Asymptotic Superluminality

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(Ongoing work)

23 July 2025, Cargese 2025 International Summer School, IESC

Motivation

Causality Bounds on EFTs

- An EFT taken beyond its regime of validity displays time advance in a gravitational background.

- This time advance can be used to bound—or rule out—EFTs.

Subtleties of Gravitational Scattering

- Notion of time delay/advance requires an asymptotic Minkowski background: well-defined in $D > 4$, ambiguous in $D = 4$.

- Gauge dependence, ambiguous asymptotic structure.

[Trincherini et al., 2022] [Maldacena et al., 2014]

Gao-Wald Theorem

Criterion for Promptness (GR)

Let (M, g_{ab}) be a **null geodesically complete spacetime** satisfying the **null energy condition**:

$$R_{ab}k^a k^b \geq 0,$$

and the **null generic condition** i.e. each null geodesic contains a point in which

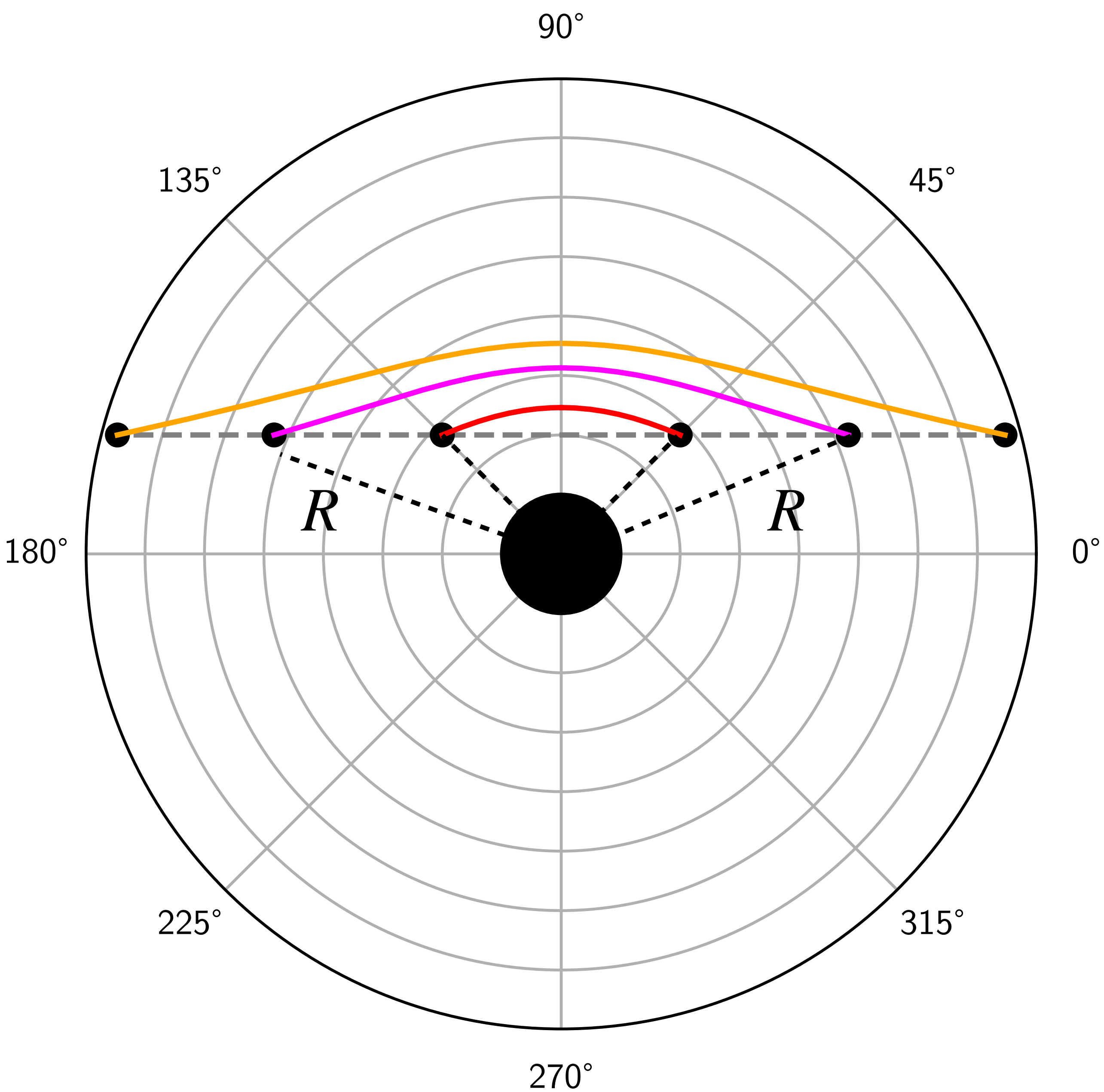
$$k_{[a}R_{b]cd[e}k_{f]}k^c k^d \neq 0,$$

