Tetraquarks at Large N?

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Motivation: Experiment and Theory

- **Experimental Evidence:**
  - Growing list of exotic hadrons near thresholds:
    - $X(3872)$ (Belle), T_{cc}^{+} (LHCb, 2021), $X(6900)$ (LHCb, 2020)
  - These states often appear near meson-meson thresholds (molecular states or compact objects?).
- **Large N QCD Predictions:**
  - In the large N limit of QCD:
    - Meson two-point functions factorize (Witten, 1979; Coleman, 1985).
  - Weinberg proposed (Weinberg, 2013):
    - Subleading connected diagrams can host tetraquark poles.
    - Although suppressed by $1/N$, such poles might be physical.
  - Later studies (Knecht and Peris, 2013; Cohen and Lebed, 2014; Maiani et al., 2016) showed non-planar diagrams could sustain physical tetraquark resonances.

Setup: Large N and BO

Regime of Interest:

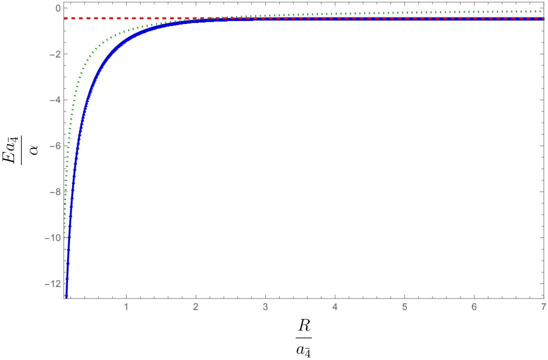
- $M_1 \geq M_{\bar{2}} \gg m_{\bar{4}} \gg m_3 \gg \Lambda_{\mathrm{QCD}}$.
- 't Hooft coupling: $\alpha = \frac{1}{2}\alpha_s N \ll 1$
- α controls both gluon exchange and relativistic corrections.
- Large mass hierarchy enables use of **Born-Oppenheimer (BO) approximation**: heavy quarks treated as static sources, light quarks adjust adiabatically.

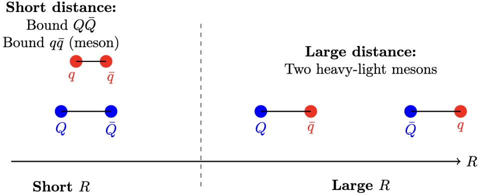
Leading Hamiltonian (NRQCD):

$$H = \sum_i \frac{\vec{p}_i^{\,2}}{2m_i} + \sum_{i<j} \frac{\alpha_s}{r_{ij}}\, T^a_{(i)} T^a_{(j)} + \text{small corrections}$$

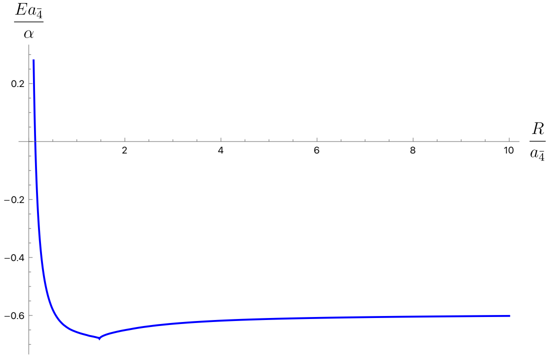
- Pairwise Coulomb-like interactions from single gluon exchange
- Valid for $r_{ij} \ll \Lambda_{\mathrm{QCD}}^{-1}$

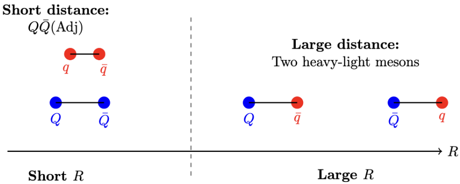
Ground State





Excited state?





Conclusion

- Experimental discoveries of exotic hadrons near meson thresholds motivate theoretical study of four-quark systems.
- Large N QCD combined with the Born-Oppenheimer approximation provides a consistent framework for heavy tetraquarks.
- The heavy quark mass hierarchy and small 't Hooft coupling $\alpha \ll 1$ justify a non-relativistic Hamiltonian with Coulomb-like interactions.
- In the ground state, at short distances the system resembles bound heavy quarkonia and light mesons; at large distances, it rearranges into two heavy-light mesons.
- A possible state with $Q\bar{Q}$ in the adjoint color configuration at short distances may exhibit a potential minimum—raising questions about its stability.