Grover search Algorithm

task: find an item in an unsorted database

Classical: requires on average $O(N)$ queries to the database

Quantum requires only $O(\sqrt{N})$ queries

In Q.C. finding an item in the d.b. means measuring the system
→ collapse with close certainty to the basis state which corresponds to required item

The algorithm

1) invert the basis of the desired basis state

2) invert all the other basis states about the average amplitude of all the states

→ this increases the amplitude of the desired basis state to unity
→ then decrease of the amplitude of the desired state to its original value

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cyclic operation with a period of $\frac{\pi}{4}\sqrt{N}$

average: $\frac{1}{N}\sum_{i=1}^{N}\alpha_i$; α_i is the ampl. of the i^{th} state

** the longer the DB, the longer the probability of finding the desired state

recall:

$$H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$\hat{I}_\phi$ = identity matrix except $i_{\phi\phi} = -1$

↳ this operator inverts the phase of the basis state $|\phi\rangle$

$N = 2^n$ (n = nb of qbits)

→ start with $|0\rangle^{\otimes n}$

→ Apply $\hat{H}$ → all states with probability $\frac{1}{\sqrt{N}}$

→ Apply $\hat{I}_z$ with $|z\rangle$ is the desired state

→ Apply the Grover diffusion operator $\hat{D}$

$$\hat{D} = \hat{H}\,\hat{I}_0\,\hat{H}$$

→ Apply $\hat{I}_z$ operator $\frac{\pi}{4}\sqrt{N}$ times

→ observe the system

** the $\hat{D}$ operator inverts all the states amplitudes around the average amplitude of all the states

for more info:

"A fast Quantum mechanical algorithm for databse search", proceedings of STOC 1996, Lov. K. Grover

example: $N = 16$, $n = 4$ qubits

$$|\tau\rangle = |0110\rangle$$

starte with $|\psi\rangle = |0 \cdots 0\rangle$

$\xrightarrow{\hat{H}} |\psi\rangle = \frac{1}{4} |1, \cdots 1\rangle$

$\frac{\pi}{4}\sqrt{N} \approx$ (rounded to 3)

1st loop

$|Z\rangle = |0110\rangle$

$\xrightarrow{\hat{I}_Z} \frac{1}{4}\left(1\,1\,1\,1\,1\,1\,1\,-1\,1\,1\,1\,1\,1\,1\,1\,1\right)$

inverts the phase

rotate all the basis states about the average

7/32

$\xrightarrow{\hat{D}} |\psi\rangle = \frac{1}{16}\left(3\,3\,3\,3\,3\,3\,11\,3\,3\,3\,3\,3\,3\,3\,3\,3\right)$

2nd loop

$\xrightarrow{\hat{I}_Z} |\psi\rangle = \frac{1}{16}\left(3\ 3\ 3\ 3\ 3\ 3\ -11\ 3\ 3\ 3\ 3\ 3\ 3\ 3\ 3\ 3\right)$

rotate about the average $(17/128)$

$\xrightarrow{\hat{D}} |\psi\rangle = \frac{1}{64}\left(5\ 5\ 5\ 5\ 5\ 5\ 61\ 5\ 5\ 5\ 5\ 5\ 5\ 5\ 5\ 5\right)$