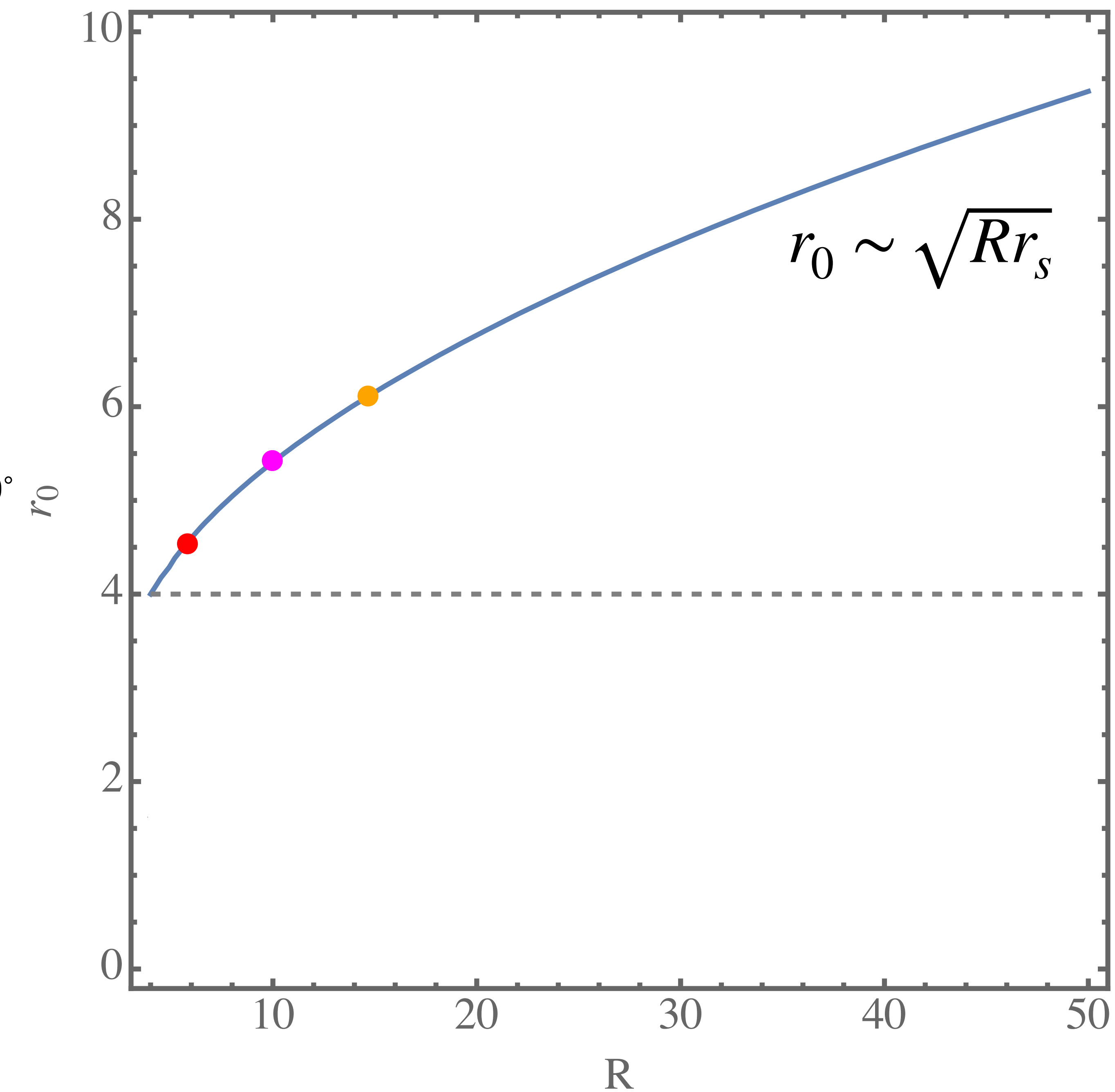
then given any compact region $K \subset M$, there exists another compact region K' such that if $q, p \notin K'$ and $q \in J^+(p) - I^+(p)$, then any causal curve γ connecting p to q cannot intersect the region K.

[Gao&Wald, 2000]

Gao-Wald Theorem

4D Schwarzschild





FFR Theory

EFT of a Maxwell Field

- The first higher-order interaction relevant in Ricci-flat ($R_{\mu\nu} = 0$) background is given by

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{M_{pl}^2}{2}R - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \alpha R_{\mu\nu\rho\sigma}F^{\mu\nu}F^{\rho\sigma}\right]$$

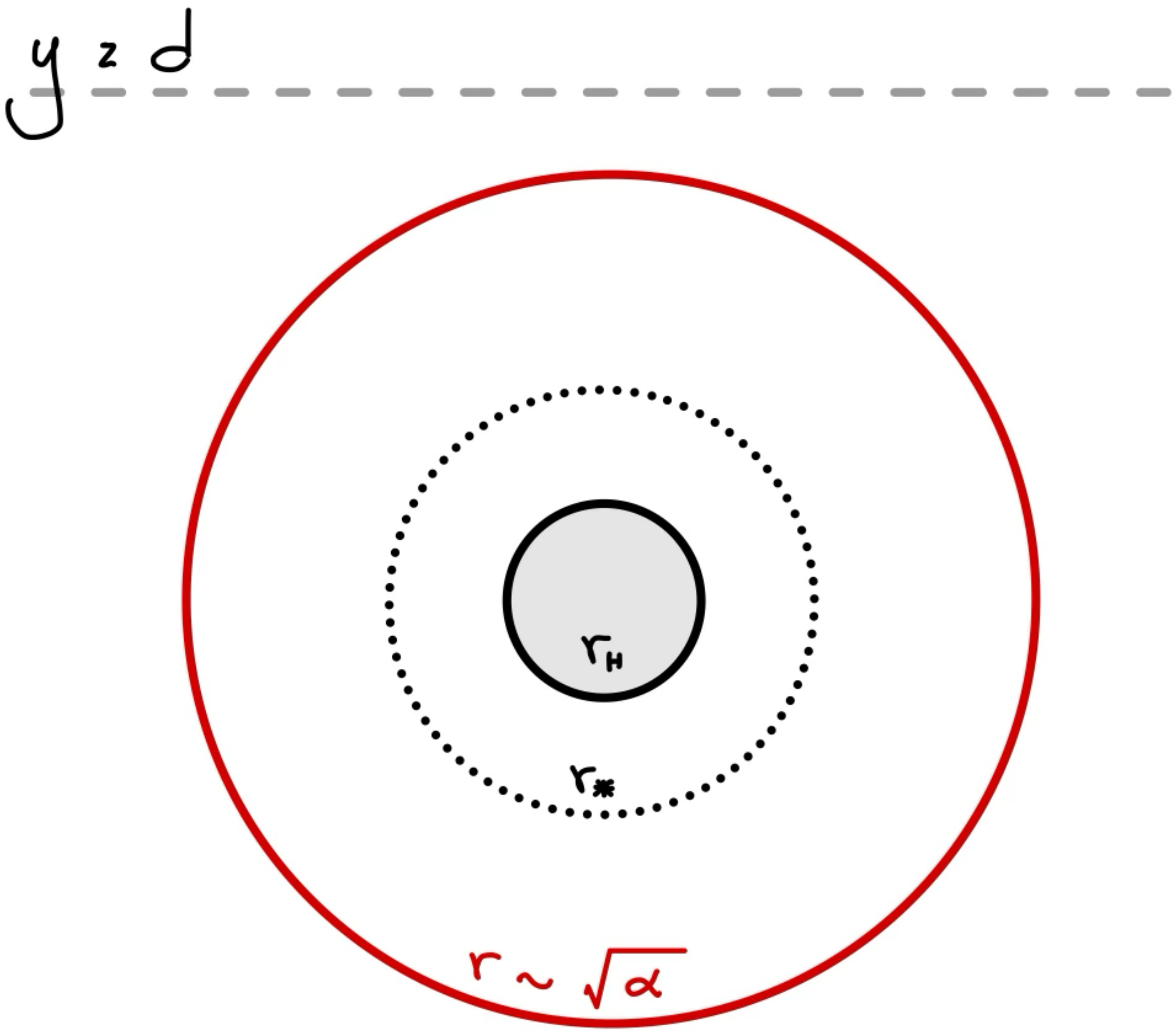
Photon Equation of Motion: $$\nabla_\nu F^{\mu\nu} + \alpha R^{\mu\nu}{}_{\rho\sigma}\nabla_\nu F^{\rho\sigma} = 0$$

