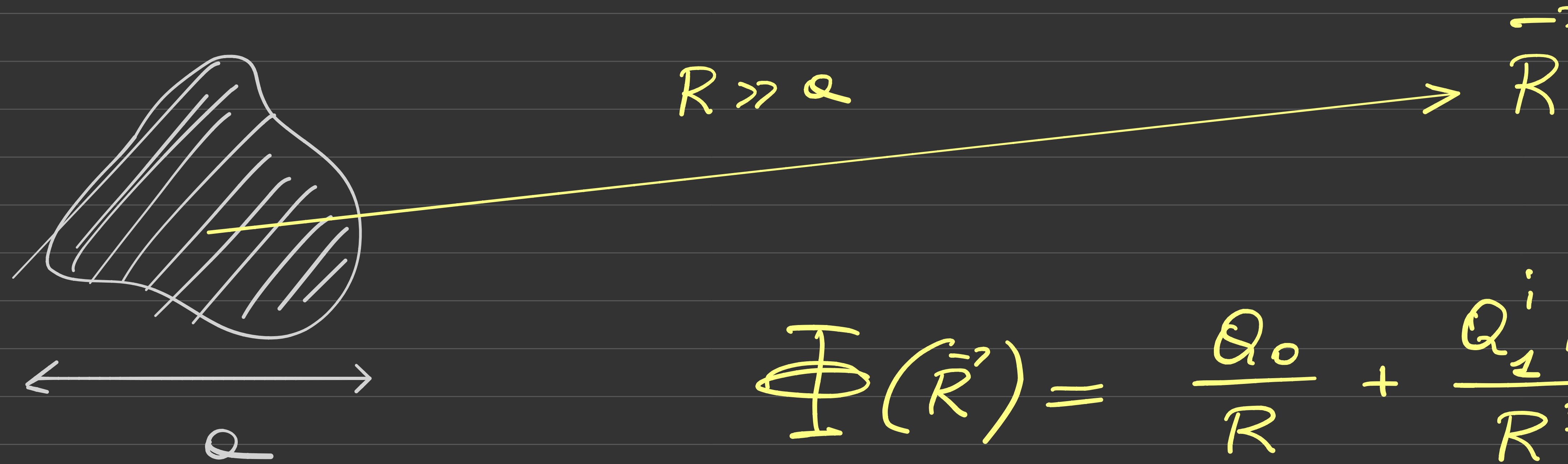
Why can we do Physics?

finite # of inputs $\Longrightarrow$ ∞ many predictions

Existence of separations of length scales

Ex Multipole expansion in Electrostatics

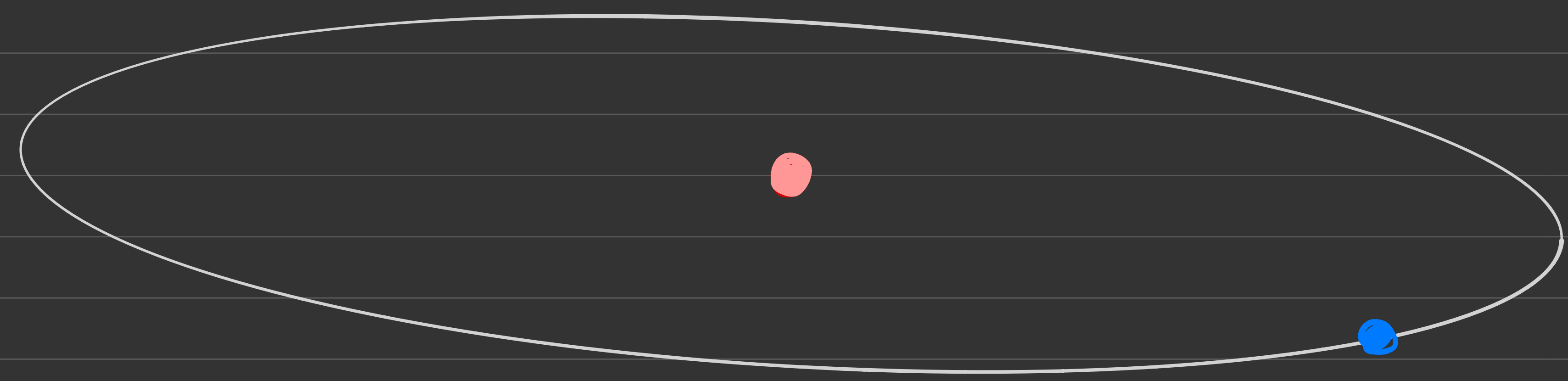


$$\Phi(\vec{R}) = \frac{Q_0}{R} + \frac{Q_1^i R^i}{R^3} + \frac{Q_2^{ij} R^i R^j}{R^5} + \cdots$$

relative size $\quad 1 + \frac{a}{R} + \frac{a^2}{R^2} + \cdots$

at fixed accuracy
- $R \to$ large : few multipoles needed — Universality
- $R \to$ small : more multipoles needed — Reductionism

Ex Celestial Mechanics See arXiv:1510.08889



$$V_{eff} = \frac{G_N m_1 m_2}{r}\left(1 + \frac{\vec{Q}_1 \cdot \vec{Q}_2}{r^2} + \# \frac{S_1 \cdot S_2}{r^2} \cdots\right)$$

Ex Classical fluids (dynamic)

$$\text{individual}\left\{\begin{array}{l}\Delta x \ll L_{coll} \ll \Delta x \\ \Delta t \ll \tau_{coll} \ll \Delta t\end{array}\right\}\ \begin{array}{l}\text{collective} \\ \text{(universal)}\end{array}$$

$$\underbrace{\rho\left(\partial_t \vec{v} + (\vec{v}\cdot\vec{\nabla})\vec{v}\right) = -\nabla p}_{\text{0th order = perfect fluid}} + \underbrace{\eta\nabla^2\vec{v} + \xi'\vec{\nabla}(\vec{\nabla}\cdot\vec{v})}_{\text{1th order = viscosity}} + \dots$$

$\rho = \rho_0 \bar{\rho}$

$p = \rho_0 v_s^2 \bar{p}$

$\eta = \rho_0 v_s^2 \tau_{coll} \bar{\eta}$

$$\rho\omega v \sim |\rho\,\partial_t v| \gg \eta\nabla^2 v \sim \eta k^2 v \sim \rho (v_s k)^2 \tau_{coll} v$$

$$\omega \gg \tau_{coll}\,\omega^2 \Longrightarrow \omega \ll \frac{1}{\tau_{coll}}$$

Effective descriptions in Q. M.

