

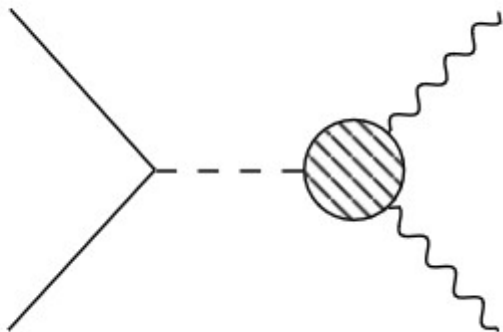
Vector Pair Production: The Onshell Way

Jared Goldberg (Technion)

Yael Shadmi (Technion)
In collaboration with Hongkai Liu (BNL)

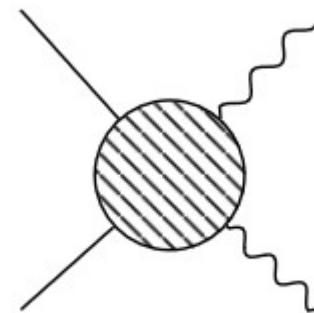
BSM Odyssey,
Cargese 2025 Intl.
Summer School





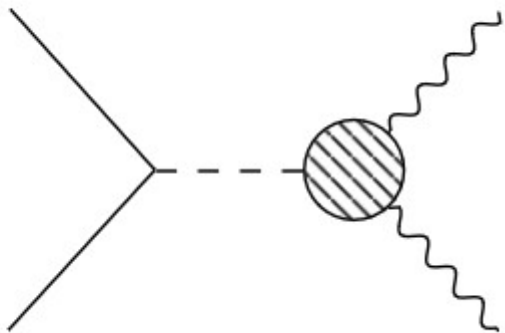
Factorize-able Terms

- Can be constructed from lower point amplitudes.



Contact Terms

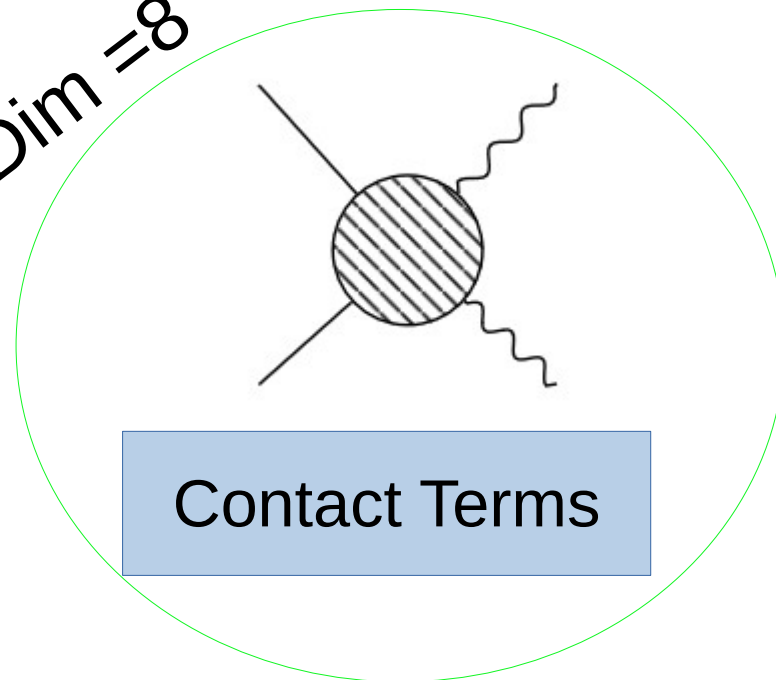
- Input for construction of higher-point amplitudes
- Parameterize NP contributions in the EFT.



Factorize-able Terms

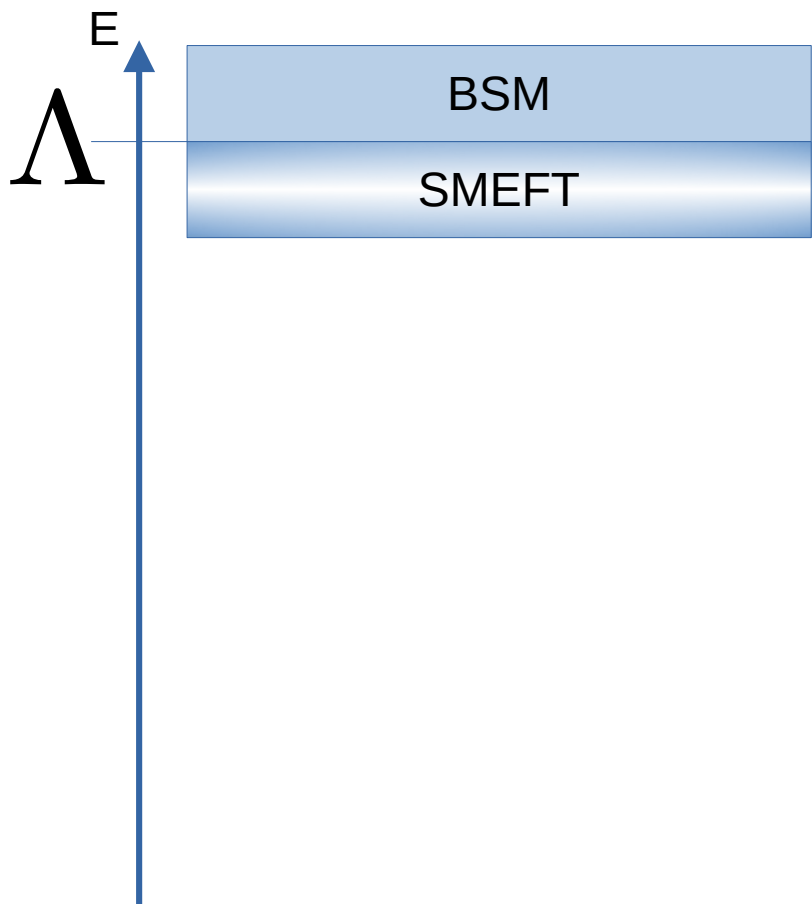
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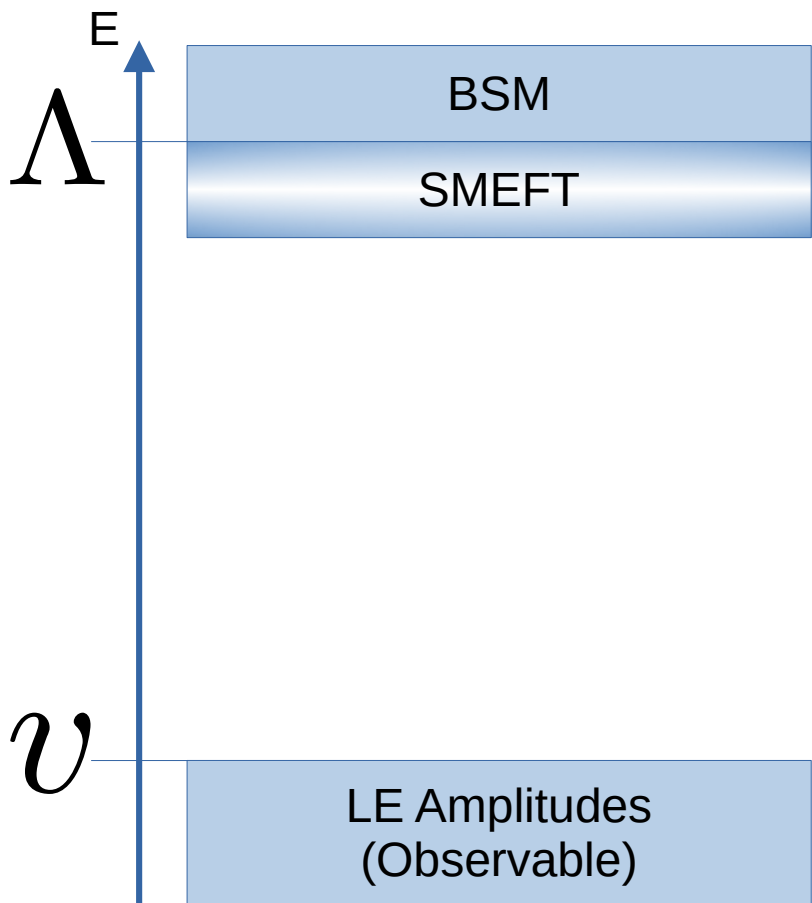
Dim = 8