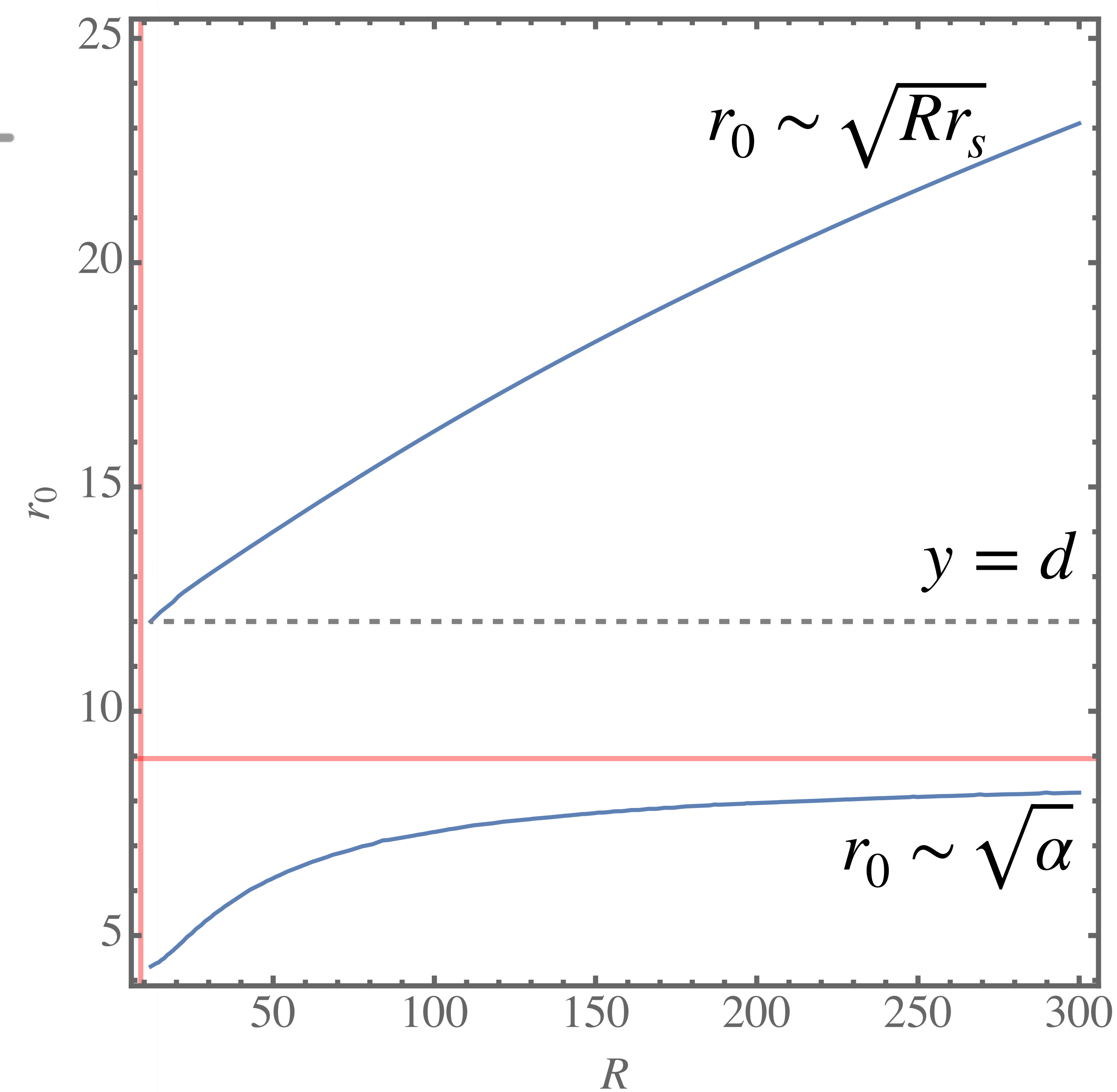
- Well known EFT that exhibits superluminality, cutoff of theory is $\alpha \gtrsim r_s^2$.

[Drummond&Hathrell, 1980] [Goon&Hinterbichler., 2016]

FFR Theory

Limit of Causal Curves





FFR Theory

Time Delay ($D = 4$)

- The gravitational time delay is log-divergent

$$\Delta t_{\text{GR}} \propto \ln\left(\frac{2R}{r_0}\right)$$

- Comparing the coordinate time of the two geodesics, we have

$$\Delta t_1 - \Delta t_2 \sim \ln\left(\frac{r_{0,2}}{r_{0,1}}\right) \sim \ln\left(\frac{\sqrt{\alpha}}{\sqrt{R}}\right) \xrightarrow{R \to \infty} \quad \Delta t_1 < \Delta t_2$$

- The geodesic that goes to infinity is prompt i.e. the asymptotic structure of this (pathological) theory is still Schwarzschild!

Towards a General Statement?

Assertion 1: This effect is general for any pathological EFTs on a 4D Schwarzschild background *provided that the time advance is bounded*.

(Can we formalise this into a 'Theorem'?)

Assertion 2: Pathologies of the theory cannot be probed by scattering at infinity. They should be probed through other local means e.g. constructing CTCs.