Path integral $\int Dq \, e^{iS[q]}$

- interested in long wavelength – low frequency q
- $\Rightarrow$ separate $q = (q_{slow}, q_{fast}) \longrightarrow \omega_* = \frac{1}{\tau_*}$

$$\langle q_{slow}(t_1) \cdots q_{slow}(t_N) \rangle = \int Dq_{slow} \underbrace{Dq_{fast} \, e^{iS[q_s, q_f]}}$$

$$= \int Dq_{slow} \, e^{iS_{eff}(q_s, \omega_*)}$$

UV cut-off

⚠ **Effective Field Theory**: imagine a scalar field theory emerging from more fundamental physics at a scale $\Lambda = \frac{1}{\tau} = \frac{1}{L}$

To be able to do physics I need scale separation: $m \ll \Lambda$. Besides this request

$\text{———}\ \Lambda$

which we may go back and debate later, I assume all other parameters controlled by $\Lambda \Longrightarrow$

$\text{———}\ m$

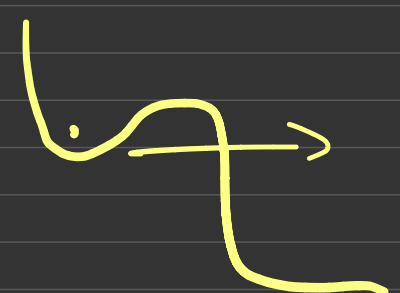
$$\mathcal{L} = \frac{1}{2}(\partial\varphi)^2 - \frac{m^2}{2}\varphi^2 + \tilde{g}_3\, m\,\varphi^3 + g_4\varphi^4 + \frac{g_5}{\Lambda}\varphi^5 + \frac{g_6}{\Lambda^2}\varphi^6 + \dots$$

$$+ \frac{\eta_3}{\Lambda}\varphi(\partial\varphi)^2 + \frac{\eta_4}{\Lambda^2}\varphi^2(\partial\varphi)^2 + \dots$$

- All $g_i, \eta_i \lesssim O(1)$ to ensure even mild perturbativity at scale Λ where we define our theory

(more precisely $\lesssim O(4\pi^{\#})$, but sophistication not needed here)

- Naively cubic $\sim g_3 \Lambda$ with $g_3 \lesssim O(1)$ but

$\varphi^3 \to$  : vacuum stability $\Longrightarrow g_3 \Lambda \lesssim m$

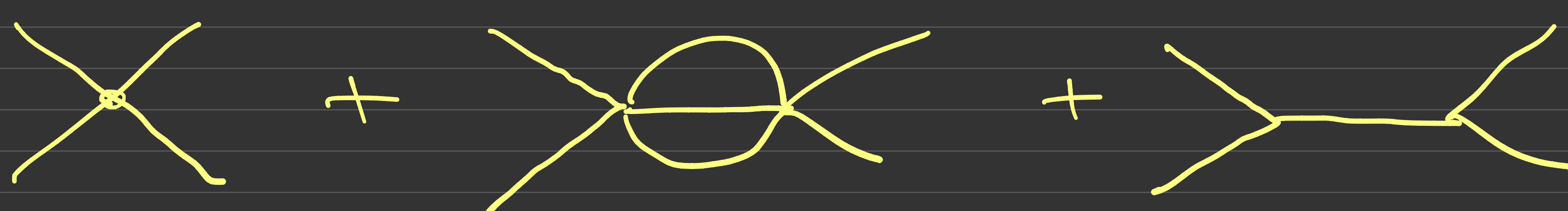
$$\Longrightarrow \quad g_3 \Lambda \longrightarrow \tilde{g}_3\, m \qquad \tilde{g}_3 \lesssim O(1)$$

$$(\partial\varphi)^2 - m^2\varphi^2 - g_3 m\varphi^3 - g_4\varphi^4 + \frac{g_5}{\Lambda}\varphi^5 + \frac{g_6}{\Lambda^2}\varphi^6 + \dots$$

$$\Delta_{\text{obs}} \sim \frac{m^2}{E^2}, \frac{g_3 m}{E} \qquad\qquad g_4 E^0 \qquad\qquad \Delta_{\text{obs}} \sim g_5 \frac{E}{\Lambda}, g_6 \frac{E^2}{\Lambda^2} \dots$$

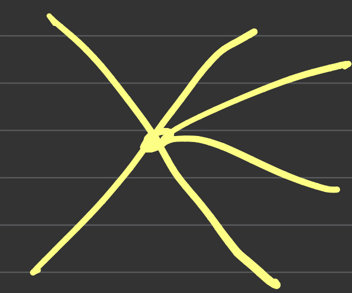
<u>relevent</u> in IR — matters equally in IR and UV $\equiv$ <u>marginal</u> — <u>irrelevent</u> in IR

- Ex: can check that in $2 \to 2$ scattering



$$g_4 \qquad + \qquad g_5^2 \frac{E^2}{\Lambda^2} \qquad + \qquad \frac{g_3^2 m^2}{E^2}$$

$2 \to 4$

 $+$  $+ \dots$

$$\frac{g_4^2}{E^2} \qquad g_6/\Lambda^2 \qquad = \frac{1}{E^2}\left(g_4^2 + \frac{g_6 E^2}{\Lambda^2} + \dots\right)$$

- $E/\Lambda \sim \frac{L_{micro}}{\lambda}$ expansion similar to $\frac{a}{R}$ in electrostatics

but in addition there are now effects that grow in IR and become important at the longest scales (relevant) and effects that are essentially the same at all scales (marginal)

Overall picture

UV $\xrightarrow{\quad\Lambda\quad}$ $\longrightarrow$ $[\mathcal{O}] > 4$

$g_5, g_6 \ldots$

$\sim$ Scale Invariance $\Longleftarrow [g_4] = 0 \Leftarrow [\mathcal{O}] \simeq 4$

IR $\xrightarrow{\quad m\quad}$ $\longrightarrow$ $[\mathcal{O}] < 4$

m^2, g_3

⚠ Separation of scales $\Longleftrightarrow$ range where QFT is approximately Scale Inv.

