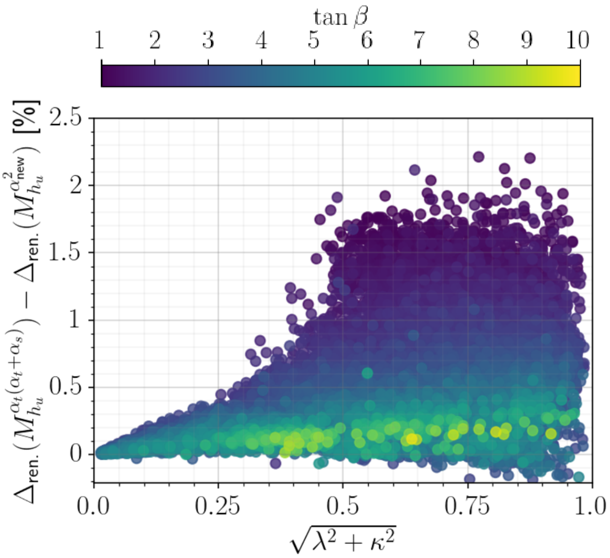
Previously at KUTS...

- prediction of m_h in the NMSSM at $\mathcal{O}((\alpha_t + \alpha_\lambda + \alpha_\kappa)^2)$

  [Dao, MG, Mühlleitner, Rzehak '21]
- estimate of the size of momentum corrections
- mixed OS-$\overline{\text{DR}}$ scheme
  - reduction of theory-uncertainty in NMSSM-specific parameter region



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  [Dao, MG, Mühlleitner, Rzehak '21]
- estimate of the size of momentum corrections
- mixed OS-$\overline{\text{DR}}$ scheme
  - OS renormalised SM VEV
  - α - scheme: $\{\alpha(0),\ M_W,\ M_Z\}$

- VEV counterterm entering at two-loops
- tree-level: $v = \frac{2M_W}{e} s_{\theta_w} = \frac{2M_W}{e}\sqrt{1 - \frac{M_W^2}{M_Z^2}}$
- $$\frac{\delta^{(2)} v}{v} = \frac{\cos\theta_w^2}{2\sin\theta_w^2}\left(\frac{\delta^{(2)} M_Z^2}{M_Z^2} - \frac{\delta^{(2)} M_W^2}{M_W^2}\right) + \frac{\delta^{(2)} M_W^2}{2M_W^2} - \frac{1}{8\sin\theta_w^4}\left[\left(\frac{\delta^{(1)} M_W^2}{M_W^2}\right)^2 - 2\cos\theta_w^2(1 + 2\sin\theta_w^2)\frac{\delta^{(1)} M_W^2}{M_W^2}\frac{\delta^{(1)} M_Z^2}{M_Z^2} + \cos\theta_w^2(1 + 3\sin\theta_w^2)\left(\frac{\delta^{(1)} M_Z^2}{M_Z^2}\right)^2\right]$$

Previously at KUTS...

Higgs mass calculation:

- VEV counterterm entering at two-loops
- $$\frac{\delta^{(2)}v}{v} = \frac{\cos\theta_w^2}{2\sin\theta_w^2}\left(\frac{\delta^{(2)}M_Z^2}{M_Z^2} - \frac{\delta^{(2)}M_W^2}{M_W^2}\right) + \frac{\delta^{(2)}M_W^2}{2M_W^2} - \frac{1}{8\sin\theta_w^4}\left[\left(\frac{\delta^{(1)}M_W^2}{M_W^2}\right)^2 - 2\cos\theta_w^2(1+2\sin\theta_w^2)\frac{\delta^{(1)}M_W^2}{M_W^2}\frac{\delta^{(1)}M_Z^2}{M_Z^2} + \cos\theta_w^2(1+3\sin\theta_w^2)\left(\frac{\delta^{(1)}M_Z^2}{M_Z^2}\right)^2\right]$$

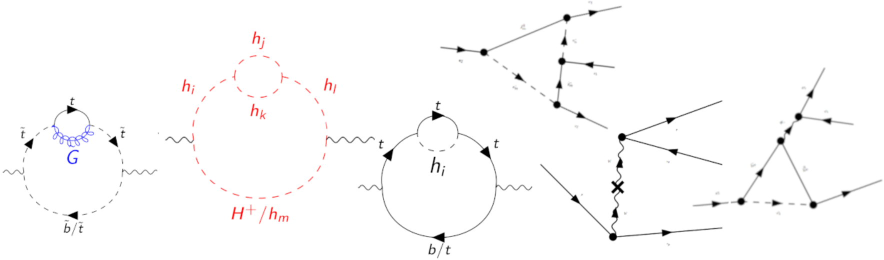
W mass calculation:

- numerically dominant contribution: electroweak rho parameter
- $$\Delta^{(2)}\rho = \left(\frac{\Sigma_{ZZ}^{(2),T}}{M_Z^2} - \frac{\Sigma_{WW}^{(2),T}}{M_W^2}\right) - \frac{\Sigma_{ZZ}^{(1),T}}{M_Z^2}\left(\frac{\Sigma_{ZZ}^{(1),T}}{M_Z^2} - \frac{\Sigma_{W}^{(1),T}}{M_W^2}\right)$$

$\rightarrow$ **All necessary corrections for a two-loop $\Delta\rho$-parameter (W-mass) prediction!(?)**

Precise W-boson mass predictions in the complex NMSSM

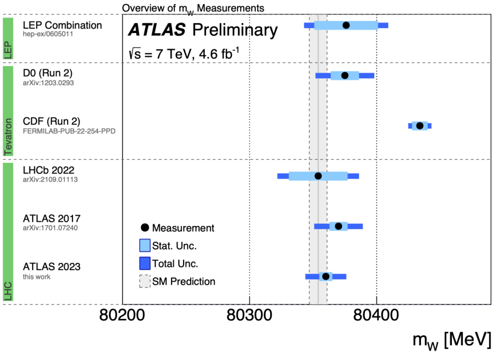
$\Delta\rho$ and M_W including two-loop SUSY corrections
based on [Dao, Gabelmann, Mühlleitner, Eur.Phys.J.C 83 (2023) 11, 1079]



Thi Nhung Dao, **Martin Gabelmann**, Margarete Mühlleitner |KUTS, June 2024

The W boson mass...

- two most-recent most-precise individual measurements
- huge discrepancy of about $\propto 7\sigma$
- also w/o CDF: small upwards tendency



[ATLAS-CONF-2023-004]

Despite what the solution for the M_W-miracle might be, it will most-likely involve a very precise number $\rightarrow$ precision predictions (for concrete BSM scenarios) required!

