

# Gauge anomalies from an on-shell perspective

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Symmetries are blueprinted  
in lagrangians



But it can happen that they are  
lost upon computing correlators



Gauge symmetry cannot  
be lost: we gear our  
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$$\partial_\mu \begin{array}{c} \nearrow \text{wavy line } \mu \\ \searrow \text{wavy line } \nu \end{array} \triangle \text{wavy line } \rho \neq 0 \quad \longrightarrow \quad \sum_n Q_n^3 = 0$$



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$$\partial_\mu \left( \begin{array}{c} \mu \\ \text{wavy line} \\ \text{triangle} \\ \text{wavy line} \\ \nu \end{array} \right) \rho \neq 0 \quad \longrightarrow \quad \sum_n Q_n^3 = 0$$

On-shell methods bypass gauge  
symmetry from the get go. What  
are anomaly cancellation  
conditions in this perspective?

## Certain amplitudes expose a tension between locality and unitarity

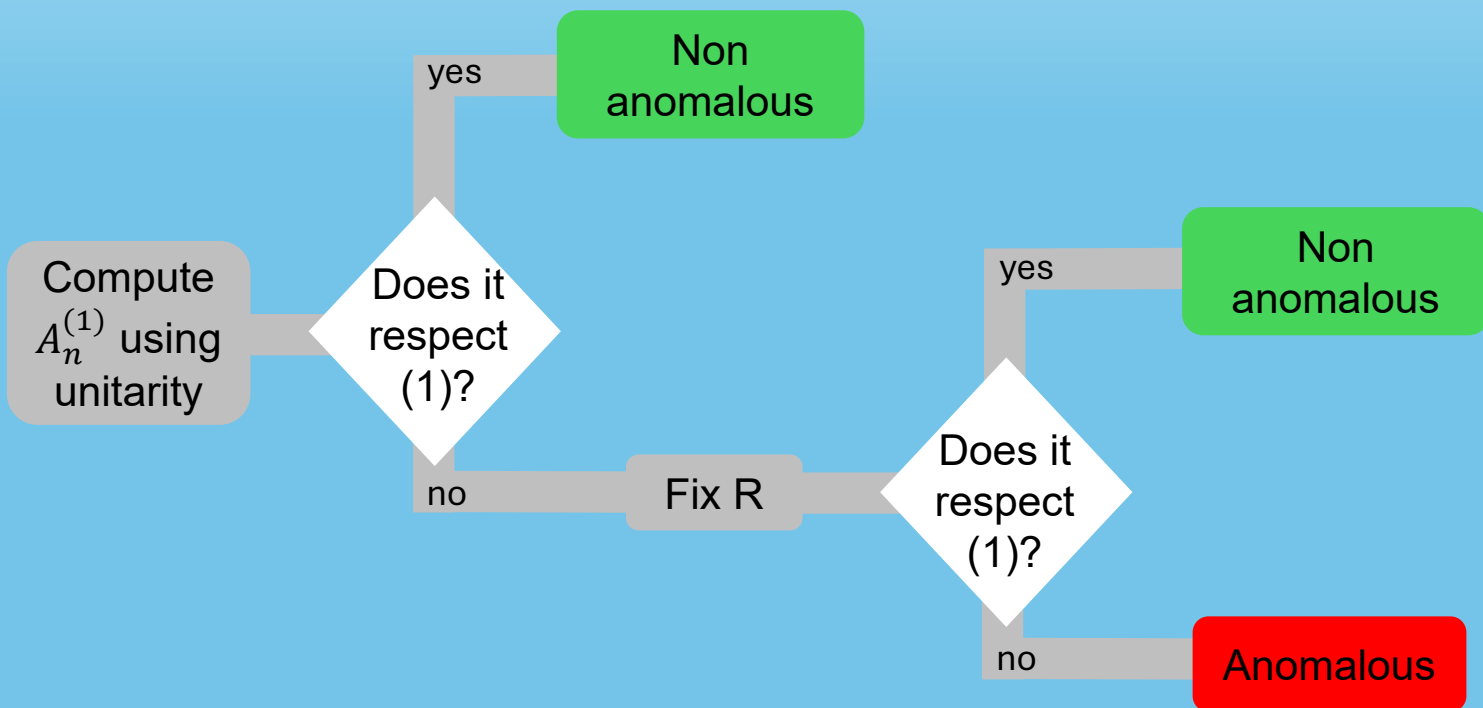
$n \geq 5$

$$\begin{array}{c}
 \text{Diagram with concentric circles and external lines } a, b \text{ and dots} \\
 \xrightarrow{s_{ab} \rightarrow 0} \\
 \text{Diagram with small circle connected to large circle} + \text{Diagram with double circle connected to large circle} \\
 \sim \frac{1}{\sqrt{s_{ab}}}
 \end{array}
 \quad (1)$$

