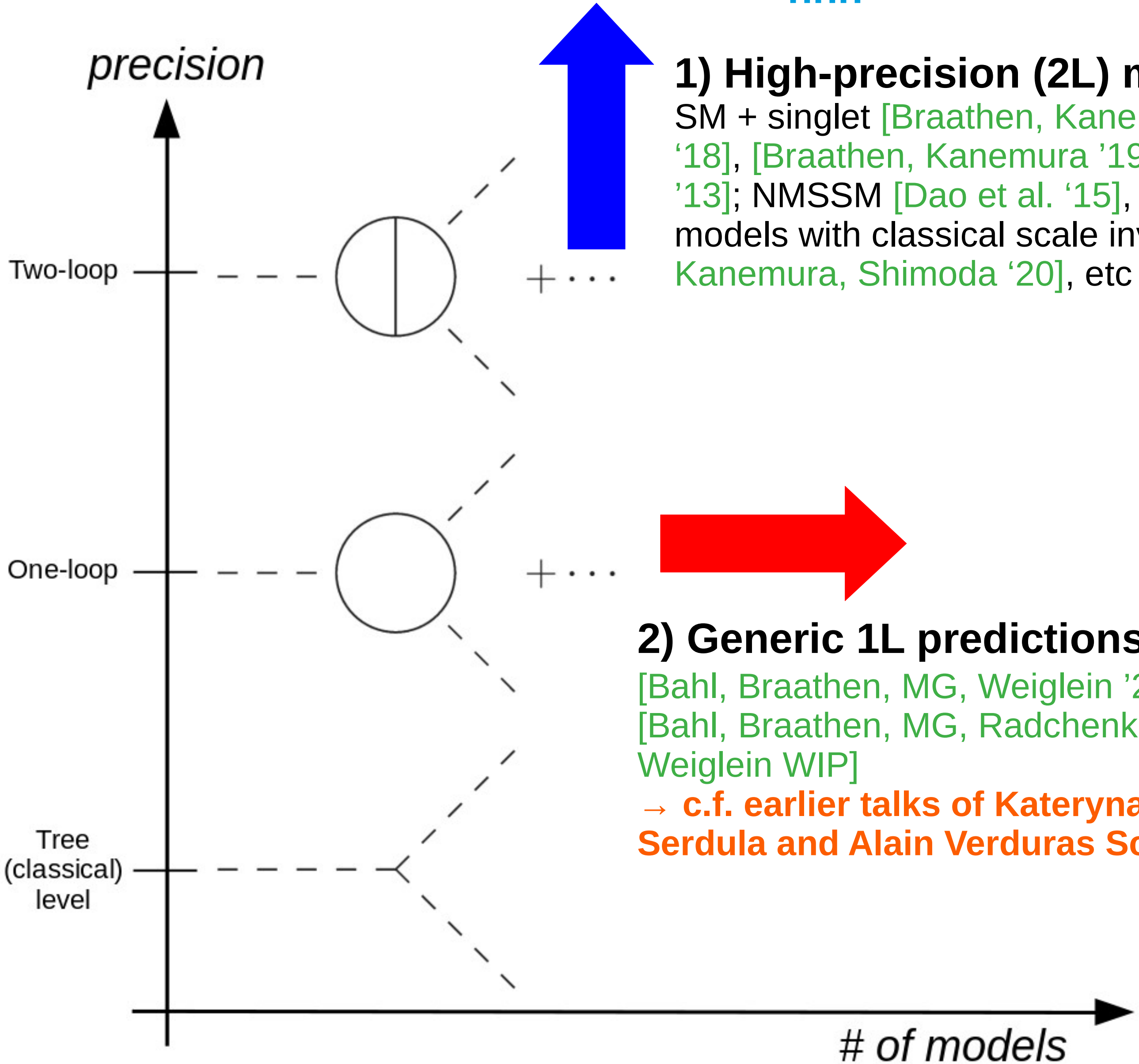

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2) Generic 1L predictions with `anyH3`

[Bahl, Braathen, MG, Weiglein '23]
[Bahl, Braathen, MG, Radchenko Serdula, Weiglein WIP]
→ **c.f. earlier talks of Kateryna Radchenko Serdula and Alain Verduras Schaeidt**

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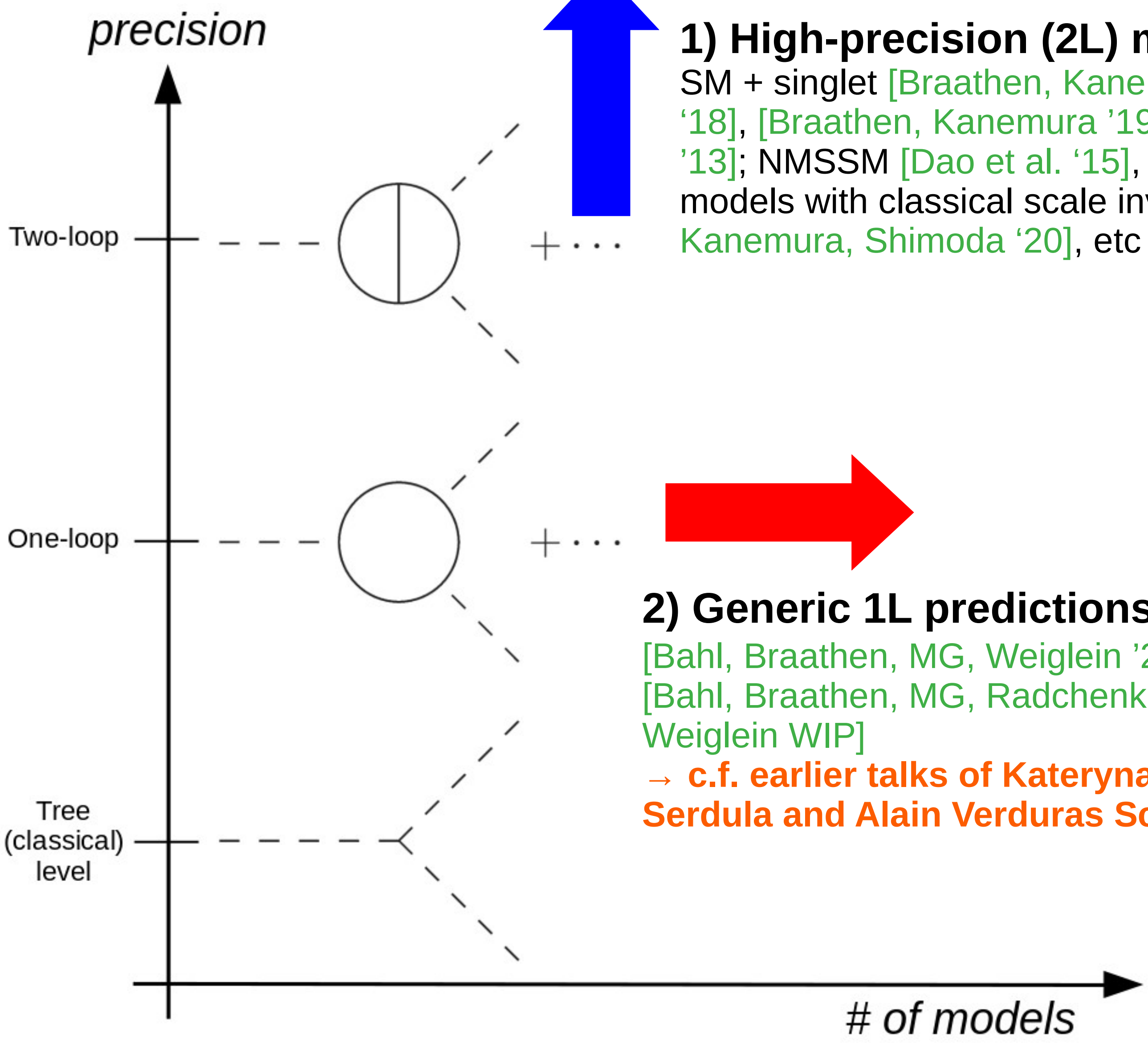

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3) High-precision (2L) generic predictions
[Bahl, Braathen, MG, Paßehr WIP]
→ this talk!

Generic predictions for λ_{hhh} and λ_{hhhh} at two loops: our setup

Computing λ_{hhh} in general renormalisable theories: method

- All 2L contributions to Higgs/scalar self-energies computed in [Goodsell, Paßehr ‘19]
- In [Bahl, Braathen, MG, Paßehr to appear] we **generalise this with**
  → **new results for full 2L corrections to λ_{iii} and λ_{iiii} (with $p^2 = 0$)**
  → **results for 0-, 1-, 2-point functions in same conventions**
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Aparté: an example of generic results

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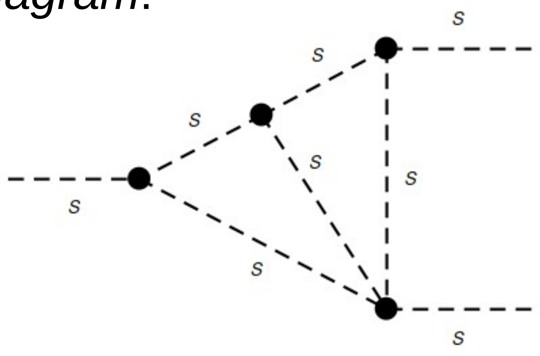
E.g. for the following 2L diagram:



Our corresponding generic expression:

```
{
  {
    {1, 1, 1},
    {edge[iv[1], v[4], S[i3]], edge[iv[2], v[5], S[i1]],
     edge[iv[3], v[6], S[i2]], edge[v[4], v[5], -S[i7]],
     edge[v[4], v[6], -S[i8]], edge[v[4], v[7], -S[i6]],
     edge[v[5], v[7], -S[i4]], edge[v[6], v[7], -S[i5]]},
    -((lfac^2*SSS[i1, i4, -i7, 1]*SSS[i2, i5, -i8, 1]*
    SSS[-i4, -i5, -i6, 1]*SSSS[i3, i6, i7, i8, 1]*
    T[Df[k1, MS[i5]]*Df[k1, MS[i8]]*Df[k3, MS[i4]]*Df[k3, MS[i7]]*
    Df[k4, MS[i6]]])/SF[1, 1/3, 1])
  }
```

*"**Edge list**" → defines diagram in terms of propagators (edges) – more details in a few slides*

Generic couplings
(here SSS & SSSS = trilinear and quartic scalar couplings)

T-integral
[Weiglein, Scharf, Böhm '93]

Symmetry factor

with $$T_{i_1\cdots i_n} = \int \frac{\mathrm{d}^d q_1\,\mathrm{d}^d q_2}{\left[\imath\,\pi^2\,(2\,\pi\,\mu)^{d-4}\right]^2}\,\frac{1}{(k_{i_1}^2 - m_{i_1}^2)\cdots(k_{i_n}^2 - m_{i_n}^2)}$$

$$k_1 = q_1\,,\quad k_2 = q_1 + p\,,\quad k_3 = q_2 - q_1\,,\quad k_4 = q_2\,,\quad k_5 = q_2 + p,$$

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- Generic diagrams generated with FeynArts, computed with TwoCalc and OneCalc [Weiglein '93+]
- Generic results then mapped to specific models via private routines (using FeynArts diagrams and amplitudes; could in principle also be done starting from UFO model)
- Topologies and diagrams reduced, taking benefit of symmetries → details in next slides

- Application of generic results to concrete models:
  (*for genuine 2L diagrams, and 2L subloop renormalisation diagrams*)

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Diagrams generated with FeynArts [Hahn '01, '09 ++]

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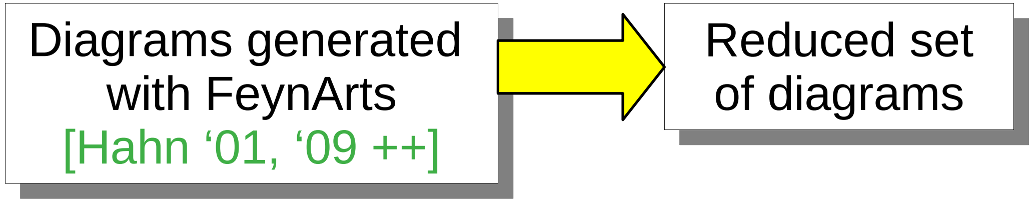

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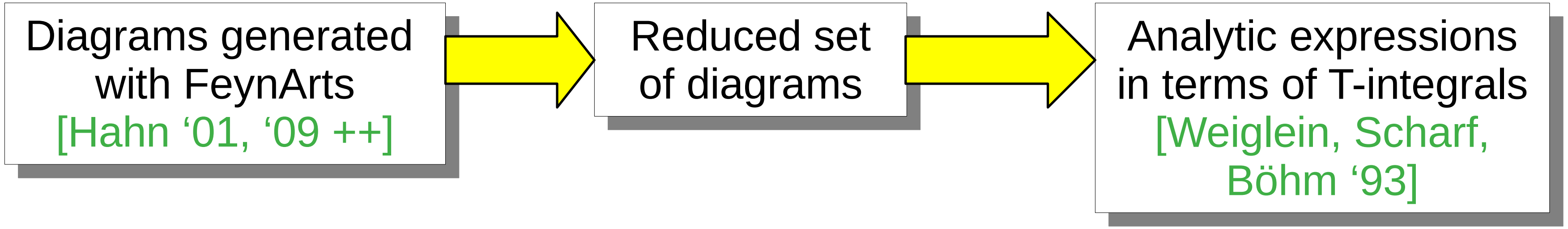