Getting started: how to "predict" M_W?

- in most models (incl. SM): M_W cannot be "predicted"
- actual prediction: relation between M_W and other observables
- e.g. in the SM the tree-level relation [Ross, Veltman '75]

$$\rho = \frac{G_{CC}(0)}{G_{NC}(0)} = \frac{M_W^2}{c_w^2 M_Z^2} = 1\,, \quad (c_w = \cos\theta_w,\; s_w = \sin\theta_w)$$

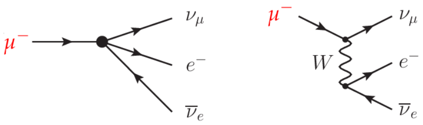
is perturbed by higher-order terms.

Strategy: use relations that involve both, theoretically and experimentally, well-defined observables (e.g. OS pole-masses) that have smallest uncertainties.
$\rightarrow$ use $\rho^{(0)} = 1$ to eliminate presence of θ_w in the relation of *some other* set of observables.

Relation between $M_W \leftrightarrow M_Z$, G_F and α_{QED}

consider muon decay: $G_F/\sqrt{2}\,(\bar{e}\gamma_\rho\nu_e)\,(\bar{\mu}\gamma^\rho\nu_\mu)$

$$G_F = \frac{\pi\alpha_{\text{QED}}}{\sqrt{2}M_W^2\,(1-M_W^2/M_Z^2)}(1+\Delta r)$$



Solve (iteratively) for M_W:

$$M_W = M_Z\left(\frac{1}{2}+\sqrt{\frac{1}{4}-\frac{\pi\alpha_{\text{QED}}}{\sqrt{2}G_F M_Z^2}(1+\Delta r)}\right)$$

- G_F: from muon life time
- $\alpha_{\text{QED}}(0)$: fine-structure constant (including $\Delta\alpha$ resummation)
- M_W and M_Z: pole-masses
- Δr: contains higher-order corrections $\Delta r \equiv \Delta r(M_W, M_Z, \alpha, \dots)$

Instead of *predicting* M_W we are asking "which (OS) value of M_W do we need to get the muon decay right?"

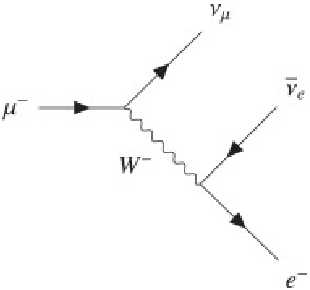
Higher-order corrections to the muon decay: Δr and $\Delta\rho$

$$\Delta r^{(1)} = 2\frac{\delta^{(1)} Z_e}{e} + \frac{\Sigma_{WW}^{(1),T}(0) - \delta^{(1)} M_W^2}{M_W^2} - \frac{\delta^{(1)} s_w^2}{s_w^2} + \delta_{\text{vertex+box}}$$

$$= \Delta\alpha - \frac{c_w^2}{s_w^2}\Delta\rho + \Delta r_{\text{remainder}}$$

ingredients:

- $\delta^{(1)} Z_e$: photon and photon-Z self-energies
- $\delta^{(1)} M_{W/Z}^2$, $\Sigma_{WW/ZZ}^{(1),T}(0)$, $\delta^{(1)} s_W^2$: (transverse) W/Z self-energies
- $\delta_{\text{vertex+box}}$: vertex/box diagrams



dominant:

- $\Delta\alpha$: light fermion contributions (SM-like)
- $\Delta\rho = \frac{\Sigma_{ZZ}^{(1),T}(0)}{M_Z^2} - \frac{\Sigma_{WW}^{(1),T}(0)}{M_W^2}$ ← sensitivity to BSM physics
  (also part of EW fits, "T" parameter)

$\Delta\rho^{\text{fit.}} \approx 3.8 \pm 2.0 \times 10^{-4}$ [PDG '22]

- this work: Δr at full one-loop (+ higher-order SM corrections).
  $\Delta\rho$ at two-loops.

The CP-Violating NMSSM

The Complex Next-to-Minimal Supersymmetric Standard Model

- Singlet extension of minimal SUSY (MSSM).
- Theoretically well-motivated (solves μ- and little-hierarchy-problem).
- Rich phenomenology in the Higgs boson sector:

$$H_d = \begin{pmatrix} \frac{v_d + h_d + i a_d}{\sqrt{2}} \\ h_d^- \end{pmatrix}, \; H_u = e^{i\varphi_u} \begin{pmatrix} h_u^+ \\ \frac{v_u + h_u + i a_u}{\sqrt{2}} \end{pmatrix}, \; S = \frac{e^{i\varphi_s}}{\sqrt{2}} (v_S + h_s + i a_s)$$

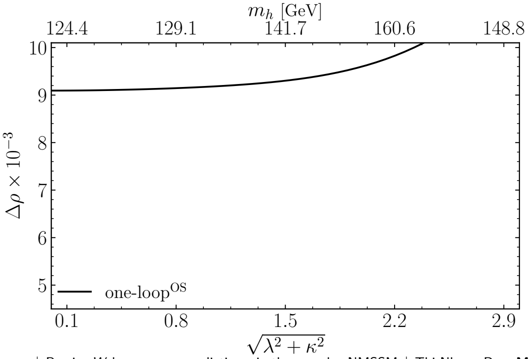
mix to

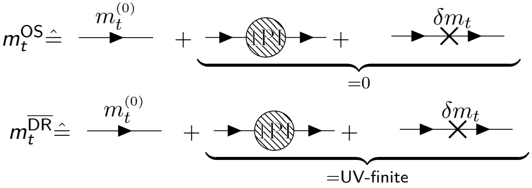
$$h_1, h_2, h_3, h_4, h_5, G^0 \text{ (mass ordered) and } h^\pm, G^\pm$$

- LHC measurements: h_1, h_2 **or** h_3 play the role of the Higgs boson h measured at LHC (h_1 or $h_{2,3}$ are "SM-like"). MSSM: no CPV at tree-level and always $h_1 = h$.
- extended electroweakino-sector: $\chi^0_{1,2,3,4,5}$, $\chi^\pm_{1,2}$
- two important parameters λ and κ: NMSSM $\xrightarrow{\lambda,\kappa\to 0}$ MSSM

$\Delta\rho$ at one-loop in the NMSSM: uncertainties?

