Supersymmetry & The Weak Scale (in a Nutshell)

- Poincaré group : $g(\Lambda, a)$ $\qquad x^\mu \longrightarrow x'^\mu = \Lambda^\mu{}_\nu x^\nu + a^\mu$

  $ISO(3,1)$

- generators : $J_{\mu\nu}$, P_μ $\qquad g = e^{i a^\mu P_\mu} e^{-i \frac{\omega^{\mu\nu}}{2} J_{\mu\nu}}$

- $\underset{\text{Poincaré}}{ISO(3,1)} = \underset{\text{Translations}}{\mathbb{R}^{3,1}} \rtimes \underset{\text{Lorentz}}{SO(3,1)}$

- Minkowsky space $\equiv \dfrac{ISO(3,1)}{SO(3,1)}$ coset

  $\{ g_1 \sim g_2 \Longleftrightarrow g_1 = g_2 \underbrace{g_\Lambda}_{\in SO(3,1)} \}$

- without loss of generality can parametrize $\mathbb{M}_4$ by pure translation $e^{ix^\mu P_\mu}$

- Q.M. implies relevence of projective representations

$$SO(3,1) \longrightarrow \underbrace{SL(2,\mathbb{C})}_{\text{universal covering}} \sim SU(2)_- \times SU(2)_+$$

- irreps $\chi = (j_-, j_+)$ $\quad$ $\dim \chi = (2j_-+1)(2j_++1)$

$$\Rightarrow \quad (0,0)\ ;\ \underbrace{(\tfrac{1}{2},0)\ ,\ (0,\tfrac{1}{2})}_{\text{quantum mechanical}}\ ;\ \underbrace{(\tfrac{1}{2},\tfrac{1}{2})\ ,\ (1,0)\ \ldots}_{\text{classical}}$$

- $P^\mu \sim x^\mu \sim \left(\frac{1}{2},\frac{1}{2}\right)$ is "almost" the smallest non-trivial irrep of $SO(3,1)$
- Can we imagine an extension of $ISO(3,1)$ where where the smallest irrep $\left(\frac{1}{2},0\right)\oplus\left(0,\frac{1}{2}\right)$ also generates a "sort of translation"
- $\underset{Q_\alpha}{\left(\frac{1}{2},0\right)} \otimes \underset{\bar{Q}_{\dot\alpha}}{\left(0,\frac{1}{2}\right)} = \underset{P_\mu}{\left(\frac{1}{2},\frac{1}{2}\right)} \Longrightarrow$ " $Q \sim \sqrt{P}$ "
- Idea: extend $x^\mu \longrightarrow X = \left(x^\mu, \theta^\alpha, \bar\theta^{\dot\alpha}\right)$

$\theta^\alpha, \theta^{\dot\alpha} \equiv$ fermionic : $(\theta^1)^2 = (\theta^2)^2 = 0$ they are "quantum mechanically small".

- correspondingly, besides ordinary $\left(\frac{1}{2},\frac{1}{2}\right)$ translations $x^\mu \to x^\mu + a^\mu$ we can imagine fermionic $\left(\frac{1}{2},0\right)$ $\left(0,\frac{1}{2}\right)$ translations $\theta^\alpha \to \theta^\alpha + \xi^\alpha$, $\bar\theta^{\dot\alpha} \to \bar\theta^{\dot\alpha} + \bar\xi^{\dot\alpha}$

However Lorentz covariance permits a more interesting intertwined option: Supersymmetry

(*) SuperTranslations

$$
\begin{cases}
x^\mu \to x^\mu + a^\mu + i\left(\xi\sigma^\mu\bar\theta - \theta\sigma^\mu\bar\xi\right) \equiv x'^\mu \\
\theta^\alpha \to \theta^\alpha + \xi^\alpha \equiv \theta'^\alpha \\
\bar\theta^{\dot\alpha} \to \bar\theta^{\dot\alpha} + \bar\xi^{\dot\alpha} \equiv \bar\theta'^{\dot\alpha}
\end{cases}
$$

Lorentz

$$
\begin{cases}
x^\mu \to \Lambda^\mu{}_\nu(\omega)\, x^\nu, \quad \theta_\alpha \to \Lambda_L(\omega)_\alpha{}^\beta\, \theta_\beta, \quad \bar\theta^{\dot\alpha} = \Lambda_R(\omega)^{\dot\alpha}{}_{\dot\beta}\, \bar\theta^{\dot\beta}
\end{cases}
$$

- SuperPoincaré $\equiv ISO(3,1|4)$ $\qquad (x, \theta, \bar{\theta}) \equiv \mathbb{R}^{3,1|4}$

- Left group action $g_1 : g \longmapsto g_1 \circ g$

- $g(x,\theta,\bar{\theta}) = e^{i(x^\mu P_\mu + \xi^\alpha Q_\alpha + \bar{\xi}_{\dot{\alpha}} \bar{Q}^{\dot{\alpha}})}$