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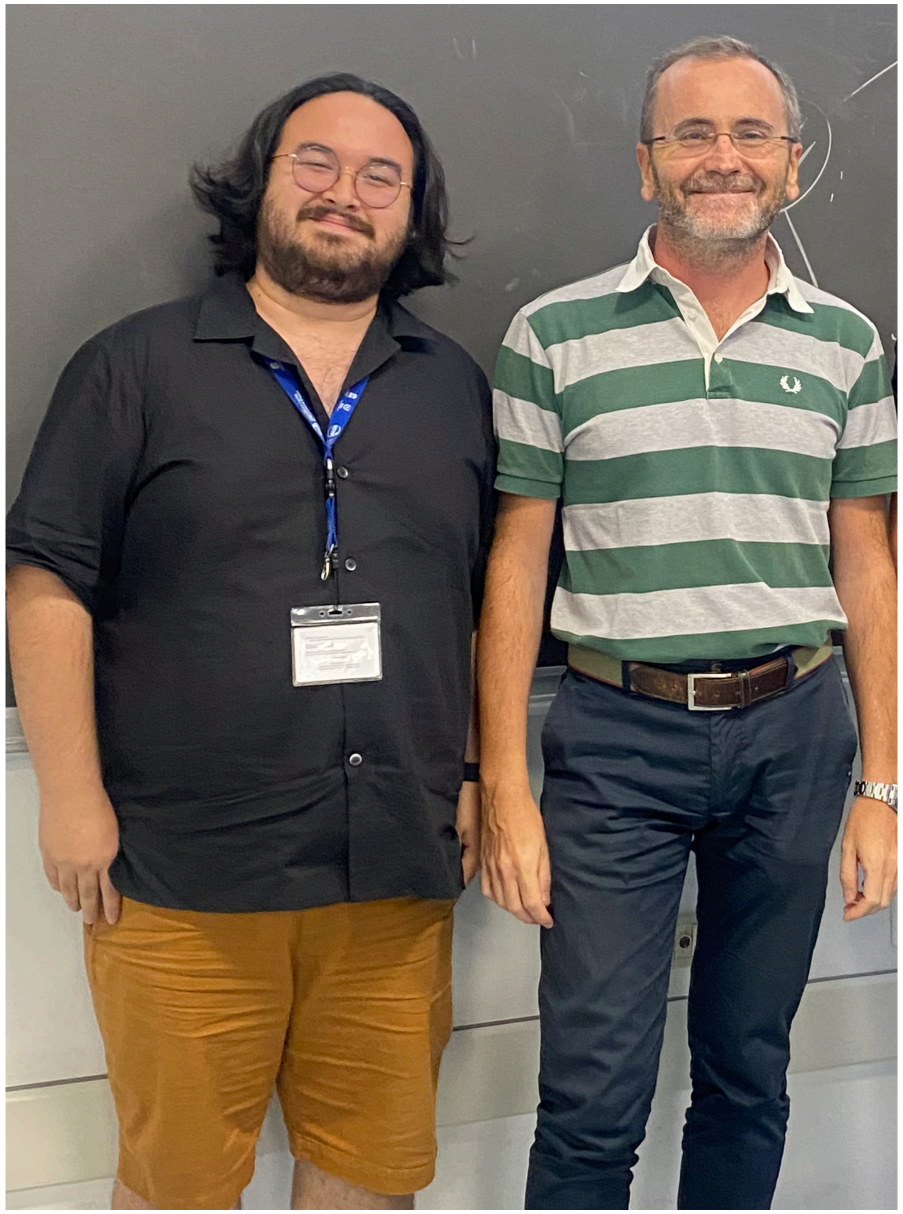
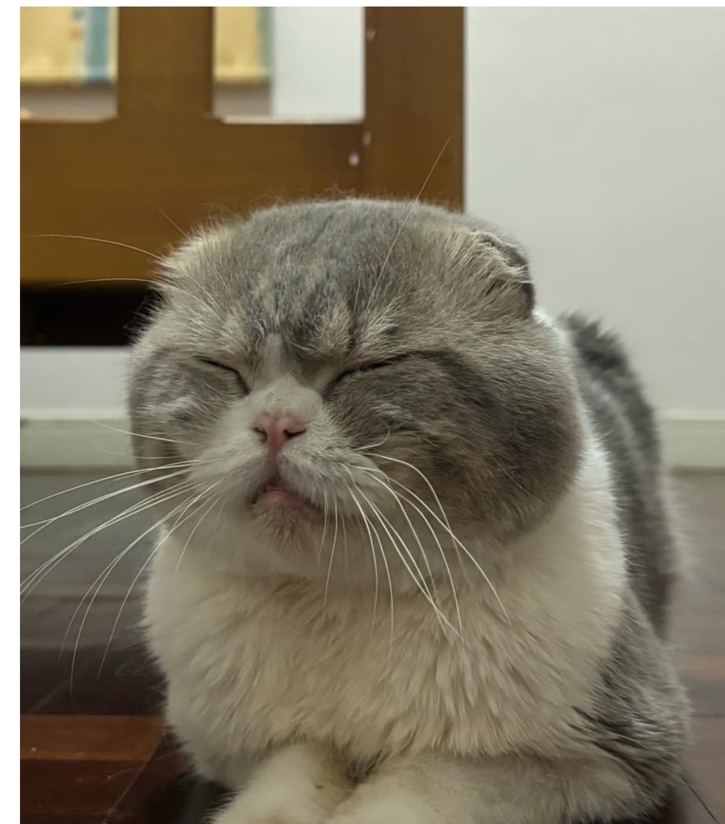


Interests in Physics: Cosmology (in general)

Interests outside Physics: Eating, DRINKING & **C A T S**



Q&A Session

Motivation

Causality Bounds on EFTs

- An EFT taken beyond its regime of validity displays time advance in a gravitational background.

Example: Modification of BHs—BH with scalar hair

$$S = \int d^4x\sqrt{-g}\ \left[\frac{M_{pl}^2}{2}R - \frac{1}{2}(\nabla_\mu\phi)^2 + M_{pl}\alpha\phi\mathscr{R}_{GB}^2 + M_{pl}\tilde{\alpha}\phi R_{\mu\nu\rho\sigma}\tilde{R}^{\mu\nu\rho\sigma}\right]$$

- The time delay of scalar-graviton scattering is given by

$$\Delta t_\pm = 2r_s\left(\ln\frac{b_0}{b} \pm \sqrt{2}\frac{|\hat{\alpha}|}{b^2}\right); \quad \hat{\alpha} = \alpha + i\tilde{\alpha}$$

- At small enough impact parameter b, Δt_- can be negative i.e. a time advance—violation of causality.

[Trincherini et al., 2022] [Maldacena et al., 2014]

Motivation

Causality Bounds on EFTs

- This time advance can be used to bound—or rule out—EFTs.