## Certain amplitudes expose a tension between locality and unitarity

$n \geq 5$

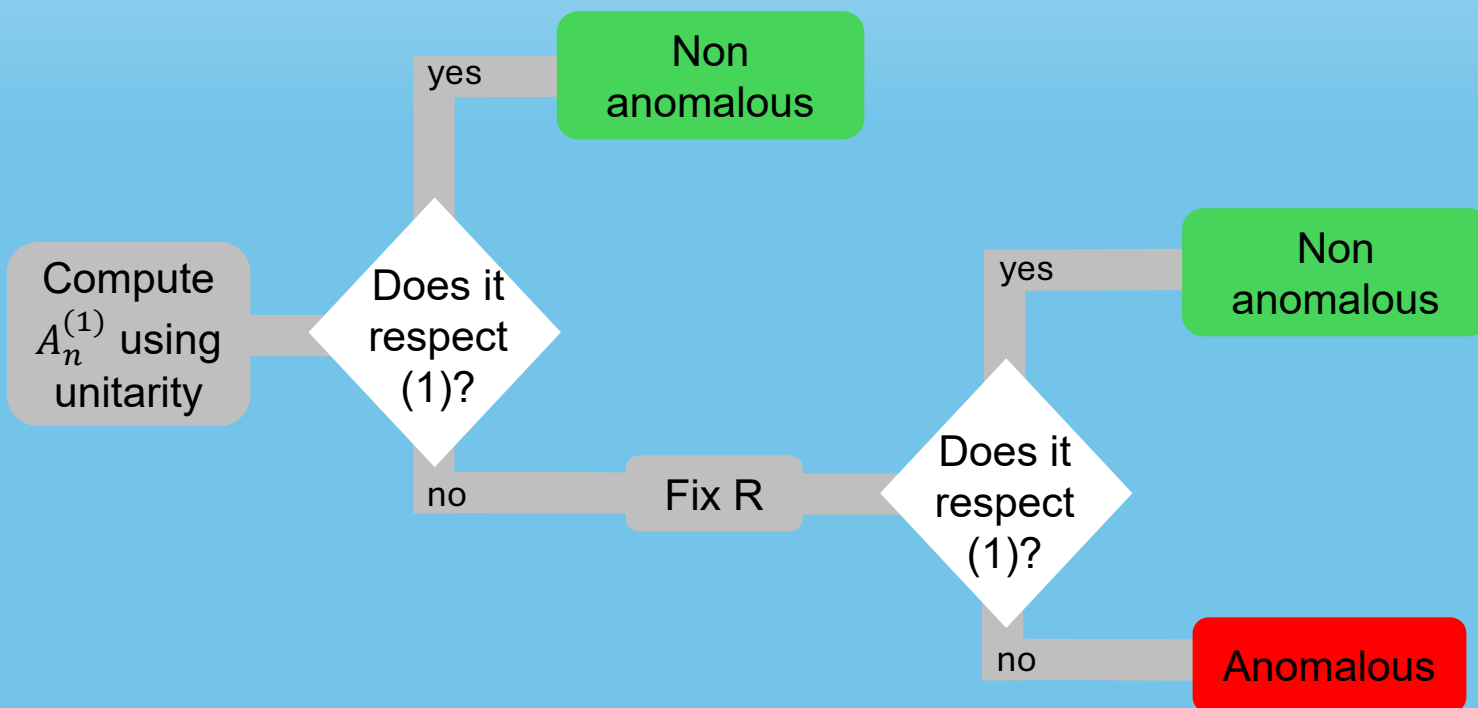
$$s_{ab} \rightarrow 0 \quad \rightarrow \quad \text{Diagram 1} + \text{Diagram 2} \quad \sim \frac{1}{\sqrt{s_{ab}}} \quad (1)$$