⚠ Almost tautological but deep consequences :

Poincaré + Scale $\longrightarrow$ Conformal Invariance $\Rightarrow$ very constraining

$ISO(3,1) + U(1)_D \longrightarrow SO(4,2)$

△ Technically in our case for $\Lambda \gg E \gg m$ QFT described by

$$\frac{1}{2}(\partial\phi)^2 - g_4 \phi^4 + (\text{relevant})$$

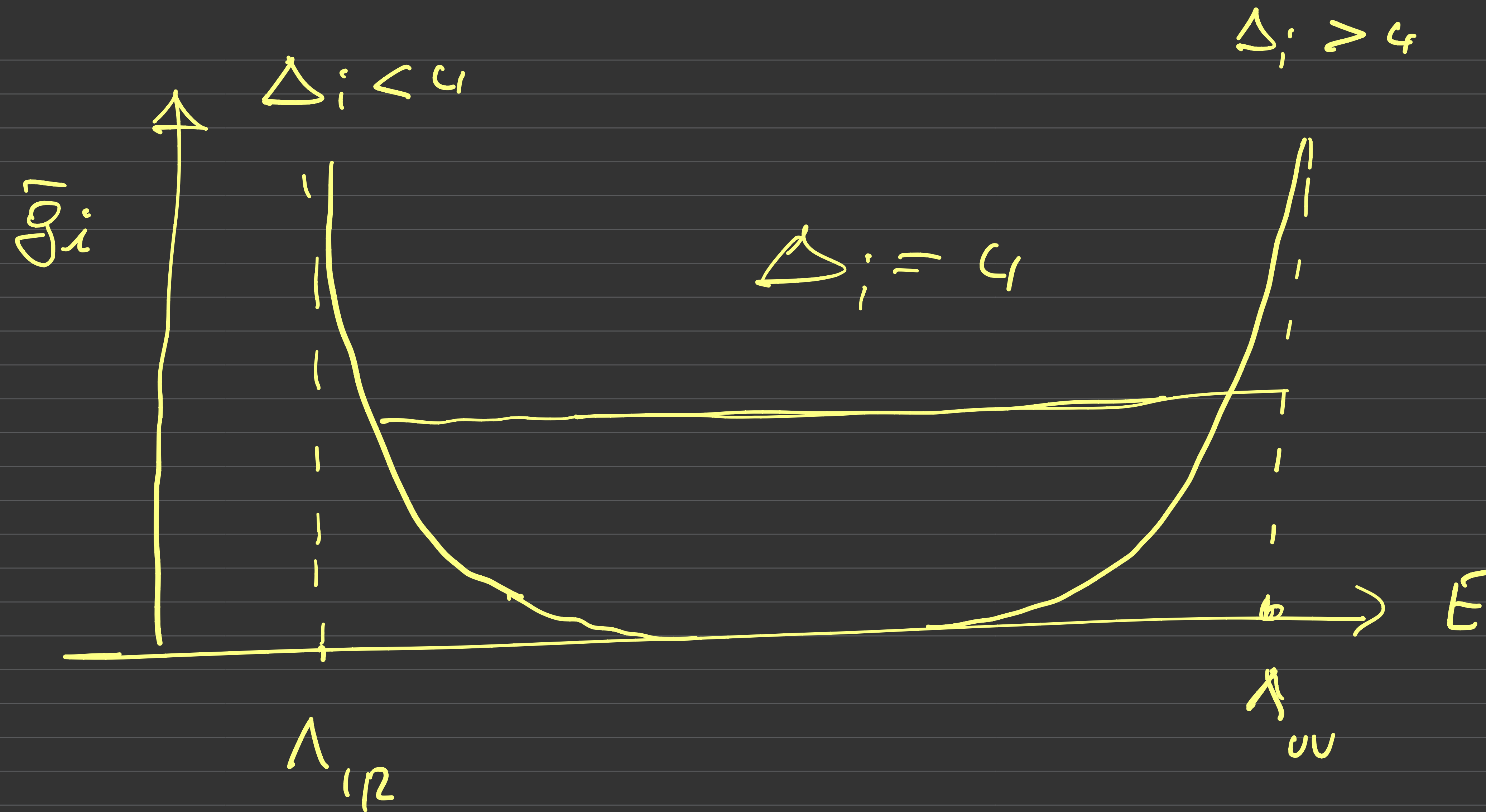
$\frac{1}{2}(\partial\phi)^2$ $\to$ free CFT