Example: Modification of BHs—BH with scalar hair

- The time delay of scattering implies

$$\Delta t_{\pm} = 2r_s\left(\ln\frac{b_0}{b} \pm \sqrt{2}\frac{|\hat{\alpha}|}{b^2}\right) \longrightarrow |\hat{\alpha}| \gtrsim b^2 \ln\frac{b_0}{b}$$

- To avoid time advances at low energies, the EFT must break down at distance scales such that this condition cannot be satisfied i.e.

$$\frac{1}{\Lambda} > b \qquad \longrightarrow \qquad |\hat{\alpha}| \gtrsim \frac{\ln(b_0\Lambda)}{\Lambda^2}$$

[Trincherini et al., 2022] [Maldacena et al., 2014]

Subtlety I: Gauge Dependency

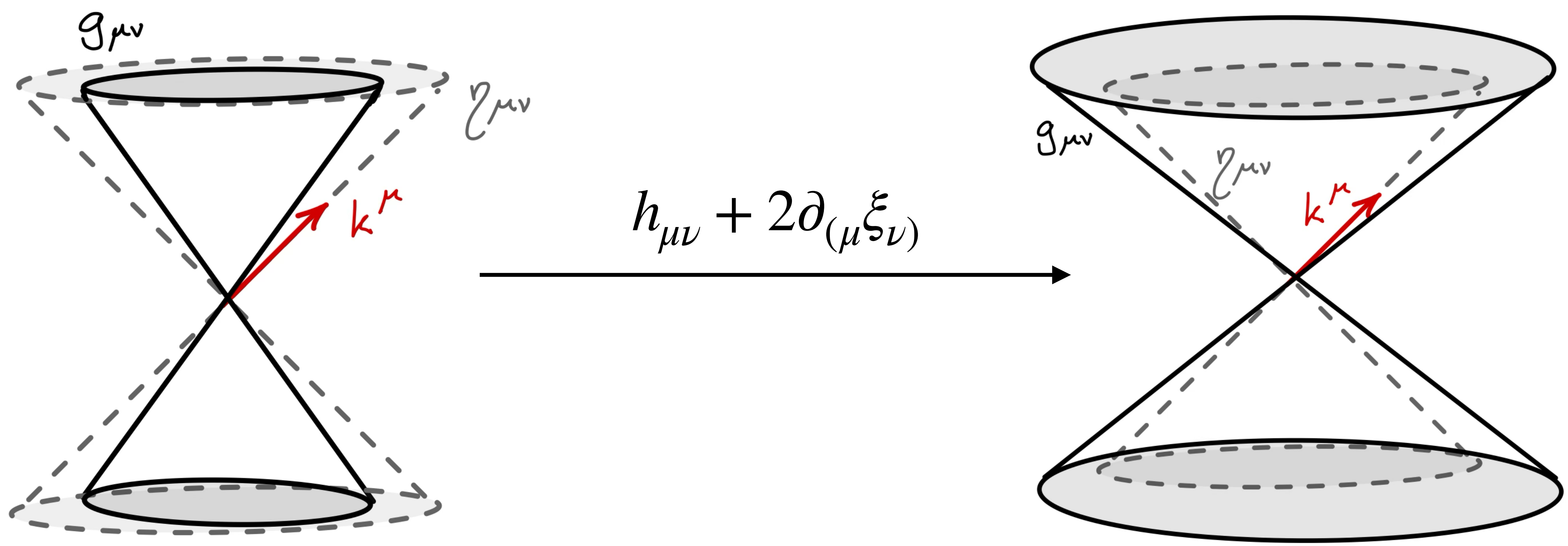
- In linearised gravity that satisfies the Null Energy Condition

$$R_{\mu\nu}k^\mu k^\nu \geq 0, \quad \eta_{\mu\nu}k^\mu k^\nu = 0,$$

the perturbation satisfies (assuming Lorenz gauge)

$$h_{\mu\nu}k^\mu k^\nu \geq 0$$

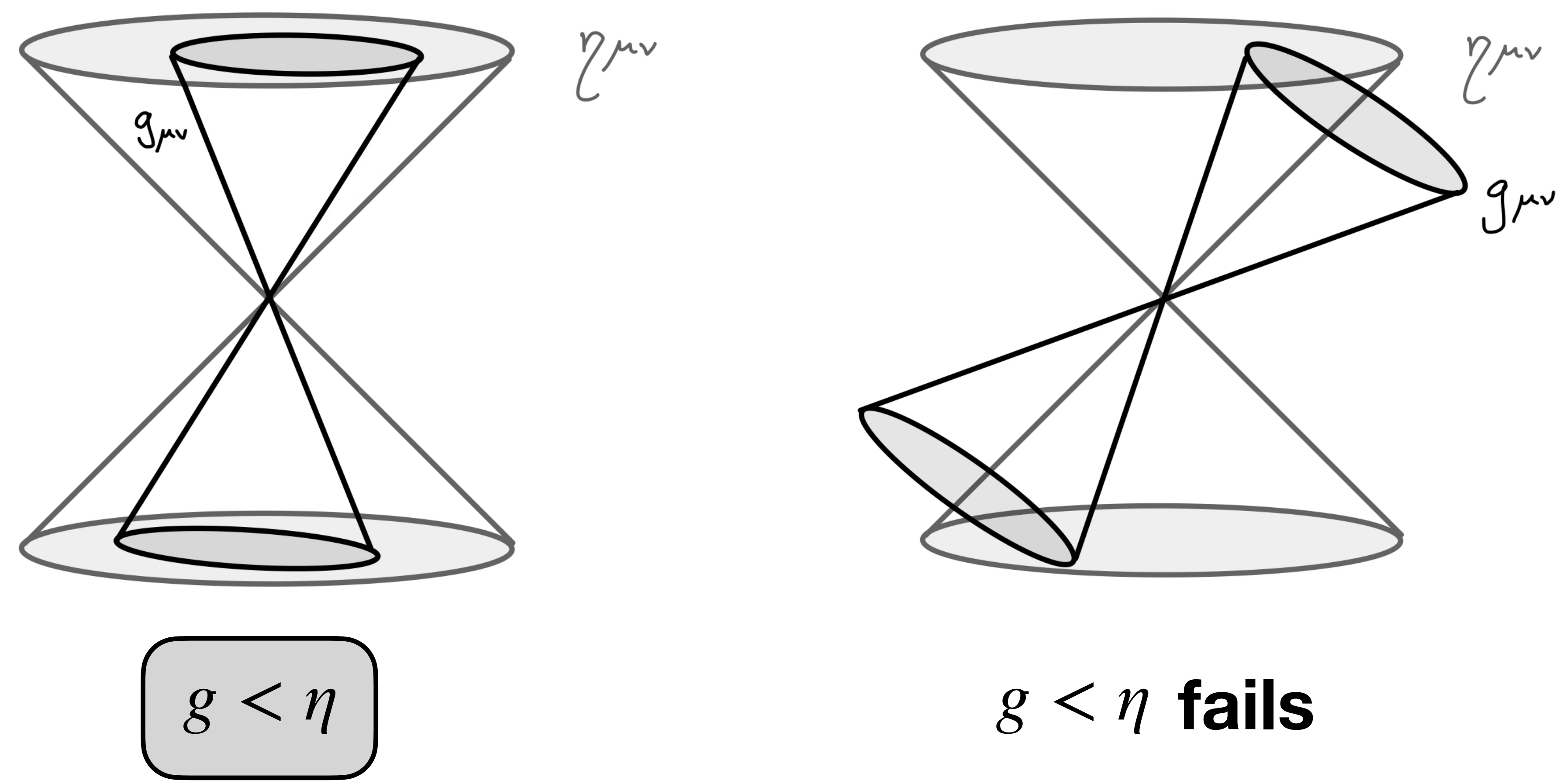
- A gauge transformation on $h_{\mu\nu}$ can reverse the above conclusion!