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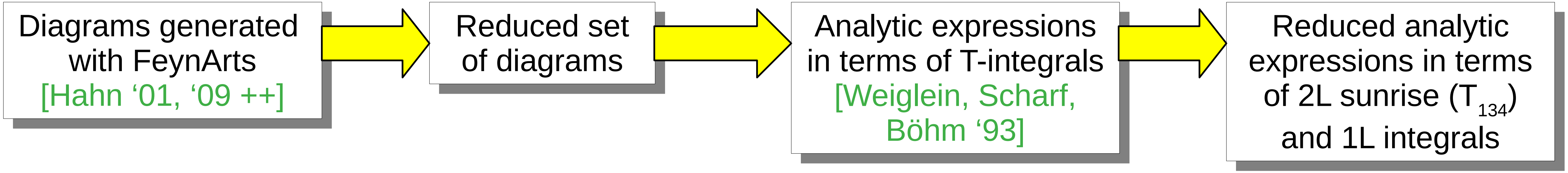
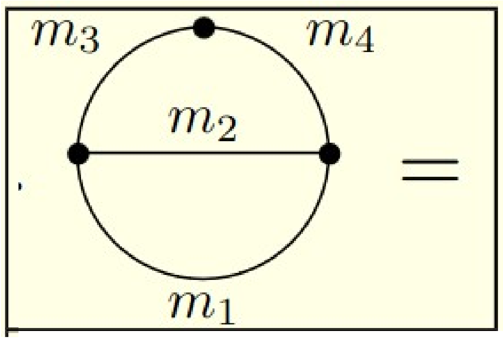


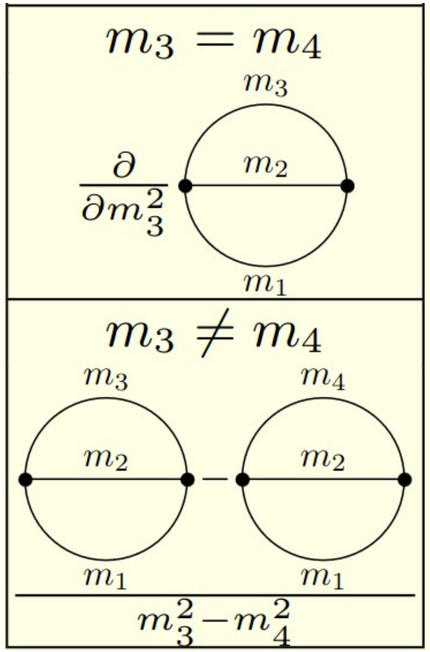
Aparté 2: Reduction of T-integrals

Diagrams generated with FeynArts [Hahn ‘01, ‘09 ++] → Reduced set of diagrams → Analytic expressions in terms of T-integrals [Weiglein, Scharf, Böhm ‘93] → Reduced analytic expressions in terms of 2L sunrise (T_{134}) and 1L integrals

E.g. reduction of T_{1134} integral



Analytically or on-the-fly during numerical evaluation



- Reduction of T-integrals to 2L sunrise (T_{134}) and products of 1L integrals
- Can be done in with our `Mathematica` routines to handle and use the generic results
- Or can be done on-the-fly during numerical evaluation in our `Python` routines (similar to what is implemented in FeynHiggs now [Paßehr ‘22])

[Figure adapted from S. Paßehr]

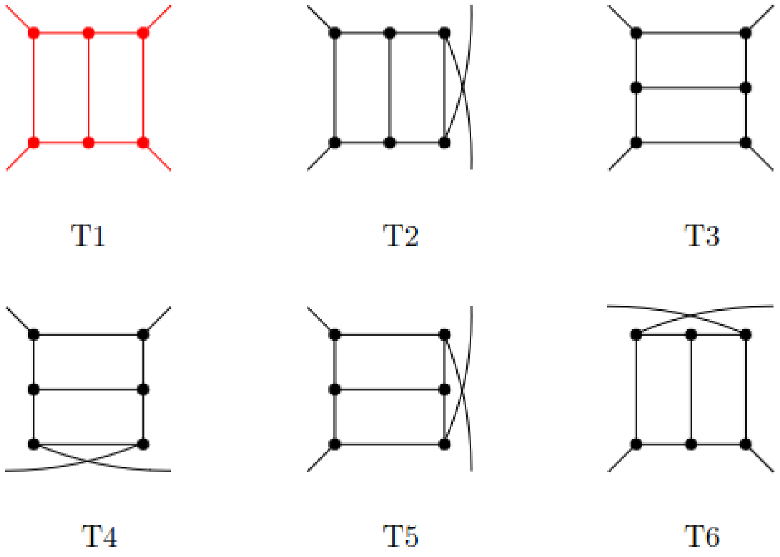
Reducing topologies and diagrams using symmetries

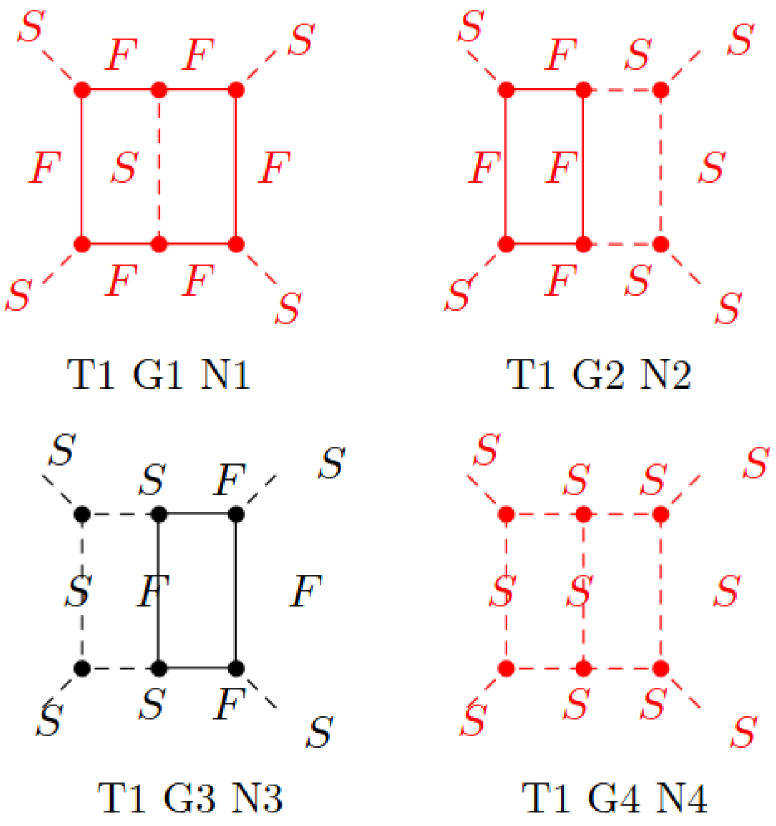
In our setting:

- External states are identical ($\rightarrow \lambda_{iii}, \lambda_{iiii}$)
- All external momenta set to zero

→ ***many diagrams are identical***

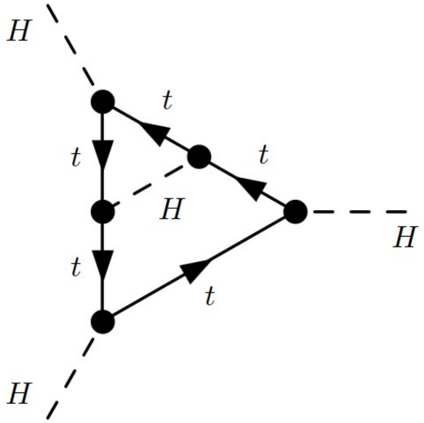
For example, double box diagram,
with fermions and scalars:
→6 topologies, 4 diagrams/topology

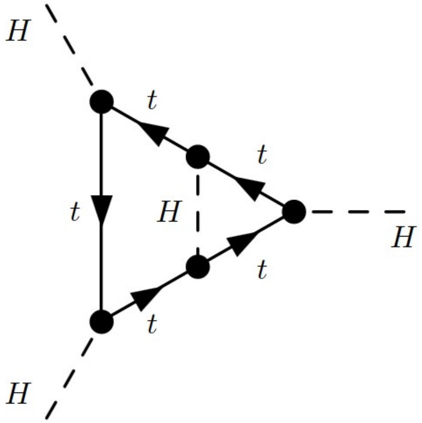




→ Only need to compute 3 diagrams, rather than 24!