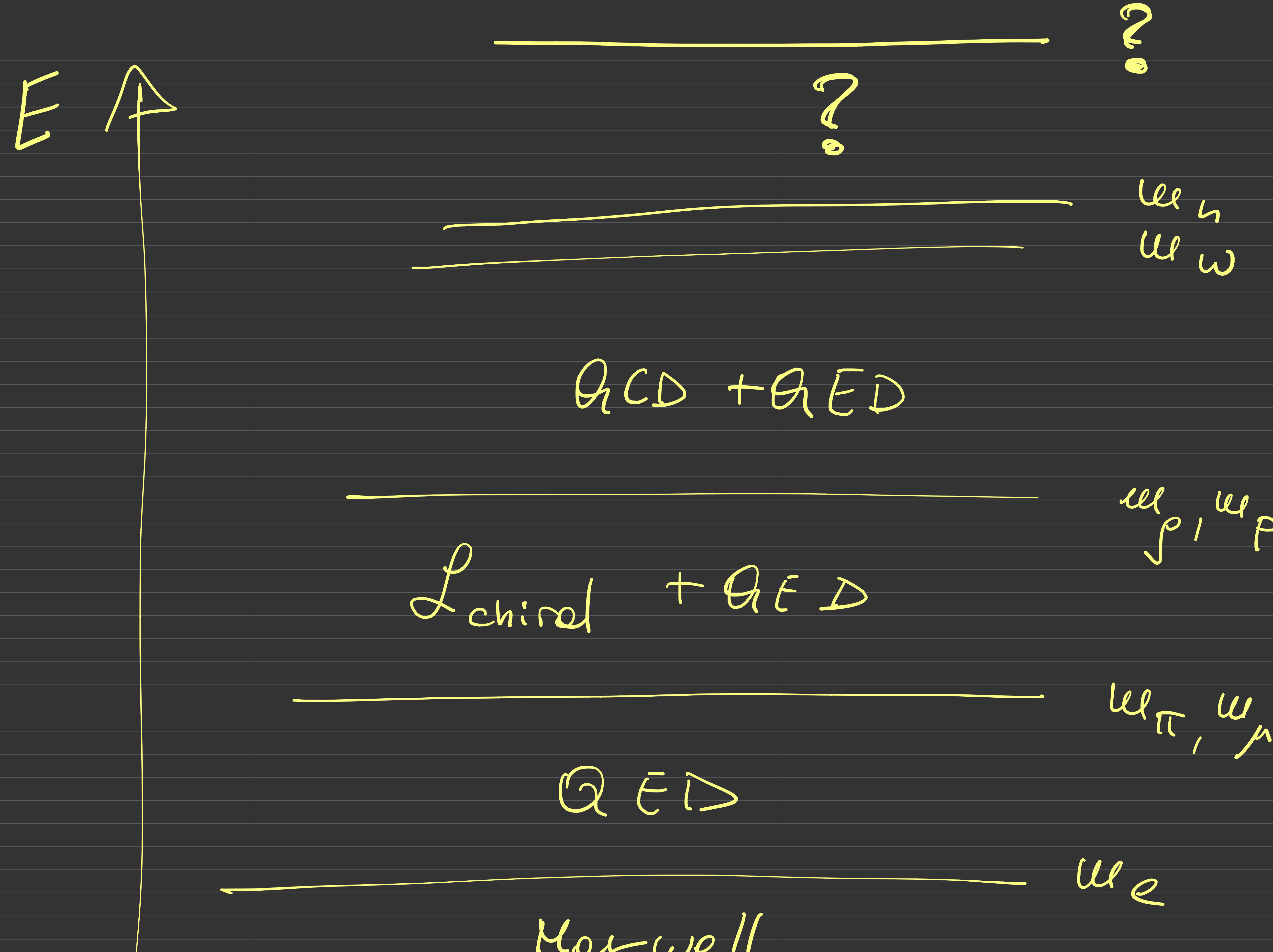
$g_4 \phi^4$ $\to$ marginal deformation

- In general can imagine situation where CFT is (strongly) interacting
- Examples abound in Cond Matt stat. phys.
- Composite Higgs models are based on such (hypothetical) situation

Sketch

$$\bar{g}_i = g_i E_i^{\Delta_i - 4}$$





E
?
?
m_n
m_ω
QCD + QED
m_ρ, m_p
$\mathcal{L}_{chiral}$ + QED
m_π, m_μ
QED
m_e
Maxwell

Optional : scale hierarchies in general CFT

$$\Lambda \qquad\qquad\qquad\qquad \to 0$$

$$\mathcal{L} = \mathcal{L}_{CFT} + \mathcal{L}_{rel} + \mathcal{L}_{irr}$$

$\mathcal{L}_{rel}$ $\to$ finite number of terms

- lowest dimensional scalar $[\mathcal{O}] = \Delta \xRightarrow{\text{expect}} \mathcal{L}_{rel} = c\Lambda^{4-\Delta}\,\mathcal{O}$

- $\Lambda_{IR} \equiv$ scale where $\mathcal{L}_{rel} \sim \mathcal{O}(1)$ perturbation $\Longrightarrow$

$$\frac{c\,\Lambda^{4-\Delta}}{E^{4-\Delta}} \sim 1 \quad \Longrightarrow \quad \Lambda_{IR} \sim c^{\frac{1}{4-\Delta}}\,\Lambda$$

if $|4-\Delta| \ll 1$ (Ex 0.1), $c \sim 0.1 \Longrightarrow \Lambda_{IR} \sim 10^{-10}\Lambda$

- for scalar mass $\Delta = 2$ $\quad c = \left(\Lambda_{IR}/\Lambda\right)^2 \xrightarrow{10^{-10}} c = 10^{-20}$

Symmetries

IR simplification of effective description

$\Longrightarrow$ accidental symmetries

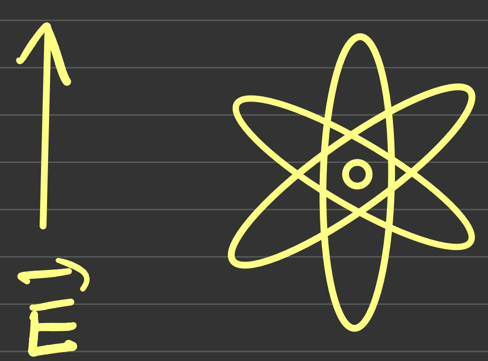
Ex electrostatics $\Phi(\vec{R}) = \underbrace{\frac{Q_0}{R}}_{SO(3)} + \underbrace{\frac{Q_1^{i} R^i}{R^3}}_{U(1)} + \underbrace{\frac{Q_2^{ij} R^i R^j}{R^5}}_{\text{nothing}} + \dots$

Ex QED : $\mathcal{L} = \underbrace{-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\psi}\left(i\not{D} - m\right)\psi}_{\text{Parity}} + \underbrace{\frac{1}{\Lambda^2} \bar{\psi}\gamma_\mu\psi\, \bar{\psi}\gamma^\mu\gamma^5\psi}_{\cancel{\text{Parity}}} + \dots$

⚠ Symmetries are important even when broken ⟹ selection rules & dimensional analysis

Ex Atom in electric field $\vec{E}$ $\Rightarrow$ $G = SO(3)$

$$H = H_0 - \vec{d}\cdot\vec{E}$$



$$* \quad U(R)\, H(\vec{E})\, U(R)^{\dagger} = H(R\cdot E) \quad \Leftrightarrow \quad \text{" E transforms like a vector "}$$

$$\lambda_\alpha |\alpha, \vec{E}\rangle = H(E)|\alpha, \vec{E}\rangle$$

$$\Rightarrow \lambda_\alpha U(R)|\alpha, E\rangle = H(RE)\, U(R)|\alpha, E\rangle$$

$$|\alpha, RE\rangle \equiv U(R)|\alpha, E\rangle$$

ground state $|0, E\rangle$: $\quad \langle 0, \vec{E}| d_i |0, \vec{E}\rangle = f_i(E) = \,?$

$$\langle 0, RE| d_i |0, RE\rangle = \langle 0, E| U^{\dagger}(R)\, d_i\, U(R) |0, E\rangle = R_{ij} \langle 0, E| d_j |0, E\rangle$$

$$\Longrightarrow f_i(RE) = R_{ij} f_j(E) \Longrightarrow f_i = E_i f(E^2)$$

Seems obvious... Point is that in 3D $\exists$ single 2-index invariant tensor $\equiv \delta^{ij}$

- in 2D $\exists$ δ^{ij} and $\underbrace{\varepsilon^{ij}}$ $\rightarrow$ P-odd

in general with P-violation in 2D

$$\langle d_i \rangle = E_i f_1(E^2) + \varepsilon_{ij} E_j f_2(E^2)$$

- Using dim analysis we can say more about $f(E^2)$

$$e/r^2 \sim \frac{e}{r_B^2} = E_0 \qquad d \sim r_B \cdot e$$

$$\langle d_i \rangle = (r_B \cdot e) \cdot \frac{E_i}{E_0} f\left(E^2/E_0^2\right) = E_i \, r_B^3 \, f\left(\frac{E^2 \cdot r_B^4}{e^2}\right)$$

expect $O(1)$

In pert. theory $\vec{d}$ from singlet triplet mixing

$$\psi_0(E) = \psi_0 + c_i |\psi_i\rangle \qquad c_i = \frac{\langle \psi_i | -E \cdot d | \psi_0 \rangle}{E_1 - E_0}$$