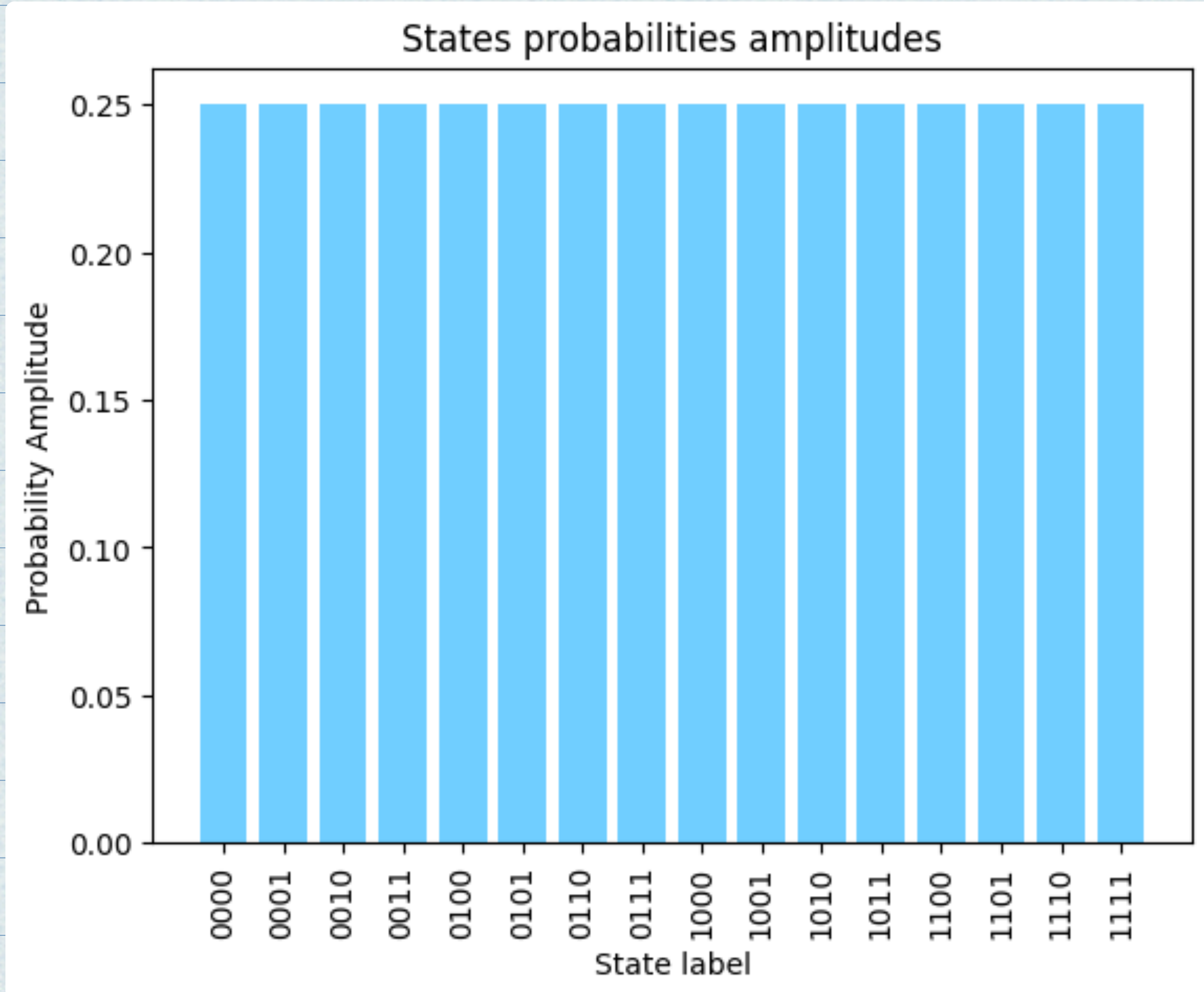
3rd loop

$\xrightarrow{\hat{I}_Z} |\psi\rangle = \frac{1}{64}\left(5\ 5\ 5\ 5\ 5\ 5\ -61\ 5\ 5\ 5\ 5\ 5\ 5\ 5\ 5\ 5\right)$