Automating the reduction






```
edgelist1 = {edge[v[1], v[4],  S[1]], edge[v[2], v[5],  S[1]],
             edge[v[3], v[6],  S[1]], edge[v[4], v[7], -F[3]],
             edge[v[4], v[8],  F[3]], edge[v[5], v[6],  F[3]],
             edge[v[5], v[8], -F[3]], edge[v[6], v[7],  F[3]],
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edgelist2 = {edge[v[1], v[4],  S[1]], edge[v[2], v[5],  S[1]],
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             edge[v[4], v[7], -F[3]], edge[v[5], v[8],  F[3]],
             edge[v[6], v[7],  F[3]], edge[v[6], v[8], -F[3]],
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```

Unique representation of diagram
→ ”*canonical edges*”

- list of ”edges” (= lines in diagram)
- identical diagrams ↔ permutations of edges
- canonical form = one particular choice of ordering

Reduction algorithm with “canonical edges” in pseudo code:

- identify internal and external indices
- generate permutations of external indices
- generate permutations of internal indices
- combine permutations of internal and external indices
- permute edge list following the combined list of permutations
- sort list of permuted edge lists
- return first element of sorted list of “edge lists”

Reducing topologies and diagrams in practice

n-point function

n	topology-level	field-level
0	$2 \to 2$	$11 \to 11$
1	$3 \to 3$	$25 \to 25$
2	$9 \to 8$	$121 \to 92\,(102)$
3	$40 \to 13$	$936 \to 229\,(291)$
4	$265 \to 29$	$10496 \to 698\,(928)$

- Count **number of two-loop diagrams before and after reduction** of diagrams using canonical edges algorithm
  - ➔ At the level of topologies
  - ➔ At the level of field insertions

  [Numbers in brackets → for models with CP violation]

- **Reduction of up to one order of magnitude!**

NB: numbers shown here at generic level, not considering model-specific particle insertions nor summation over generation indices

Example applications

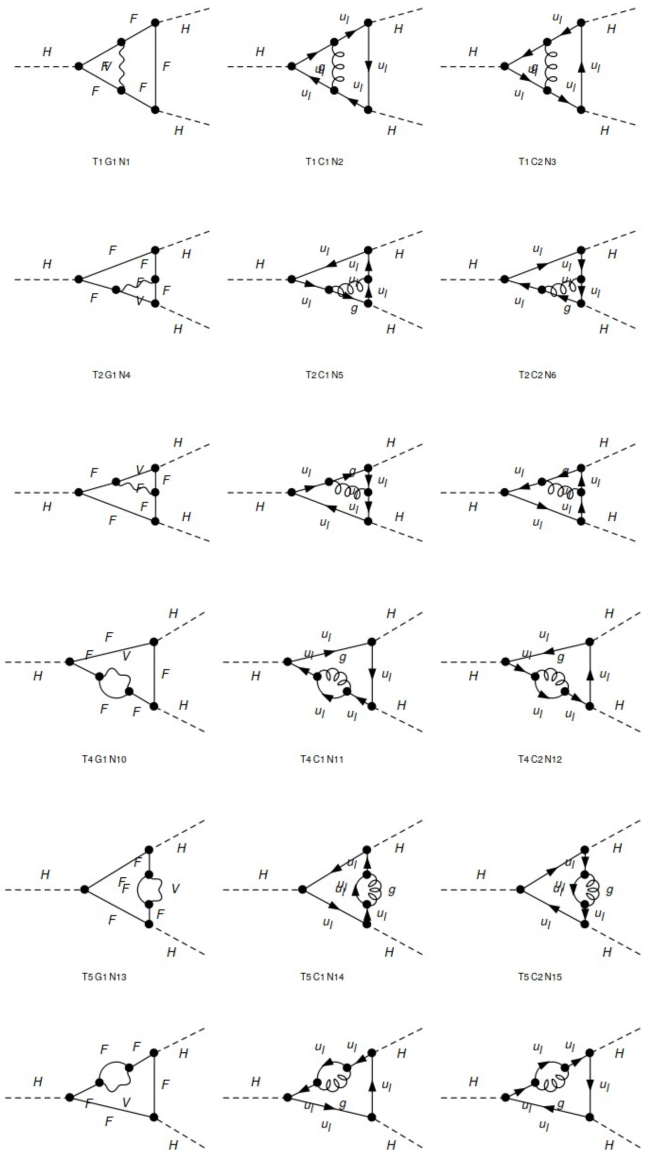
[Bahl, Braathen, MG, Paßehr to appear]

1) Cross-checks with existing results → today in SM

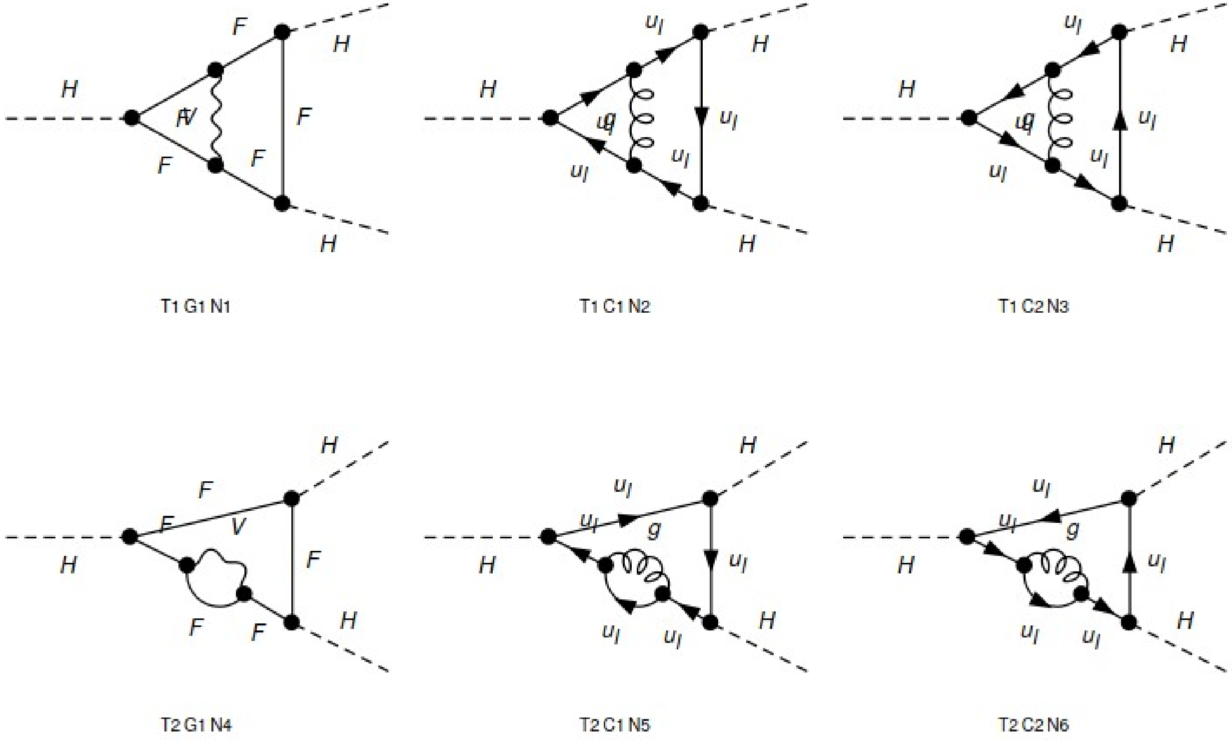
(in paper also for Z_2SSM, 2HDM, NMSSM)

2) New results → today in singlet-extended SM

Leading two-loop $O(\alpha_s \alpha_t)$ and $O(\alpha_t^2)$ corrections in the SM



Reduction of 3-point diagrams, shown here at $O(\alpha_s \alpha_t)$



Leading two-loop $O(\alpha_s\alpha_t)$ and $O(\alpha_t^2)$ corrections in the SM

- ***Genuine 2L contributions at $O(\alpha_s\alpha_t)$***

$$\Gamma_h^{(2)}\Big|^{\text{gen.}}_{\mathcal{O}(\alpha_s\alpha_t)} = \frac{\alpha_s m_t^4}{\pi^3 v}\left[-\frac{3}{2\epsilon^2}+\frac{3\overline{\ln}m_t^2-2}{\epsilon}-4-\frac{\pi^2}{4}+4\overline{\ln}m_t^2-3\overline{\ln}^2 m_t^2\right],$$

$$\Gamma_{hh}^{(2)}(p^2=0)\Big|^{\text{gen.}}_{\mathcal{O}(\alpha_s\alpha_t)} = \frac{\alpha_s m_t^4}{\pi^3 v^2}\left[-\frac{9}{2\epsilon^2}+\frac{9\overline{\ln}m_t^2}{\epsilon}-4-\frac{3\pi^2}{4}-9\overline{\ln}^2 m_t^2\right],$$

$$\Gamma_{hhh}^{(2)}(0,0,0)\Big|^{\text{gen.}}_{\mathcal{O}(\alpha_s\alpha_t)} = \frac{\alpha_s m_t^4}{\pi^3 v^3}\left[-\frac{9}{\epsilon^2}+\frac{18}{\epsilon}(1+\overline{\ln}m_t^2)-8-\frac{3\pi^2}{2}-18\overline{\ln}m_t^2(2+\overline{\ln}m_t^2)\right],$$