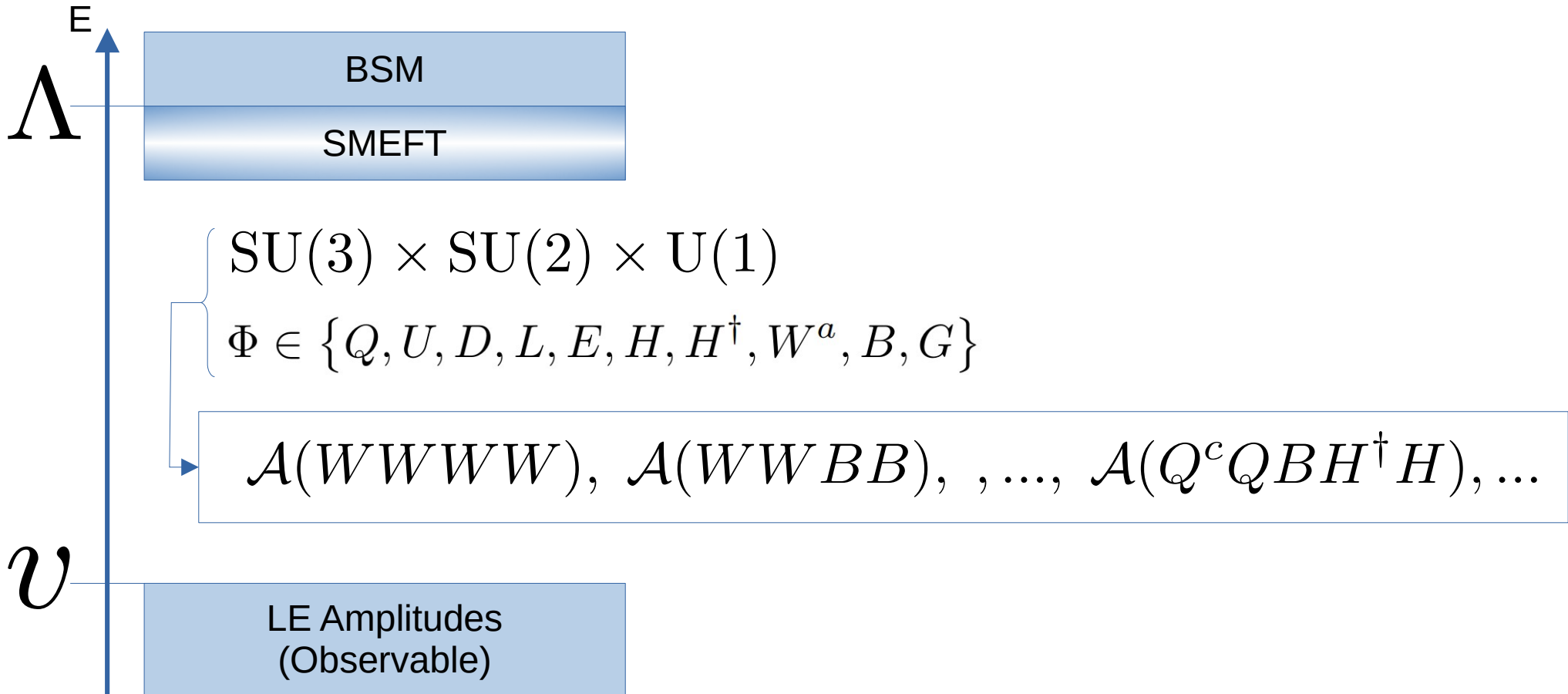


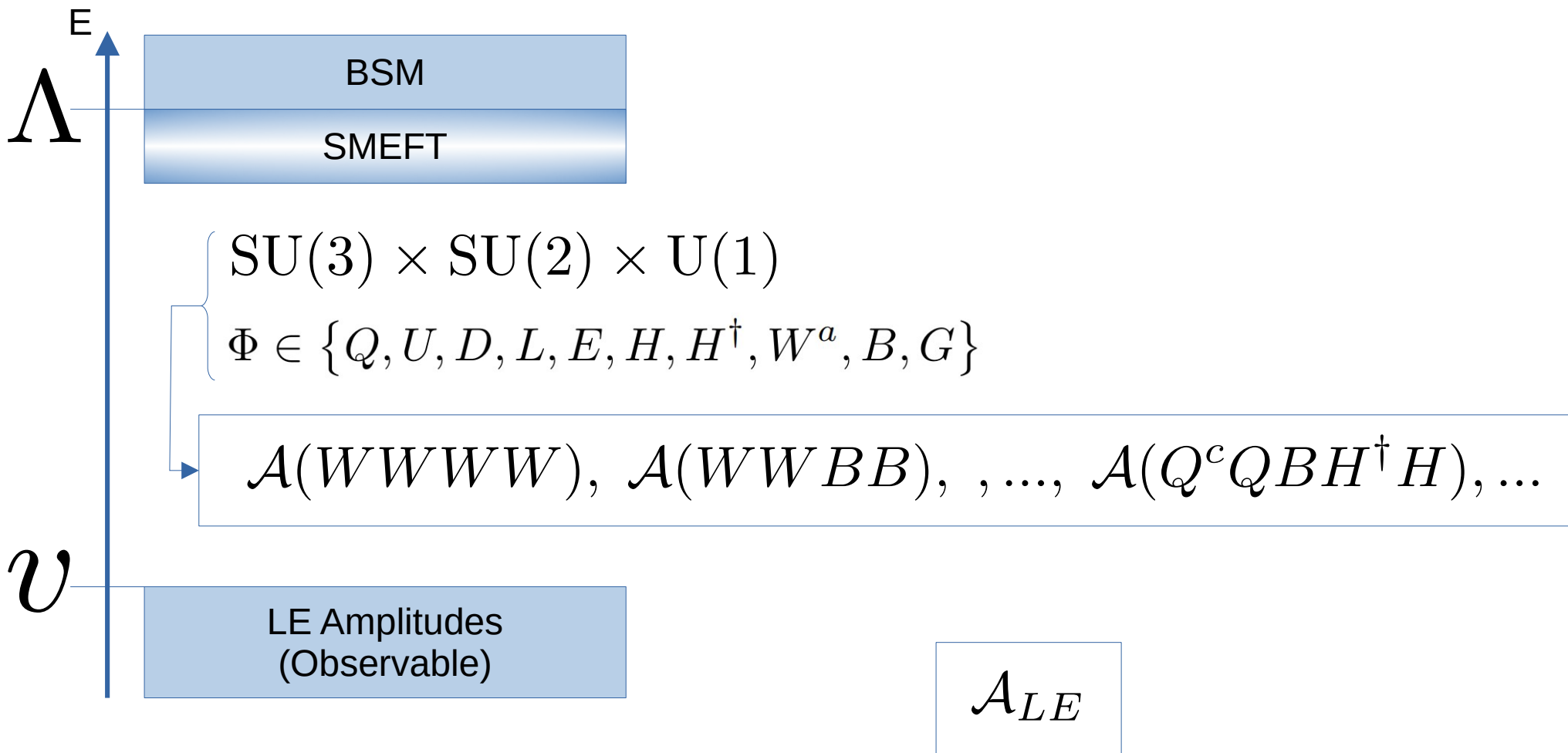
Contact Terms

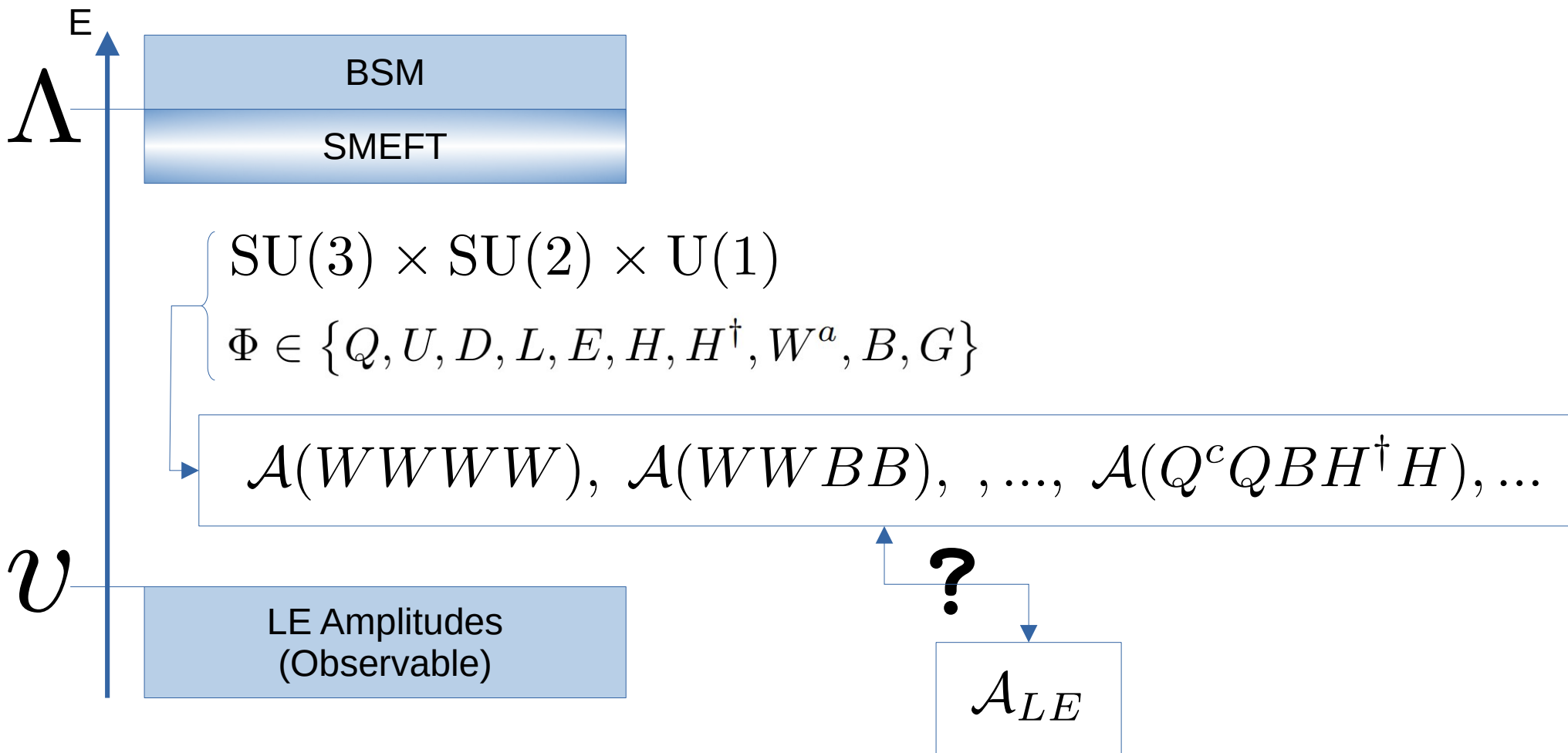
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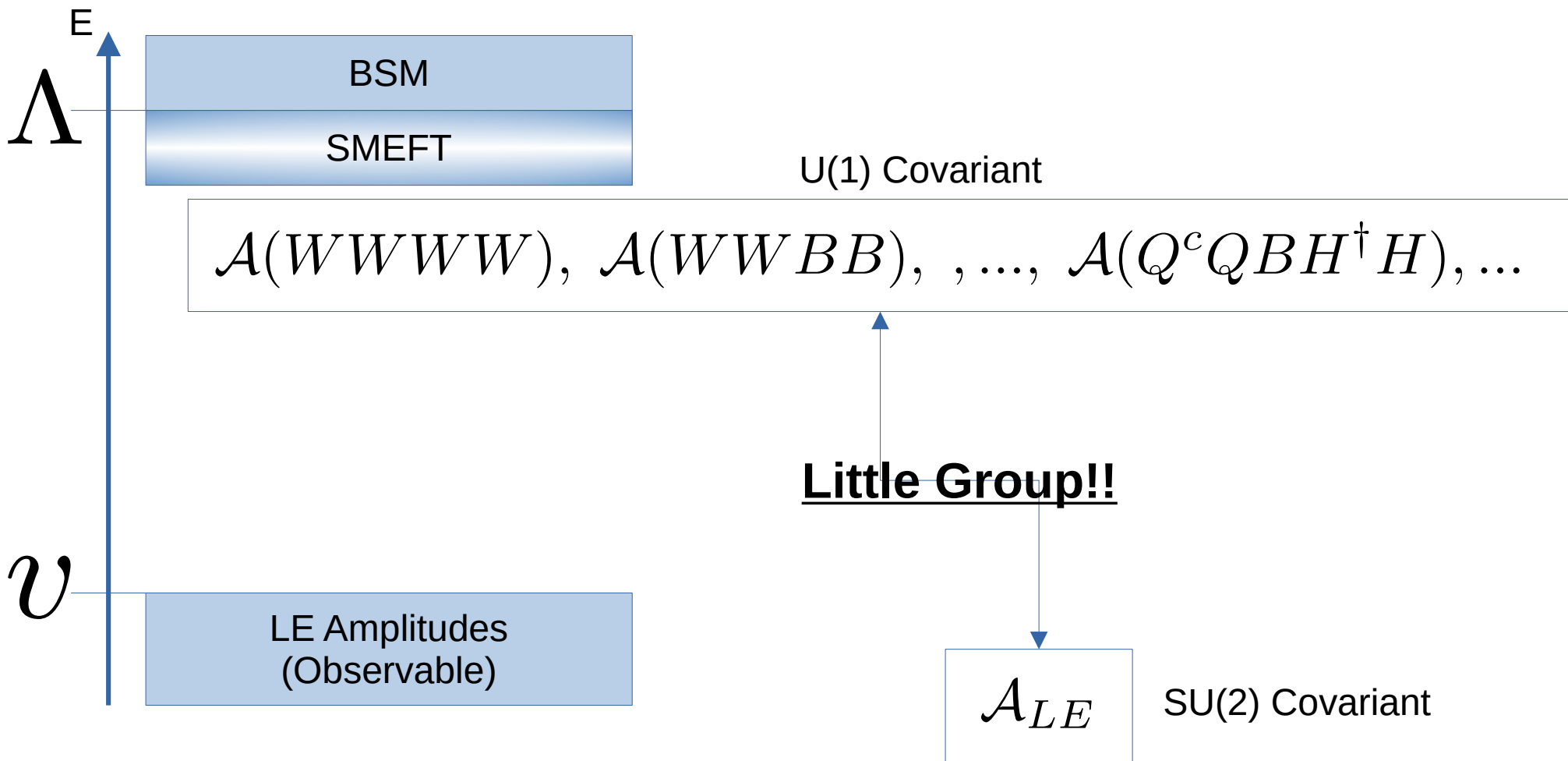