# Certain amplitudes expose a tension between locality and unitarity

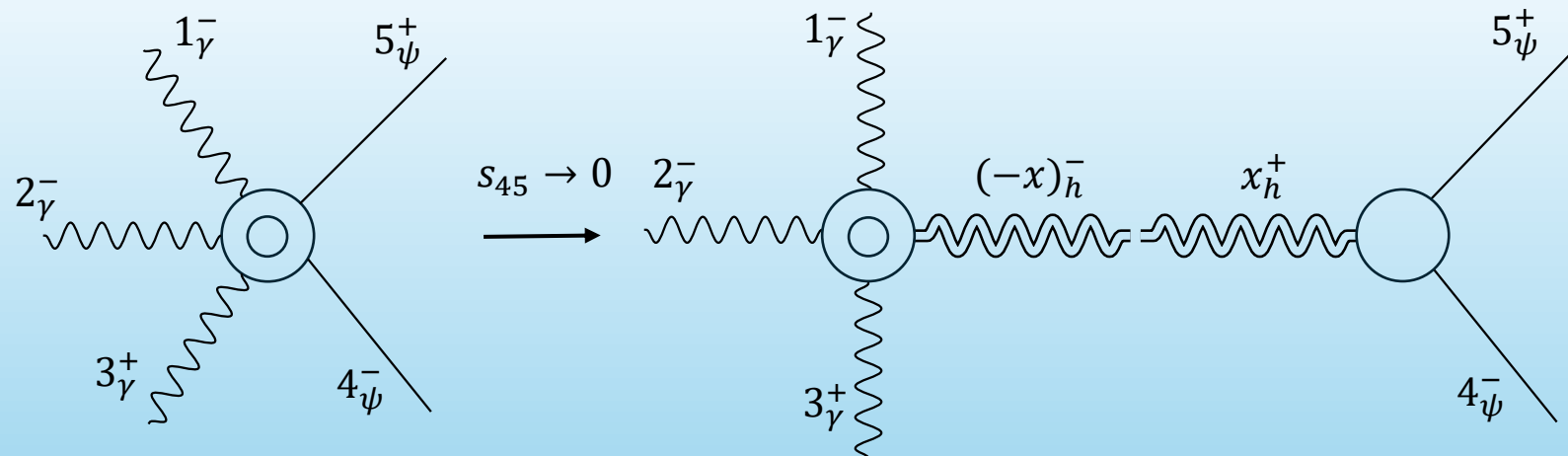
$n \geq 5$

$$\xrightarrow{s_{ab} \rightarrow 0} \text{Diagram 1} + \text{Diagram 2} \sim \frac{1}{\sqrt{s_{ab}}} \quad (1)$$



$$\sim \sum_n Q_n^3$$





$$A = \frac{-\sqrt{2}e^3}{16\pi^2 M_{Pl}^2}$$

$$S_{QED} = \frac{\langle 12 \rangle^2 [35]^3 \langle 3|p_2\sigma|4 \rangle \langle 45 \rangle^2}{s_{12}s_{45}}$$

$$\sim \sqrt{s_{12}} + O(s_{12})$$

$s_{13} \rightarrow 0$

$s_{12} \rightarrow 0$

$\sim \frac{1}{\sqrt{s_{12}}}$

$s_{12} \rightarrow 0$

$$A S_{QED} \left( \frac{\sum_n Q_n^3}{s_{12}s_{13}} + O(s_{12}^0, s_{45}^1) \right) + R$$

Inconsistent with locality: any possible factorization channel is zero or proportional to  $Q_i$  or  $Q_i^3$ , not  $\sum_n Q_n^3$ .