$$\Gamma_{hhhh}^{(2)}(0,0,0,0)\Big|^{\text{gen.}}_{\mathcal{O}(\alpha_s\alpha_t)} = \frac{\alpha_s m_t^4}{\pi^3 v^4}\left[-\frac{9}{\epsilon^2}+\frac{18}{\epsilon}(3+\overline{\ln}m_t^2)-80-\frac{3\pi^2}{2}-108\overline{\ln}m_t^2-18\overline{\ln}^2 m_t^2\right]$$

and $O(\alpha_t^2)$

$$\Gamma_h^{(2)}\Big|^{\text{gen.}}_{\mathcal{O}(\alpha_t^2)} = \frac{3m_t^6}{16\pi^4 v^3}\left[\frac{3}{4\epsilon^2}+\frac{1}{\epsilon}\left(2-\frac{3}{2}\overline{\ln}m_t^2\right)+\frac{9}{2}+\frac{5\pi^2}{24}-4\overline{\ln}m_t^2+\frac{3}{2}\overline{\ln}^2 m_t^2\right],$$

$$\Gamma_{hh}^{(2)}(p^2=0)\Big|^{\text{gen.}}_{\mathcal{O}(\alpha_t^2)} = \frac{3m_t^6}{16\pi^4 v^4}\left[\frac{9}{4\epsilon^2}+\frac{3}{\epsilon}\left(1-\frac{3}{2}\overline{\ln}m_t^2\right)+\frac{11}{2}+\frac{5\pi^2}{8}-6\overline{\ln}m_t^2+\frac{9}{2}\overline{\ln}^2 m_t^2\right],$$

$$\Gamma_{hhh}^{(2)}(0,0,0)\Big|^{\text{gen.}}_{\mathcal{O}(\alpha_t^2)} = \frac{3m_t^6}{16\pi^4 v^5}\left[\frac{9}{2\epsilon^2}-\frac{3}{\epsilon}\left(1+3\overline{\ln}m_t^2\right)-1+\frac{5\pi^2}{4}+6\overline{\ln}m_t^2+9\overline{\ln}^2 m_t^2\right],$$

$$\Gamma_{hhhh}^{(2)}(0,0,0,0)\Big|^{\text{gen.}}_{\mathcal{O}(\alpha_t^2)} = \frac{3m_t^6}{16\pi^4 v^6}\left[\frac{9}{2\epsilon^2}-\frac{3}{\epsilon}\left(7+3\overline{\ln}m_t^2\right)+11+\frac{5\pi^2}{4}+42\overline{\ln}m_t^2+9\overline{\ln}^2 m_t^2\right].$$

- ***Subloop renormalisation***

$$\Gamma_h^{(2)}\Big|^{\text{subloop}} = \frac{3m_t^2}{8\pi^2 v}\left[\frac{\delta^{(1)}_{\text{CT}}v^2}{v^2}-\delta^{(1)}_{\text{CT}}Z_h\right]\mathbf{A}_0(m_t^2) - \frac{3m_t\delta^{(1)}_{\text{CT}}m_t}{2\pi^2 v}\left[\mathbf{A}_0(m_t^2)+m_t^2\mathbf{B}_0(0,m_t^2,m_t^2)\right],$$

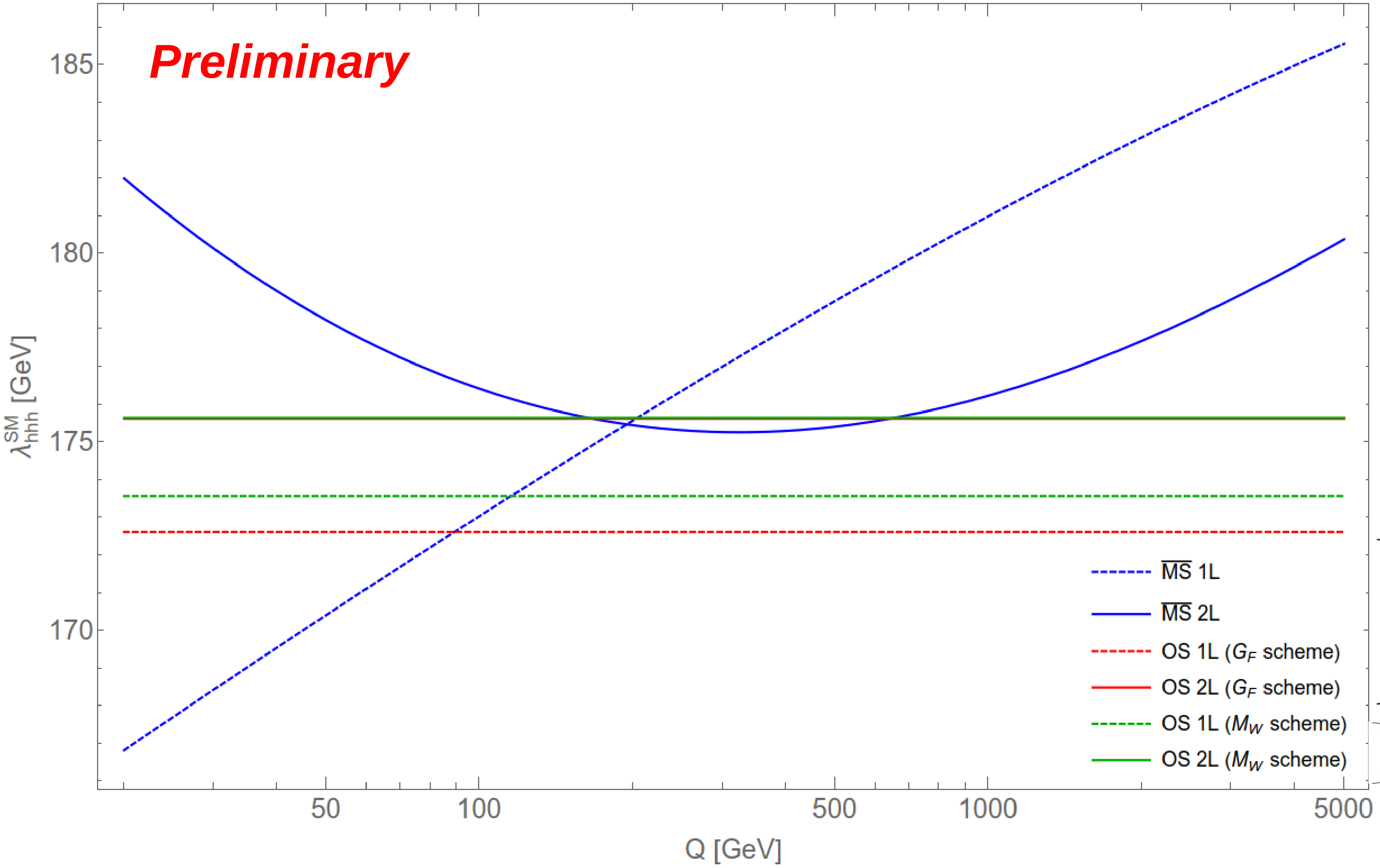
$$\Gamma_{hh}^{(2)}(p^2=0)\Big|^{\text{subloop}} = \frac{3m_t^2}{4\pi^2 v^2}\left[\frac{\delta^{(1)}_{\text{CT}}v^2}{v^2}-\delta^{(1)}_{\text{CT}}Z_h\right]\left[\mathbf{A}_0(m_t^2)+2m_t^2\mathbf{B}_0(0,m_t^2,m_t^2)\right] - \frac{3m_t\delta^{(1)}_{\text{CT}}m_t}{2\pi^2 v^2}\left[\mathbf{A}_0(m_t^2)+m_t^2\left(5\mathbf{B}_0(0,m_t^2,m_t^2)+4m_t^2\mathbf{C}_0(m_t^2,m_t^2,m_t^2)\right)\right],$$

$$\Gamma_{hhh}^{(2)}(0,0,0)\Big|^{\text{subloop}} = \frac{9m_t^4}{4\pi^2 v^3}\left[\frac{\delta^{(1)}_{\text{CT}}v^2}{v^2}-\delta^{(1)}_{\text{CT}}Z_h\right]\left[3\mathbf{B}_0(0,m_t^2,m_t^2)+4m_t^2\mathbf{C}_0(m_t^2,m_t^2,m_t^2)\right] - \frac{18m_t^3\delta^{(1)}_{\text{CT}}m_t}{\pi^2 v^3}\left[\mathbf{B}_0(0,m_t^2,m_t^2)+m_t^2\left(3\mathbf{C}_0(m_t^2,m_t^2,m_t^2) + 2m_t^2\mathbf{D}_0(m_t^2,m_t^2,m_t^2,m_t^2)\right)\right],$$

$$\Gamma_{hhhh}^{(2)}(0,0,0,0)\Big|^{\text{subloop}} = \frac{9m_t^4}{\pi^2 v^4}\left[\frac{\delta^{(1)}_{\text{CT}}v^2}{v^2}-\delta^{(1)}_{\text{CT}}Z_h\right] \times\left[\mathbf{B}_0(0,m_t^2,m_t^2)+8m_t^2\left(\mathbf{C}_0(m_t^2,m_t^2,m_t^2)+m_t^2\mathbf{D}_0(m_t^2,m_t^2,m_t^2,m_t^2)\right)\right] - \frac{18m_t^3\delta^{(1)}_{\text{CT}}m_t}{\pi^2 v^4}\times\left\{\mathbf{B}_0(0,m_t^2,m_t^2)+m_t^2\left[13\mathbf{C}_0(m_t^2,m_t^2,m_t^2) + 4m_t^2\left(7\mathbf{D}_0(m_t^2,m_t^2,m_t^2,m_t^2) + 4m_t^2\mathbf{E}_0(m_t^2,m_t^2,m_t^2,m_t^2,m_t^2)\right)\right]\right\},$$