$$\sim \frac{E \cdot r_B \cdot e}{e^2/r_B} \sim E \cdot r_B^2/e$$

$$d = c_i \langle \psi_0 | d | \psi_i \rangle$$

$$\simeq E \cdot r_B^3$$

Ex dimensional analysis: rescaling of units $G = \otimes \mathbb{R}^+$

In general: $q'_e(t') = f_e(q(t), \alpha)$ $t' = g(t, \alpha)$

$$\Rightarrow \mathcal{L}(q, \dot{q}, \lambda_i)\, dt = \mathcal{L}'(q', \dot{q}', \lambda_i)\, dt' = c(\alpha)\, \mathcal{L}(q', \dot{q}', \lambda'_i)\, dt'$$

with $\lambda'_i = f_i(\lambda, \alpha)$

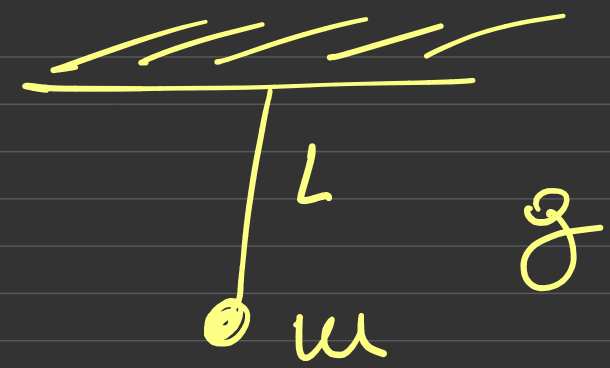
$\Rightarrow$

$$\text{eom}(q, \dot{q}, \lambda) \Longleftrightarrow \text{eom}(q', \dot{q}', \lambda') \equiv \underline{\text{covariance}}$$

Ex: rescaling
$$\begin{cases} q' = K_x q \\ t' = K_t t \\ \lambda' = K_x^{d_1} K_t^{d_2} \lambda \end{cases} \Rightarrow$$

Covariance $\equiv$ Dimensional Analysis

Ex pendulum



$$L \to K_x L$$

$$g \to \frac{K_x}{K_t^2}\, g$$

$$m \to K_m\, m$$

$$\left.\begin{aligned} \omega &= f(L, g, m) \\ &= c\sqrt{\frac{g}{L}} \end{aligned}\right\}$$

rescaling covariance $\equiv$ dimensional analysis

$$L = \frac{m}{2}\left[\dot{\underline{x}}^2 + \frac{(\underline{x}\,\dot{\underline{x}})^2}{L^2 - \underline{x}^2} - 2g\sqrt{L^2 - \underline{x}^2}\right] \qquad \underline{x} = (x^1, x^2)$$

Ex π-mass

QCD + QED

1 GeV ————————————— $\Longrightarrow \mathcal{L}_{eff}$

mesons, photon

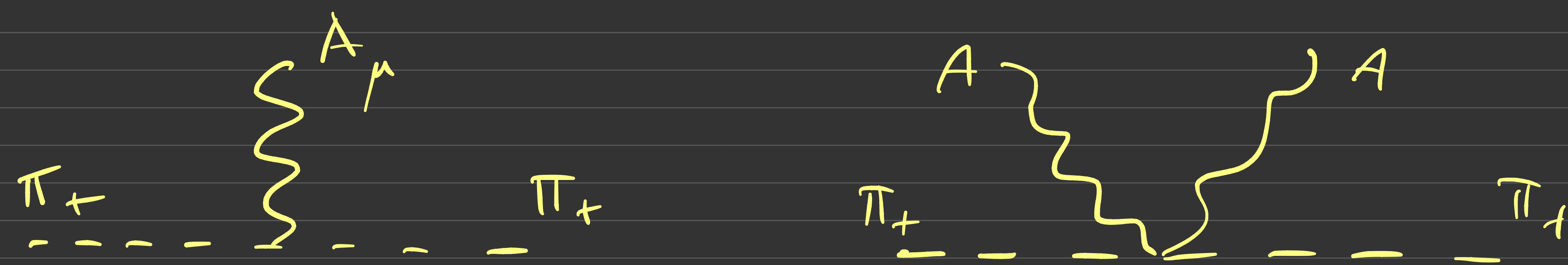
$\left\{ \begin{array}{l} \cdot\ \Lambda_{QCD},\ m_u,\ m_d,\ \alpha \\ \cdot\ SU(2)_L \times SU(2)_R \to SU(2)_I \\ \quad \Rightarrow \pi^\pm, \pi^0 \text{ are } \sim \text{Goldstones} \end{array} \right.$

$m_{u,d}$, α break χ-symm $\Longrightarrow$ m_π^2 controlled by selection rules

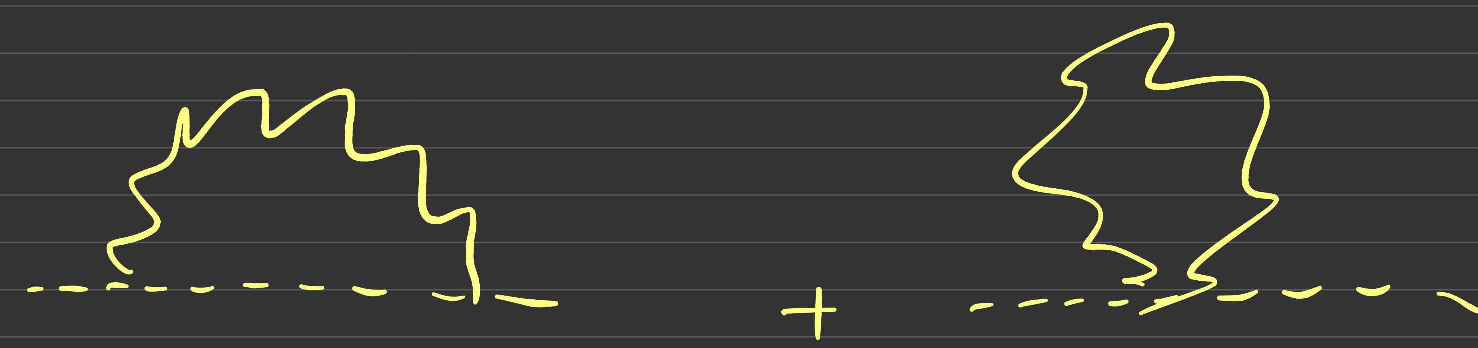
- leading $\quad m_{\pi^+}^2 = m_{\pi^0}^2 = c_2 \Lambda_{QCD}(m_u + m_d)$
- sub-lead $\quad m_{\pi^+}^2 - m_{\pi^0}^2 = c_3 (m_u - m_d)^2 + c_4 \dfrac{e^2}{16\pi^2} \Lambda_{QCD}^2$

isospin $\qquad$ dilation

- $\frac{e^2}{16\pi^2}\Lambda_{QCD}^2$ term correctly estimated by naive 1-loop computation within EFT



none for π_0



$\Longrightarrow \delta m^2_{\pi_+}$

$$\simeq e^2 \int \frac{d^4p}{16\pi^4}\frac{1}{p^2} \sim \frac{e^2}{16\pi^2}\int d^2p \sim \frac{e^2}{16\pi^2}\Lambda_{QCD}^2$$

dependence $\delta m_\pi^2 \propto e^2$ also "simply" fixed by selection rules of "amazing" symmetry of free fields