- all one-loop contributions are well-known (in CP-conserving case) [Stål, Weiglein, Zeune '15]
- how to estimate the uncertainty without calculating higher-orders?
  $\rightarrow$ using different renormalisation conditions
- $\overline{\text{DR}}$ or OS renormalisation of top & stop sector [Graf et al. '12]
  $\rightarrow$ estimate uncertainty due to missing higher-orders
- one-loop:





$$\Delta\rho(m_t^{\text{OS}}) \overset{\text{all-orders}}{=} \Delta\rho(m_t^{\overline{\text{DR}}})$$

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  $\rightarrow$ using different renormalisation conditions
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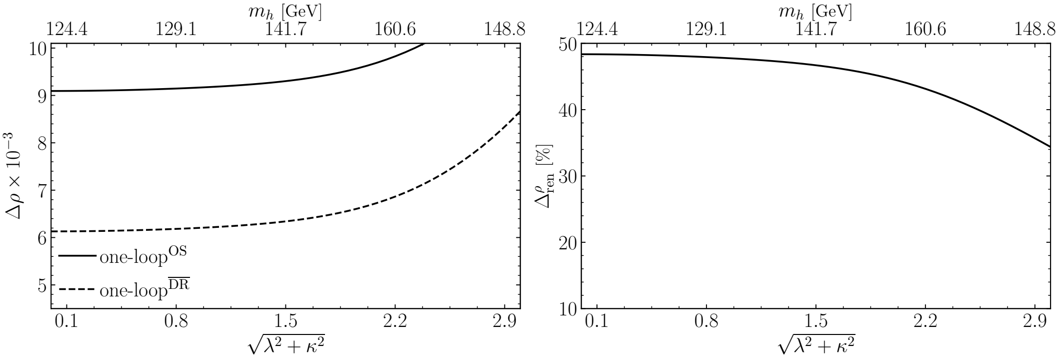
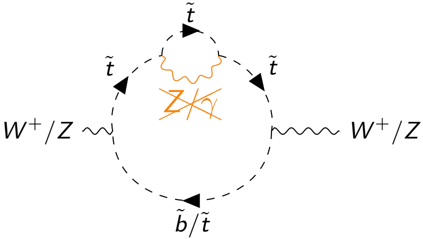
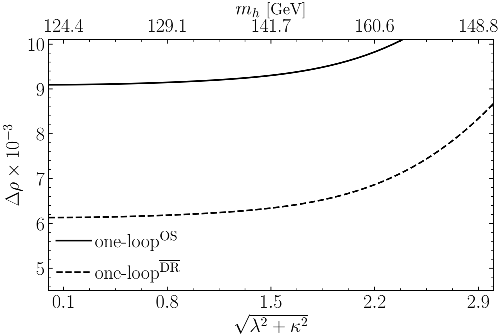
  $\rightarrow$ estimate uncertainty due to missing higher-orders
- one-loop: huge uncertainty $\rightarrow$ higher-orders required

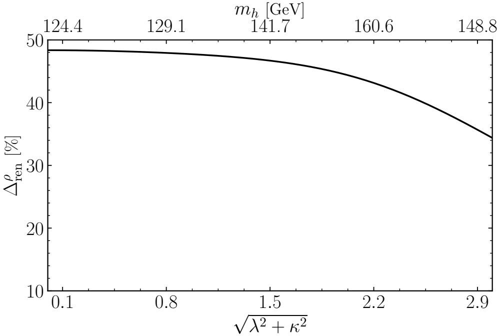


$\Delta\rho$ beyond one-loop

> approximation: gaugeless limit $g_1, g_2 \to 0$

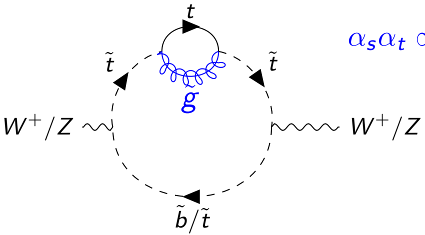




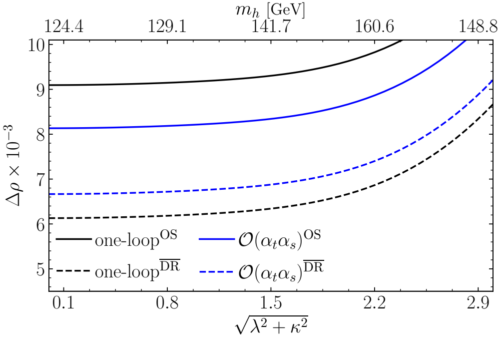


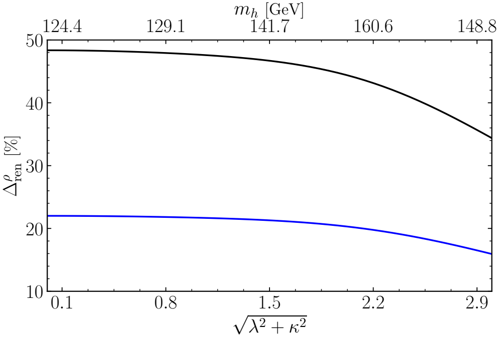
$\Delta\rho$ beyond one-loop

- approximation: gaugeless limit $g_1, g_2 \to 0$
- leading QCD $\mathcal{O}(\alpha_s\alpha_t)$ mixed OS/$\overline{\text{DR}}$



$\alpha_s\alpha_t \propto \frac{g_3^2 y_t^2}{16\pi^2}$

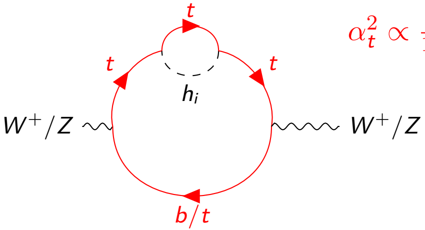


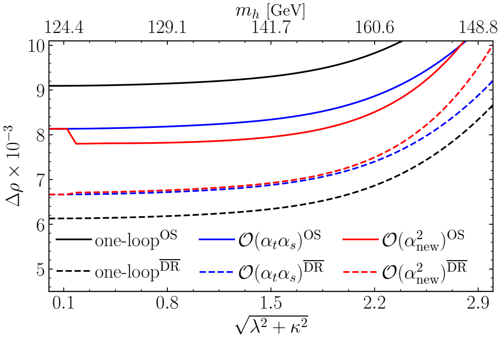


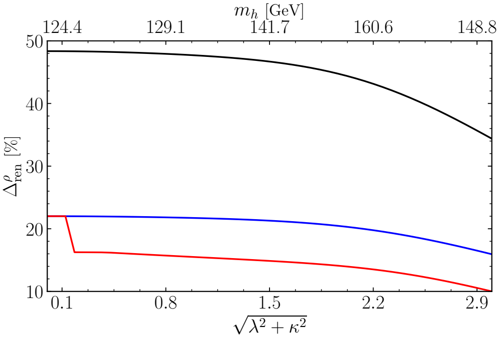
$\Delta\rho$ beyond one-loop

- approximation: gaugeless limit $g_1, g_2 \to 0$
- leading QCD $\mathcal{O}(\alpha_s\alpha_t)$ mixed OS/$\overline{\text{DR}}$
- $\mathcal{O}(\alpha_t^2)$ very sensitive to top mass