→ combined with OS/$\overline{\text{MS}}$ counterterms, as desired

Leading two-loop $O(\alpha_s \alpha_t)$ and $O(\alpha_t^2)$ corrections in the SM



Results obtained for 2 electroweak schemes:

→ G_F scheme

→ $M_W/M_Z/\alpha_{em}$ scheme

[Difference is a 3L effect]

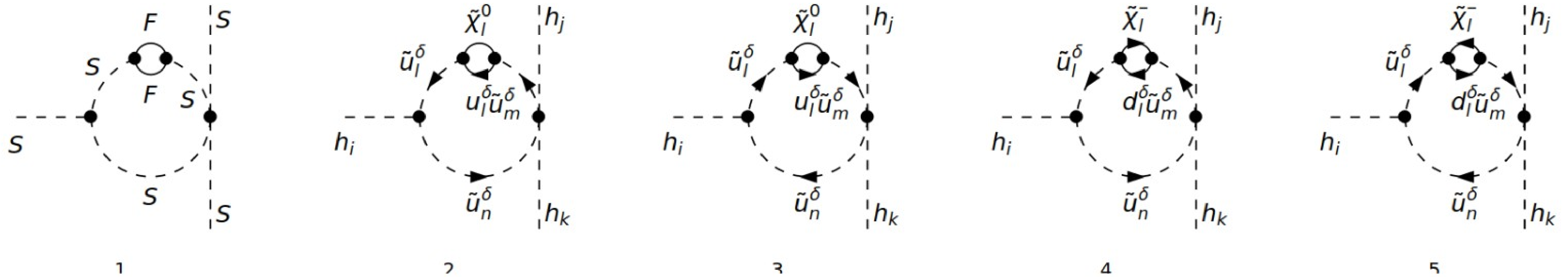
→ reproduces result of [Braathen, Kanemura '19]

→ new result

→ Remarkable agreement (to ~20 MeV)

Cross-check: CP-violating NMSSM

- $\lambda_{hhh}^{\mathcal{O}(\alpha_t^2)}$ first computed in [Borschensky et al. '22] (see talk by MG@KUTS23)
- w/o symmetry-reduction: check on diagram-by-diagram level



(YES) Topology 12: my: $\left\{-1.71058 - \frac{0.098447}{\epsilon^2} + \frac{0.487837}{\epsilon}, -1.71058 - \frac{0.098447}{\epsilon^2} + \frac{0.487837}{\epsilon}, -0.710459 - \frac{0.0492235}{\epsilon^2} + \frac{0.213257}{\epsilon}, -0.710459 - \frac{0.0492235}{\epsilon^2} + \frac{0.213257}{\epsilon}\right\}$

new: $\left\{-1.71058 - \frac{0.098447}{\epsilon^2} + \frac{0.487837}{\epsilon}, -1.71058 - \frac{0.098447}{\epsilon^2} + \frac{0.487837}{\epsilon}, -0.710459 - \frac{0.0492235}{\epsilon^2} + \frac{0.213257}{\epsilon}, -0.710459 - \frac{0.0492235}{\epsilon^2} + \frac{0.213257}{\epsilon}\right\}$ my-new: {0, 0, 0, 0}

- full numerical agreement for all genuine 2L diagrams
- w/ symmetry reduction:

diagrams	topology-level	field-level
genuine two-loop	$39 \to 12$	$213 \to 67(32)$
sub-loop	$15 \to 5$	$36 \to 12(7)$

Example applications

[Bahl, Braathen, MG, Paßehr to appear]

1) Cross-checks with existing results → today in SM

(in paper also for Z_2SSM, 2HDM, NMSSM)

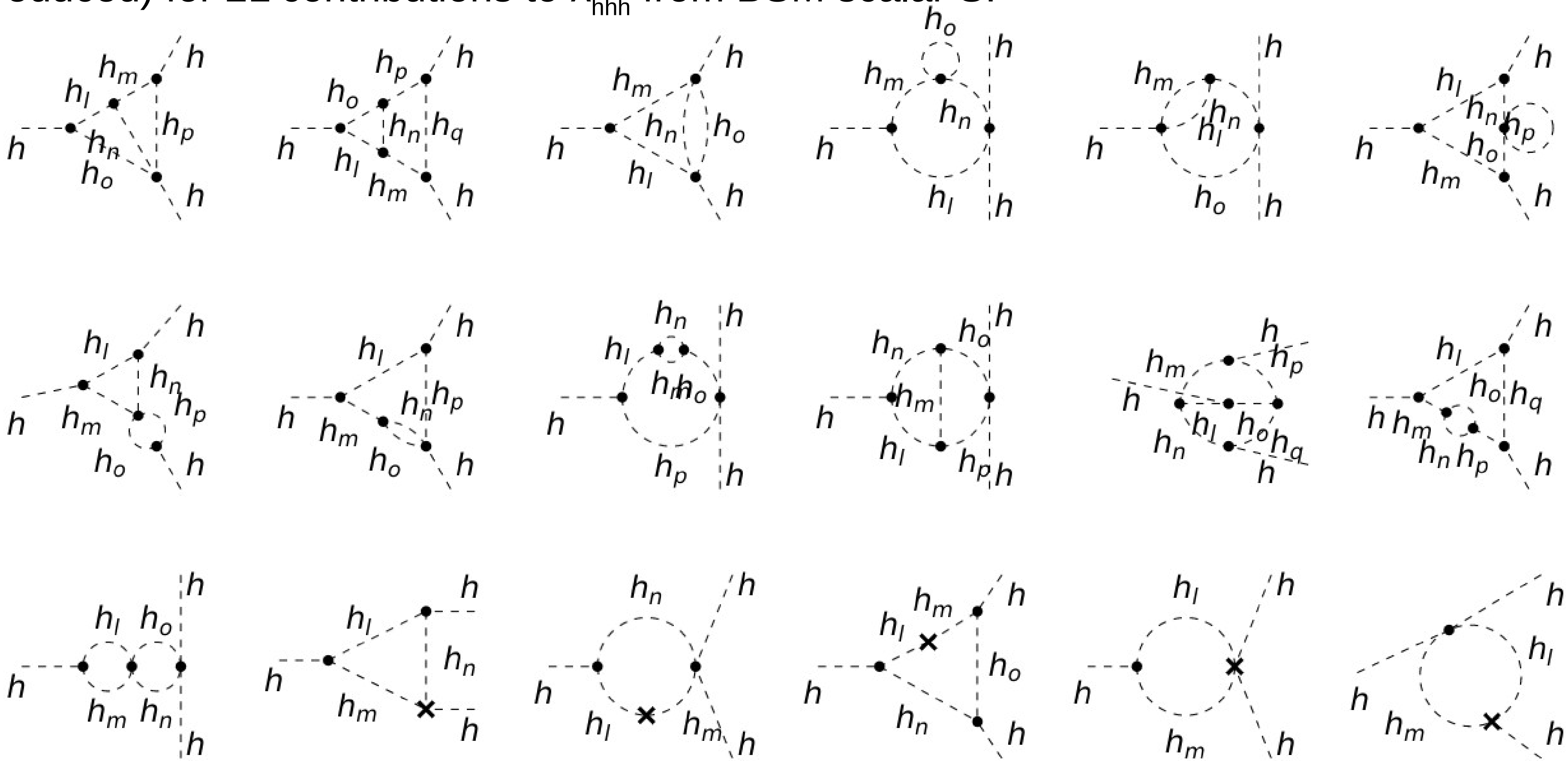
2) New results → today in singlet-extended SM

New results: Leading two-loop corrections in the SSM

- **SSM** = Real-singlet extension of SM, without Z_2 symmetry

$$V_{\text{SSM}} = \mu^2|\Phi|^2 + \frac{\lambda_H}{2}|\Phi|^4 + \frac{m_S^2}{2}S^2 + \frac{\kappa_S}{3}S^3 + \frac{\lambda_S}{2}S^4 + \kappa_{SH}S|\Phi|^2 + \frac{\lambda_{SH}}{2}S^2|\Phi|^2$$

- Diagrams (reduced) for 2L contributions to λ_{hhh} from BSM scalar S:



New results: Leading two-loop corrections in the SSM

$$V_{\text{SSM}} = \mu^2|\Phi|^2 + \frac{\lambda_H}{2}|\Phi|^4 + \frac{m_S^2}{2}S^2 + \frac{\kappa_S}{3}S^3 + \frac{\lambda_S}{2}S^4 + \kappa_{SH}S|\Phi|^2 + \frac{\lambda_{SH}}{2}S^2|\Phi|^2$$

- Neglect light Higgs mass before BSM scalar mass, $m_s \gg m_h$
- $\overline{\text{MS}}$ result first:

$$\begin{aligned}(4\pi)^4\delta^{(2)}\lambda_{hhh}^{\overline{\text{MS}}} =& -\frac{3}{8}\frac{\kappa_{SH}^2 v}{v_S^5}\Big[6\kappa_S v_S^2(3\overline{\ln}m_s^2 + \overline{\ln}^2 m_s^2 - 3) + 8\kappa_{SH}v_S^2(-2\overline{\ln}m_s^2 + \overline{\ln}^2 m_s^2 + 1) \\ &+ \kappa_{SH}v^2(-23\overline{\ln}m_s^2 - 3\overline{\ln}^2 m_s^2 + 35)\Big] \\ &- \frac{1}{m_s^2}\frac{3\kappa_{SH}^2 v}{16 v_S^6}\Big[\kappa_{SH}^2 v^4(35 - 17\overline{\ln}m_s^2) - 4\kappa_S^2 v_S^4(\overline{\ln}m_s^2 - 1) \\ &+ 4\kappa_{SH}v_S^2 v^2\left(\kappa_S(3\overline{\ln}m_s^2 - 8) - 6\kappa_{SH}(\overline{\ln}m_s^2 - 1)\right)\Big] \\ &+ \frac{1}{m_s^4}\frac{\kappa_{SH}^3 v^3(\overline{\ln}m_s^2 - 2)}{16 v_S^7}\Big[4\kappa_S^2 v_S^4 + 4\kappa_{SH}v_S^2 v^2(2\kappa_{SH} - 3\kappa_S) + 9\kappa_{SH}^2 v^4\Big] \\ &+ m_s^2\frac{9\kappa_{SH}^2 v}{4 v_S^4}\left(4\overline{\ln}m_s^2 + \overline{\ln}^2 m_s^2 - 4\right).\end{aligned}$$