[Gao&Wald, 2000] [Visser, Basset & Liberati, 1999]

Subtlety II: Asymptotic Structure

- One could imagine that in a sensible theory, the fields in the theory should propagate according to its effective metric $g_{\mu\nu}$ in a way that also obeys causality set by $\eta_{\mu\nu}$.



- However, this condition fails for many examples in GR.

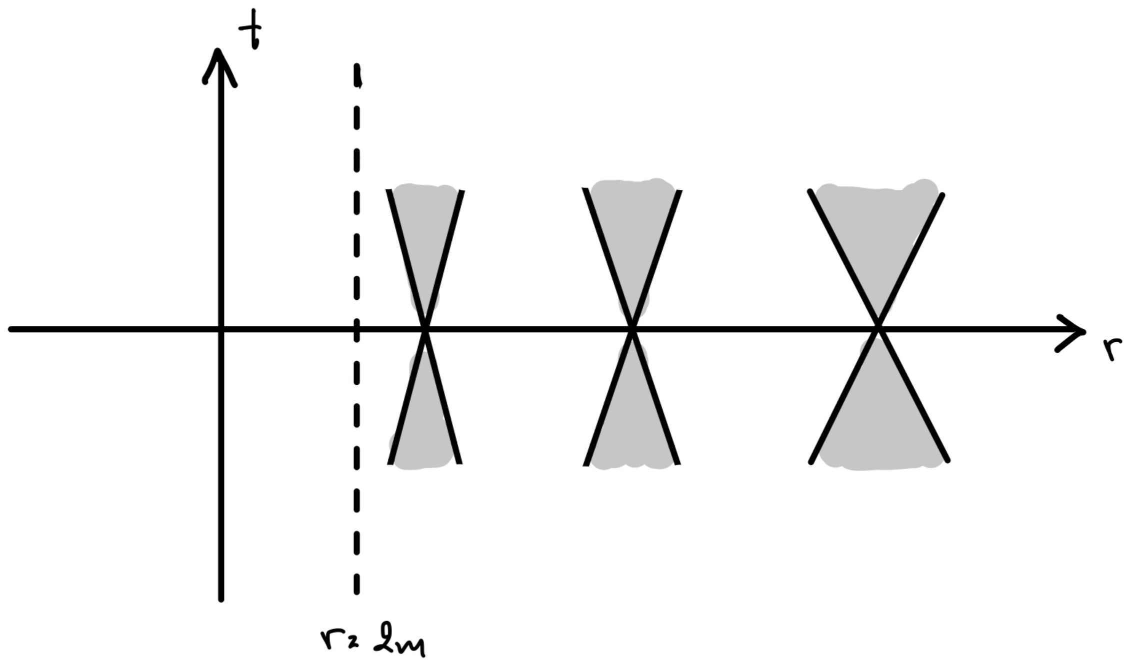
[Penrose (Taub Festschrift), 1980] [Cameron&Dunajski, 2020]

Subtlety II: Asymptotic Structure

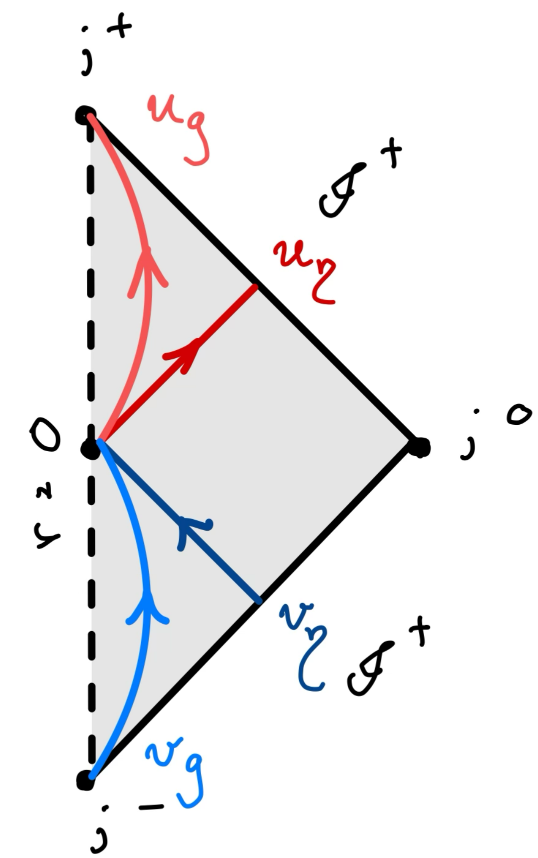
4D Schwarzschild, standard coordinates:

$$ds^2 = -\left(1 - \frac{r_s}{r}\right)dt^2 + \left(1 - \frac{r_s}{r}\right)^{-1}dr^2 + r^2 d\Omega^2$$

$$d\sigma^2 = -dt^2 + dr^2 + r^2 d\Omega^2$$



$$\frac{dt}{dr} = \pm \frac{1}{1 - r_s/r}, \quad g < \eta$$



$$u_g = t - r - r_s \ln(r - r_s)$$
$$v_g = t + r + r_s \ln(r - r_s)$$

- Not optimal for scattering problems in Schwarzschild spacetimes.

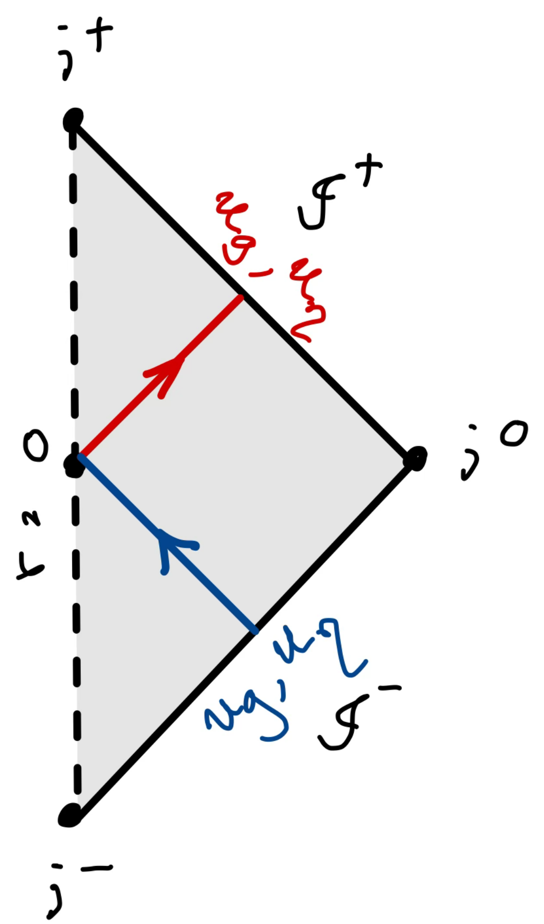
[Penrose (Taub Festschrift), 1980] [Cameron&Dunajski, 2020]