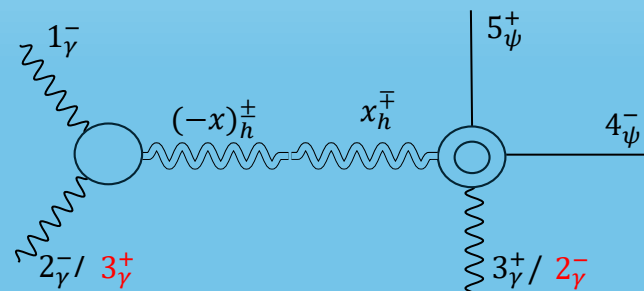
fix

$$R = A \sum_n Q_n^3 S_{QED} \frac{1}{s_{12}^2 s_{13}}$$

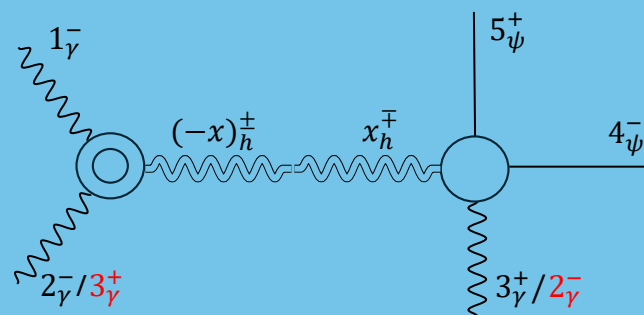
to eliminate sick behavior; holds outside of  $s_{12}$  limit, but introduces a pole in  $s_{13}$ , again inconsistent with factorization.

Only way to keep locality preserved is requiring  $\sum_n Q_n^3 = 0$ .

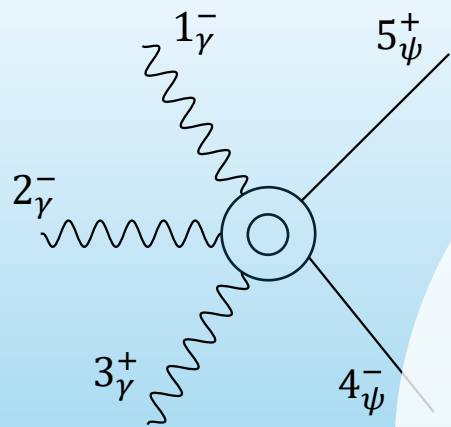
$Q_i^3 \sim$



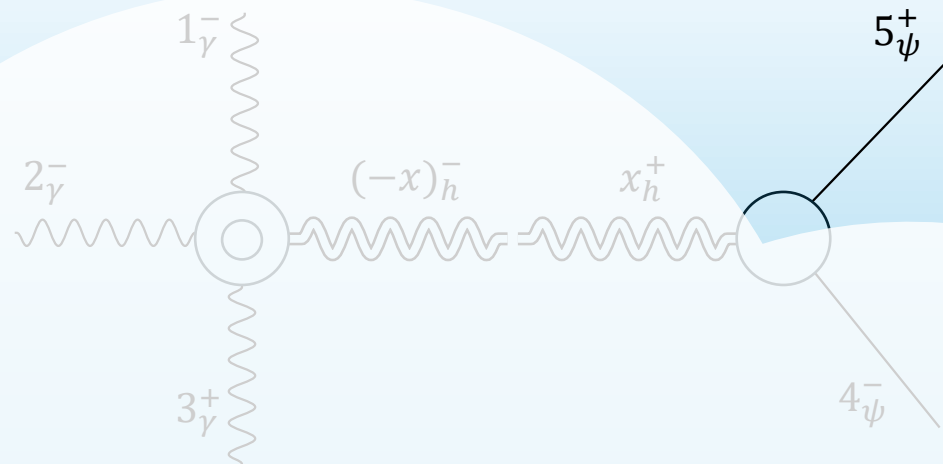
+







$$s_{45} \rightarrow 0$$



$$A = \frac{-\sqrt{2}e^3}{16\pi^2 M_{Pl}^2}$$

$$S_{QED} = \frac{\langle 12 \rangle^2 [35]^3 \langle 3|p_2\sigma|4\rangle \langle 45 \rangle^2}{s_{12}s_{45}}$$

$$\sim \sqrt{s_{12}} + \mathcal{O}(s_{12})$$

$$s_{13} \rightarrow 0$$

$$s_{12} \rightarrow 0$$

$$\sim \frac{1}{\sqrt{s_{12}}}$$

$$s_{12} \rightarrow 0$$

# Thank you!

Inconsistent with locality: any possible factorization channel is zero or proportional to  $Q_i$  or  $Q_i^3$ , none of  $Q_n^3$ .

$$R = A \sum_n Q_n^3 S_{12} \frac{1}{s_{12}^2 s_{13}}$$

to eliminate sick behavior; holds outside of  $s_{12}$  limit, but introduces a pole in  $s_{13}$ , again inconsistent with factorization.

Only way to keep locality preserved is requiring  $\sum_n Q_n^3 = 0.c$

$$\sum_n Q_n^3 = 0.c$$