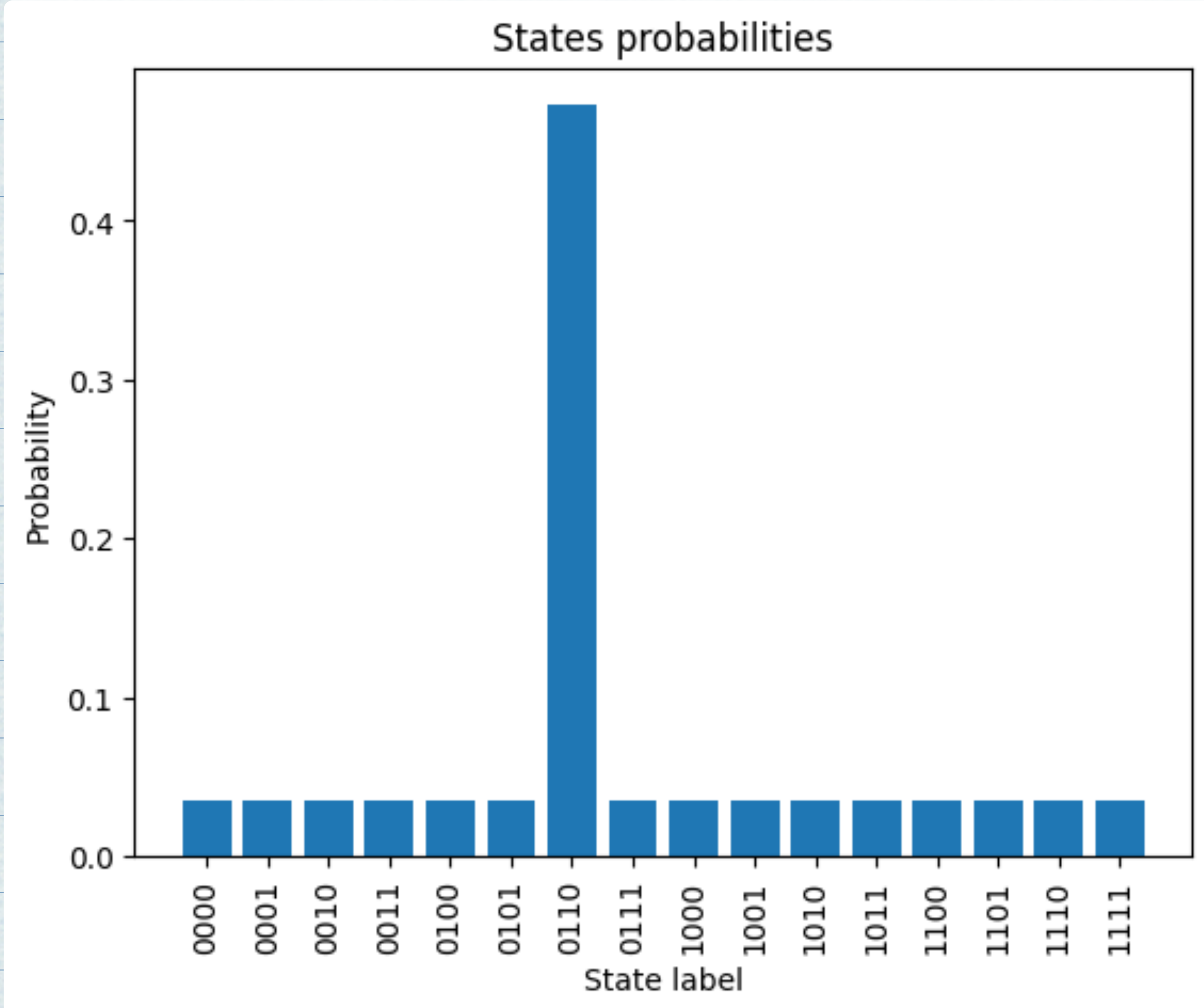
$\xrightarrow{\hat{D}} |\psi\rangle = \frac{1}{256}\left(-13\ \ldots\ 251\ -13\ \ldots\ -13\right)$

squaring the coefficients → probability of collapsing into the corresponding state

After applying the $\hat{H}$ operator: equal prob.