→ **decoupling only apparent when scaling $v_s \sim m_s$ and $m_s \to \infty$**

New results: Leading two-loop corrections in the SSM

$$V_{\mathrm{SSM}} = \mu^2|\Phi|^2 + \frac{\lambda_H}{2}|\Phi|^4 + \frac{m_S^2}{2}S^2 + \frac{\kappa_S}{3}S^3 + \frac{\lambda_S}{2}S^4 + \kappa_{SH}S|\Phi|^2 + \frac{\lambda_{SH}}{2}S^2|\Phi|^2$$

- Neglect light Higgs mass before BSM scalar mass, $m_s >> m_h$
- Provide result in alignment limit (i.e. α = 0), but subloop renormalisation still requires a counterterm for mixing angle α
- Renormalisation scheme chosen as $\underbrace{m_h,\, m_s,\, \alpha,\, t_s,\, t_h,\, v,}_{\text{on-shell}}\ \underbrace{v_S,\, \kappa_S,\, \kappa_{SH}}_{\overline{\text{MS}}}$

OS conditions

$$\delta^{(1)} m_s^2 = -\Sigma_s(p^2 = m_s^2)$$

$$\delta^{(1)} Z_{ij} = -\delta^{(1)} Z_{ij} = -2\Sigma_{ij}(p^2 = 0)/m_s^2,\ \ i \neq j,$$

$$\delta^{(1)} Z_{ii} = \frac{\partial}{\partial p^2}\Sigma_{ii}(p^2)\Big|_{p^2=0},\ \ i,j = s,h\,,$$

$$\delta^{(2)} t_h = -t_h^{(2)} - \frac{1}{2}\delta^{(1)} Z_{hs}\delta^{(1)} t_s - \frac{1}{2}\delta^{(1)} Z_{hh}\delta^{(1)} t_h,$$

$$\delta^{(1)} m_{hs}^2 = (m_h^2 - m_s^2)\,\delta^{(1)}\alpha = -m_s^2\,\delta^{(1)}\alpha = \Sigma_{hs}(p^2 = 0)\,.$$

$\overline{\text{MS}}$ conditions

$$(4\pi)^2\delta^{(1)}\kappa_S^{\overline{\text{MS}}} = \frac{3}{\epsilon}\left(6\kappa_S\lambda_S + \kappa_{SH}\lambda_{SH}\right)\,,$$

$$(4\pi)^2\delta^{(1)}\kappa_{SH}^{\overline{\text{MS}}} = \frac{\lambda_{SH}}{\epsilon}\left(\kappa_S + 2\kappa_{SH}\right)\,.$$

New results: Leading two-loop corrections in the SSM

$$V_{\rm SSM} = \mu^2|\Phi|^2 + \frac{\lambda_H}{2}|\Phi|^4 + \frac{m_S^2}{2}S^2 + \frac{\kappa_S}{3}S^3 + \frac{\lambda_S}{2}S^4 + \kappa_{SH}S|\Phi|^2 + \frac{\lambda_{SH}}{2}S^2|\Phi|^2$$

- Neglect light Higgs mass before BSM scalar mass, $m_s >> m_h$
- Provide result in alignment limit (i.e. α = 0), but subloop renormalisation still requires a counterterm for mixing angle α
- Renormalisation scheme chosen as $\underbrace{m_h,\, m_s,\, \alpha,\, t_s,\, t_h,\, v,}_{\text{on-shell}}\ \underbrace{v_S,\, \kappa_S,\, \kappa_{SH}}_{\overline{\rm MS}}$

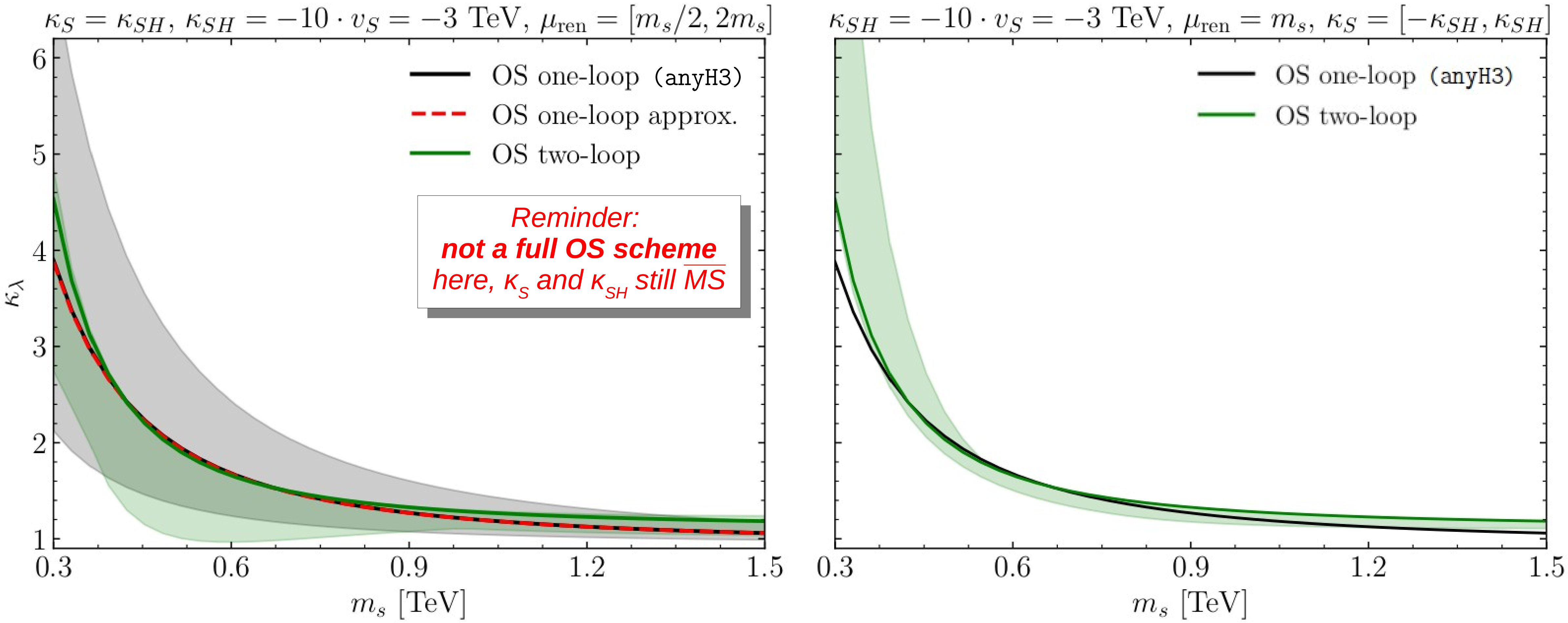
$$(4\pi)^4\delta^{(2)}\lambda^{\rm OS}_{hhh} = -\frac{9\kappa_{SH}^3v^3}{2v_S^5} - \frac{3\kappa_{SH}^3v^3}{2m_s^2v_S^4}\left[(\kappa_S+2\kappa_{SH})\overline{\ln}m_s^2 - 2(\kappa_S-\kappa_{SH}) - 3\kappa_{SH}\frac{v^2}{v_S^2}\right]$$
$$- \frac{\kappa_{SH}^3v^3}{8m_s^4v_S^3}\left[4\kappa_S^2 + \kappa_{SH}(5\kappa_{SH}-12\kappa_S)\frac{v^2}{v_S^2} + 9\kappa_{SH}^2\frac{v^4}{v_S^4}\right],$$
$$\approx -\frac{9\kappa_{SH}^3v^3}{2v_S^5} + \mathcal{O}\left(\frac{m_h^2}{m_s^2}, \frac{\kappa_{SH}^2}{m_s^2}, \frac{\kappa_S^2}{m_s^2}\right)$$

while

$$(4\pi)^2\lambda^{(1),\,\rm OS}_{hhh} = -\frac{\kappa_{SH}^3v^3}{2v_S^3m_s^2} + \mathcal{O}(m_h^2/m_s^2)$$