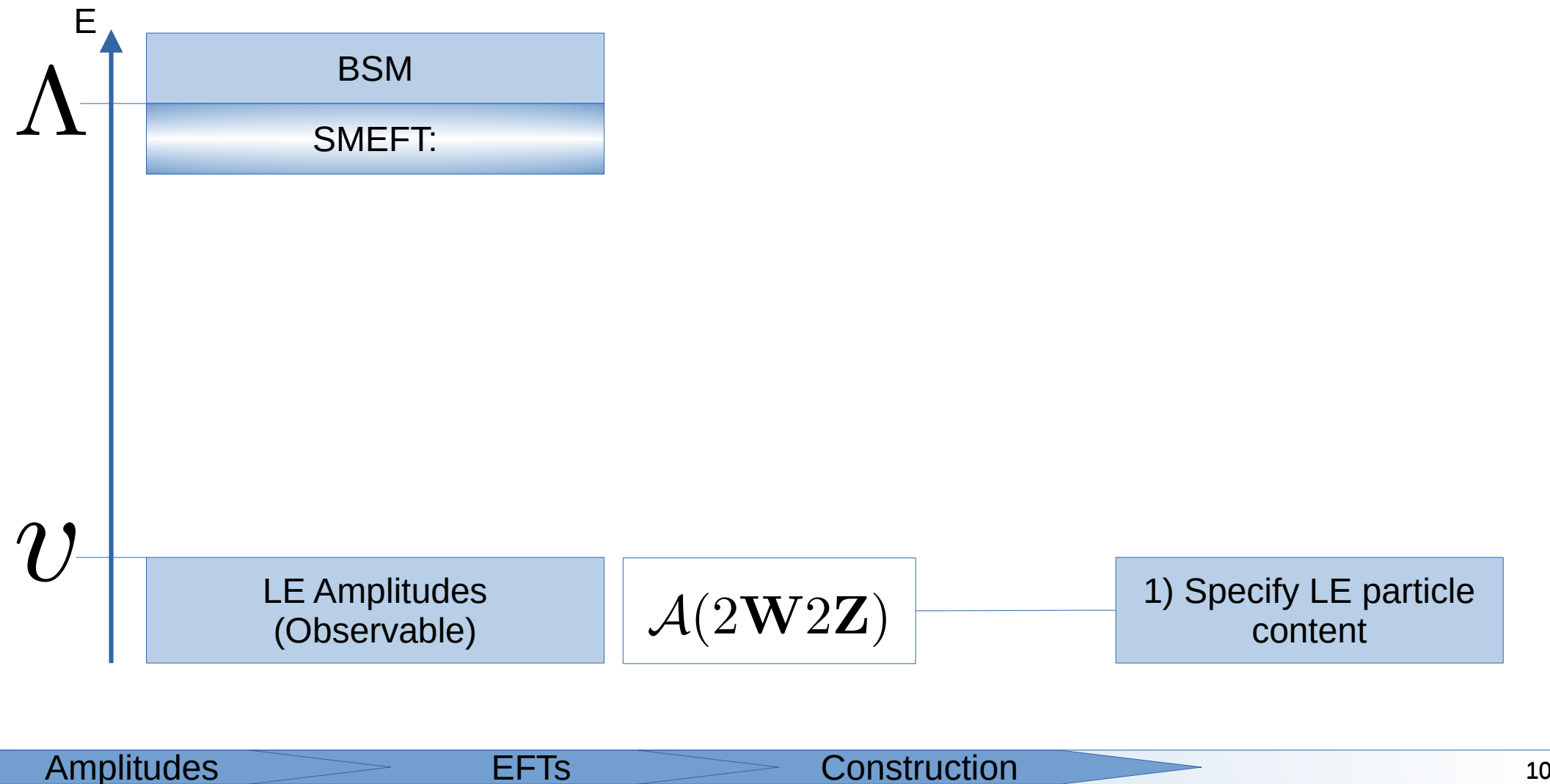


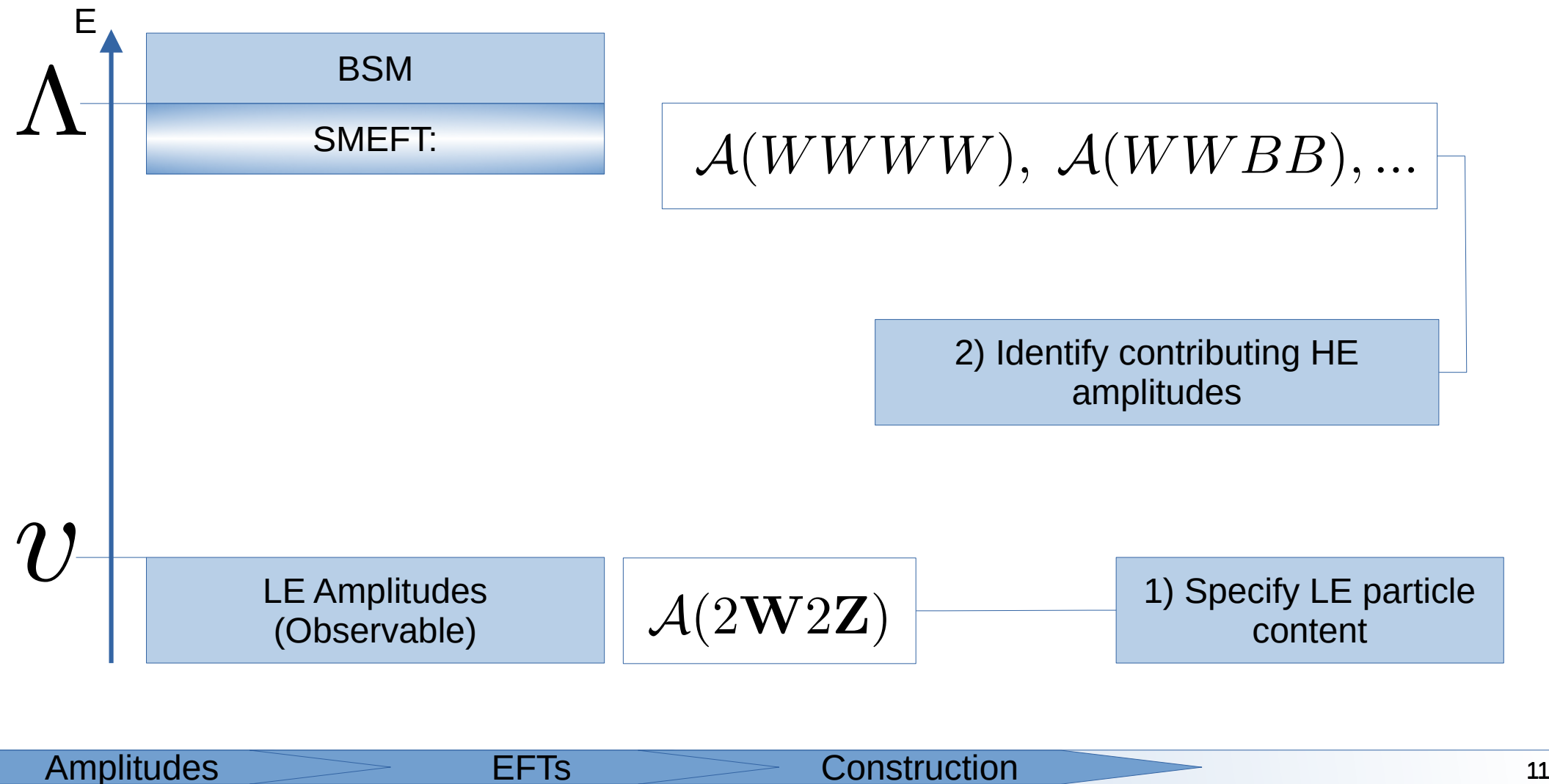


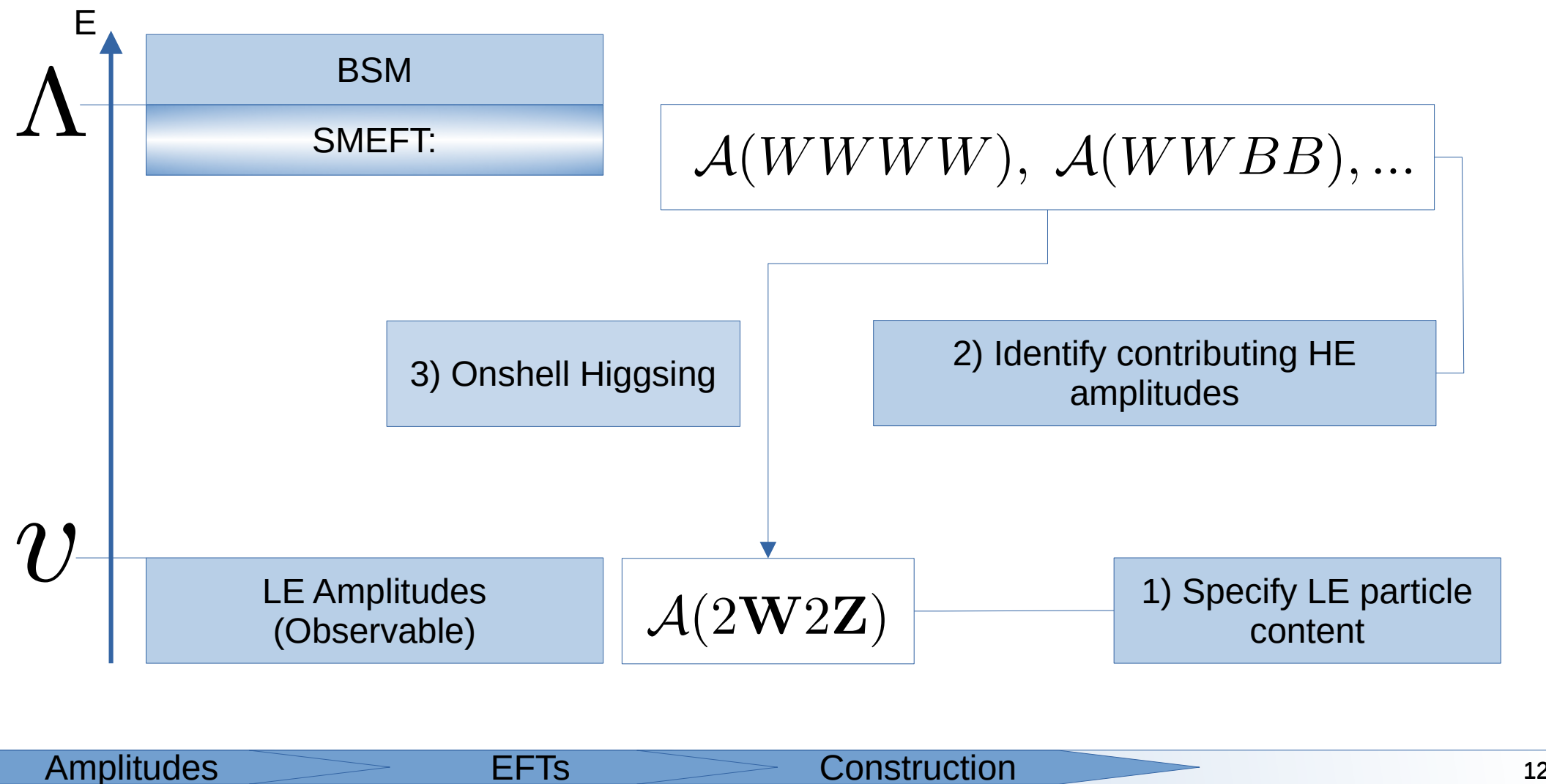


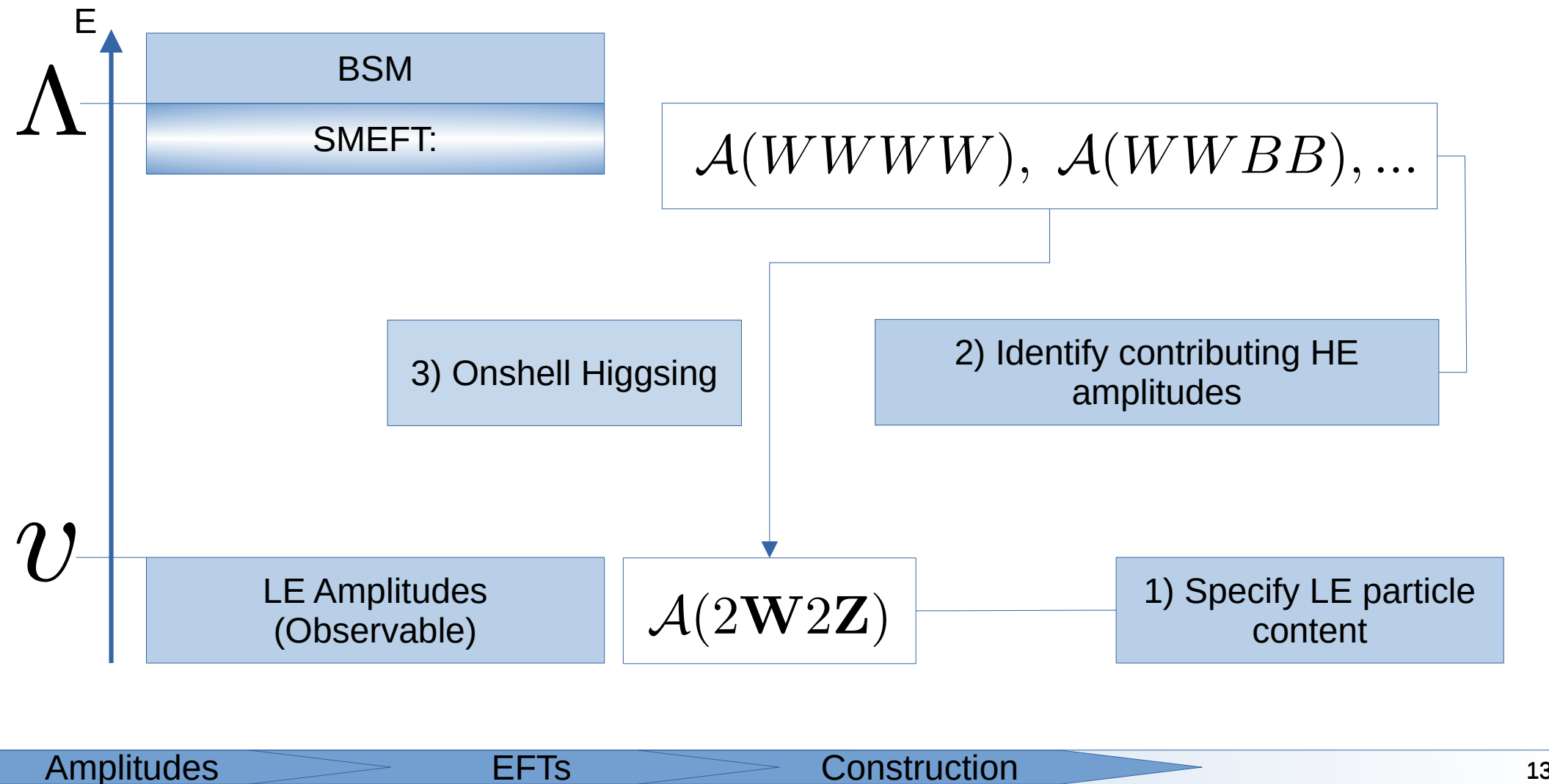












- Full parameterization of LE (massive) $VVVV$ and $ffVV$ contact term amplitudes generated by dim-8 SMEFT
- Leading order (no dim=6 contributions for these amplitudes) Liu, Ma, Shadmi, Waterbury '23
- Selection rules on kinematic configurations of observable amplitudes
- Comparison to HEFT

Thanks for Listening!

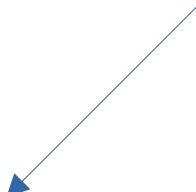
Backups

The Standard Model EFT (SMEFT)

$$\mathcal{L} = \mathcal{L}_{SM} + \sum_i \sum_{d>4} c_i \Lambda^{4-d} \mathcal{O}^i$$


$$SU(3)_C \times SU(2)_L \times U(1)_Y + \Phi \in \{Q, U, D, L, E, H, H^\dagger, W^a, B, G\}$$

Effective Field Theories

$$\mathcal{L} = \mathcal{L}_{SM} + \sum_i \sum_{d>4} c_i \Lambda^{4-d} \mathcal{O}^i$$


Constructing higher dim operators is a significant challenge due to:

- IBP
- EOMS
- Field Redefinitions

The Standard Model EFT (SMEFT)

e.g. for the SM Effective Field Theory (SMEFT):

Dim 6 operators, 1 fermion gen: 59

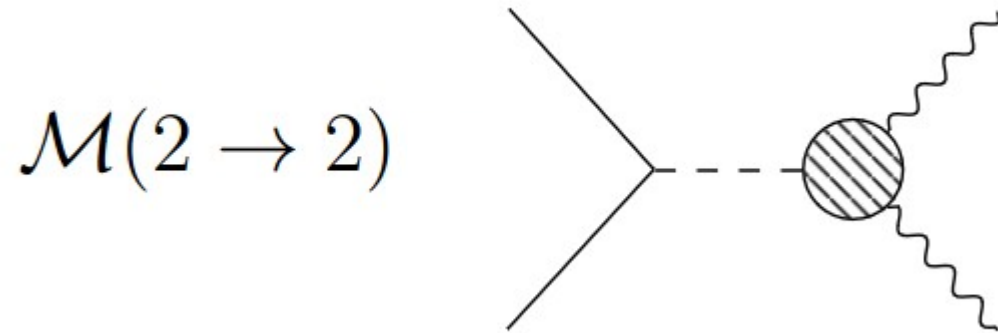
Dim 8 operators, 1 fermion gen: 889

Dim 8 operators, 3 fermion gen: >40000

Amplitude Structure

$$\mathcal{M}(2 \rightarrow 2)$$

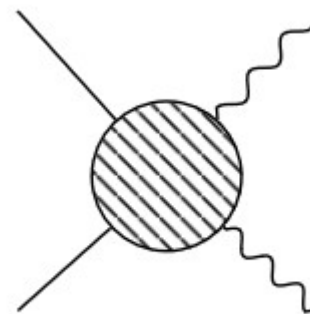
Amplitude Structure



Factorizeable

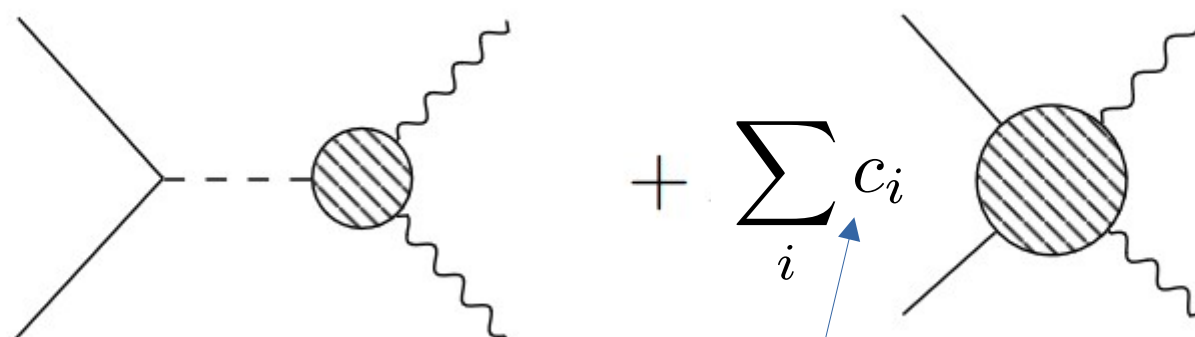
Amplitude Structure

$$\mathcal{M}(2 \rightarrow 2)$$



Contact Terms:
4-pt and higher

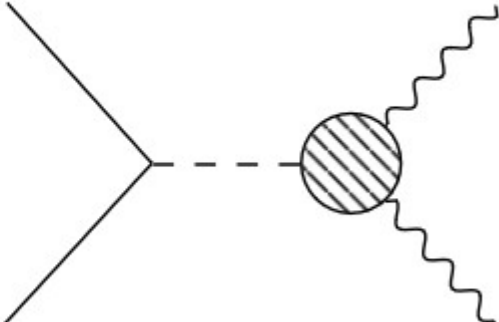
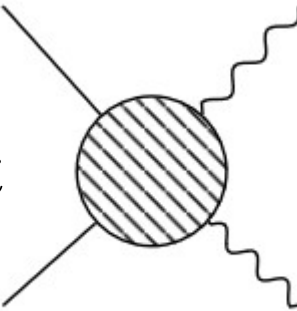
Amplitude Structure

$$\mathcal{M}(2 \rightarrow 2) =$$


The diagram shows the amplitude structure for a 2 to 2 scattering process. On the left, the expression $\mathcal{M}(2 \rightarrow 2) =$ is followed by a Feynman diagram representing a t-channel exchange. Two solid lines on the left meet at a vertex, and two wavy lines on the right meet at another vertex, connected by a dashed line. To the right of this is a plus sign, followed by a summation $\sum_i c_i$. A blue arrow points from the text box below to the coefficient c_i . To the right of the summation is another Feynman diagram representing a contact term. Two solid lines and two wavy lines all meet at a single central shaded circular vertex.

Each independent contact-term comes with a coefficient

Amplitude Structure

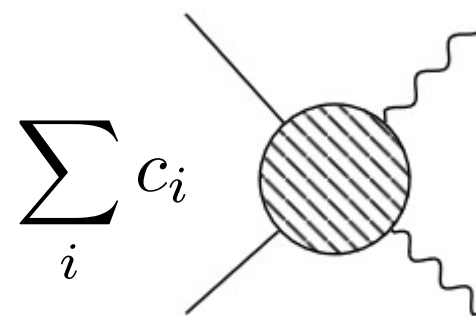
$$\mathcal{M}(2 \rightarrow 2) =$$

$$+ \sum_i c_i$$


Combining all possible contact term contributions gives the most general EFT amplitude.

Contact with SMEFT

Standard Model EFT

$$\mathcal{L} = \mathcal{L}_{SM} + \sum_i \sum_{d>4} c_i \Lambda^{4-d} \mathcal{O}^i$$



$$\sum_i c_i$$

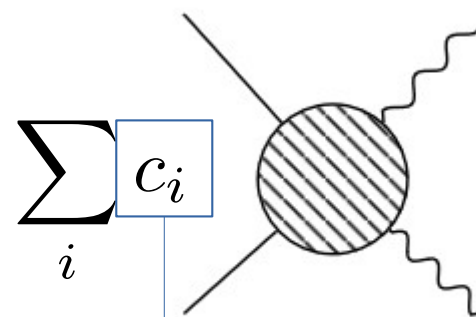
There is a one-to-one correspondence between the massless EFT amplitudes and SMEFT Lagrangian operators

Shadmi, Weiss '18

Contact with SMEFT

Standard Model **EFT**

$$\mathcal{L} = \mathcal{L}_{SM} + \sum_i \sum_{d>4} \boxed{c_i} \Lambda^{4-d} \mathcal{O}^i$$



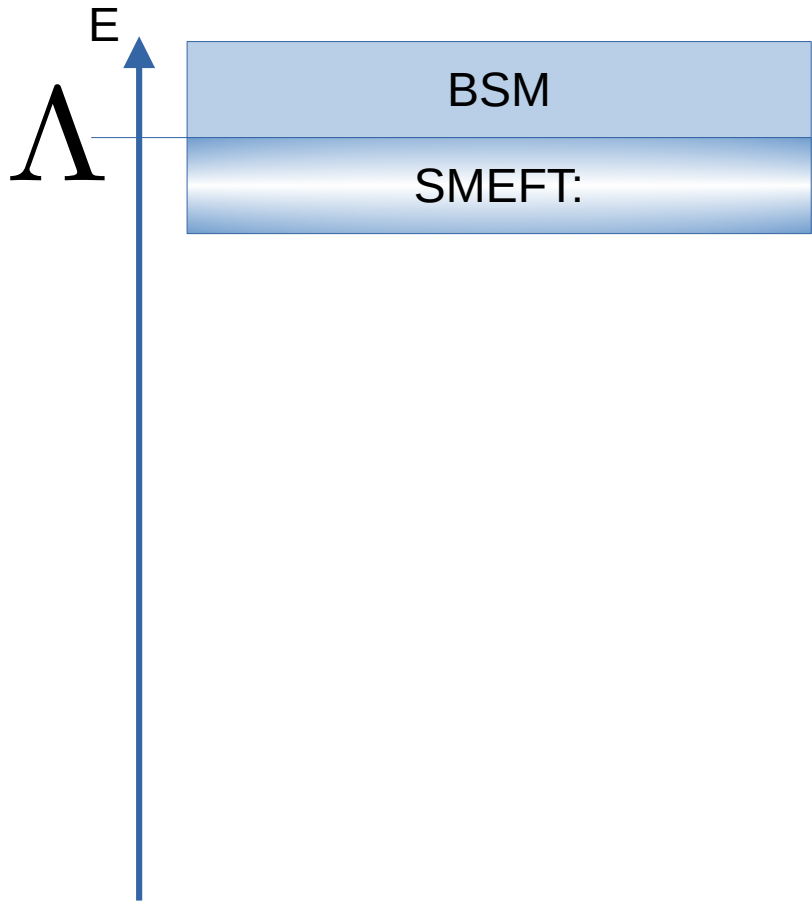
Wilson Coefficients: Parameterization of New Physics effects

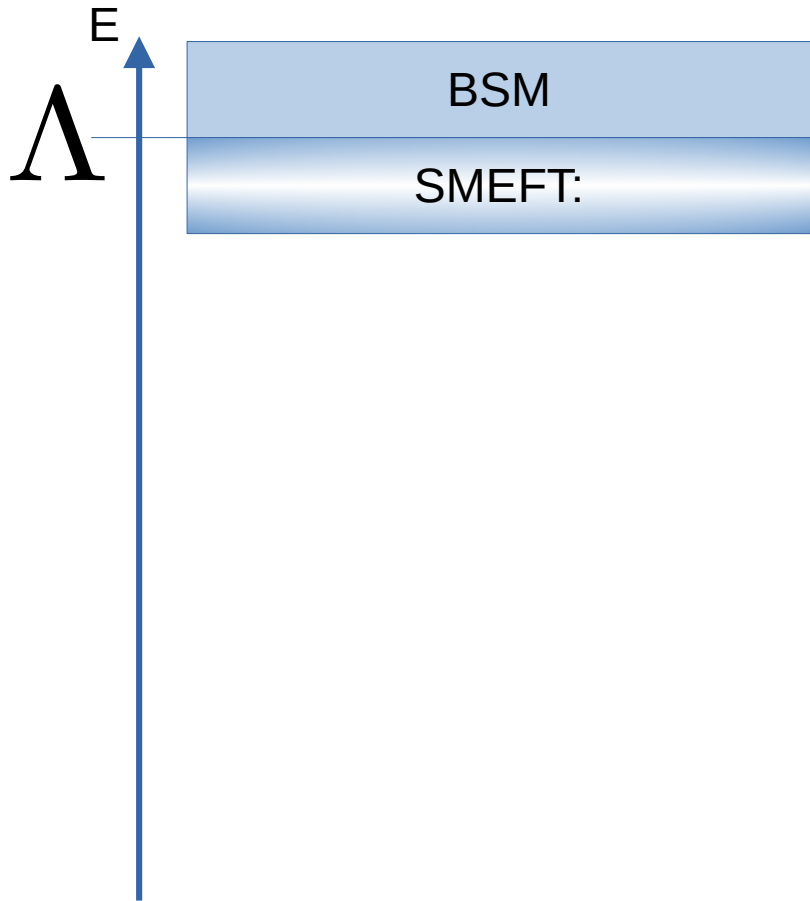
Contact with SMEFT

Standard Model **EFT**

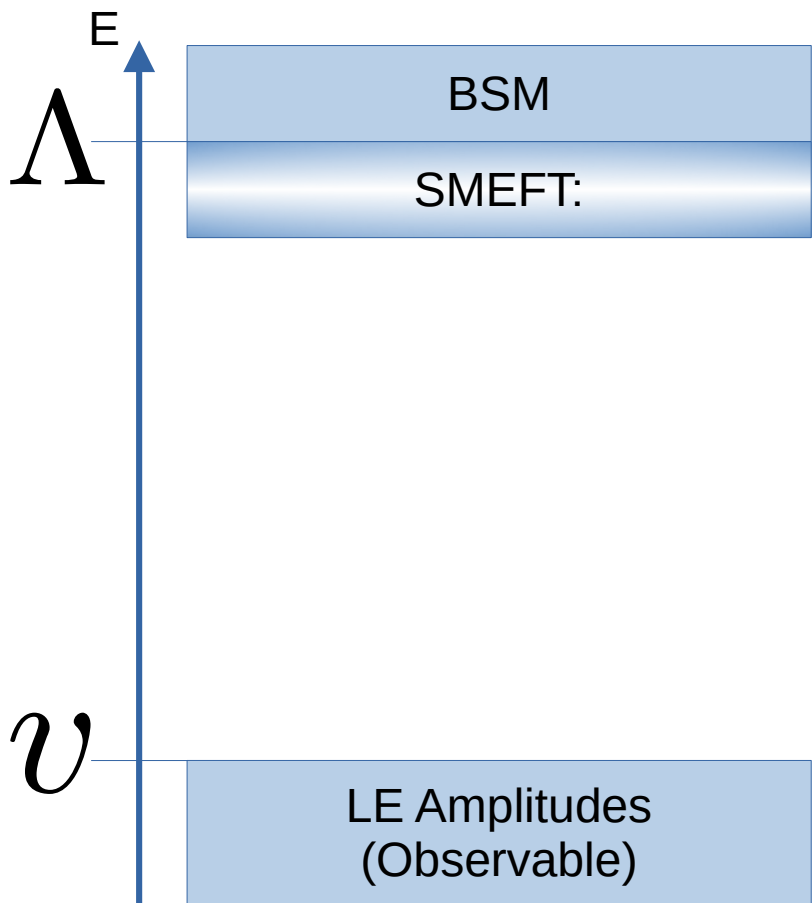
$$\mathcal{L} = \mathcal{L}_{SM} + \sum_i \sum_{d>4} c_i \Lambda^{4-d} \mathcal{O}^i$$