<u>Ex</u> $\mathcal{L}_{free} = \int \phi(x)\left(-\partial^2 - m^2\right)\phi(x)\, d^4x$

$$= \int \frac{d^4p}{(2\pi)^4}\, \hat{\phi}(p)^*\left(p^2 - m^2\right)\hat{\phi}(p) \qquad \hat{\phi}^*(p) = \hat{\phi}(-p)$$

$$\begin{cases} \hat{\phi}(p) \longrightarrow e^{i\theta(p)}\,\phi(p) & \text{is a symmetry} \\ \theta(p) = -\theta(-p) \end{cases}$$

$$\theta = \underbrace{Q_1^\mu p_\mu}_{\text{translation}} + \underbrace{Q_3^{\mu\nu\rho} p_\mu p_\nu p_\rho}_{\text{spin 4 current}} + \underbrace{\cdots}_{\text{spin 6}} \quad - - -$$

interaction $\int \lambda_3 \phi^3(x) = \int \lambda_3 \, \delta^3(p_1+p_2+p_3) \, \hat{\phi}(p_1) \hat{\phi}(p_2) \hat{\phi}(p_3)$

- breaks symmetry down to translations
- can formally restore it $\lambda_3 \delta^3(p_1, p_2, p_3) \longrightarrow \hat{\lambda}_3(p_1, p_2, p_3)$

$$(*) \qquad \hat{\lambda}_3(p_1, p_2, p_3) \longrightarrow \hat{\lambda}_3(p_1, p_2, p_3) \, e^{-i(\theta(p_1)+\theta(p_2)+\theta(p_3))}$$

fixes the way λ_3 can affect n-point functions

Ex 2pt $\quad \Gamma \xrightarrow{1P1} \int dp_1 dp_2 \, \hat{\phi}(p_1) \hat{\phi}(p_2) \, \Gamma^2(p_1, p_2)$

$$\Gamma^2(p_1, p_2) \propto \hat{\lambda}_3(p_1, p_3, p_4) \, \hat{\lambda}(p_2, -p_3, -p_4)$$

of course this is just 

but conceptually important that result can be cast in terms of symmetry like all else

(Atom in $\vec{E}$, pendulum, ... etc)

Naturalness

In general, given couplings $\{\lambda_a\}$, an observable $\mathcal{O}$ is given by

$$\langle \mathcal{O} \rangle = \sum_a \langle \mathcal{O} \rangle_a \qquad \langle \mathcal{O} \rangle_a = \underbrace{c_a}_{O(1)\ \text{coeff.}} \underbrace{\lambda_1^{n_{1a}} \ldots \lambda_N^{n_{Na}}}_{\text{symm. \& dim.}}$$

If $\quad |\langle \mathcal{O} \rangle_{exp}| \ll \max |\langle \mathcal{O} \rangle_a| \quad$ it seems we are missing something

natural

✦ we overlooked a ***symmetry*** implying cancellations among the $\langle \mathcal{O} \rangle_a$

✦ there is a ***fine-tuning*** of parameters not related to symmetry

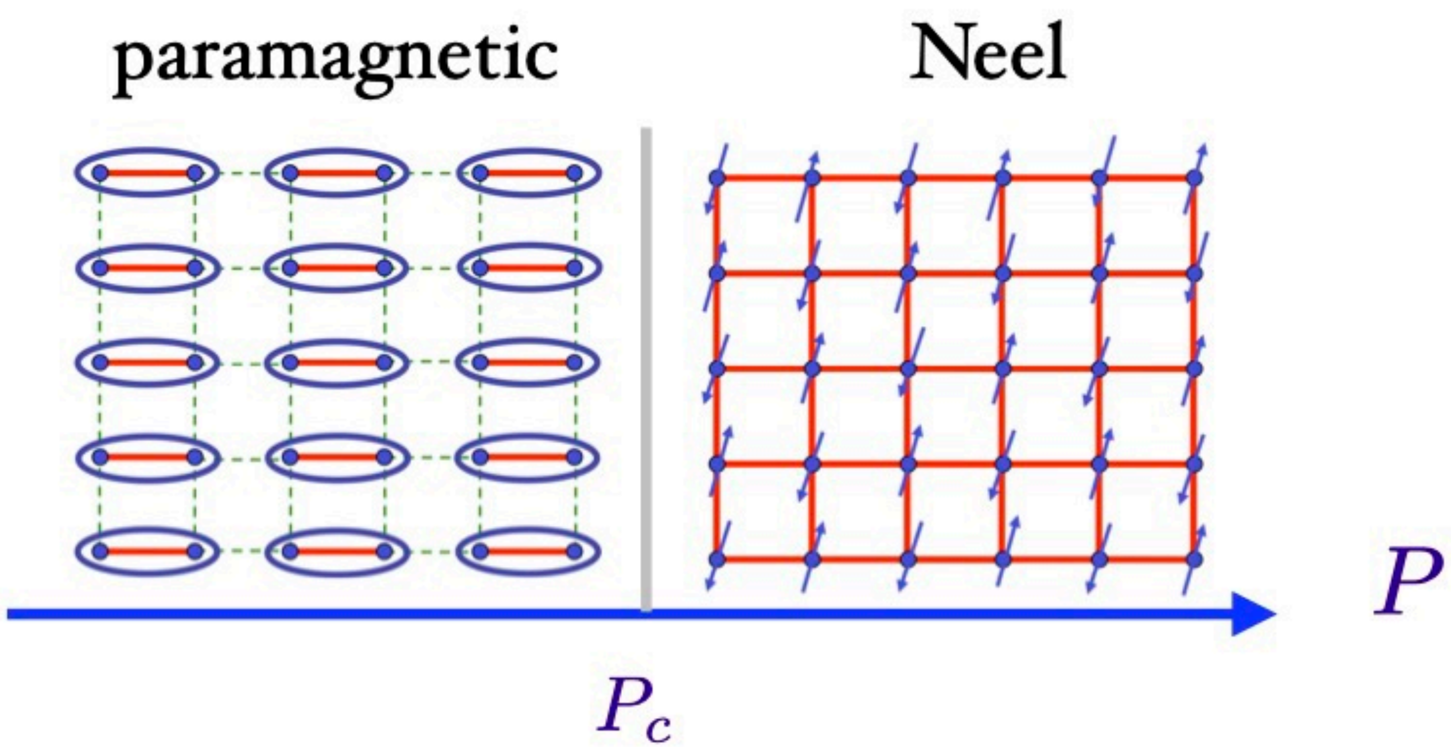
un-natural

In the normal (lab) practice extreme fine tunings are associated to the existence of a large set of options (a landscape) from which we can pick