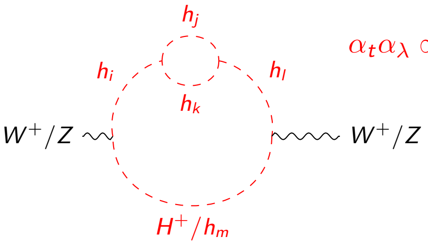
$$\alpha_t^2 \propto \frac{y_t^4}{16\pi^2}$$





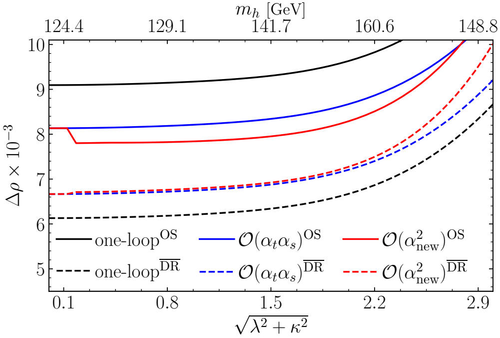


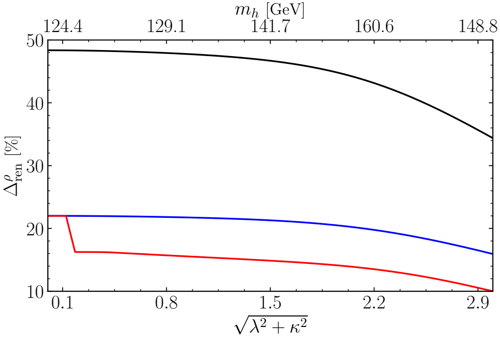
$\Delta\rho$ beyond one-loop



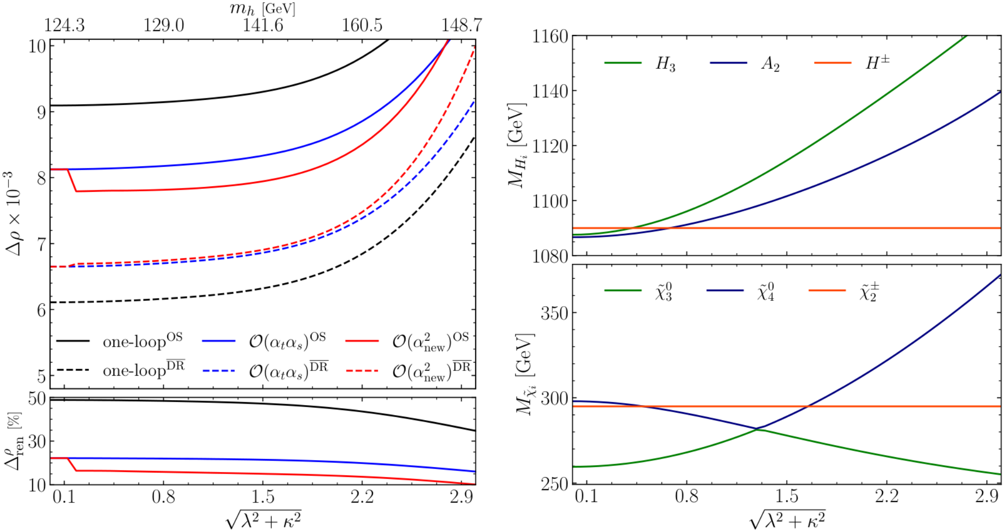
$\alpha_t\alpha_\lambda \propto \frac{y_t^2\lambda^2}{16\pi^2}$

- approximation: gaugeless limit $g_1, g_2 \to 0$
- leading QCD $\mathcal{O}(\alpha_s\alpha_t)$ mixed OS/$\overline{\text{DR}}$
- $\boldsymbol{\mathcal{O}(\alpha_t^2)}$ very sensitive to top mass
- $\boldsymbol{\mathcal{O}((\alpha_t+\alpha_\lambda+\alpha_\kappa)^2)}$ can be large for large λ, κ
  technical difficulty: intermediate infrared-divergences (backup slides).
  Final result turns out to be IR-finite (using dim.-reg. or mass-reg.)!





$SU(2)_L$ mass splittings $\to \Delta\rho$ (custodial symmetry)



corrections driven by mass splitting ($SU(2)_V$-breaking), not by large couplings!

M_W via Δr: Combination with known higher-order SM-results

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Reminder: $$M_W = M_Z\left(\frac{1}{2}+\sqrt{\frac{1}{4}-\frac{\pi\alpha_{\text{QED}}}{\sqrt{2}G_F M_Z^2}(1+\Delta r(M_W, M_Z, M_t, M_{H_i}, M_{\chi^\pm}, M_{\tilde{f}}, \ldots))}\right)$$

- $$\Delta r = \Delta^{(4)} r\big|_{\text{SM}} + \Delta r^{(2)}\big|_{\text{SUSY}} = \Delta^{(4)} r\big|_{\text{SM}} - \frac{c_w^2}{s_w^2}\Delta\rho^{(2)}\big|_{\text{SUSY}} + \Delta r^{(2)}_{\text{remainder}}\Big|_{\text{SUSY}}$$
  $$\approx \Delta r^{(4)}\big|_{\text{SM}} + \Delta r^{(1)}\big|_{\text{SUSY}} - \frac{c_w^2}{s_w^2}\Delta^{(2)}\rho\big|_{\text{SUSY}}$$
- $\Delta^{(n)} X\big|_{\text{SUSY}} = \Delta^{(n)} X\big|_{\text{NMSSM}} - \Delta^{(n)} X\big|_{\text{SM}}, \quad X = \{\rho, r\},\ n = \{1, 2\}$ ← take with care!
- $\Delta^{(4)} r\big|_{\text{SM}} = \Delta^{(1)} r + \Delta^{(\alpha\alpha_s)} r + \Delta^{(\alpha\alpha_s^2)} r + \Delta^{(\alpha^2)} r + \Delta^{(G_\mu^2 m_t^4 \alpha_s)} r + \Delta^{(G_\mu^2 m_t^6)} r + \Delta^{(G_\mu^2 m_t^2 \alpha_s^3)} r$
  with full two-loop and partial three & four loop results **obtained in the OS scheme**
  [Awramik, Chakraborti, Chen, Chetyrkin, Czakon, Degrassi, Djouad, Fleischer, Freitas, Giardino, Gambino, Heinemeyer, Hollik, Jegerlehner, Kühn, Kniehl, Saha, Sirlin, Steinhauser, Tarasov, Weiglein, ...]