Subtlety II: Asymptotic Structure

4D Schwarzschild, tortoise coordinates:

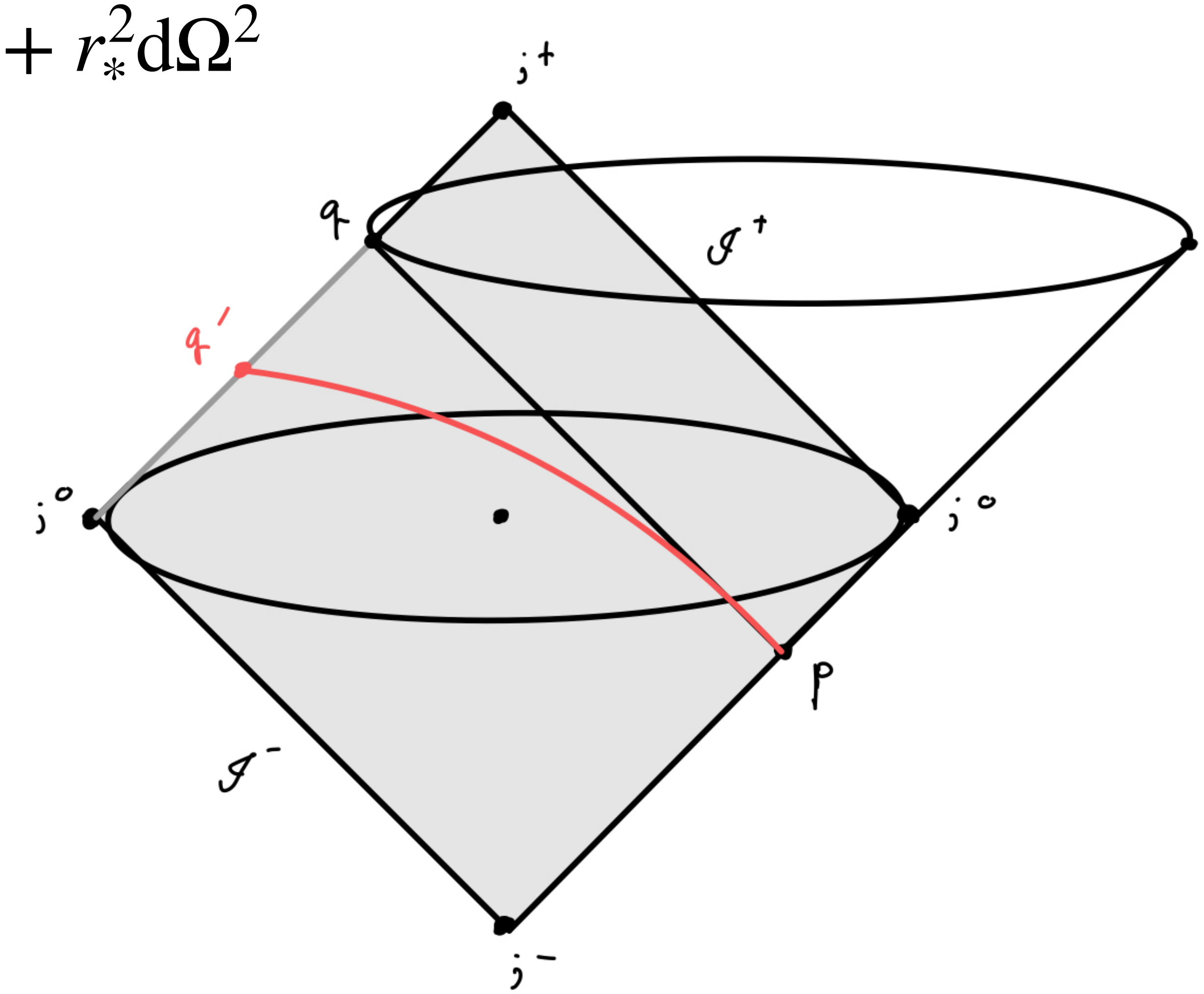
$$\mathrm{d}s^2 = -\left(1 - \frac{r_s}{r}\right)[\mathrm{d}t^2 + \mathrm{d}r_*^2] + r^2\mathrm{d}\Omega^2$$

$$\mathrm{d}\sigma^2 = -\mathrm{d}t^2 + \mathrm{d}r_*^2 + r_*^2\mathrm{d}\Omega^2$$