$$\dim [\mathcal{O}] = 8$$



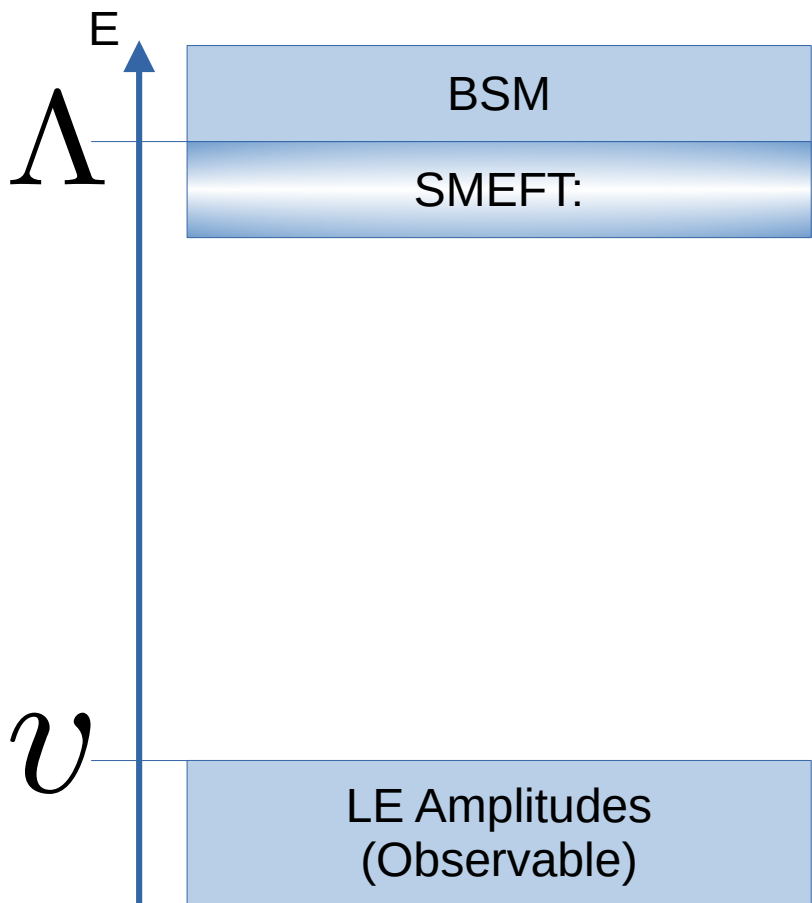


$$\mathcal{A}_{\text{SMEFT}} : \quad SU(3) \times SU(2) \times U(1)$$

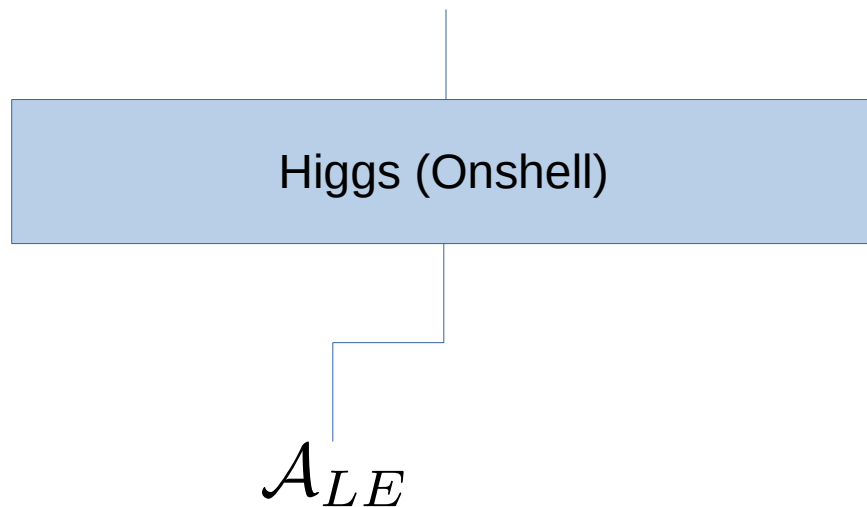


$$\mathcal{A}_{\text{SMEFT}} : \quad SU(3) \times SU(2) \times U(1)$$

$$\mathcal{A}_{LE}$$



$$\mathcal{A}_{\text{SMEFT}} : \quad SU(3) \times SU(2) \times U(1)$$



Onshell Methods: Spinor Variables

The Little Group

States in QFT

$$|p, \sigma\rangle = U(L(p; k)) |k, \sigma\rangle$$

The Little Group

States in QFT

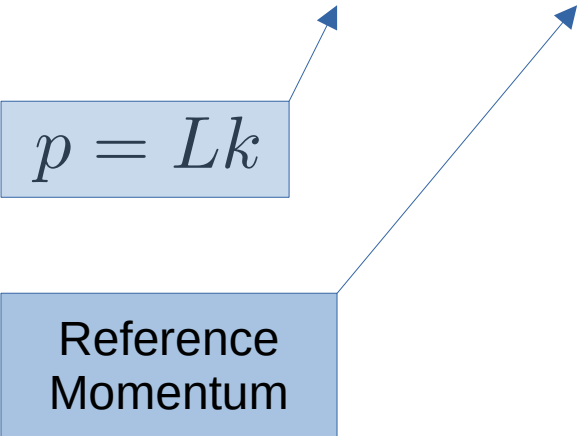
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$$p = Lk$$


The Little Group

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Reference
Momentum

The Little Group

States in QFT

$$|p, \sigma\rangle = U(L(p; k)) |k, \sigma\rangle$$

Λ — Generic Lorentz Transf

$$|p, \sigma\rangle \rightarrow U(\Lambda) |p, \sigma\rangle = D_{\sigma\sigma'}(W(\Lambda, p; k)) |\Lambda p, \sigma'\rangle$$

The Little Group

States in QFT

$$|p, \sigma\rangle = U(L(p; k)) |k, \sigma\rangle$$

Λ — Generic Lorentz Transf

$$|p, \sigma\rangle \rightarrow U(\Lambda) |p, \sigma\rangle = D_{\sigma\sigma'}(W(\Lambda, p; k)) |\Lambda p, \sigma'\rangle$$

Representation of the “Little Group,” leaves

From the Lorentz invariance of the S matrix:

$$\mathcal{M}(p_a, \sigma_a) = \delta^{(D)}(p_1^\mu, \dots, p_n^\mu) M(p_a, \sigma_a)$$

$$M(p_a, \sigma_a) \xrightarrow{\Lambda} \prod_a (D_{\sigma_a, \sigma'_a}(W)) M(\Lambda p_a, \sigma'_a)$$

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► Amplitudes transform with representations of the LG!

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$$M(p_a, \sigma_a) \xrightarrow{\Lambda} \prod_a (D_{\sigma_a, \sigma'_a}(W)) M(\Lambda p_a, \sigma'_a)$$

Amplitudes transform with representations of the LG!

Admit construction as symmetric tensors

The Lorentz Group: $SO(3,1) \cong SU(2) \times SU(2)$

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- Basic building blocks are 2-component spinors

Spinors!

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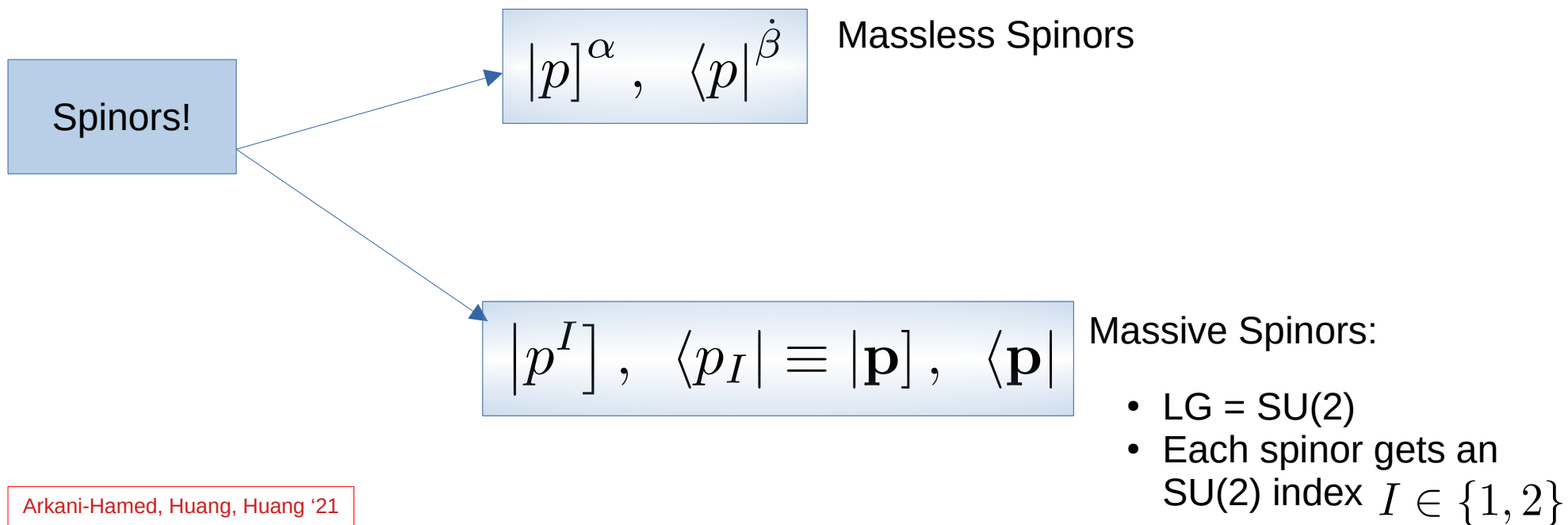
$$|p]^\alpha, \quad \langle p|^\dot{\beta}$$

Massless Spinors

- LG = U(1)
- Bra-kets represent *helicity weights*:
 - Square = +1
 - Angle = -1

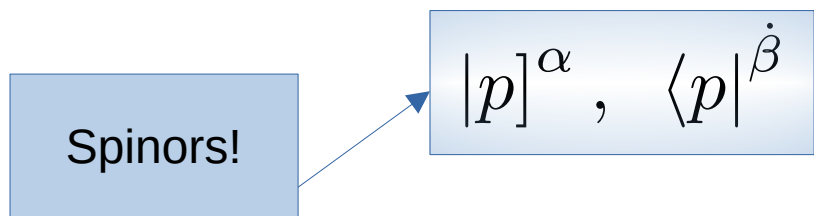
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For Massless Spinors

- Contracting the spinor indices defines spinor bra-kets
- These are LG covariant objects!

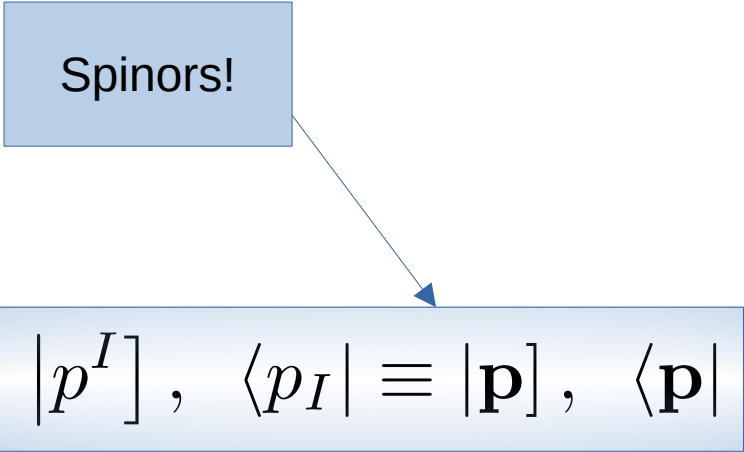
$$[p_1 p_2] \equiv [p_1]^\alpha [p_2]_\alpha$$

$$\langle p_1 p_2 \rangle \equiv \langle p_1 |^{\dot{\beta}} |p_2 \rangle_{\dot{\beta}}$$

The Lorentz Group: $SO(3,1) \cong SU(2) \times SU(2)$

- Basic building blocks are 2-component spinors

Spinors!



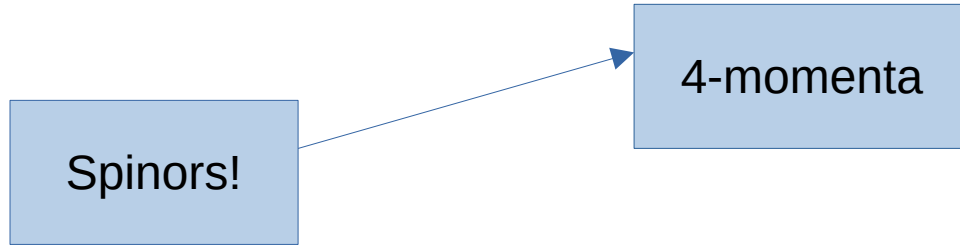
$$|p^I], \quad \langle p_I| \equiv |\mathbf{p}], \quad \langle \mathbf{p}|$$

For Massive Spinors

- Amplitudes are represented by *symmetric* tensors
- SU(2) indices must be symmetrized $\rightarrow$ Bold notation

$$[\mathbf{p}q][\mathbf{p}r] \equiv \begin{cases} [p_1^I q] [p_1^J r] & I = J \\ [p_1^I q] [p_1^J r] + [p_1^J q] [p_1^I r] & I \neq J \end{cases}$$

Onshell Methods: Amplitude Construction



Spinors!

4-momenta

$$p^\mu \rightarrow p^{\alpha\dot{\beta}} \equiv p^\mu \sigma_\mu^{\alpha\dot{\beta}}$$

Spinors!

4-momenta

$$p^\mu \rightarrow p^{\alpha\dot{\beta}} \equiv p^\mu \sigma_\mu^{\alpha\dot{\beta}} \longrightarrow |p]^\alpha \langle p|^{\dot{\beta}} \quad \text{Massless: rank(1)}$$

Spinors!