M_W via Δr: Combination with known higher-order SM-results

Reminder: $$M_W = M_Z \left(\frac{1}{2} + \sqrt{\frac{1}{4} - \frac{\pi \alpha_{\text{QED}}}{\sqrt{2} G_F M_Z^2} (1 + \Delta r(M_W, M_Z, M_t, M_{H_i}, M_{\chi^\pm}, M_{\tilde{f}}, \ldots))} \right)$$

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  with full two-loop and partial three & four loop results **obtained in the OS scheme**
  two-loop $\mathcal{O}(\alpha\alpha_s)$: m_b-effects missing but in principle available (pre-2000s) analytically (thanks to Georg!), but not yet implemented...

$$\Delta^{(n)} X\big|_{\text{SUSY}} = \Delta^{(n)} X\big|_{\text{NMSSM}} - \Delta^{(n)} X\big|_{\text{SM}}\,, \quad X = \{\rho, r\},\ n = \{1, 2\}$$

> ensure "$|_{\text{SM}}$" and "$|_{\text{NMSSM}}$" treat input-parameters consistently!

$$\Delta^{(\mathrm{n})} X\Big|_{\mathrm{SUSY}} = \Delta^{(\mathrm{n})} X\Big|_{\mathrm{NMSSM}} - \Delta^{(\mathrm{n})} X\Big|_{\mathrm{SM}}, \quad X = \{\rho, r\}, \; n = \{1, 2\}$$

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- natural choice: OS scheme (already in `NMSSMCALC`)

$\Delta^{(n)}X\big|_{\text{SUSY}} = \Delta^{(n)}X\big|_{\text{NMSSM}} - \Delta^{(n)}X\big|_{\text{SM}}, \;\; X = \{\rho, r\}, \;\; n = \{1,2\}$

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- independent calculation of full $\Delta^{(1)}r|_{\text{SM}}$ and $\Delta^{(2)}r|_{\text{SM}}$ up to $\mathcal{O}(\alpha_t(\alpha_t+\alpha_s))$
  - $\Delta^{(2)}r|_{\text{NMSSM}} \xrightarrow{m_{\text{SUSY}}\to\infty} \Delta^{(2)}r|_{\text{SM}}$ ✓
  - $M_W|_{\text{NMSSM}} \xrightarrow{m_{\text{SUSY}}\to\infty} \Delta^{(2)}M_W|_{\text{SM}}$ ✓
  - $M_W \equiv M_W(m_h^{(0),\,\text{NMSSM}}) \rightarrow$ substitute with $m_h^{(2),\,\text{NMSSM}} \approx 125\,\text{GeV}$ *only in* $\Delta r|_{\text{SM}}$

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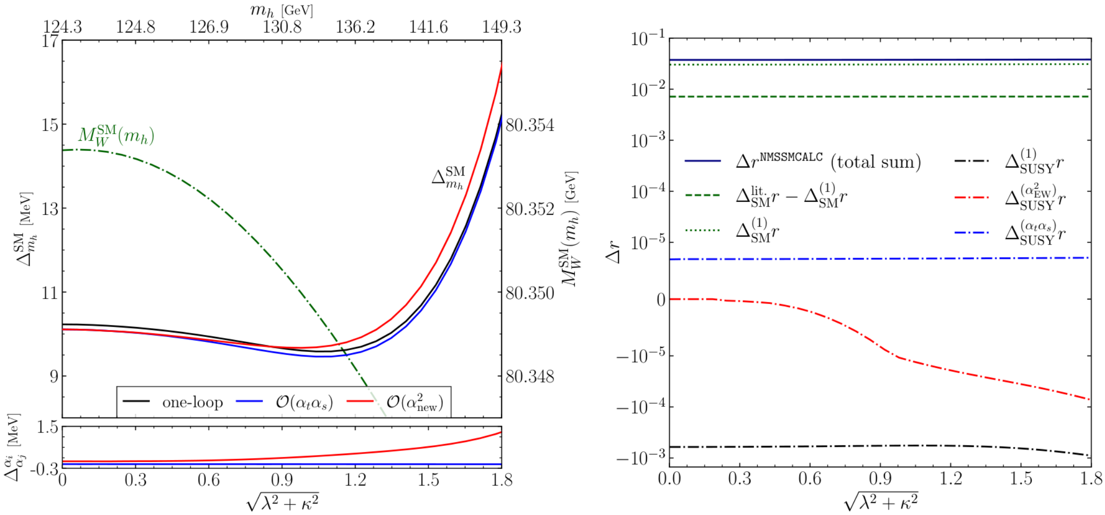
  - $M_W|_{\text{NMSSM}} \xrightarrow{m_{\text{SUSY}}\to\infty} \Delta^{(2)}M_W|_{\text{SM}}$ ✓
  - $M_W \equiv M_W(m_h^{(0),\,\text{NMSSM}}) \to$ substitute with $m_h^{(2),\,\text{NMSSM}} \approx 125\,\text{GeV}$ *only in* $\Delta r|_{\text{SM}}$

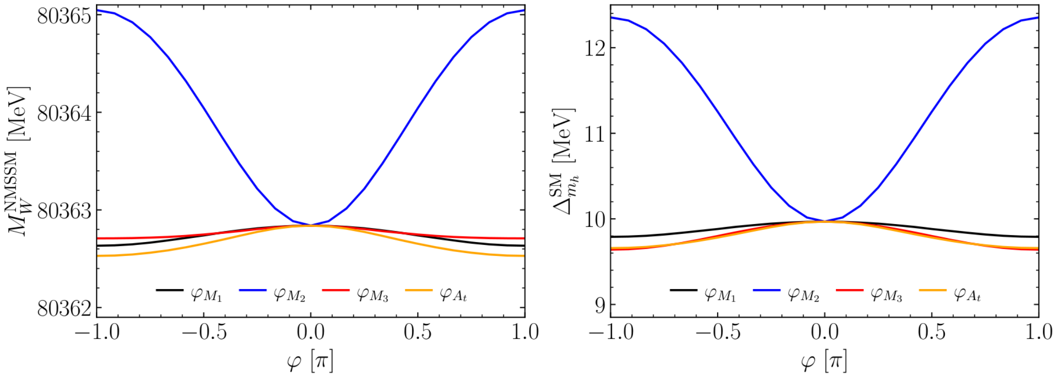
Caveat: cannot easily "switch" to $\overline{\text{DR}}$ scheme to estimate theory-uncertainty for M_W (todo...)