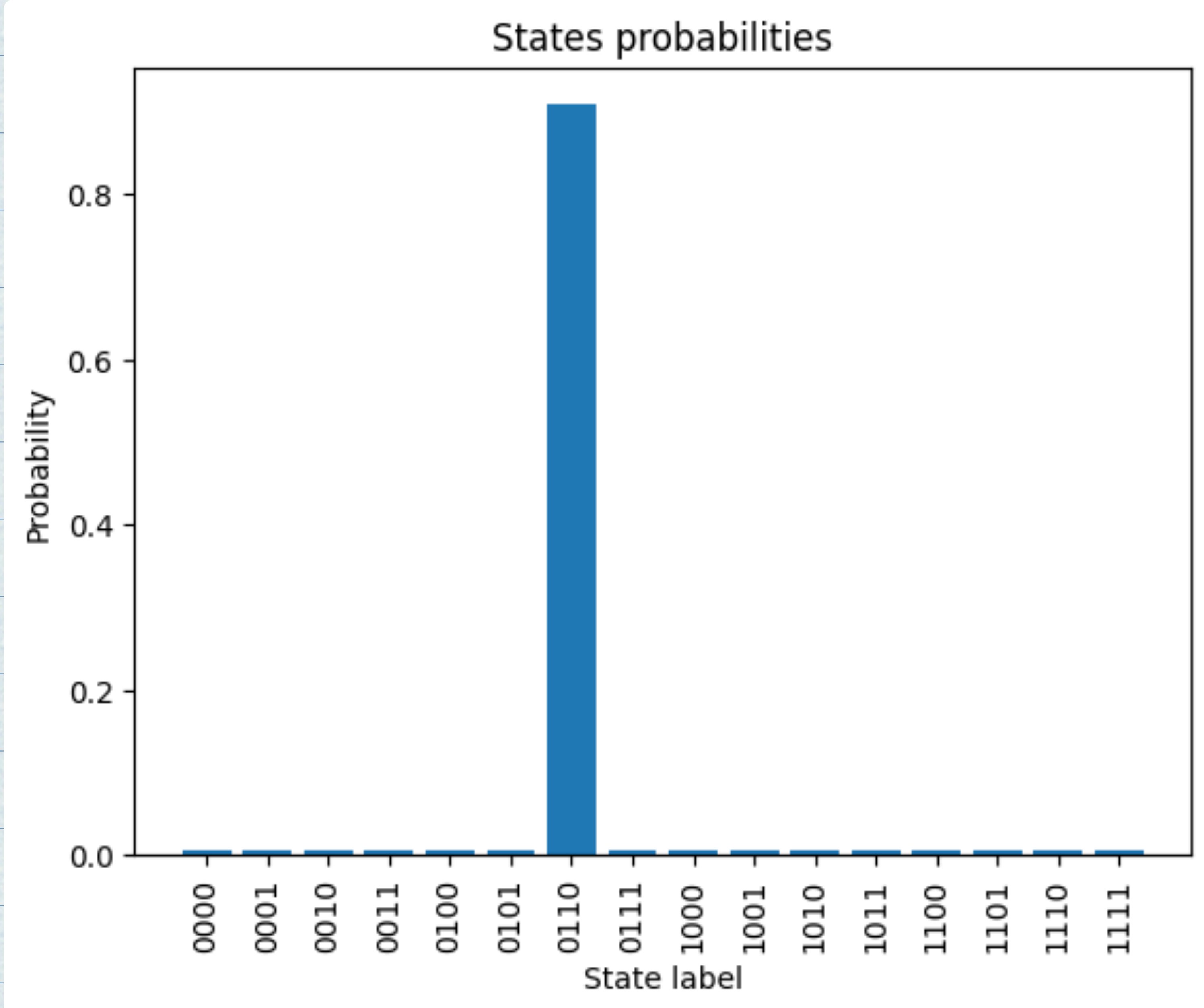


1) iteration

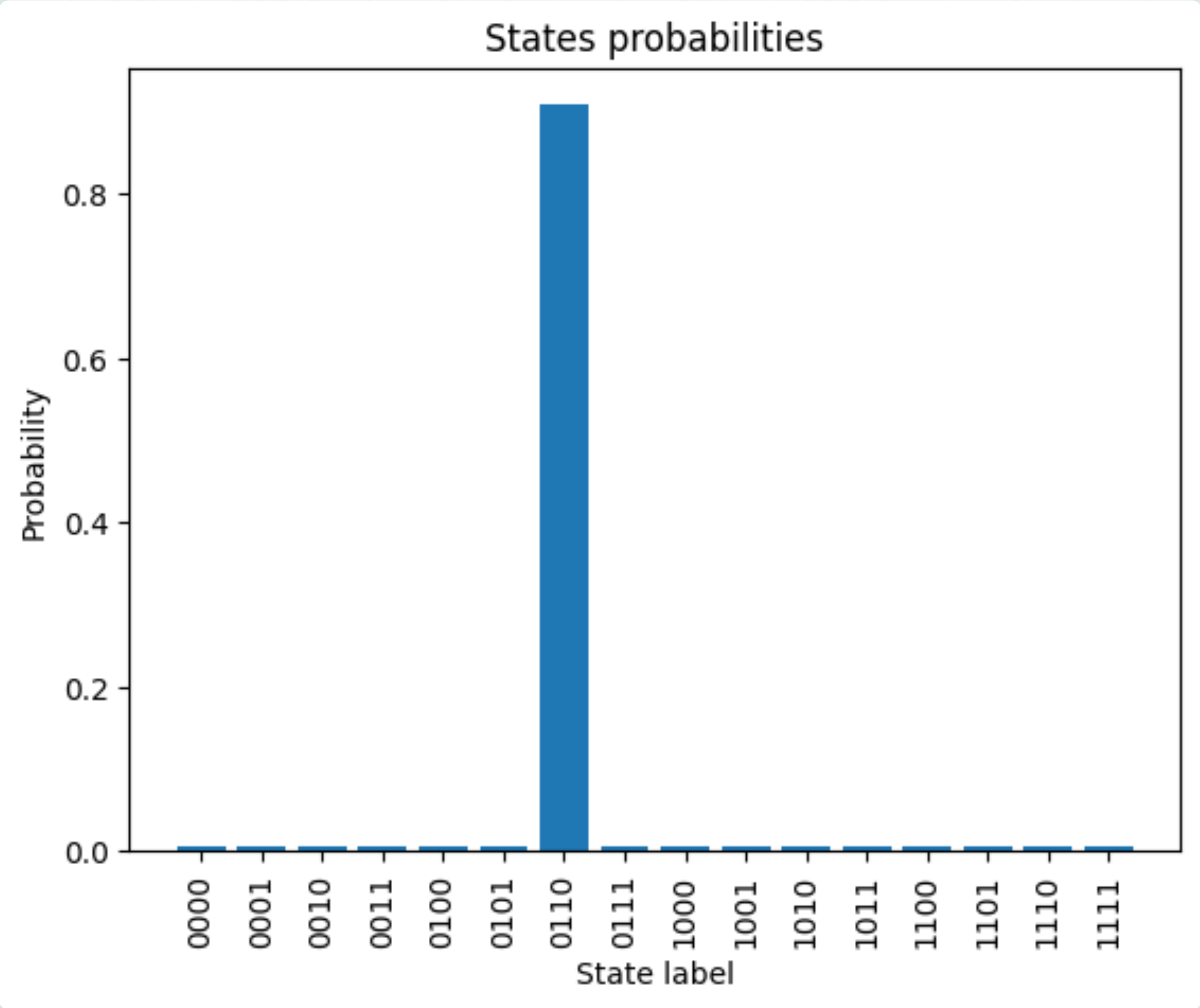


States probabilities
Probability
0.0
0.1
0.2
0.3
0.4
0000
0001
0010
0011
0100
0101
0110
0111
1000
1001
1010
1011
1100
1101
1110
1111
State label

2 iterations



3 iterations



9 iterations