4-momenta

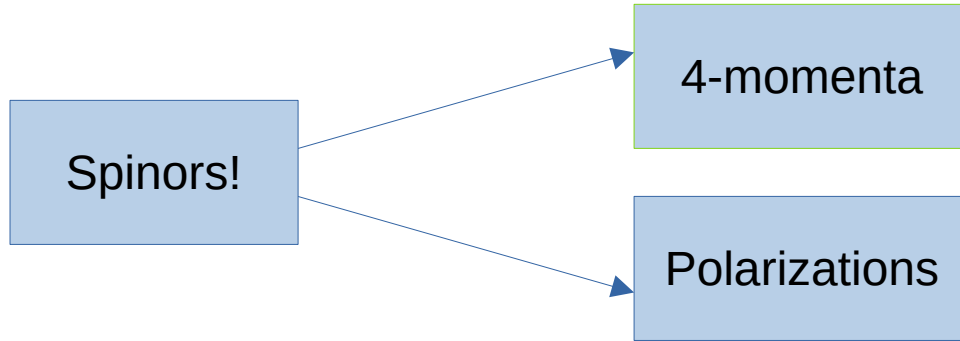
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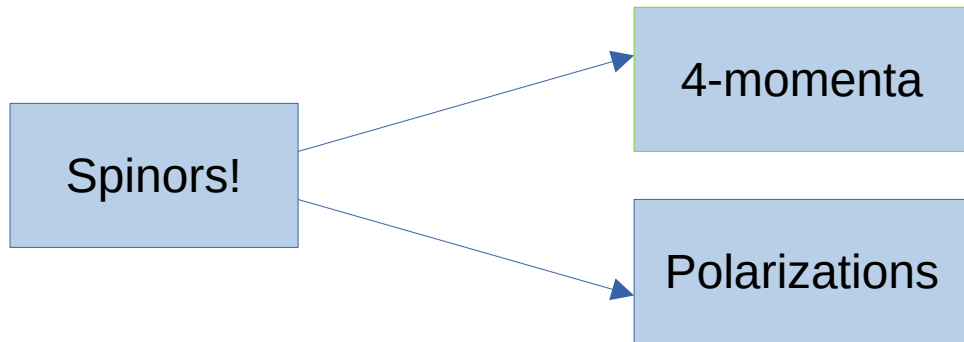
$$|p]^\alpha \langle p|^\dot{\beta}$$

Massless: rank(1)

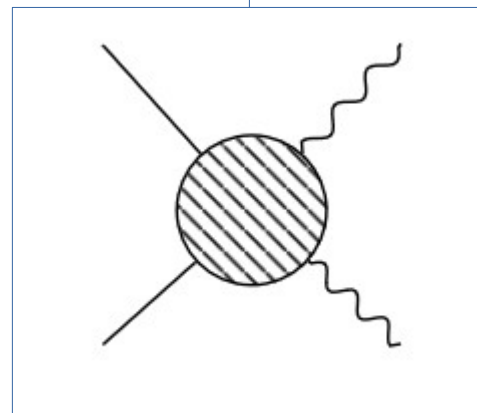
$$|p^I] \langle p_I| \equiv |\mathbf{p}] \langle \mathbf{p}|$$

Massive: rank(2)

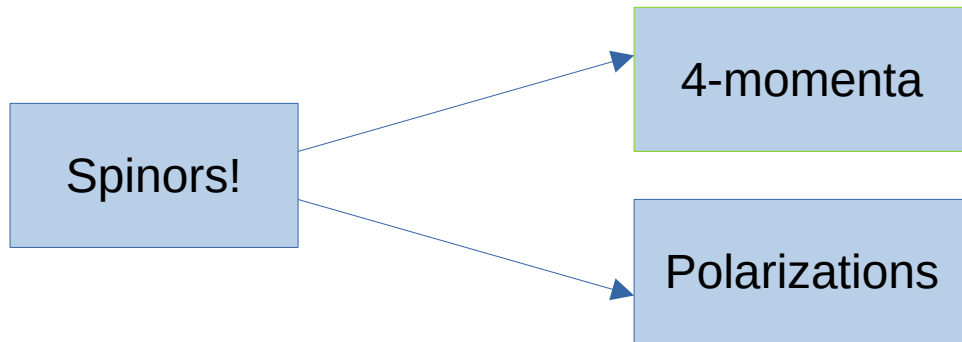




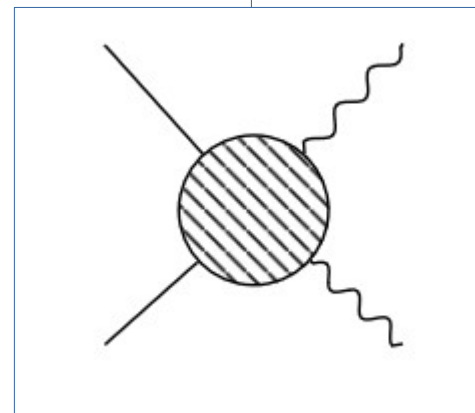
Massless Fermions:
 $|p]$ or $|p\rangle$



Massless Vectors
 $|p] |p]$ or $|p\rangle |p\rangle$



Massive Fermions:
 $|p^I]$ or $|p^I\rangle$



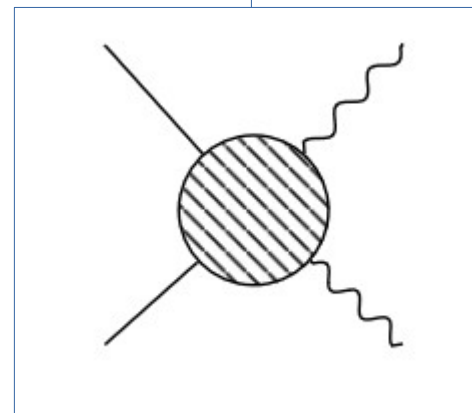
Massive Vectors
 $|p]^{I} |p]^{J}\}$ or $|p\rangle^{I} |p\rangle^{J}\}$

Spinors!

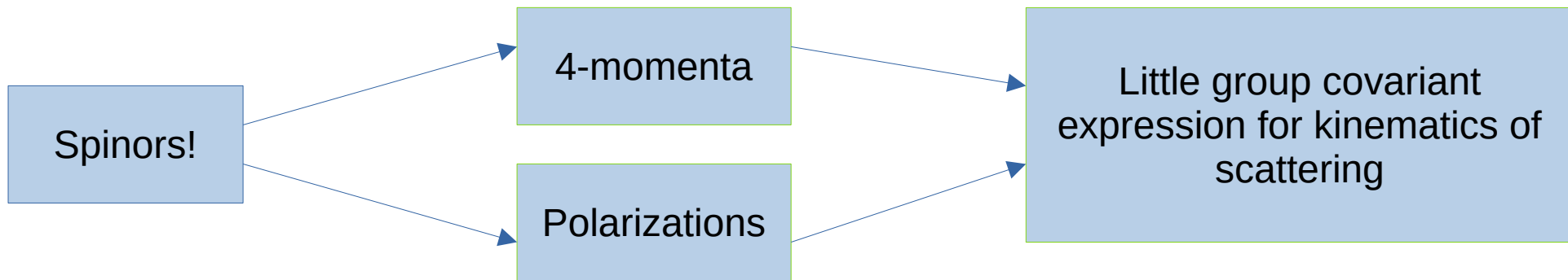
4-momenta

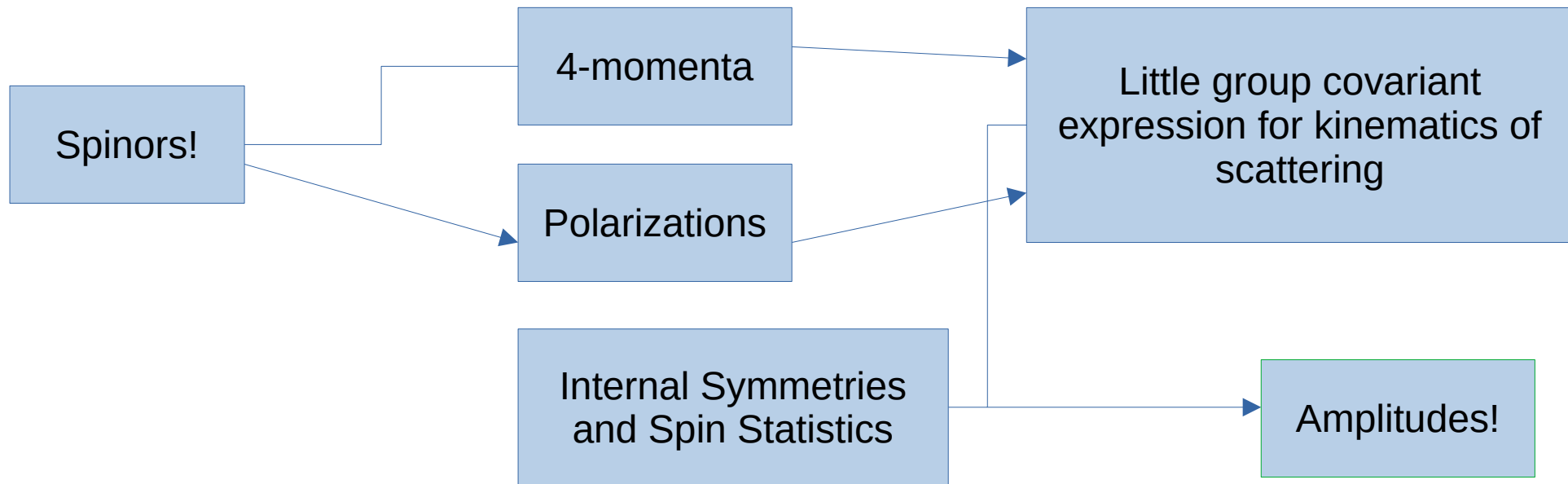
Polarizations

Massive Fermions:
 $|p^I]$ or $|p^I\rangle \rightarrow |\mathbf{p})$



Massive Vectors
 $|p]^{\{I} |p\}^{J\}}$ or $|p\rangle^{\{I} |p\rangle^{J\}} \rightarrow |\mathbf{p}) | \mathbf{p})$





Example:

$$W_+^a(1)W_+^b(2)W_+^c(3)W_+^d(4)$$

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HE fields

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1) Kinematic Terms

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← SMEFT amplitudes are for the
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$$|p] \times 2, \quad p \in \{1, 2, 3, 4\}$$

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SMEFT amplitudes are for the
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1) Kinematic Terms

$$|p] \times 2, \quad p \in \{1, 2, 3, 4\}$$

Contract spinors

$$\begin{aligned} & [12]^2 [34]^2 \\ & [13]^2 [24]^2 \\ & [14]^2 [23]^2 \end{aligned}$$

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SMEFT amplitudes are for the
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$[12]^2 [34]^2$

$[13]^2 [24]^2$

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Terms like:

$$[12] [23] [34] [14]$$

Are linearly dependent on
the three listed terms

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2) Group Theory Terms

$$\delta^{ab}\delta^{dc}, \quad \delta^{ac}\delta^{bd}, \quad \delta^{ad}\delta^{bc}$$

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Four identical particles, **symmetrize** over 1a,2b,3c,4d:



Example:

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← SMEFT amplitudes are for the HE fields

1) Kinematic Terms

$$[12]^2 [34]^2 \quad [13]^2 [24]^2 \quad [14]^2 [23]^2$$

2) Group Theory Terms

$$\delta^{ab}\delta^{dc}, \delta^{ac}\delta^{bd}, \delta^{ad}\delta^{bc}$$

Four identical particles, **symmetrize** over 1a,2b,3c,4d:

$$[12]^2 [34]^2 \delta^{ab}\delta^{cd} + \text{Permutations}(1a, 2b, 3c, 4d)$$

$$([13]^2 [24]^2 + [14]^2 [23]^2)\delta^{ab}\delta^{cd} + \text{Permutations}(1a, 2b, 3c, 4d)$$

Example:

$$W_+^a(1)W_+^b(2)W_+^c(3)W_+^d(4)$$

← SMEFT amplitudes are for the HE fields

1) Kinematic Terms

$$[12]^2 [34]^2 \quad [13]^2 [24]^2 \quad [14]^2 [23]^2$$

2) Group Theory Terms

$$\delta^{ab}\delta^{dc}, \delta^{ac}\delta^{bd}, \delta^{ad}\delta^{bc}$$

Four identical particles, **symmetrize** over 1a,2b,3c,4d:

Each term
gets an
independent
coefficient



$$c_1 [12]^2 [34]^2 \delta^{ab} \delta^{cd} + \text{Permutations}(1a, 2b, 3c, 4d)$$

$$c_2 ([13]^2 [24]^2 + [14]^2 [23]^2) \delta^{ab} \delta^{cd} + \text{Permutations}(1a, 2b, 3c, 4d)$$

C,C,P

In the Lagrangian formalism, operators must be hermitian, e.g.:

$$\mathcal{O}_1 \equiv c_1 \bar{u} \gamma^\nu u H^\dagger \vec{D}^\mu H B_{\mu\nu}$$

 $c_1 = c_1^* \Rightarrow c_1 \in \mathbb{R}$

C,C,P

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$$\mathcal{O}_1 \equiv c_1 \bar{u} \gamma^\nu u H^\dagger \vec{D}^\mu H B_{\mu\nu}$$

 $c_1 = c_1^* \Rightarrow c_1 \in \mathbb{R}$

Or come with hermitian conjugates, e.g.:

$$\mathcal{O}_2 \equiv c_2 \bar{u} \gamma^\nu d \tilde{H}^\dagger \vec{D}^\mu H B_{\mu\nu} + c_3 \bar{d} \gamma^\nu u H^\dagger \vec{D}^\mu \tilde{H} B_{\mu\nu}$$

 $\Rightarrow c_3 = c_2^*$

C,C,P

This hermiticity is not built into amplitude construction, so must be imposed:

$$c_1 [23] [3(4 - 5)1\rangle$$

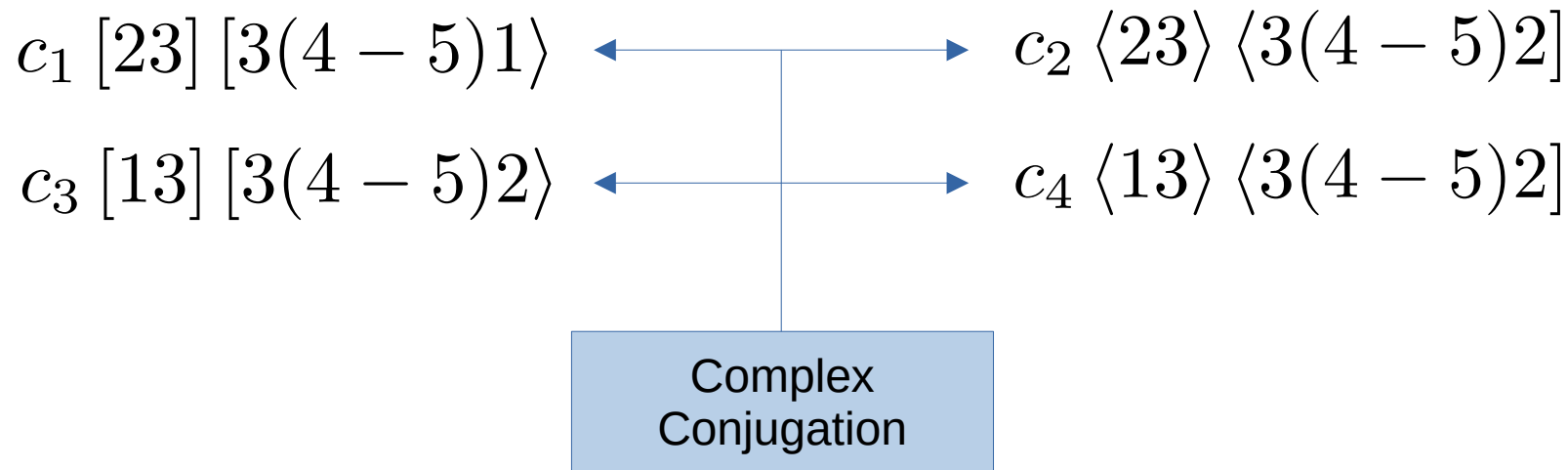
$$c_2 \langle 23 \rangle \langle 3(4 - 5)2]$$

$$c_3 [13] [3(4 - 5)2\rangle$$

$$c_4 \langle 13 \rangle \langle 3(4 - 5)2]$$

C,C,P

This hermiticity is not built into amplitude construction, so must be imposed:



C, C, P

This hermiticity is not built into amplitude construction, so must be imposed:

$$c_1 [23] [3(4-5)1] \longleftrightarrow c_2 \langle 23 \rangle \langle 3(4-5)2 \rangle$$

$$c_3 [13] [3(4-5)2] \longleftrightarrow c_4 \langle 13 \rangle \langle 3(4-5)2 \rangle$$

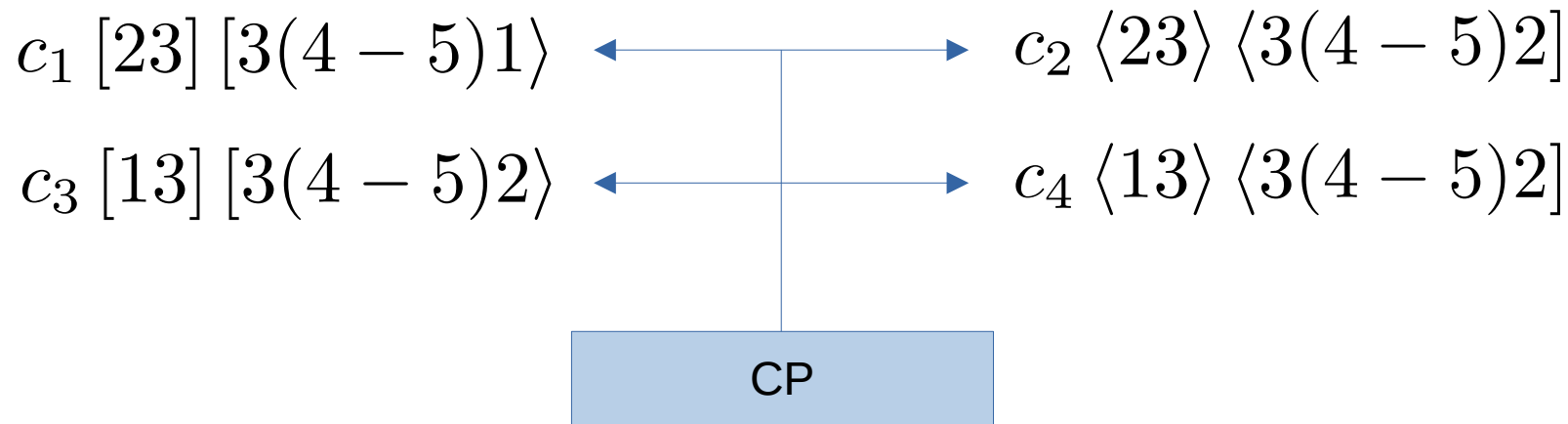
Complex
Conjugation

$$\Rightarrow c_2 = c_1^*, \quad c_4 = c_3^*$$

4 complex coefficient $\rightarrow$ 2 complex coefficients. This counting applies to all amplitudes.