- **Nice bonus:** $\Delta r \leftrightarrow \{G_F, M_Z, M_W\}$-OS scheme $\leftarrow$ new in NMSSMCALC (see m_h-talk by Christoph)

Numerical results for M_W and Δr



CP-violating effects



- Phases in gluino- (φ_{M_3}) and stop-sector (φ_{A_t}) typically have a small effect if the particles have large ($\gtrsim 1\,\mathrm{TeV}$) masses
- electroweakinos ($\varphi_{M_{1,2}}$) can be light and can have sizeable effects

Comparison with other tools

Comparison with other tools: General considerations

- FeynHiggs: same approach as in NMSSMCALC:
  - $\Delta r = \Delta^{(4)} r|_{\text{SM}} + \Delta^{(2)} r|_{\text{SUSY}}$
  - found perfect agreement in decoupling limit!
  - full dependence of Δr on actual value of M_W

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  - $\Delta r = \Delta^{(4)} r|_{\text{SM}} + \Delta^{(2)} r|_{\text{SUSY}}$
  - found perfect agreement in decoupling limit!
  - full dependence of Δr on actual value of M_W
- generic spectrum generators: FlexibleSUSY and SARAH/SPheno
  - fit formula for M_W^{SM} (in $\overline{\text{MS}}$ scheme)

$$M_W^{\substack{\texttt{FlexibleSUSY}\\ \texttt{SARAH/SPheno}}} = \sqrt{M_W^{\text{SM fit.}\,2}(m_h, m_t, \alpha, \alpha_s)\left[1 + \frac{s_W^2}{c_W^2 - s_W^2}\Delta r^{(n)}_{\text{SUSY}}\right]},$$

  - $M_W^{\text{SM fit.}}$ does not depend on M_W!

Comparison with other tools: General considerations

- FeynHiggs: same approach as in NMSSMCALC:
  - $\Delta r = \Delta^{(4)} r|_{\text{SM}} + \Delta^{(2)} r|_{\text{SUSY}}$
  - found perfect agreement in decoupling limit!
  - full dependence of Δr on actual value of M_W
- generic spectrum generators: FlexibleSUSY and SARAH/SPheno
  - fit formula for M_W^{SM} (in $\overline{\text{MS}}$ scheme)

$$M_W^{\substack{\texttt{FlexibleSUSY}\\\texttt{SARAH/SPheno}}} = \sqrt{M_W^{\text{SM fit.}\,2}(m_h, m_t, \alpha, \alpha_s)\left[1 + \frac{s_W^2}{c_W^2 - s_W^2}\Delta r_{\text{SUSY}}^{(n)}\right]},$$

  - $M_W^{\text{SM fit.}}$ does not depend on M_W!
- NMSSMTools:
  - solves $G_F(M_W, \Delta r)$ for Δr using $M_W^{\text{SM fit.}}$

$$\Delta^{\texttt{NMSSMTools}} r(M_W) = \Delta_{\text{SUSY}}\, r(M_W) + \Delta_{\text{SM}}\, r(M_W^{\text{SM fit.}}) + \sum_{n=1}^{3} a_n \left(M_W - M_W^{\text{SM fit.}}\right)^n$$

$$\approx \Delta_{\text{SUSY}}\, r(M_W) + \Delta_{\text{SM}}^{\text{lit.}} r(M_W)$$

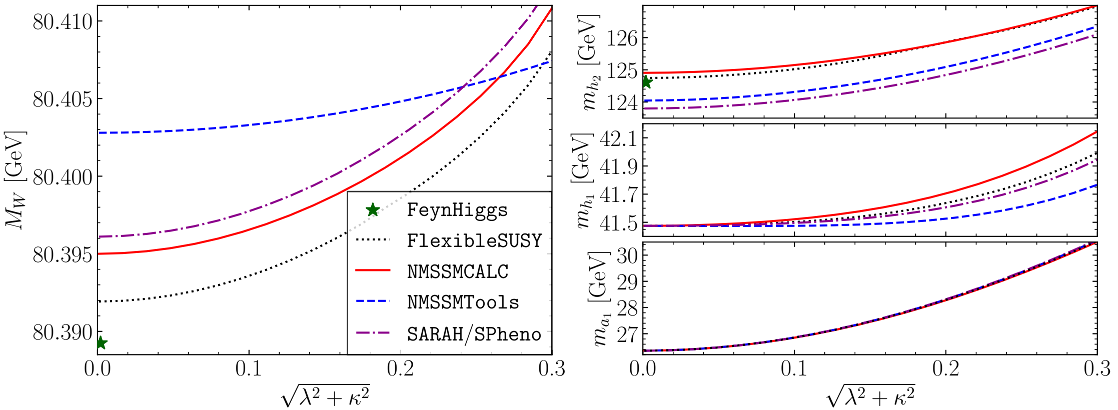
Comparison with other tools: Hacks/Details

SUSY/SM inputs are treated quite different:

- Hacked codes to compute M_W at M_{SUSY} rather than M_Z
- Turn-off loop-corrections to sfermions and electroweakinos
- SUSY sector: all inputs defined at M_{SUSY} at $\overline{\text{DR}}$
- SM sector: different schemes and corrections are not accounted for
- different workarounds for tachyonic tree-level masses (not accounted for)
- make sure $M_W(m_h) \equiv M_W(m_h^{\text{SM-like}})$ is used!