New results: Leading two-loop corrections in the SSM

Numerical results, in decoupling limit $m_s \to \infty$



→ inclusion of 2L corrections significantly reduces the renormalisation-scale dependence
→ for the future: implement OS scheme for κ_S and κ_{SH} ? (c.f. talk of Alain Verduras Schaeidt)

Summary

- λ_{hhh} plays a crucial role to understand the **shape of the Higgs potential**, and probe indirectly **signs of New Physics**
- Two-loop corrections to λ_{hhh} can be significant, even for points allowed by theoretical (esp. perturbative unitarity) and experimental constraints → **inclusion of two-loop corrections is important for reliable interpretation of bounds on κ_λ**
- **Full two-loop corrections to λ_{hhh} and λ_{hhhh} computed for general renormalisable theories**
  - Generic results mapped onto diagrams generated by FeynArts for model(s) to consider – *could also be done from UFO model rather than FeynArts outputs*
  - Extensive cross-checks performed, and new results obtained for λ_{hhh}
  - EFT-like matching of λ_{hhhh} also possible
- Many applications ongoing, countless more possible!

Thank you very much for your attention!

Contact

DESY. Deutsches Elektronen-Synchrotron

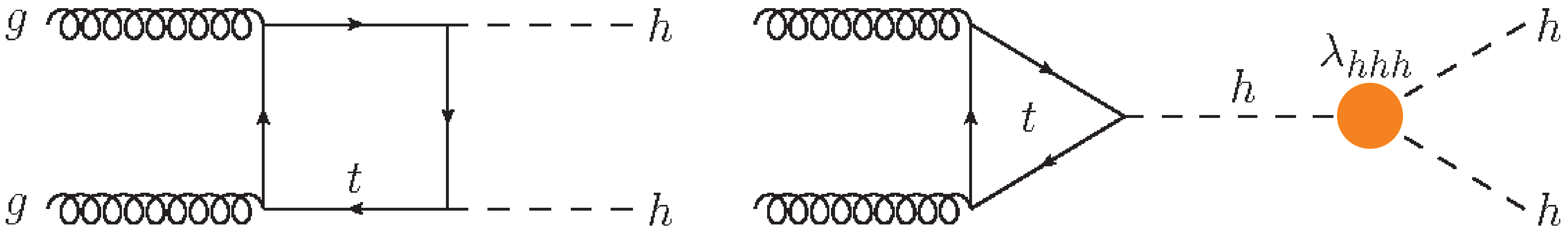
www.desy.de

Martin Gabelmann
DESY Theory group

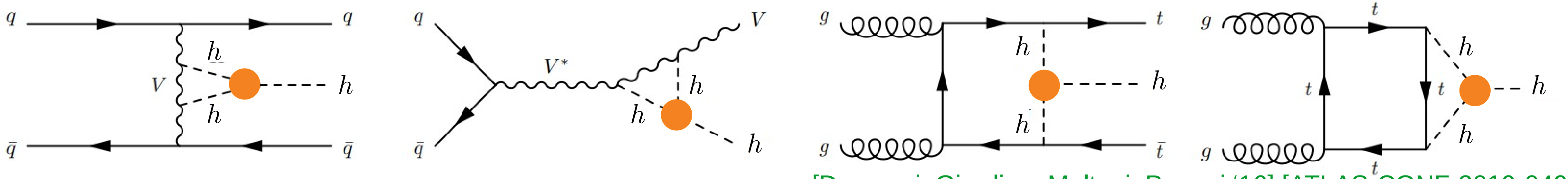
Backup

Experimental probes of λ_{hhh}

- **Double-Higgs production** → λ_{hhh} enters at leading order (LO) → **most direct probe!**

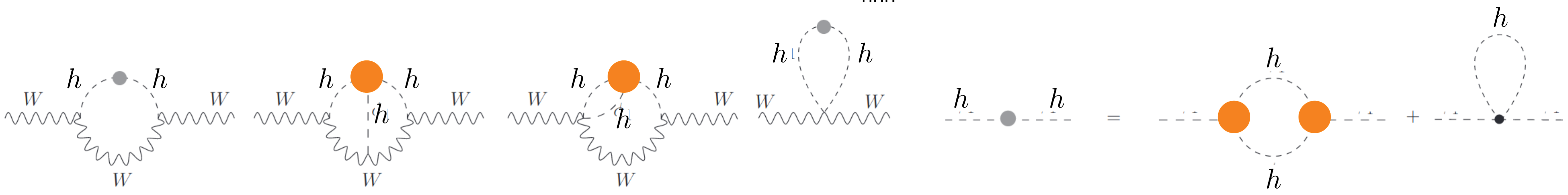


- **Single-Higgs production** → λ_{hhh} enters at NLO



[Degrassi, Giardino, Maltoni, Pagani '16] [ATLAS-CONF-2019-049]

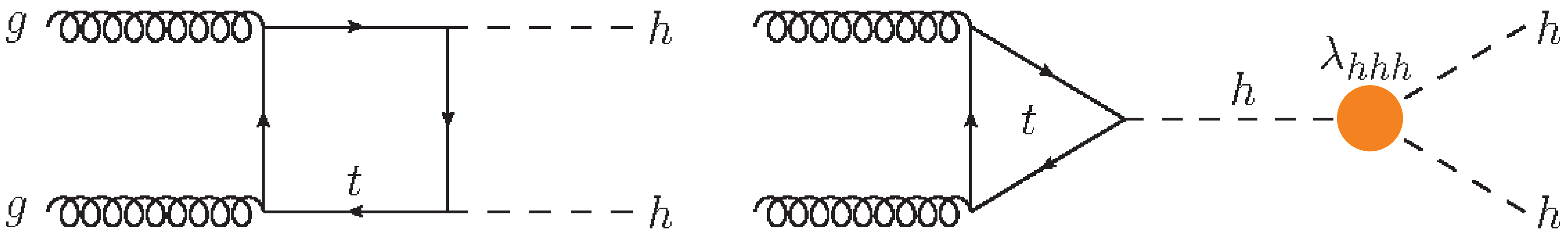
- **Electroweak Precision Observables** (EWPOs) → λ_{hhh} enters at NNLO



[Degrassi, Fedele, Giardino '17]

Accessing λ_{hhh} experimentally

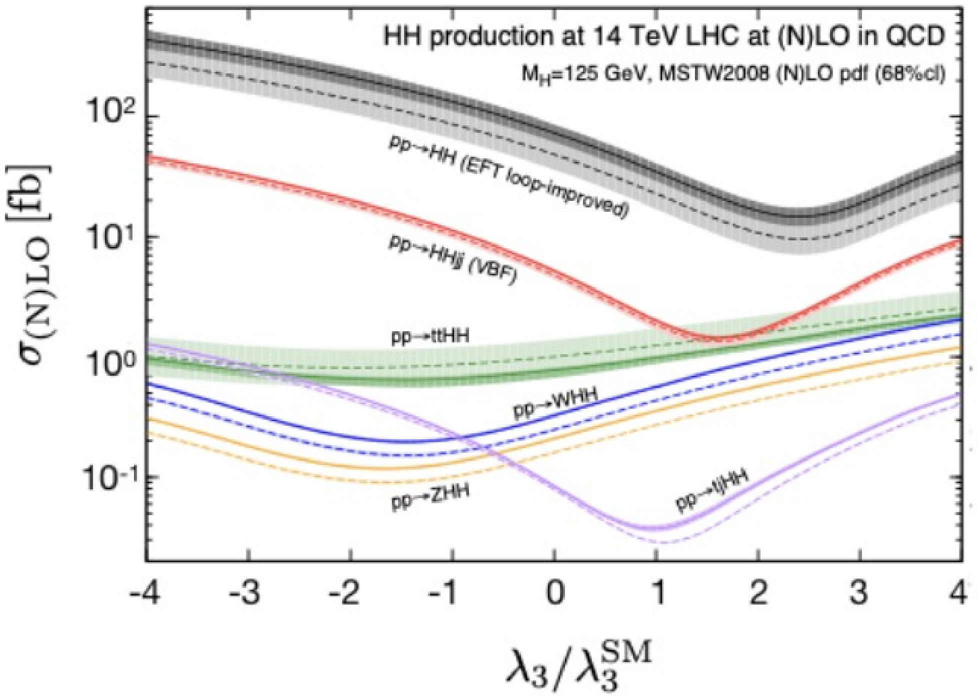
- **Double-Higgs production** → λ_{hhh} enters at LO → **most direct probe of λ_{hhh}**



- Box and triangle diagrams **interfere destructively**
  → small prediction in SM
  → BSM deviation in λ_{hhh} can **significantly alter double-Higgs production!**

- Upper limit on double-Higgs production cross-section
  → **limits on $\kappa_\lambda \equiv \lambda_{hhh}/(\lambda_{hhh}^{(0)})^{SM}$**

- κ_λ as an effective coupling → $\mathcal{L} \supset -\kappa_\lambda \times \frac{3m_h^2}{v^2} \cdot h^3 + \cdots$



[Frederix et al., '14]

Accessing λ_{hhh} via double-Higgs production

- **Double-Higgs production** → λ_{hhh} enters at LO → **most direct probe of λ_{hhh}**

Recent results from ATLAS hh-searches [ATLAS-CONF-2022-050]
yield the limits:

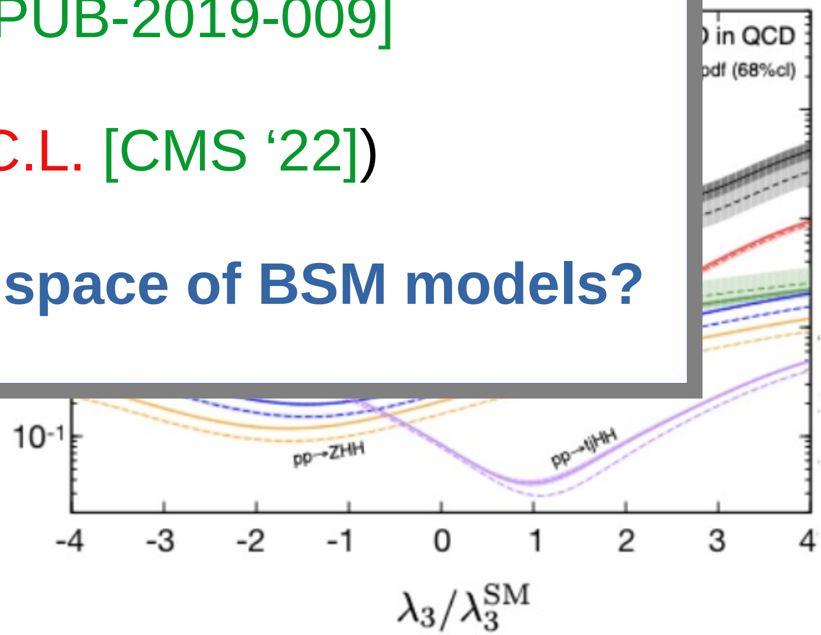
-0.4 < κ_λ < 6.3 at 95% C.L.

→ factor ~2 improvement compared to
pre-2021 best ATLAS limits (from single-h prod.)
-3.2 < κ_λ < 11.9 at 95% C.L. [ATLAS-PHYS-PUB-2019-009]

(CMS recently gave **-1.2 < κ_λ < 6.5** at 95% C.L. [CMS '22])

→ **Can κ_λ now be used to constrain the parameter space of BSM models?**

- Box
  → sı
  → B
  hh-p
- Uppe
  $\kappa_\lambda \equiv \lambda$
- κ_λ as an effective coupling → $\mathcal{L} \supset -\kappa_\lambda \times \frac{3m_h^2}{v^2} \cdot h^3 + \cdots$