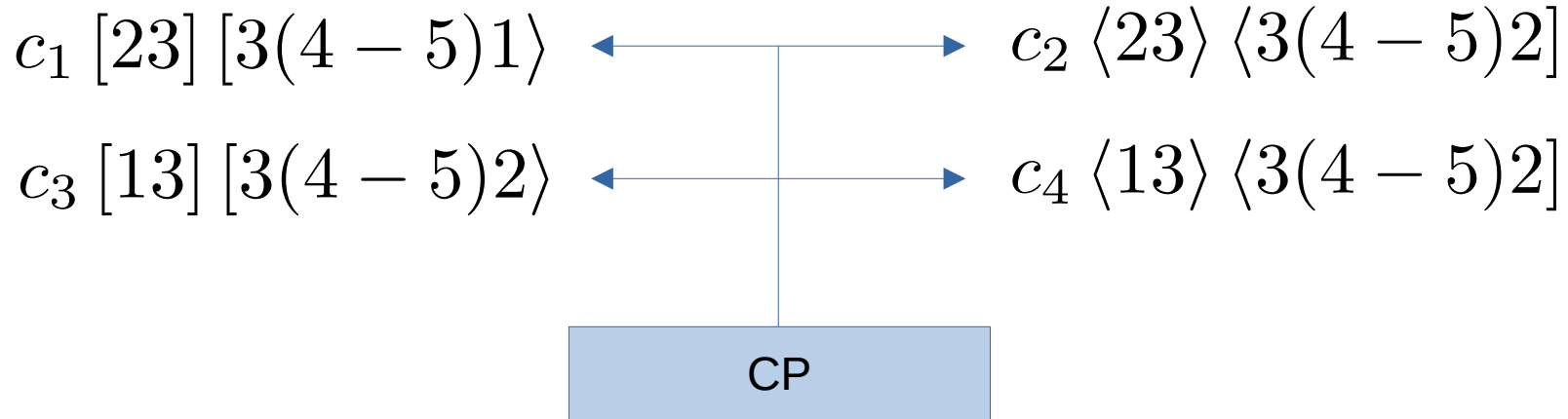
C, C, P

These amplitudes are also related by CP transformations (not a CP eigenbasis):



C, C, P

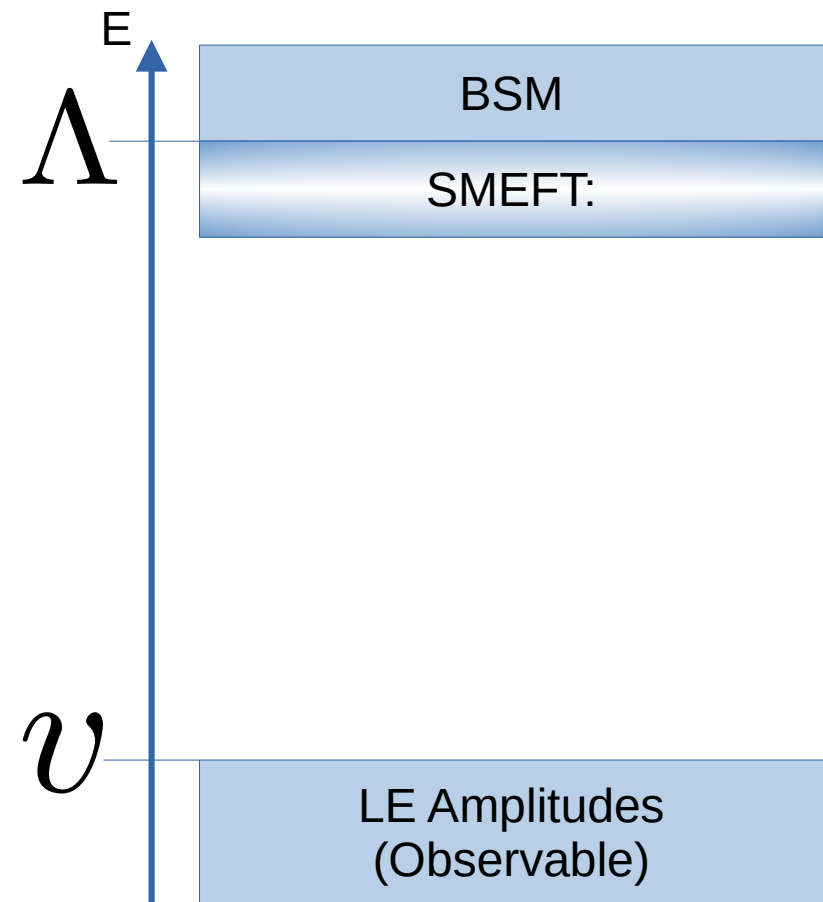
These amplitudes are also related by CP transformations:



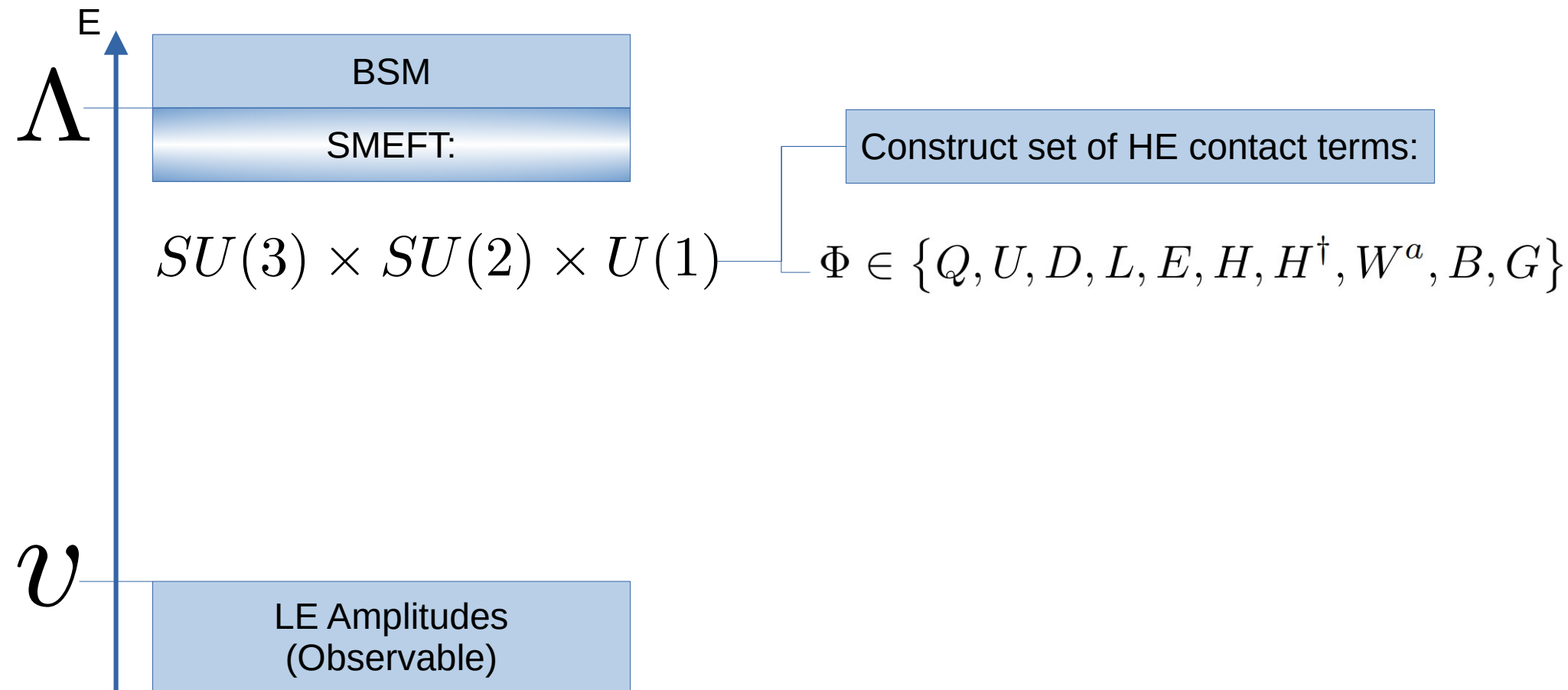
$$\Rightarrow c_2 = c_1, \quad c_4 = c_3$$

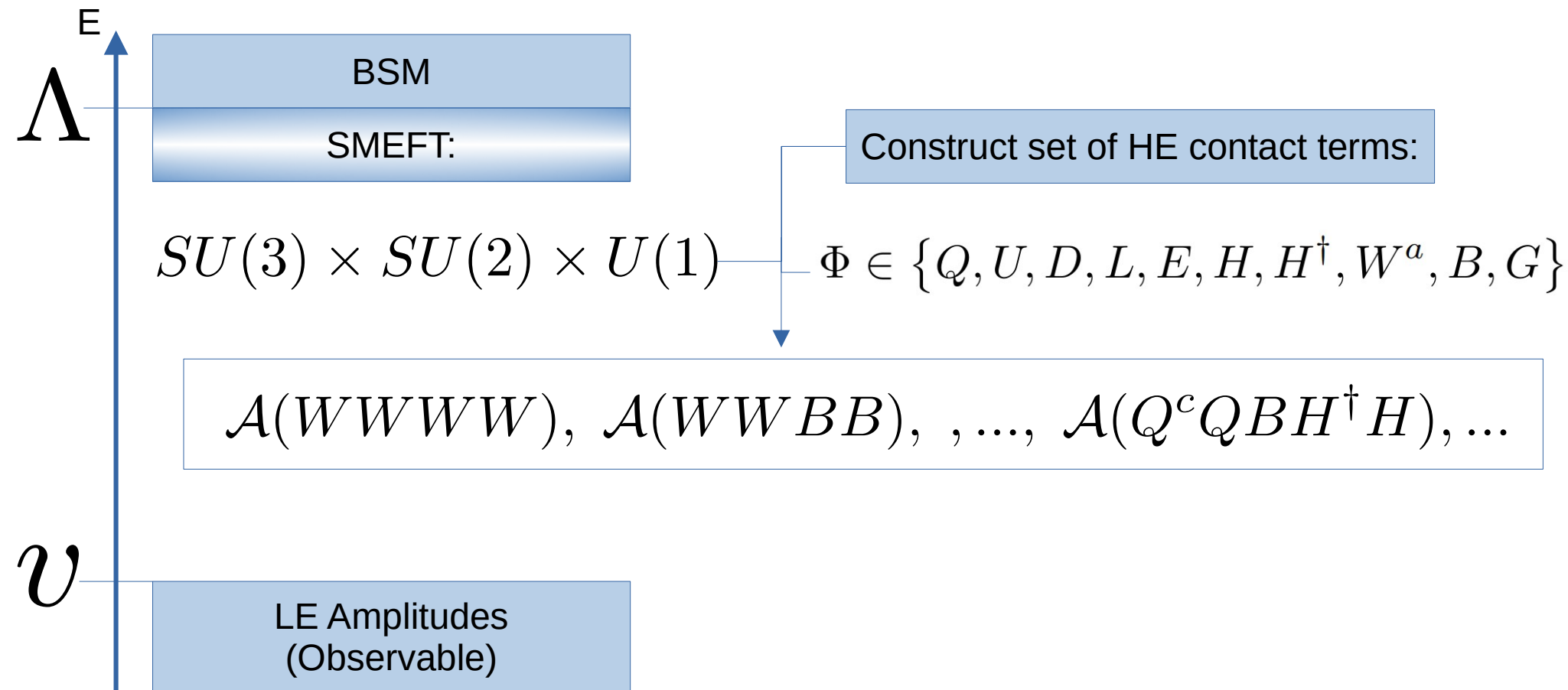
Demanding CP invariance: 2 complex coefficients $\rightarrow$ two real coefficients

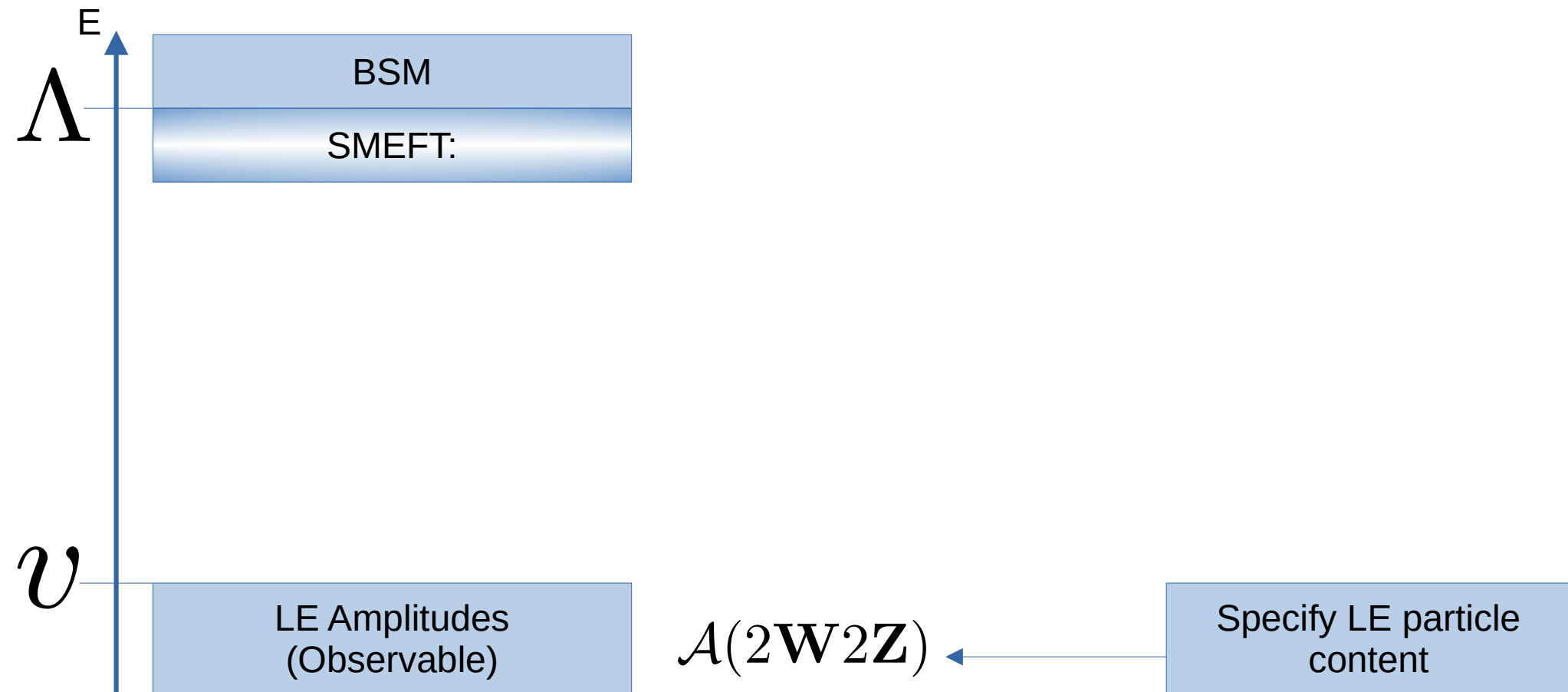
Construction of LE (Massive) SMEFT Amplitudes