Comparison with other tools: Results

BP with very light sleptons $\mathcal{O}(100\,\mathrm{GeV})$ and light singlet:



→ agreement within SM uncertainty band

Implementation in NMSSMCALC [(see itp.kit.edu/~maggie/NMSSMCALC for references)]

NMSSMCALC is more than a *spectrum generator* for the CP-violating NMSSM:

- takes parameter point using SLHA and calculates:
- Higgs boson masses ($m_{H_i^0}$ and $m_{H^\pm}$) up to two-loop $\mathcal{O}(\alpha_s\alpha_t + (\alpha_t + \alpha_\lambda + \alpha_\kappa)^2)$
- Higgs boson self-couplings $\lambda^{\text{eff}}_{hhh}$ up to two-loop $\mathcal{O}(\alpha_s\alpha_t + \alpha_t^2)$
- $H_i^{0,\pm} \to X_j X_k$ decays at NLO including SUSY QCD+EW corrections (or using $\lambda^{\text{eff}}_{hhh}$)
- electric dipole moments (EDMs) of e and various bound-states
- $(g-2)_e$ and $(g-2)_\mu$
- **New:** m_h^{EFT}, M_W and $\Delta\rho$!
- also for inverse seesaw scenario (NMSSMCALC-nuSS)

example: compile and run NMSSMCALC

```
wget https://www.itp.kit.edu/\~maggie/NMSSMCALC/nmssmcalc.tar
tar xf nmssmcalc.tar
cd nmssmcalc-C
make
./run inp.dat
```

Summary

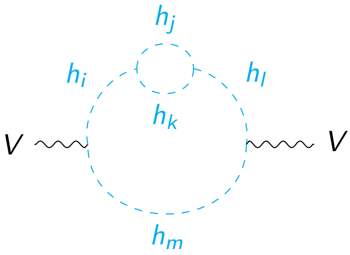
- W boson mass M_W and $\Delta\rho$ are important precision observables!
- studied two-loop corrections to $\Delta\rho$ in the NMSSM
  - uncertainties significantly reduced
- combined with full one-loop correction to Δr (muon decay)
  $\rightarrow$ precise M_W prediction

Outlook

- more detailed pheno studies
- influence of ren. scheme of charged Higgs mass (OS / $\overline{\text{DR}}$ scheme)
- uncertainty estimate for M_W
- two-loop corrections w/ sleptons (can have large one-loop shifts)

Backup

Comparison with MSSM/previous Results

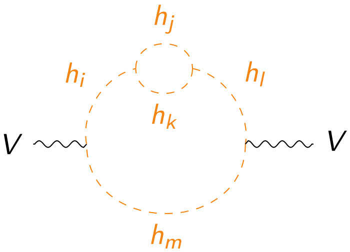


$$\xrightarrow{g_1, g_2 \to 0} 0$$

In the MSSM, Higgs-self couplings are given by gauge couplings:

$$V_{\text{MSSM}}^{\text{quartic}} \propto \boldsymbol{g_1}^2(|H_u|^2 - |H_d|^2)^2 + \boldsymbol{g_2}^2(H_u\sigma_a H_u + H_d\sigma_a H_d)^2 \xrightarrow{g_1, g_2 \to 0} 0$$

Comparison with MSSM/previous Results



$$\xrightarrow{g_1, g_2 \to 0} \neq 0$$

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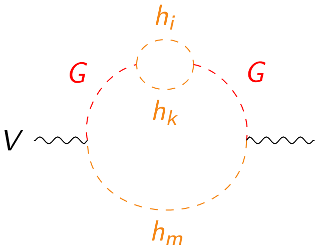
In the NMSSM, there are additional non-zero self-couplings:

$$V^{\text{quartic}}_{\text{NMSSM}} \propto V^{\text{quartic}}_{\text{MSSM}} + |\boldsymbol{\lambda} H_u H_d + \boldsymbol{\kappa} S^2|^2 \xrightarrow{g_1, g_2 \to 0} \neq 0$$

→ Many new **two-loop diagrams with Higgs self-couplings**.

Comparison with MSSM/previous Results



$$\propto \log m_G^2 \xrightarrow{m_G^2 \to 0} -\infty$$

In the MSSM, Higgs-self couplings are given by gauge couplings:

$$V^{\text{quartic}}_{\text{MSSM}} \propto \boldsymbol{g_1}^2(|H_u|^2 - |H_d|^2)^2 + \boldsymbol{g_2}^2(H_u\sigma_a H_u + H_d\sigma_a H_d)^2 \xrightarrow{\boldsymbol{g_1},\boldsymbol{g_2}\to 0} 0$$

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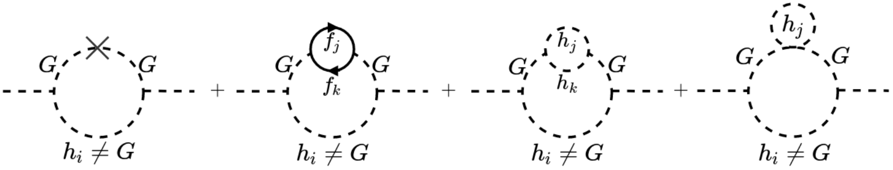
→ Many new **two-loop diagrams with Higgs self-couplings**.
Massless Goldstones → appearance of intermediate IR divergences (final result IR-finite).

IR-finite two-loop self-energies

Example of an IR-finite subset with intermediate IR-divergences:



Careful isolation of divergences using mass regulator or dimensional regularisation shows:

- IR-divergence of first diagram cancels against the other three
- cancellation happens only if $M^{\text{1-loop}}_{\text{Goldstone}} \equiv 0$
- $\rightarrow$ working at the *tree-level* minimum is sufficient [this work] or alternatively using an OS-condition for the Goldstone mass [Braathen, Goodsell, '16]

Parameter point P1 for the shown plots

$$\texttt{P1}: \quad m_{\tilde{t}_R} = 2002 \text{ GeV}, \; m_{\tilde{Q}_3} = 2803 \text{ GeV}, \; m_{\tilde{b}_R} = 2765 \text{ GeV},$$
$$m_{\tilde{L}_{1,2}} = 565 \text{ GeV}, \; m_{\tilde{e}_R,\tilde{\mu}_R} = 374 \text{ GeV},$$
$$m_{\tilde{L}_3} = 575 \text{ GeV}, \; m_{\tilde{\tau}_R} = 981 \text{ GeV},$$
$$|A_{u,c,t}| = 2532 \text{ GeV}, \; |A_{d,s,b}| = 1885 \text{ GeV}, \; |A_{e,\mu,\tau}| = 1170 \text{ GeV},$$
$$|M_1| = 133 \text{ GeV}, \; |M_2| = 166 \text{ GeV}, \; |M_3| = 2300 \text{ GeV},$$
$$\lambda = 0.301 \text{ GeV}, \; \kappa = 0.299 \text{ GeV}, \; \tan\beta = 4.42 \text{ GeV},$$
$$\mu_{\text{eff}} = 254 \text{ GeV}, \; \text{Re}A_\kappa = -791 \text{ GeV}, \; M_{H^\pm} = 1090 \text{ GeV},$$
$$\varphi_{A_{e,\mu,\tau}} = 0, \; \varphi_{A_{d,s,b}} = \pi, \; \varphi_{A_{u,c,t}} = \varphi_{M_1} = \varphi_{M_2} = \varphi_{M_3} = 0 \,.$$