Ex. quantum criticality in anti-ferromagnet ($TlCuCl_3$)

paramagnetic

Sachdev '09



$$\vec{\varphi} \equiv (-1)^{x+y} \vec{S}(x,y)$$

$$V(\vec{\varphi}) = m^2(P)\vec{\varphi}\cdot\vec{\varphi} + \lambda(\vec{\varphi}\cdot\vec{\varphi})^2 \qquad m^2(P) = m_0^2\left(1 - \frac{P}{P_c}\right)$$

Can undo ***natural*** expectation from atomic physics by ***tuning*** the pressure at a ***critical*** value in a ***landscape*** of possibilities

The Naturalness Criterion 't Hooft 1979

- Under what condition is a QFT natural?

Λ

G_a

g_i

$$\Longrightarrow \quad \underbrace{g_i \equiv g_i(G_a)}_{\text{covariant}}$$

covariant $\equiv$ respects selection rules

- Imagine $[g_{i_1}] = [F_{i_1}(g_i)] = [F_{i_1}(g_i(G_a))]$

$g_i \neq g_{i_1}$

given the g_i's this offers a way to assess the naturalness of any pattern of this set $\{g_i\}$

1975

$\Rightarrow$ original 't Hooft formulation

a coupling g is naturally small if $g \to 0$ implies the presence of an additional symmetry

Exercises

$\nu_{1,2}$ Weyl bispinors $\nu^{\alpha}_{1,2}$

1) $$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - \frac{m^2\phi^2}{2} + i\bar{\nu}_1\bar{\partial}\nu_1 + i\bar{\nu}_2\bar{\partial}\nu_2$$

$$\phi\left(g_{11}\nu_1^2 + g_{12}\nu_1\nu_2 + g_{22}\nu_2^2\right)$$

$U(1)_1 : \nu_1 \to e^{i\alpha_1}\nu_1$

$U(1)_2 : \nu_2 \to e^{i\alpha_2}\nu_2$

- classify couplings according to χ-symm
- starting at $\Lambda \gg m$ what is the minimum of $g_{11}(m)$ implied by the values of g_{12}, g_{22}?

(hint assume couplings are weak enough to work at 1-loop RG)

2) $\frac{1}{2}(\partial\phi_1)^2 + \frac{1}{2}(\partial\phi_2)^2 - g_{11}\phi_1^4 - g_{12}\phi_1^2\phi_2^2 - g_{22}\phi_2^4$

- characterize couplings on the basis of their transf properties under the symmetries of free field theory

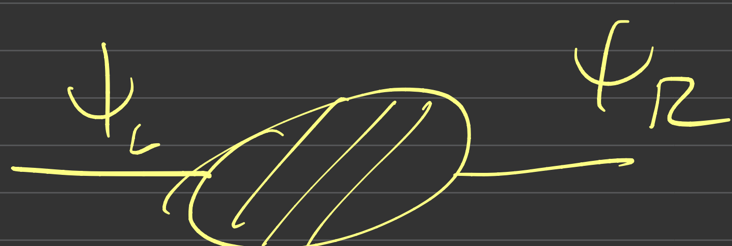
- Considering a generic $\mu \ll \Lambda$, what does Naturalness suggest as upper bound for $g_{11}(\mu)$ in terms of g_{12}, g_{22} (work at 1-loop like in Exercise 1)

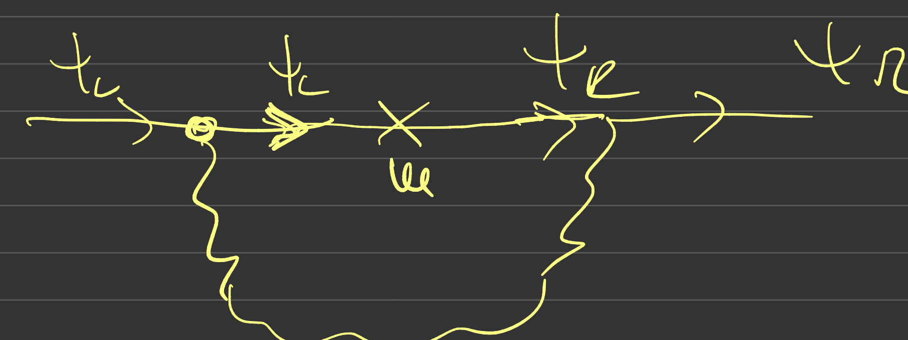
Ex electron mass in QED

- $\psi = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix}$ $\qquad D_\mu \equiv \partial_\mu - i e A_\mu$

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + i \psi_L^\dagger \bar{\sigma} \cdot D \psi_L + i \psi_R^\dagger \sigma D \psi_R - m \psi_L^\dagger \psi_R - m^* \psi_R^\dagger \psi_L$$

$$U(1)_A \longrightarrow \boxed{\begin{array}{l} \psi_L \to e^{-i\theta} \psi_L \\ \psi_R \to e^{i\theta} \psi_R \\ m \to m\, e^{2i\theta} \\ e \to e \end{array}} \Longrightarrow \delta m_e \Big|_{\text{loops}} \propto m_e$$