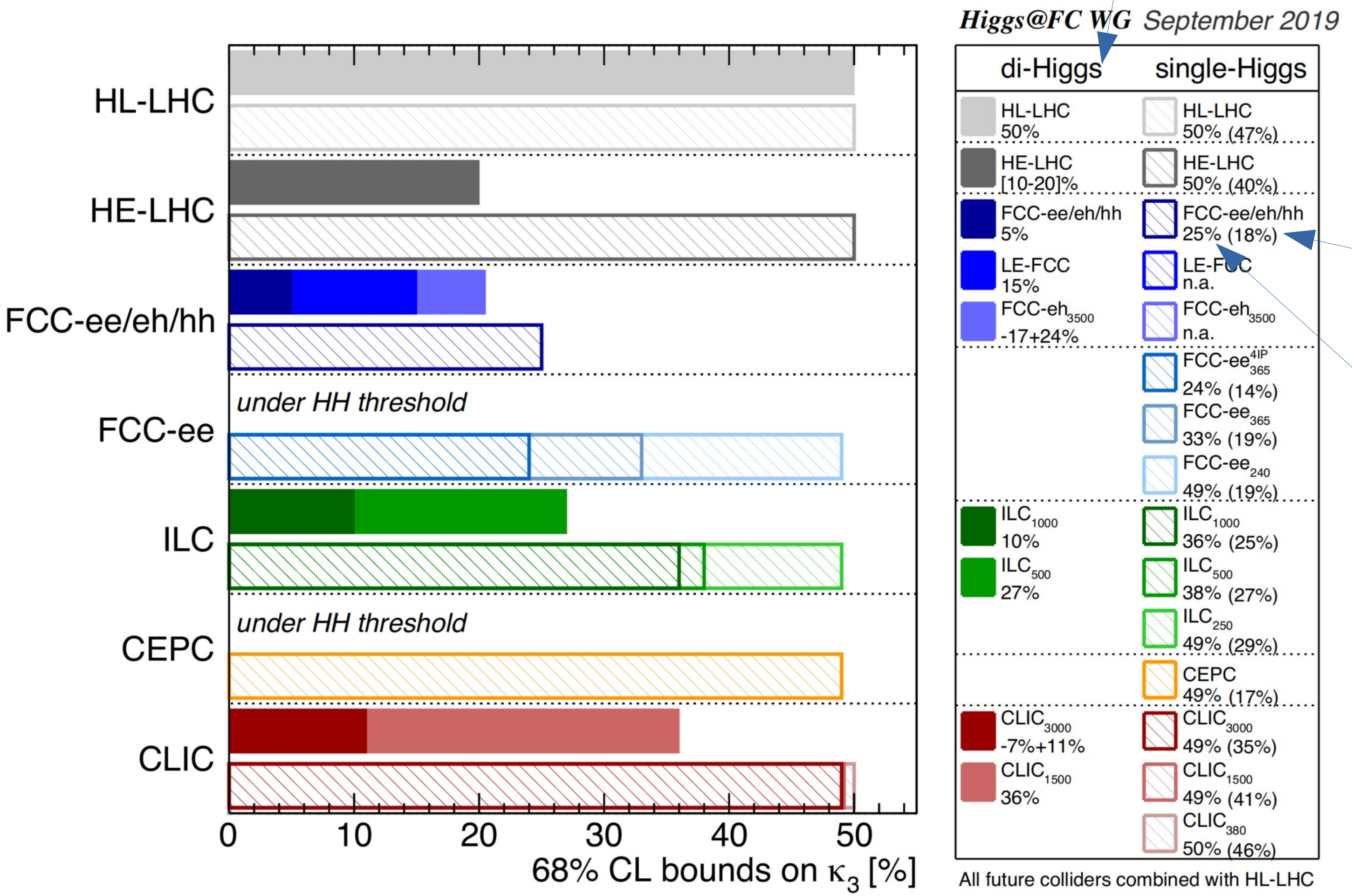
[Frederix et al., '14]

Future determination of λ_{hhh}

Expected sensitivities in literature, assuming $\lambda_{hhh} = (\lambda_{hhh})^{SM}$

Plot taken from
[de Blas et al., 1905.03764]

di-Higgs exclusive result



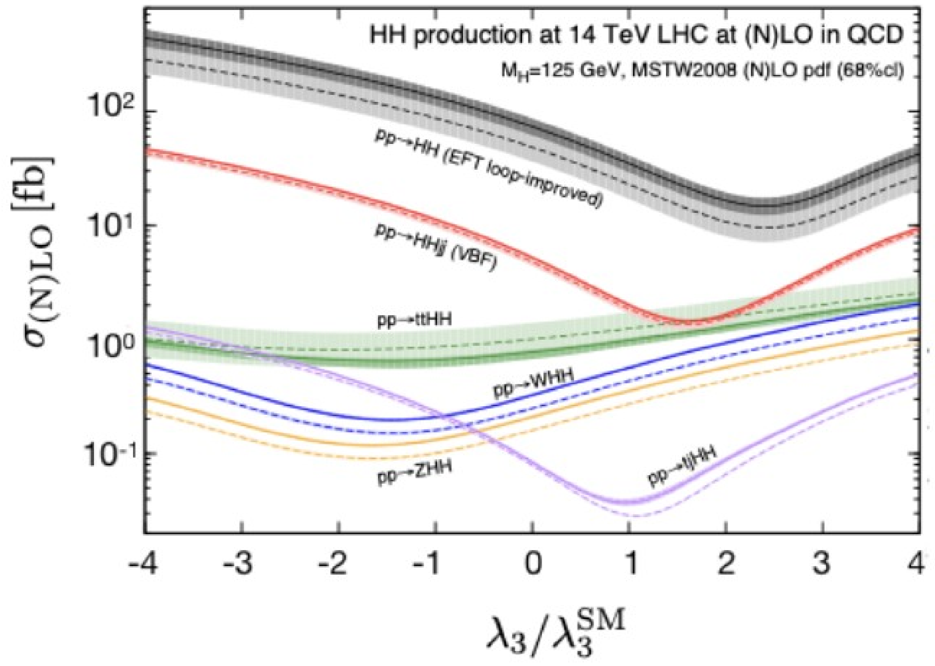
single-Higgs exclusive

single-Higgs global

see also [Cepeda et al., 1902.00134], [Di Vita et al.1711.03978], [Fujii et al. 1506.05992, 1710.07621, 1908.11299], [Roloff et al., 1901.05897], [Chang et al. 1804.07130,1908.00753], *etc.*

Future determination of λ_{hhh}

Higgs production cross-sections (here double Higgs production) depend on λ_{hhh}



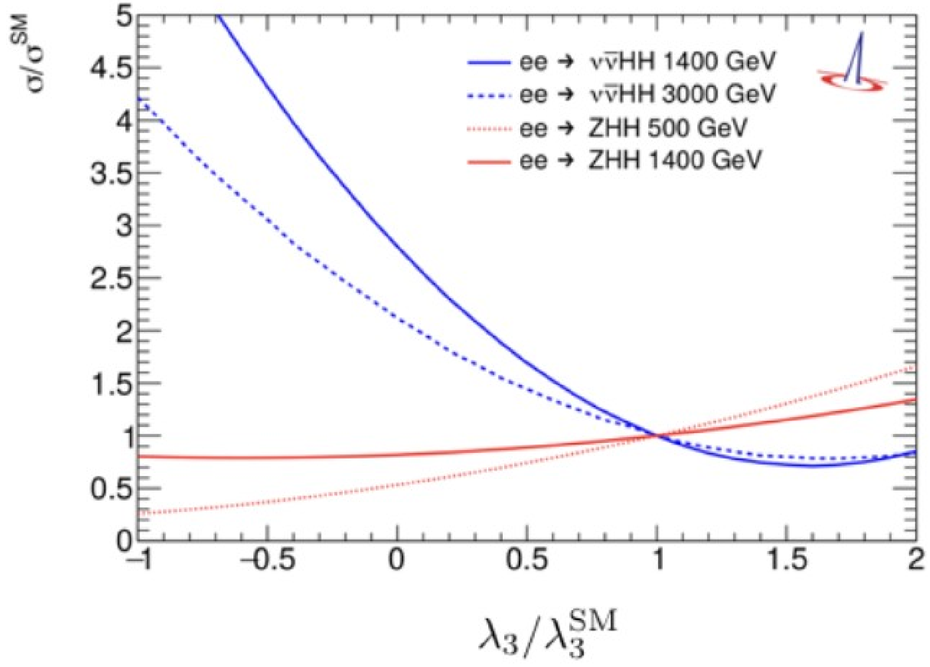


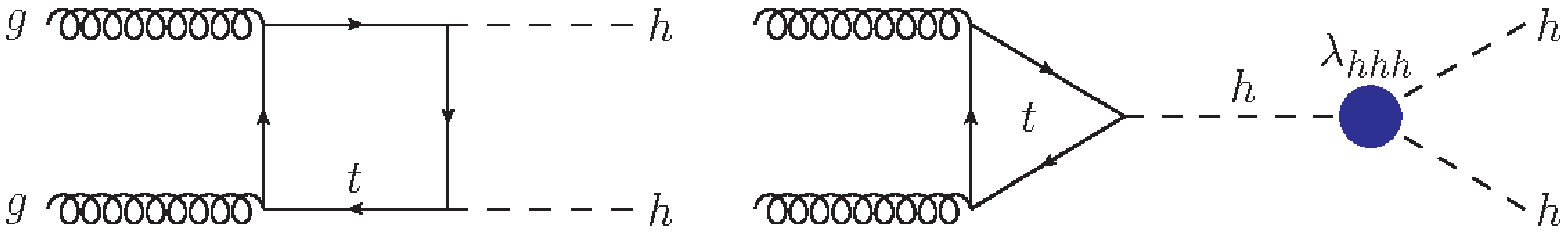
Figure 10. Double Higgs production at hadron (left) [65] and lepton (right) [66] colliders as a function of the modified Higgs cubic self-coupling. See Table 18 for the SM rates. At lepton colliders, the production cross sections do depend on the polarisation but this dependence drops out in the ratios to the SM rates (beam spectrum and QED ISR effects have been included).

[Frederix et al., 1401.7340]

Plots taken from
[de Blas et al., 1905.03764]

Experimental situation for λ_{hhh}

- **Double-Higgs production** → λ_{hhh} enters at LO → **most direct probe of λ_{hhh}**



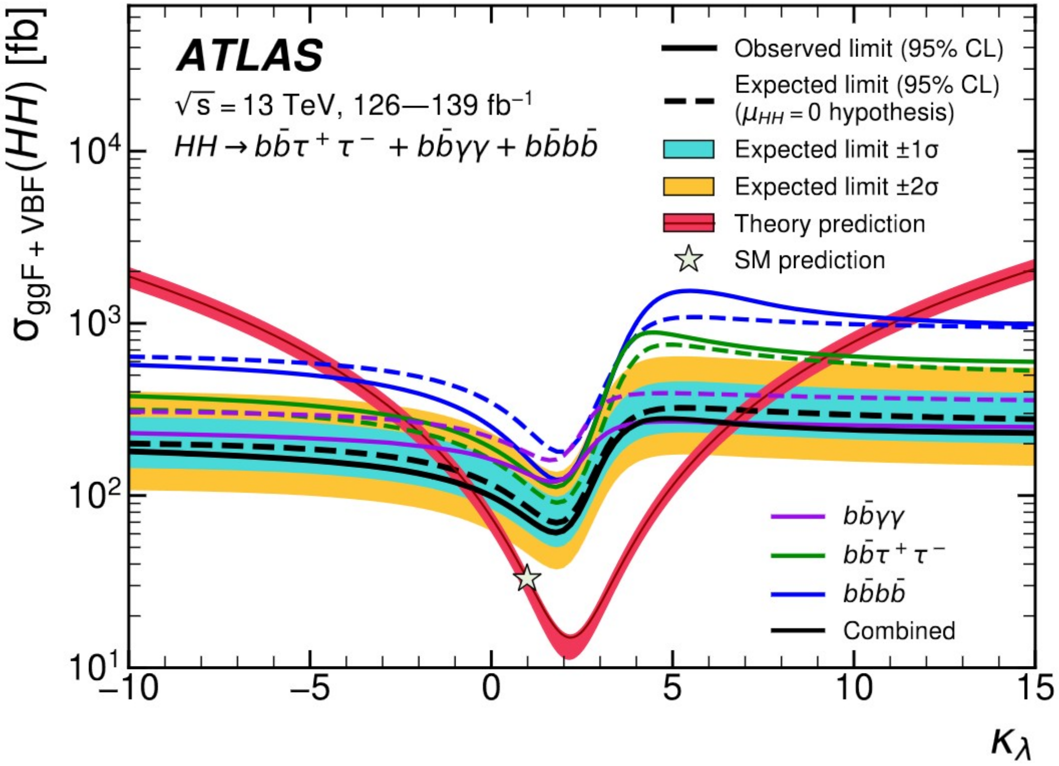
[Note: Single-Higgs production (EW precision observables) → λ_{hhh} enters at NLO (NNLO)]

- Box and triangle diagrams **interfere destructively**
  → small prediction in SM
  → BSM deviation in λ_{hhh} can **significantly enhance double-Higgs production!**

- Search limits on double-Higgs production
  → **limits on effective coupling $\kappa_\lambda \equiv \lambda_{hhh}/(\lambda_{hhh}^{(0)})^{SM}$**

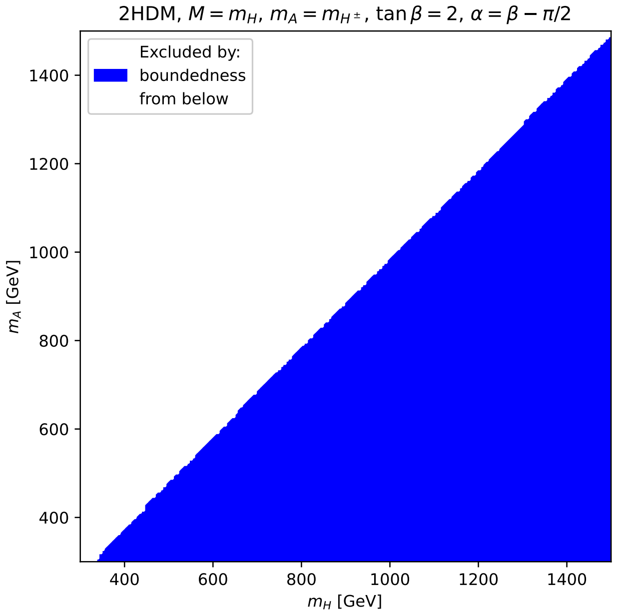
$-0.4 < \kappa_\lambda < 6.3$

[ATLAS PLB ‘23]

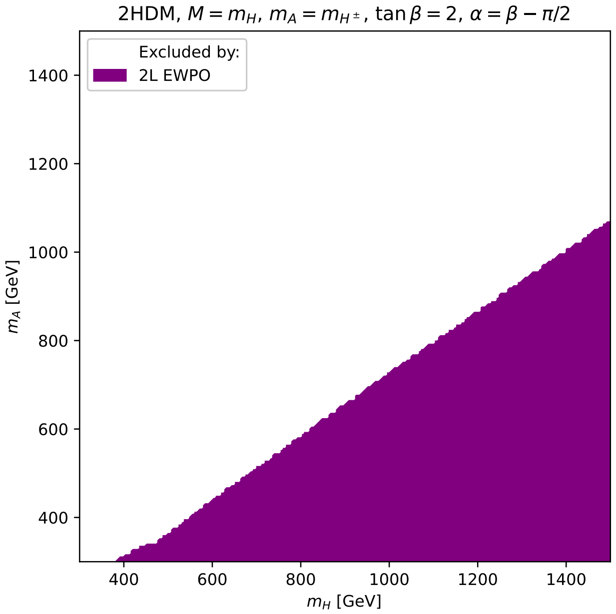


2HDM benchmark plane – individual theoretical constraints

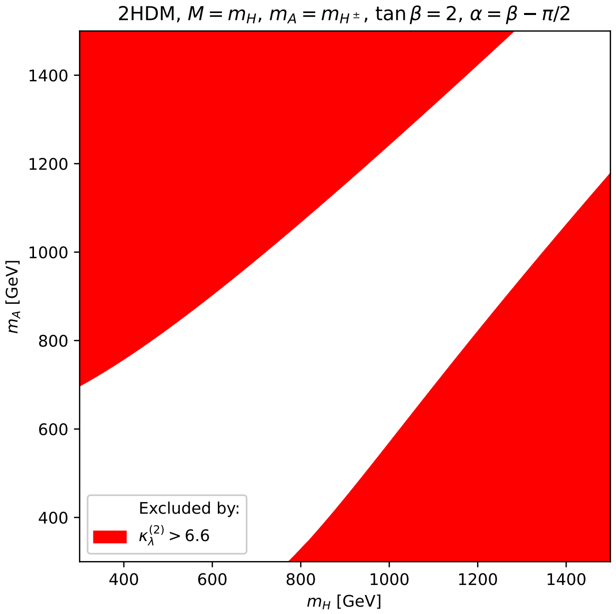
Constraints shown below are independent of 2HDM type



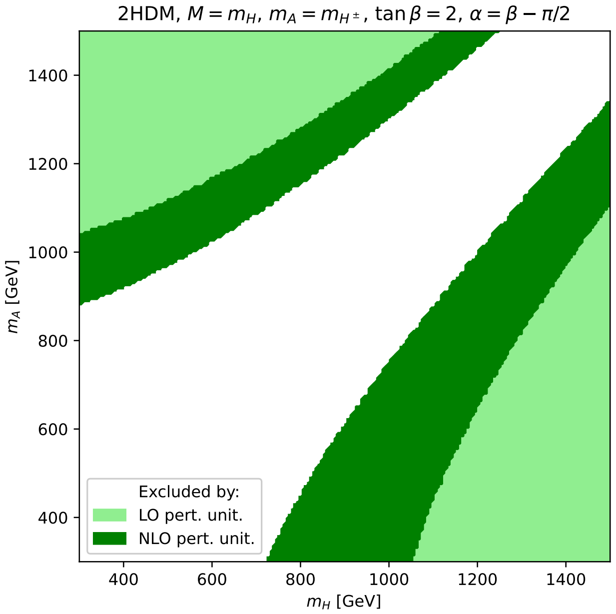
Boundedness from below



EW precision observables computed at 2L



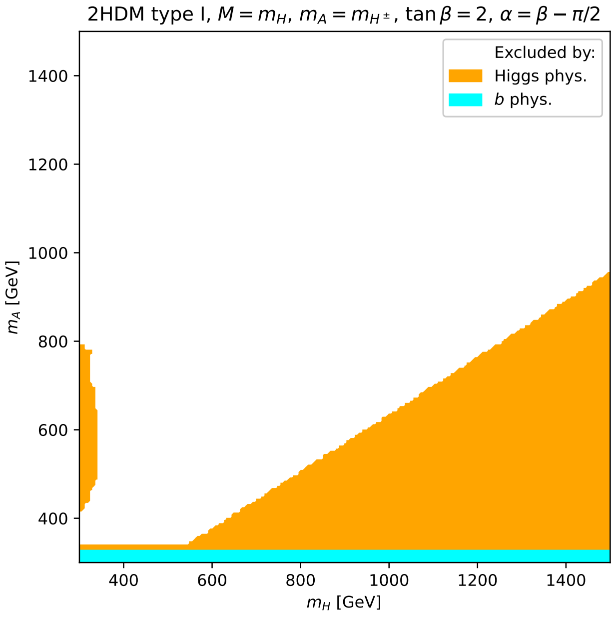
$\kappa_\lambda^{(2)} > 6.6$



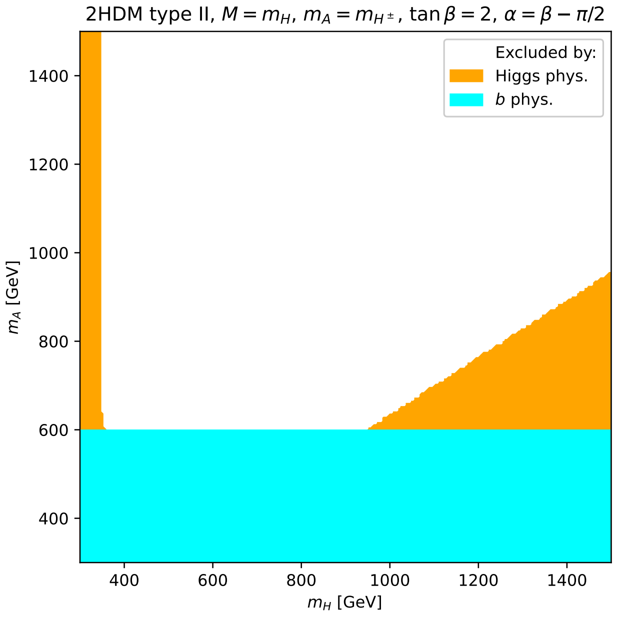
Perturbative unitarity at (N)LO

2HDM benchmark plane – experimental constraints

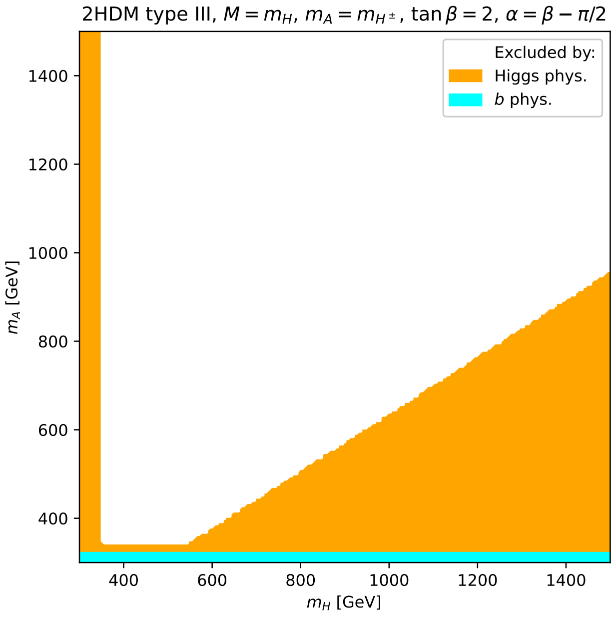
i.e. Higgs physics (via HiggsBounds and HiggsSignals) and *b* physics (from [Gfitter group 1803.01853])



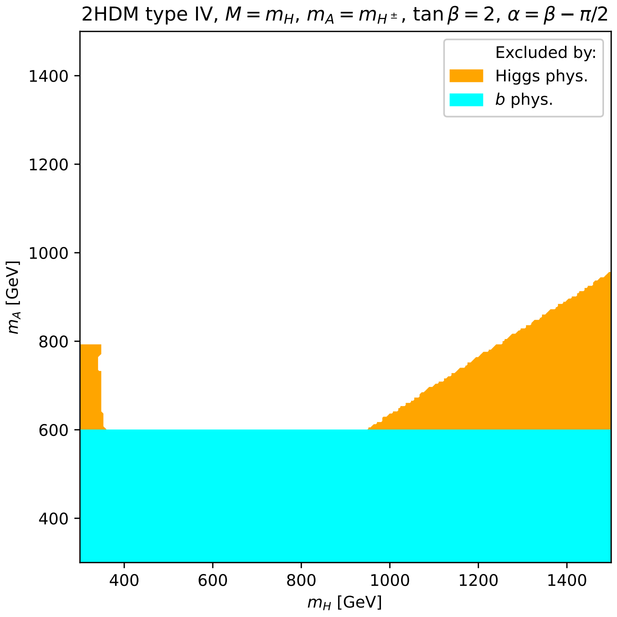
Type-I



Type-II



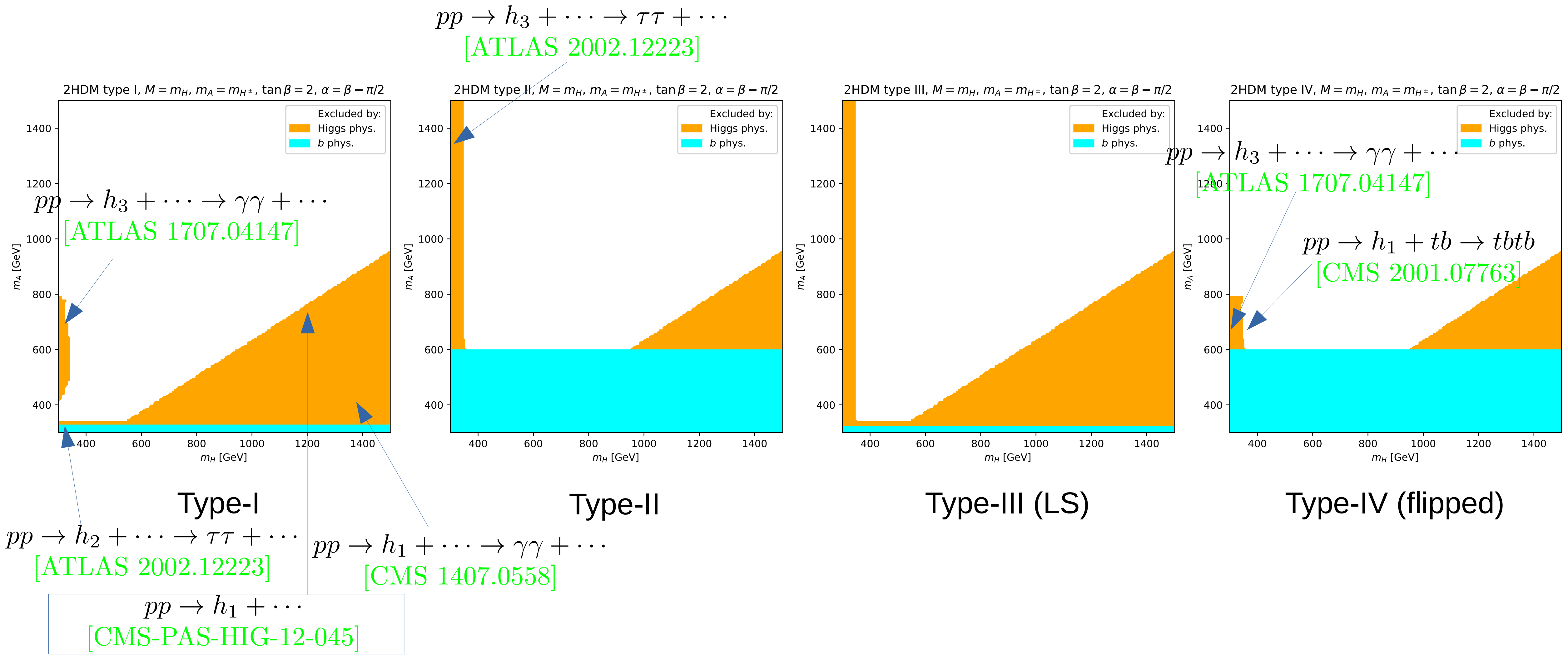
Type-III (LS)



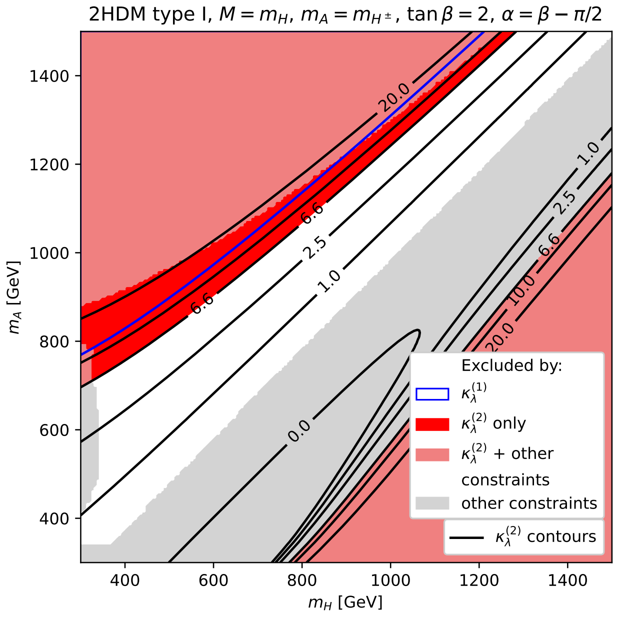
Type-IV (flipped)

2HDM benchmark plane – experimental constraints

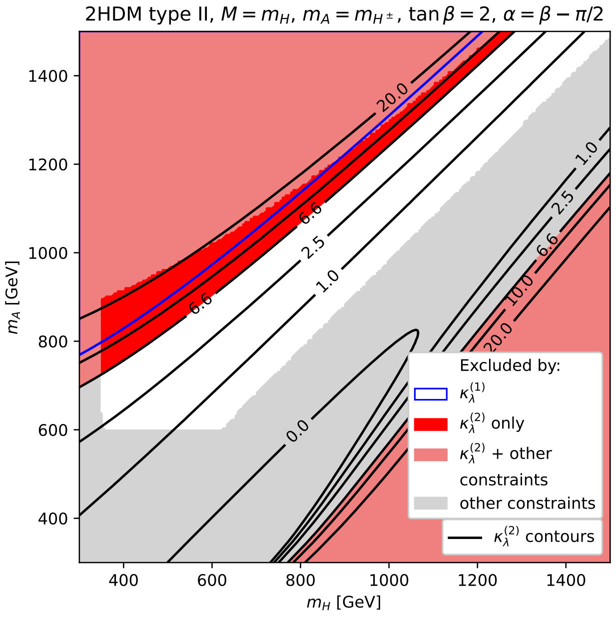
i.e. Higgs physics (via HiggsBounds and HiggsSignals) and *b* physics (from [Gfitter group 1803.01853])



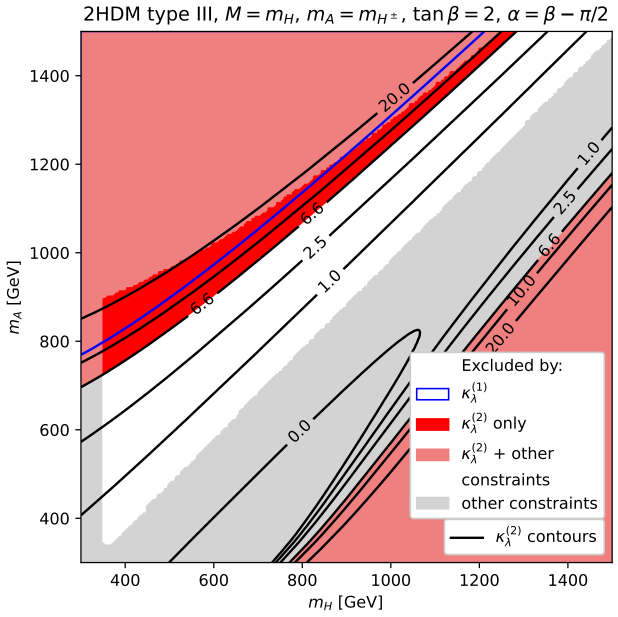
2HDM benchmark plane – results for all types



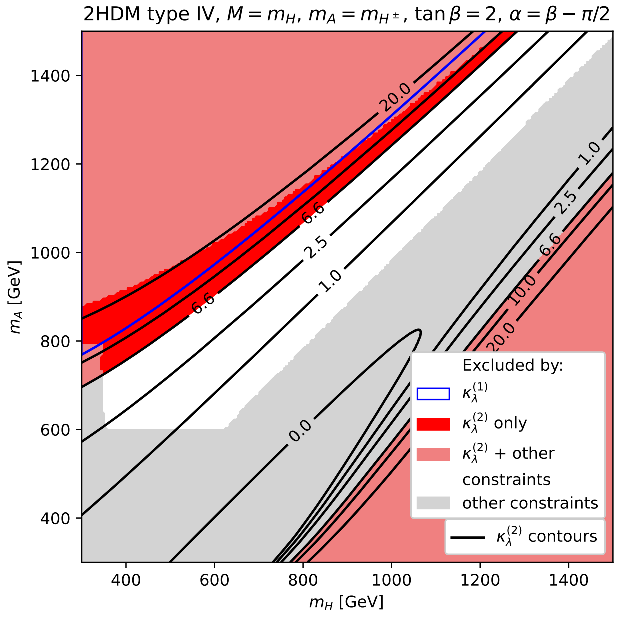
Type-I



Type-II



Type-III (LS)



Type-IV (flipped)