H_1	H_2	H_3	H_4	H_5
125.4	230.6	770.5	1088.0	1090.1
h_u	h_s	a_s	a	h_d

$\tilde{\chi}_1^0$	$\tilde{\chi}_2^0$	$\tilde{\chi}_3^0$	$\tilde{\chi}_4^0$	$\tilde{\chi}_5^0$	$\tilde{\chi}_1^+$	$\tilde{\chi}_2^+$
113.9	145.07	261.95	295.74	509.49	132.93	294.98

Parameter point BP3 for the tool-comparison

$$
\begin{aligned}
\texttt{BP3}: \quad & m_{\tilde{t}_R} = 2144\ \text{GeV}\,,\ m_{\tilde{Q}_3} = 1112\ \text{GeV}\,,\ m_{\tilde{b}_R} = 1539\ \text{GeV}\,,\\
& m_{\tilde{L}_{1,2}} = 131.9\ \text{GeV}\,,\ m_{\tilde{e}_R,\tilde{\mu}_R} = 103.6\ \text{GeV}\,,\\
& m_{\tilde{L}_3} = 205.2\ \text{GeV}\,,\ m_{\tilde{\tau}_R} = 238.6\ \text{GeV}\,,\ |A_{u,c,t}| = 3971.2\ \text{GeV}\,,\\
& |A_{d,s,b}| = 1210.3\ \text{GeV}\,,\ |A_{e,\mu}| = 3643\ \text{GeV}\,,\ |A_\tau| = 2052.4\ \text{GeV}\,,\\
& |M_1| = 178.3\ \text{GeV},\ |M_2| = 128.6\ \text{GeV}\,,\ |M_3| = 1757.6\ \text{GeV}\,,\\
& \lambda = 0.1229\ \text{GeV},\ \kappa = 0.0128\ \text{GeV}\,,\ \tan\beta = 8.7199\ \text{GeV}\,,\\
& \mu_{\text{eff}} = 212\ \text{GeV},\ \text{Re}A_\kappa = -10.48.\ \text{GeV}\,,\ \text{Re}A_\lambda = 2245\ \text{GeV}\,,\\
& \varphi_{A_{e,\mu,\tau}} = 0\,,\ \varphi_{A_{u,c,t}} = \pi\,,\ \varphi_{A_{d,s,b}} = \varphi_{M_1} = \varphi_{M_2} = \varphi_{M_3} = 0\,.
\end{aligned}
$$

Comparison with other tools: results for m_h, M_W and a_μ

		FlexibleSUSY	NMSSMCALC	NMSSMTools	SARAH
P1	m_h [GeV]	119.77	119.19	118.61	118.95
	M_W [MeV]	80366.3	80365.7	80370.8±23	80366.2
	$a_\mu \times 10^9$	0.29±0.01	0.256	0.329±0.03	0.33
BP3	m_h [GeV]	125.60	125.63	124.63	123.97
	M_W [MeV]	80396.9	80400.0	80404.2±22	80401.3
	$a_\mu \times 10^9$	2.98±0.45	2.89	3.19±0.34	3.70

Fixed and Running width

- theory: chose δM_V such that M_V corresponds to real part of complex pole of "Breit-Wigner" propagator

$$\frac{1}{p^2 - M_V^{\text{fw},\,2} + i M_V^{\text{fw}} \Gamma_V}$$

- experiment: chooses different parametrisation:

$$\frac{1}{p^2 - M_V^{\text{run},\,2} + i p^2 \frac{\Gamma_V}{M_V^{\text{run}}}}$$

- use $M_Z^{\text{fw}} = M_Z^{\text{run}} - \Gamma_Z^2/(2M_Z^{\text{run}})$ before calculation using experimental input
- for M_W: use theory input $\Gamma_W^{\text{SM}}|_{\text{theo}} = 3G_\mu M_W^3/(2\sqrt{2\pi})(1+2\alpha_s/(3\pi))$
- this introduces a model-dependence!
- SM: shift in M_W by $\mathcal{O}(27\,\text{MeV})$

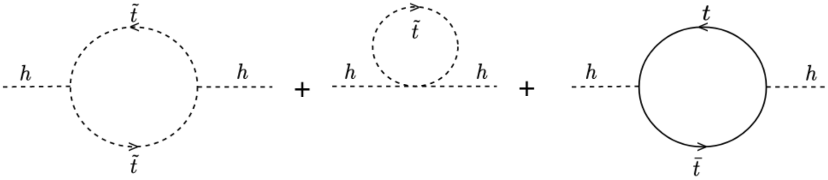
The SM-like neutral Higgs boson mass in the NMSSM

$$\left(m_h^{\text{tree}}\right)^2 \approx \underbrace{m_Z^2 \cos^2 2\beta}_{\text{MSSM}} + \underbrace{\lambda v^2 \sin^2 2\beta}_{\text{NMSSM}}$$

$\rightarrow$ SUSY connects scalar- with gauge- and Yukawa-sector!

> MSSM: $m_h^{\text{tree}} \leq m_Z < 125\,\text{GeV}$
> NMSSM: $\lambda < 0.7$ (assuming perturbative unitarity below m_{GUT})

$\rightarrow$ In either case: Higher-order corrections must shift m_h to the measured Higgs mass. At one-loop, the leading contributions to $\delta^{(1)} m_h^2$ from the top/stop sector are:



$$\approx \frac{1}{(4\pi)^2} \frac{m_t^4}{v^2} \log \frac{m_t^2}{M_{\tilde{t}}^2}$$

> $M_{\tilde{t}} = m_t + m_{\text{SUSY}} \Rightarrow$ in the SUSY-restoring limit: $\delta^{(1)} m_h^2 \xrightarrow{m_{\text{SUSY}} \to 0} 0$
> but we need $\delta m_h^2 \approx \mathcal{O}(20 - 40\,\text{GeV})$! $\rightarrow$ higher-orders required