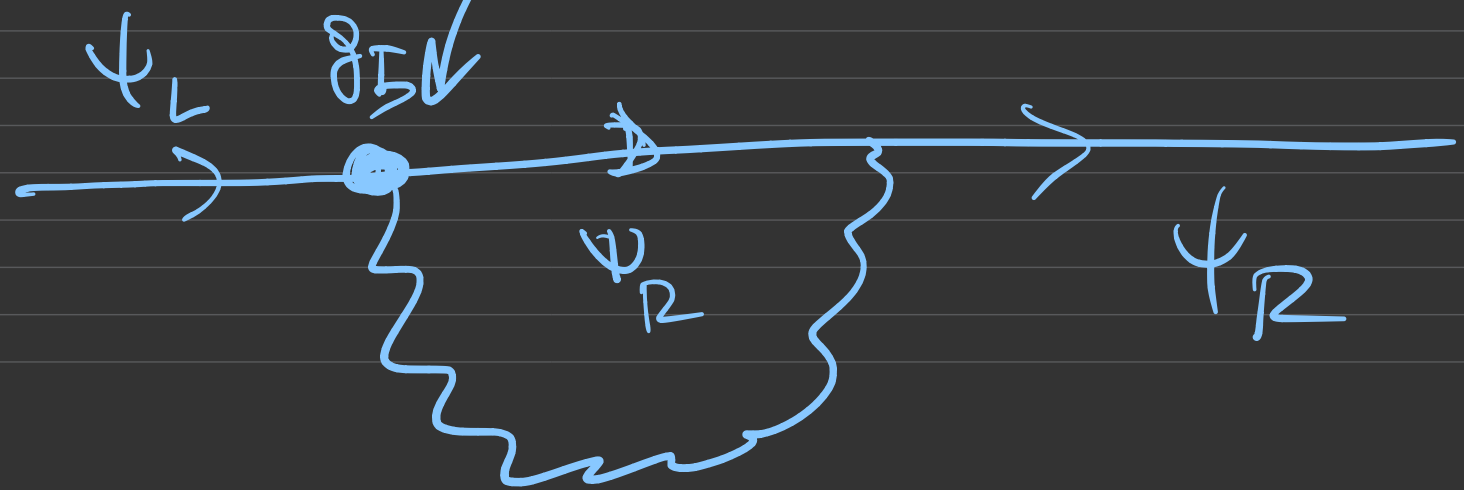


- EFT perspective : must assume $\mathcal{O}_{\Delta>4}$ satisfy this approx symm

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + i \psi_L^\dagger \bar{\sigma} \cdot D \psi_L + i \psi_R^\dagger \sigma D \psi_R - m \psi_L^\dagger \psi_R - m^* \psi_R^\dagger \psi_L +$$

$$+ \frac{g_5}{\Lambda} \psi_L^\dagger \sigma_{\mu\nu} \psi_R F^{\mu\nu} + \frac{g_6}{\Lambda^2} (\psi_L^\dagger \bar{\sigma}_\mu \psi_L)(\psi_R^\dagger \sigma^\mu \psi_R) + \cdots$$

$$U(1)_A : \frac{g_5}{\Lambda} \to e^{2i\theta} \frac{g_5}{\Lambda} \qquad g_5 \sim \frac{m_e}{\Lambda}$$



$$\delta m_e \sim \frac{1}{16\pi^2} \frac{g_5}{\Lambda} \int \frac{d^4 p}{p^2}$$

$$\approx \frac{1}{16\pi^2} g_5 \cdot \Lambda$$

Λ ———————————— $\longrightarrow$ approximate $U(1)_A$ could be accidental

m ————————————

Ex

- EFT above Λ $U(1)_A$ is gauged
- $U_A(1)$ broken by fermion condensate $\langle T_L^\dagger T_R \rangle \sim \Lambda^3$

$$\mathcal{L}_{mass} = \frac{1}{\Lambda_*^2} T_L^\dagger T_R \, e_R^\dagger e_L$$

$$\Longrightarrow \quad m_e \approx \frac{\Lambda^3}{\Lambda_*^2} \ll \Lambda$$

|Optional|

⚠ Natural & Un-Natural mass hierarchies

G_a

Λ_{UV}

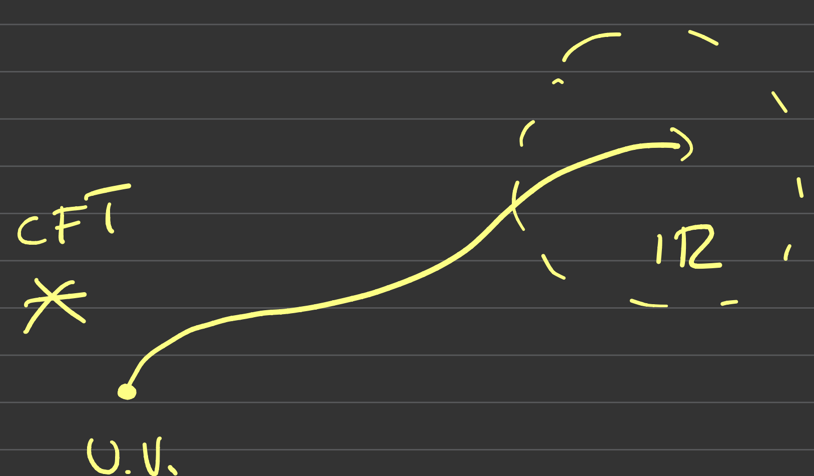
$$\text{CFT} + \{g_\alpha^R\}$$

Λ_{IR}

- Λ_{IR} determined by $\{g_\alpha^R\}$:

under what condition is $\Lambda_{IR} \ll \Lambda_{UV}$ natural **?**

Lg



$$\Lambda_{IR} \Longleftrightarrow \bar{g}_\alpha^R(\Lambda_{IR}) \sim O(1)$$

$$\Lambda_{IR} \ll \Lambda_{UV} \Longleftarrow \text{very slow evolution}$$

- Natural hierarchy $\Longleftrightarrow$ naturally slow RG evolution

△ Two canonical options

I. Marginality $\Longleftrightarrow$ CFT does not possess "strongly relevant" operators

$\Longrightarrow$ lowest scalar $\mathcal{O}$ has $\Delta = 4 - \varepsilon$
$\varepsilon \ll 1$

- Ex 1 YM $\Longrightarrow$ $\mathcal{O} = G_{\mu\nu} G^{\mu\nu}$ $\quad \Delta_{\mathcal{O}} = 4$ at $g = 0$

def $\lambda \equiv \frac{g^2}{16\pi^2} \frac{11 N_c}{3}$ $\quad \xrightarrow{RG}$ $\quad E \frac{d\lambda}{dE} = -2\lambda^2$

$$\lambda = \frac{\lambda_{UV}}{1 + 2\lambda_{UV} \ln E/\Lambda}$$

$\Lambda_{IR} \longleftrightarrow \lambda \sim O(1) \Longrightarrow \Lambda_{IR} = \Lambda_{UV}\, e^{-\frac{1}{2\lambda_{UV}}}$

- to generate hierarchy enough algebraically small λ_{UV} (say $1/20$). Moreover $\lambda \to 0$ is natural because of free field theory is natural.

II. Symmetry: Relevant operators do exist but they can all be associated with the breaking of a global symmetry

$\Longrightarrow$ their coefficients can all be naturally assumed to be small

Ex QCD $\rightarrow$

$$\mathcal{L} = -\frac{1}{4g^2} G_{\mu\nu}^2 + i\bar{\psi}\not{D}\psi - M_{ij}\bar{\psi}_i\psi_j + \text{irrelevant}$$

χ-symm $SU(N_F)_L \times SU(N_F)_R$: $M \rightarrow U_L M U_R^\dagger$

- $M \rightarrow 0$ χ-symm exact
- $M \ll \Lambda_{UV}$ natural