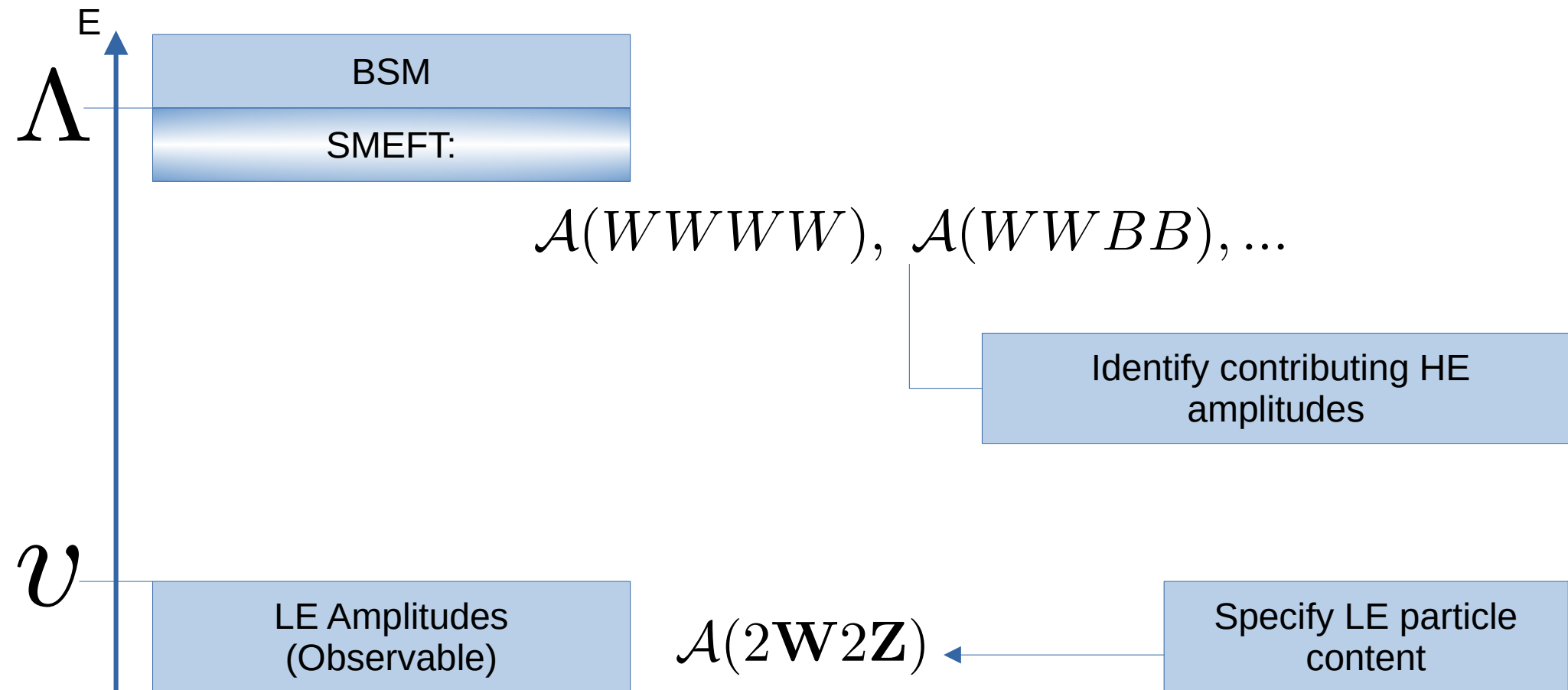


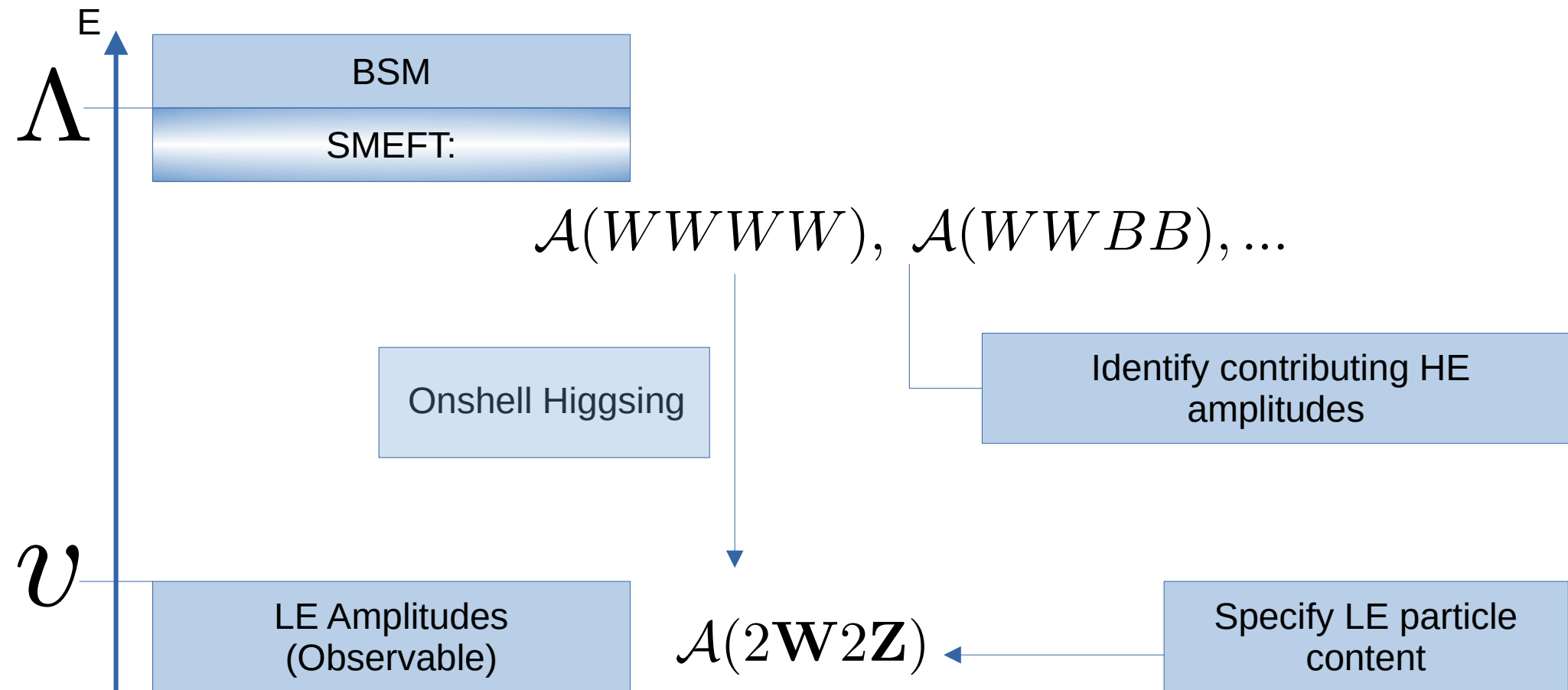
Construct set of HE contact terms:



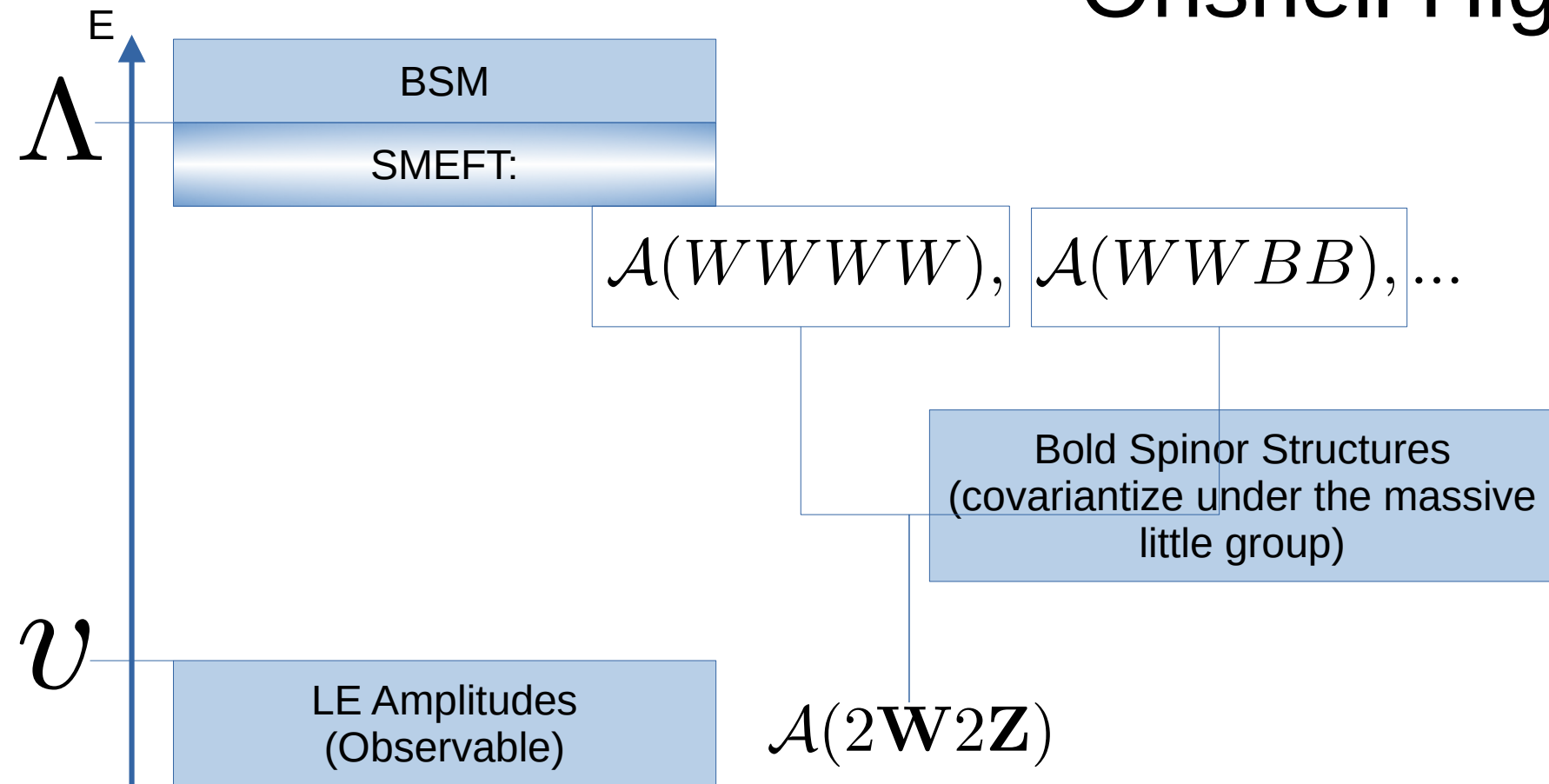




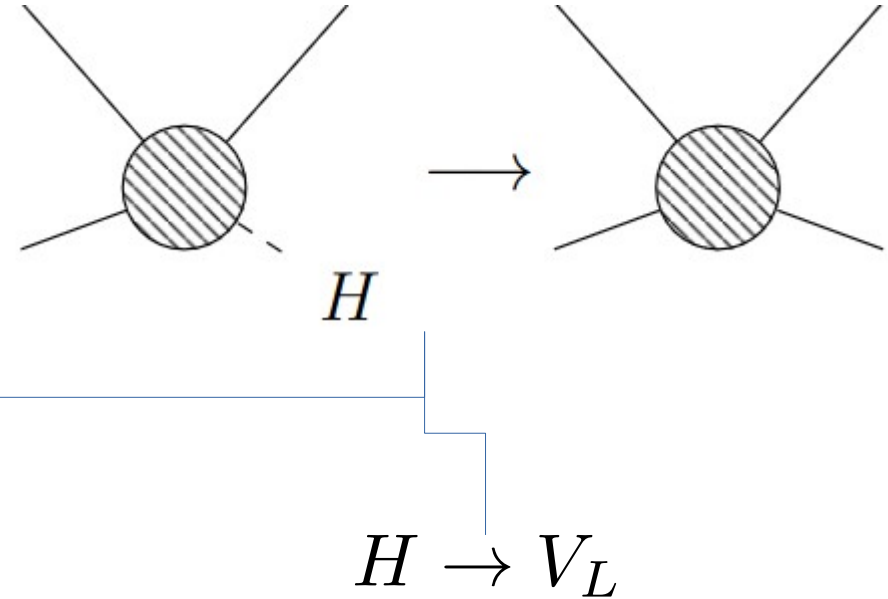
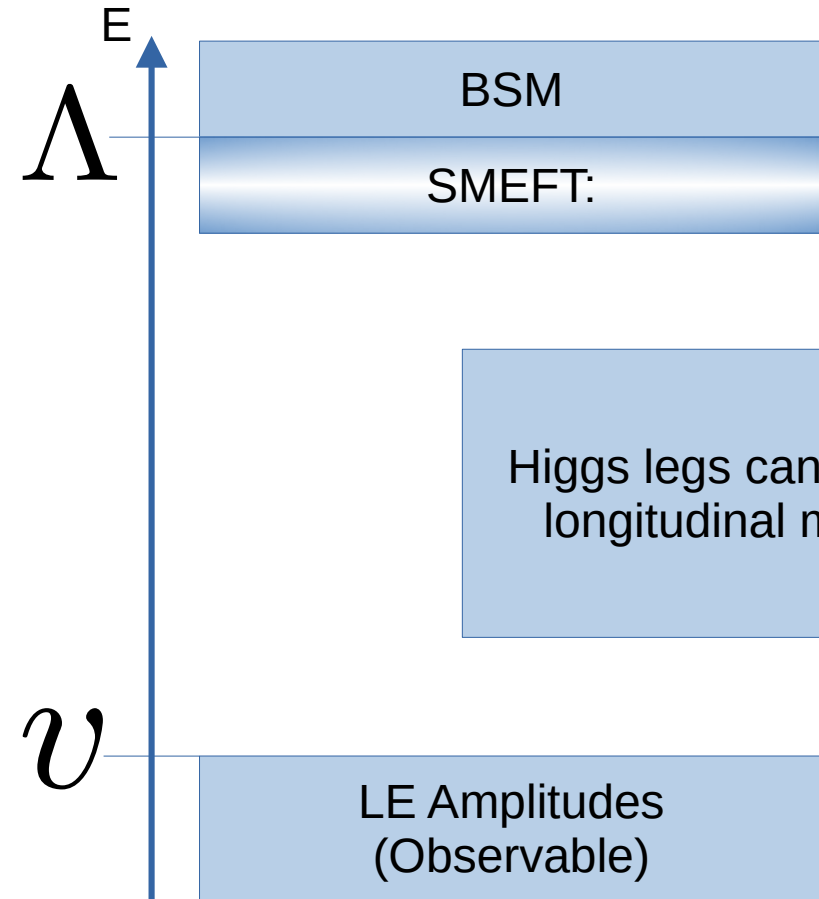




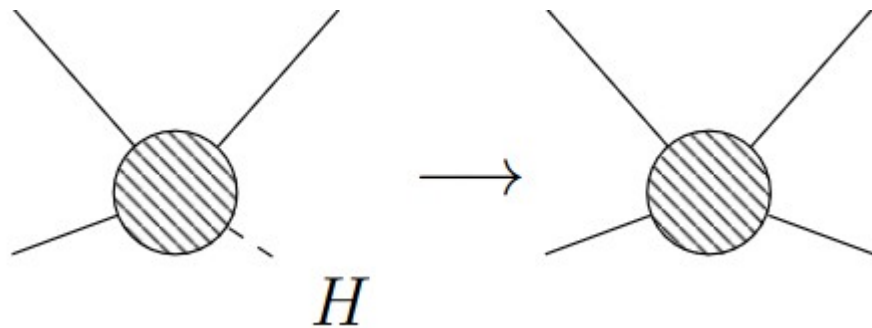
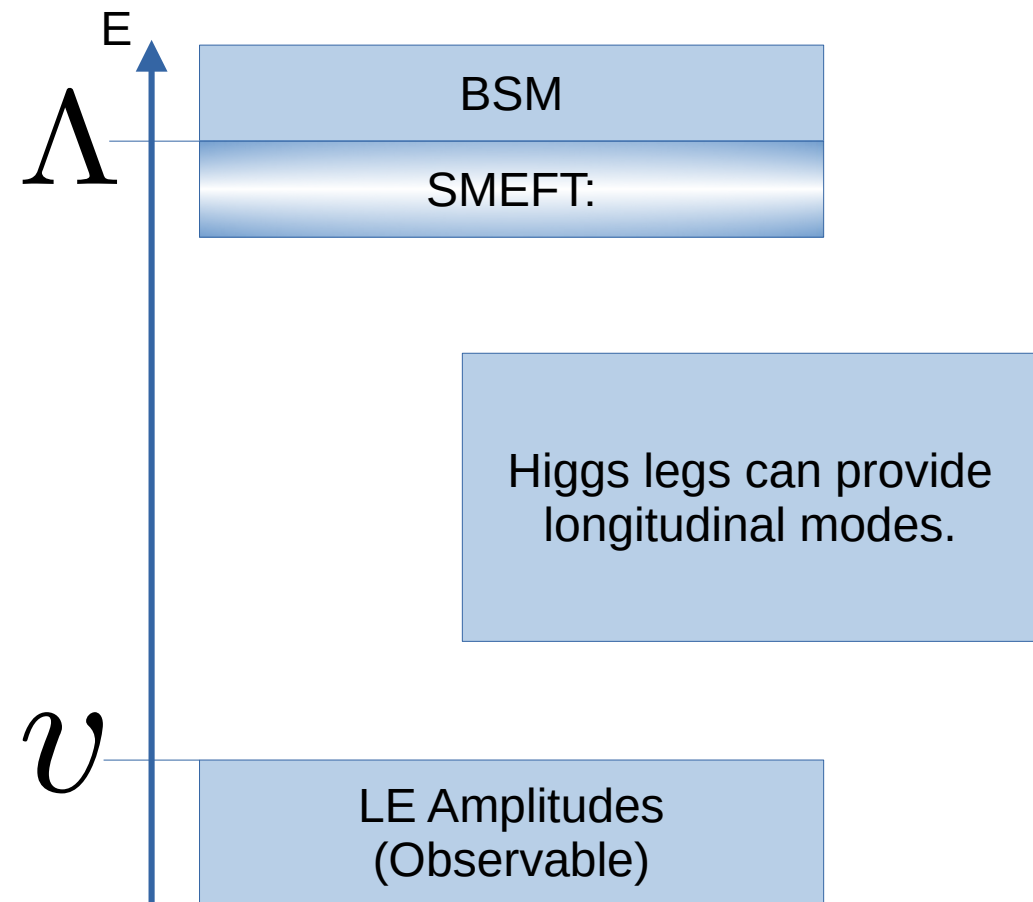
Onshell Higgsing