$$u_g = u_\eta = t - r_*$$

$$v_g = v_\eta = t + r_*$$

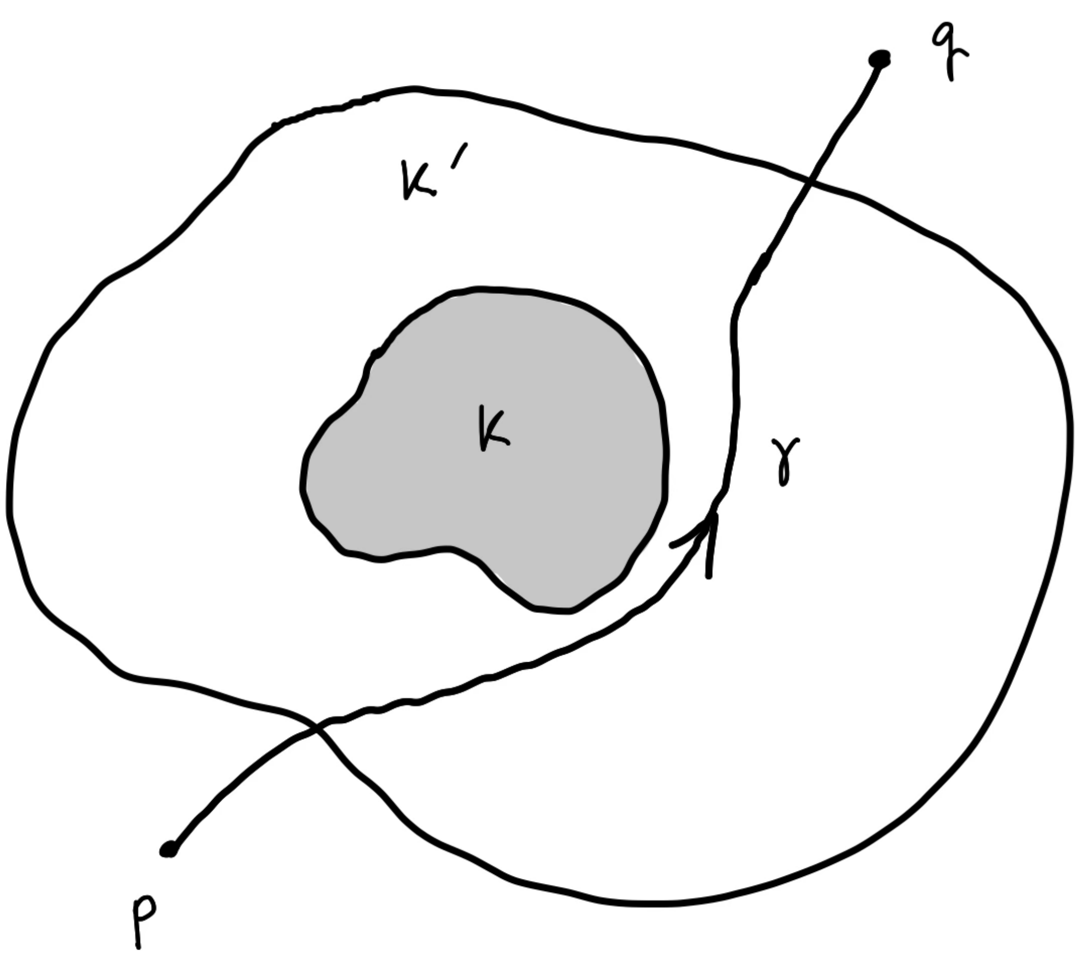


$g < \eta$ **fails**—a causal curve that reaches points outside Minkowski can be constructed.

[Penrose (Taub Festschrift), 1980] [Cameron&Dunajski, 2020]

Gao-Wald Theorem

Criterion for Promptness



- Gauge-invariant, dimension independent criterion for promptness in GR.
- For asymptotically flat spacetimes, this theory implies that if $p, q \to \infty$ then $\gamma \to \infty$ as well.
- If 'time advance' can be produced in K then one expects the prompt null geodesic to enter K.

[Gao&Wald, 2000]

FFR Theory

'Drummond&Hathrell' Problem

- Start from QED in curved spacetimes.

$$S = \int d^4x\sqrt{-g}\left[\frac{M_{pl}^2}{2}R - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \bar{\psi}(\not{D} + m)\psi\right]$$

Integrating out fermions to 1-loop order on Ricci-flat backgrounds ($R_{\mu\nu} = R = 0$):

$$S = \int d^4x\sqrt{-g}\left[\frac{M_{pl}^2}{2}R - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \alpha R_{\mu\nu\rho\sigma}F^{\mu\nu}F^{\rho\sigma}\right]; \quad \alpha \sim \frac{e^2}{m_e^2}$$

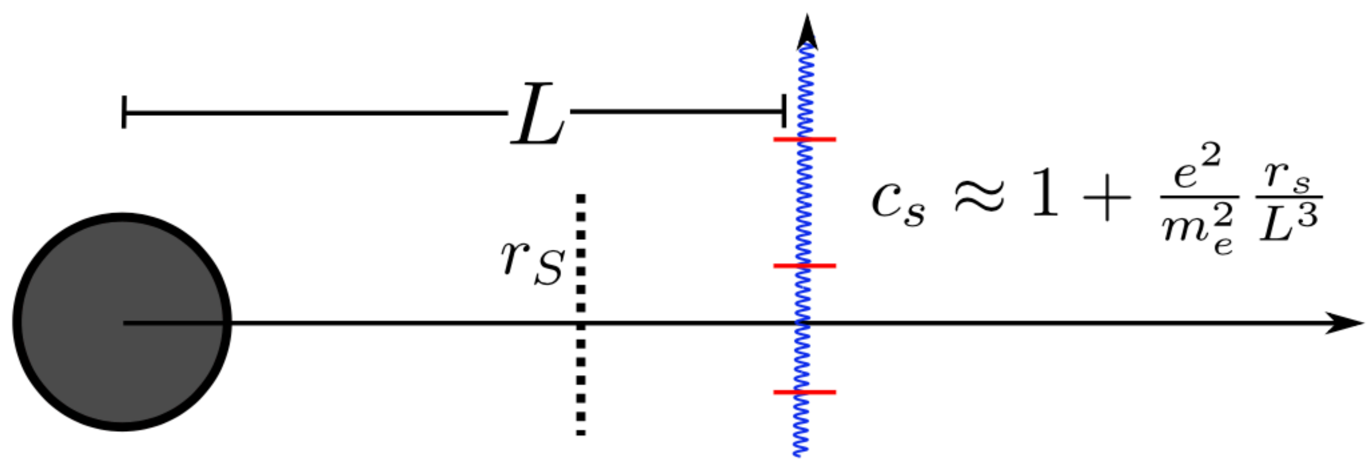
Photon Equation of Motion:

$$\nabla_\nu F^{\mu\nu} + \alpha R_{\mu\nu\rho\sigma}F^{\mu\nu}F^{\rho\sigma} = 0$$

[Drummond&Hathrell, 1980] [Goon&Hinterbichler., 2016]

FFR Theory

Resolution to ‘Drummond&Hathrell’ Problem



- One resolution of this problem is that the effect is small and parametrically smaller than the cutoff of the EFT.

- A superluminal photon will generate a cumulative effect of order

$$\Delta d \sim \delta c_s \times L \sim m_e^{-1}\left(\frac{e^2}{m_e}\frac{r_s^2}{L}\right) \ll m_e^{-1}$$

- For SM electron and solar mass BH we have

$m_e^{-1} \sim 10^{-13}$ m $\qquad$ $\Delta d \lesssim 10^{-31}$ m

[Drummond&Hathrell, 1980] [Goon&Hinterbichler., 2016]

FFR Theory

Time Delay ($D > 4$)

Working in the perturbative regime [$r_0 \gg r_s$, $r_0 \gg (\alpha r_s^{D-3})^{\frac{1}{D-1}}$]

- The usual gravitational time delay has no divergences

$$\Delta t_{\text{Sch}} \propto \frac{1}{r_0^{D-4}}$$

- With FFR included, calculation shows that there is a region where time advance is possible$-r_0 \sim \sqrt{\alpha}$.

- Expectation: the asymptotic structure in this theory should be modified.