Onshell Higgsing



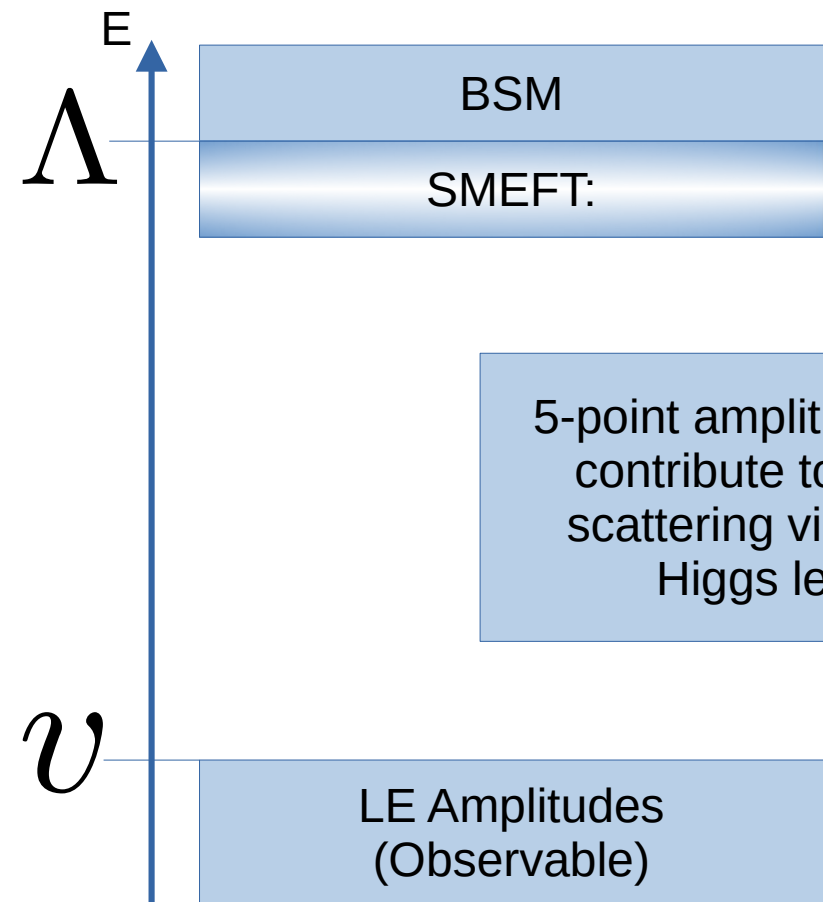
Onshell Higgsing



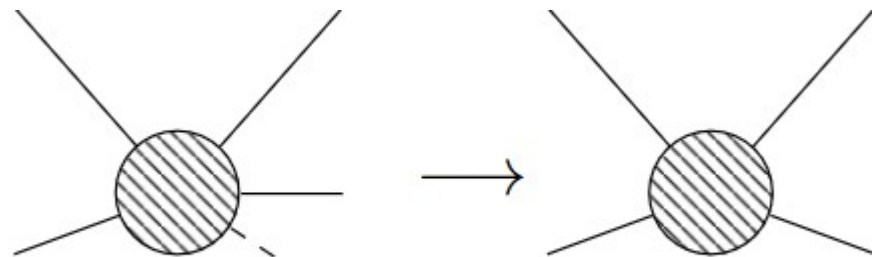
$$H \rightarrow V_L$$

$$|p_H] \langle p_H| \rightarrow |\mathbf{p}_H] \langle \mathbf{p}_H|$$

Onshell Higgsing

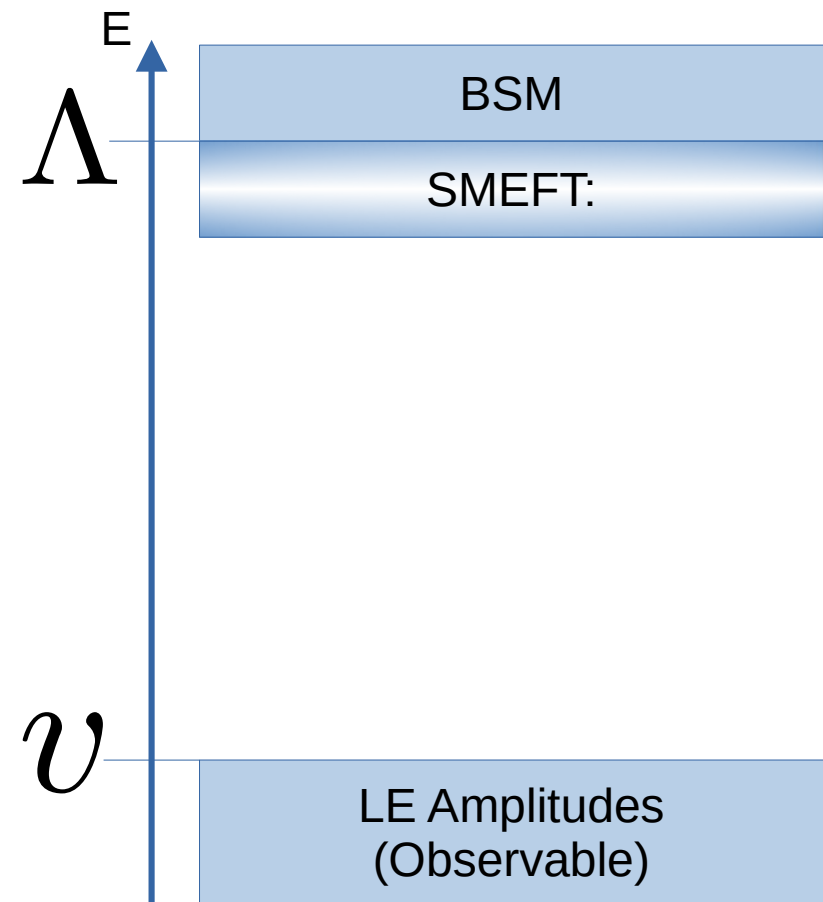


5-point amplitudes can contribute to 2-to2 scattering via extra Higgs legs



$$H \rightarrow \frac{1}{\sqrt{2}}(h + v)$$

$$p_h \rightarrow 0$$

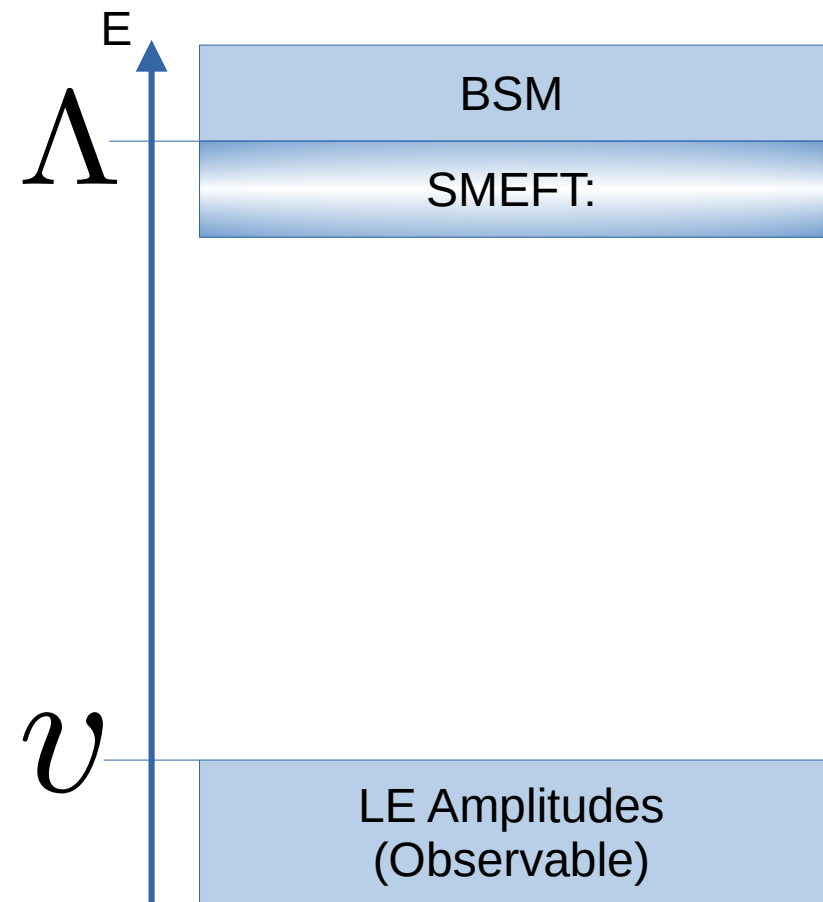


$$\mathcal{A}(Q^c Q B H^\dagger H)$$

5-point amplitudes can contribute to 2-to2 scattering via extra Higgs legs

Higgs legs can provide longitudinal modes.

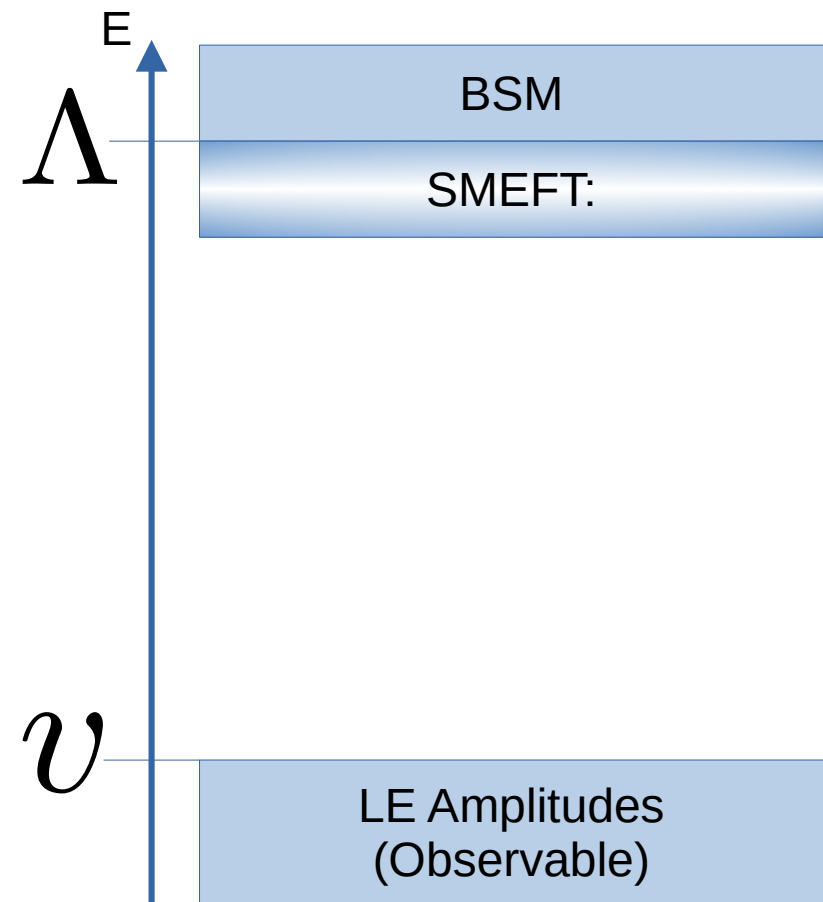
$$\mathcal{A}(u^c d 2\mathbf{Z})$$



$$\mathcal{A}(Q^c Q B H^\dagger H)$$

Transverse Z from B
field

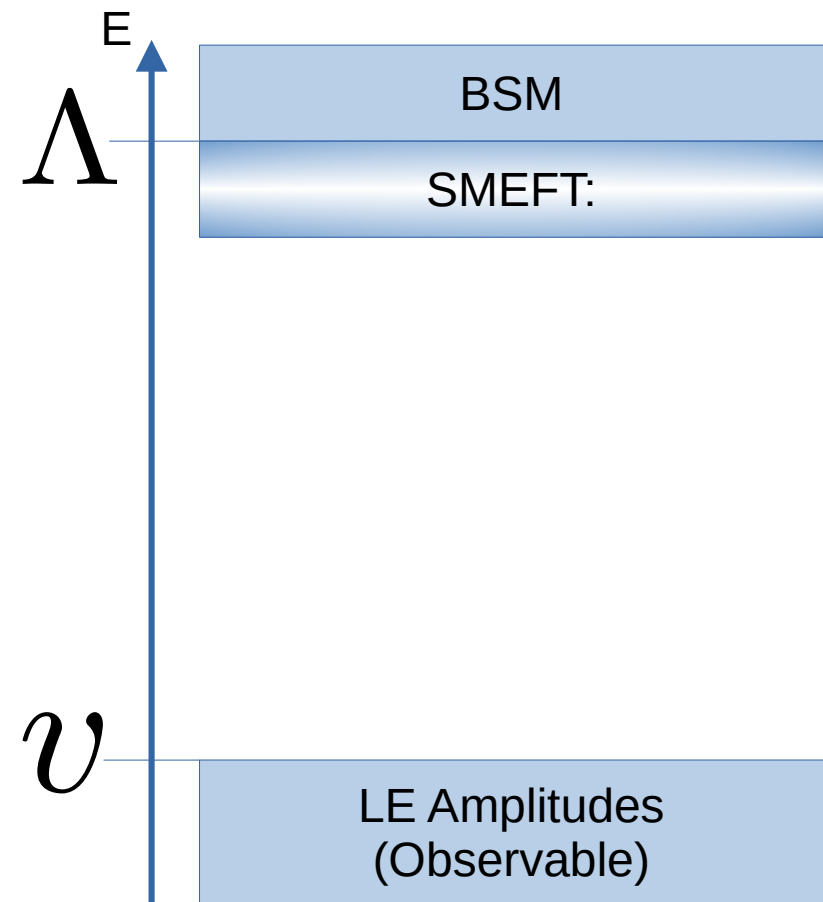
$$\mathcal{A}(u^c d 2\mathbf{Z})$$



$$\mathcal{A}(Q^c Q B H^\dagger H)$$

One of the two
Higgs legs maps to
the longitudinal
mode of the Z

$$\mathcal{A}(u^c d 2\mathbf{Z})$$



$$\mathcal{A}(Q^c Q B H^\dagger H)$$

The remaining Higgs leg is “soft,” incurs power of the VeV

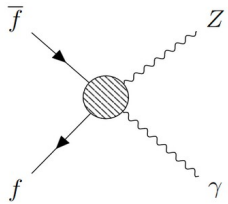
$$\mathcal{A}(u^c d 2Z) \cdot v$$

Results

Results

- Full parameterization of LE (massive) $VVVV$ and $ffVV$ contact term amplitudes generated by dim-8 SMEFT

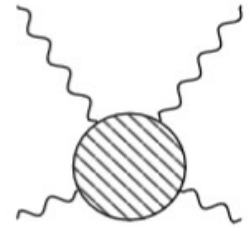
- Selection rules on kinematic configurations of observable amplitudes



$$\mathcal{A}(\bar{f}f \rightarrow Z\gamma)$$

$$[\mathbf{34}]^2 ([\mathbf{231}] - [\mathbf{241}])$$

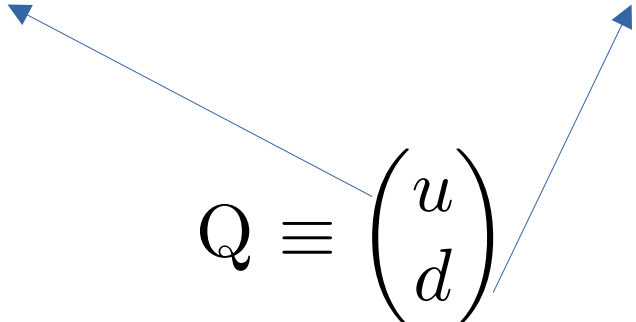
$$[\mathbf{12}]^2 [\mathbf{34}] \langle \mathbf{34} \rangle$$



$$\mathcal{A}(WW \rightarrow WW)$$

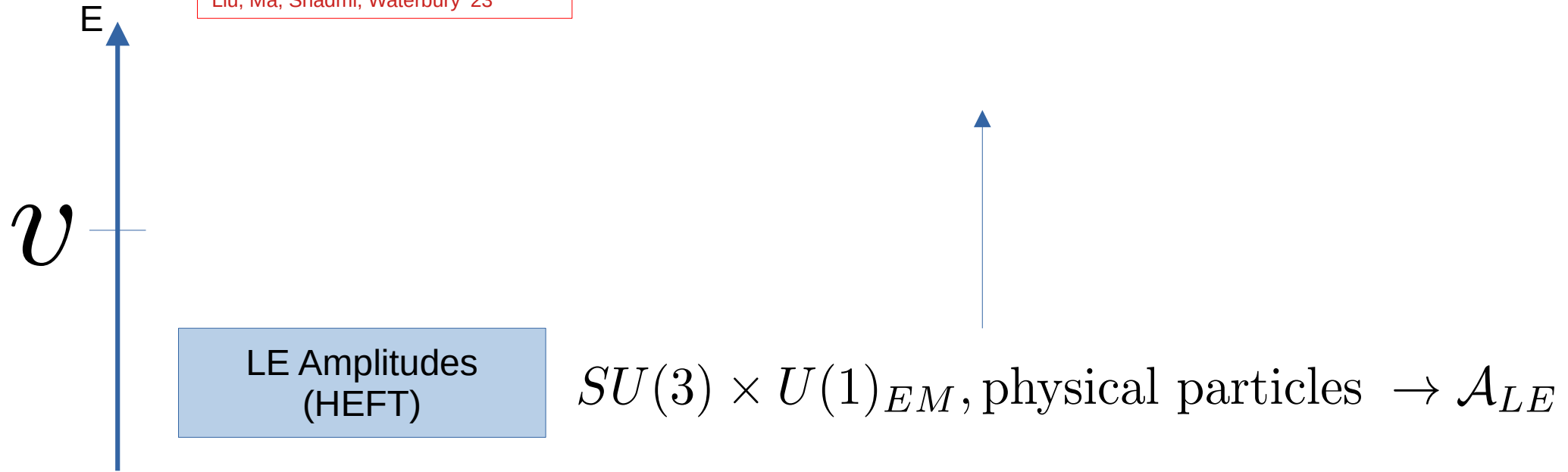
- SU(2) Symmetry Breaking pattern in the amplitudes, e.g:

Structure	$\bar{u}uW^+W^-$	$\bar{d}dW^+W^-$
$[1\mathbf{3}][\mathbf{3}\mathbf{4}]\langle\mathbf{4}\mathbf{2}\rangle$	$\frac{v}{\sqrt{2}}(c_2^{Q^c QW+2H} + c_3^{Q^c QW+2H})$	$-\frac{v}{\sqrt{2}}c_2^{Q^c QW+2H}$

$$Q \equiv \begin{pmatrix} u \\ d \end{pmatrix}$$


- Distinguish the SMEFT from bottom-up amplitude construction (HEFT):

Shadmi, Weiss '18
 Durieux, Kitahara, Shadmi, Weiss '19
 Liu, Ma, Shadmi, Waterbury '23



Summary

- Derived LE dim=8 SMEFT amplitudes using onshell Higgsing:
 - Model independent parameterization of low energy observables
 - Complete basis for specific processes: no redundancies!
 - Identification of theoretically interesting observables for study at colliders.