- $g(a,\xi,\bar{\xi}) : \quad g(x,\theta,\bar{\theta}) \mapsto g(a,\xi,\bar{\xi})\, g(x,\theta,\bar{\theta}) \equiv g(x', \theta', \bar{\theta}')$

- $(*) \Longrightarrow$ commutation relations

$$g^{-1}(0,\xi_1,\bar{\xi}_1)\, g^{-1}(0,\xi_2,\bar{\xi}_2)\, g(0,\xi_1,\bar{\xi}_1)\, g(0,\xi_2,\bar{\xi}_2) = g(2i(\xi_1\sigma\bar{\xi}_2 - \xi_2\sigma\bar{\xi}_1), 0, 0)$$

$$x^\mu \to x^\mu + 2i(\xi_1\sigma^\mu\bar{\xi}_2 - \xi_2\sigma^\mu\bar{\xi}_1)$$
$$\theta^\alpha \to \theta^\alpha$$
$$\bar{\theta}^{\dot{\alpha}} \to \bar{\theta}^{\dot{\alpha}}$$

$$e^{-iA}\,e^{-iB}\,e^{iA}\,e^{iB} = e^{-[A,B]}$$

$$\Rightarrow \{Q_\alpha, \bar{Q}_{\dot\beta}\} = 2\sigma^\mu_{\alpha\dot\beta} P_\mu$$

$$\{Q_\alpha, Q_\beta\} = 0$$

$$\{Q_\alpha, \xi_\beta\} = 0$$

... etc.

- Notice

$$\left.\begin{aligned} \{Q_1, \bar{Q}_{\dot 1}\} &= 2(P_0 + P_3) \\ \{Q_2, \bar{Q}_{\dot 2}\} &= 2(P_0 - P_3) \end{aligned}\right\} \Rightarrow \{Q_1, \bar{Q}_{\dot 1}\} + \{Q_2, \bar{Q}_{\dot 2}\} = 4P_0$$

$$\langle\psi| P_0 |\psi\rangle = \frac{1}{4}\left(\|Q_1|\psi\rangle\|^2 + \|\bar{Q}_{\dot 1}|\psi\rangle\|^2 + \|Q_2|\psi\rangle\|^2 + \|\bar{Q}_{\dot 2}|\psi\rangle\|^2\right)$$

$$\geq 0$$

⚠ Representation on fields

- Poincaré: $x' = \Lambda x + a$

  $Obs' \leftrightarrow Obs$

- fields $\qquad \phi_A(x)$

  $\hookrightarrow$ $SL(2,\mathbb{C})$ representation

- irrep $\qquad A \rightarrow r \equiv (j_-, j_+)$

$$\phi'_B(x') = \Lambda^r(\Lambda)_B{}^A \, \phi_A(x)$$

$$\phi'_B(x) = \Lambda^r(\Lambda)_B{}^A \, \phi_A(\Lambda^{-1}x)$$

- Supersymmetry $\longrightarrow$ Superfields $\Phi_A(x,\theta,\bar{\theta}) \equiv \Phi_A(X)$

Lorentz irrep

- $(*)$ $X \longrightarrow X' = g_{a,\xi,\bar{\xi}}(X) \Longrightarrow$

- $\Phi'_A(X) = \Phi_A \circ g^{-1}_{a,\xi,\bar{\xi}}(X)$

- $\delta\Phi_A = \left[\Phi_A'(X) - \Phi_A(X)\right]_{\text{linearized}} = i\left(a\cdot P + \xi\cdot Q + \bar{\xi}\cdot\bar{Q}\right)\Phi$

$$\Longrightarrow \begin{cases} P_\mu = i\,\partial_\mu \\ Q_\alpha = i\left(\frac{\partial}{\partial\theta^\alpha} + i(\sigma^\mu\bar{\theta})_\alpha\,\partial_\mu\right) \\ \bar{Q}_{\dot{\alpha}} = -i\left(\frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}} + i(\theta\sigma^\mu)_{\dot{\alpha}}\,\partial_\mu\right) \end{cases}$$

Convention

$$\frac{\partial}{\partial\theta^\alpha}\theta^\beta = \delta_\alpha^\beta$$

$$\frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}}\bar{\theta}^{\dot{\beta}} = \delta_{\dot{\alpha}}^{\dot{\beta}}$$

Ex "Vector superfield" $V(x, \theta, \bar{\theta})$

$$V = \varphi + \chi^\alpha \theta_\alpha + \bar{\chi}_{\dot{\alpha}} \bar{\theta}^{\dot{\alpha}} + M\theta^2 + N\bar{\theta}^2 + \bar{\theta}\bar{\sigma}_\mu \theta A^\mu +$$
$$+ \theta^2 \bar{\theta}_{\dot{\alpha}} \bar{\lambda}^{\dot{\alpha}} + \bar{\theta}^2 \theta^\alpha \eta_\alpha + \theta^2 \bar{\theta}^2 D$$

$$\delta V \Rightarrow \begin{cases} \delta\varphi = -\chi\xi - \bar{\chi}\bar{\xi} \\ \delta\chi^\alpha = -2M\xi^\alpha - i(\bar{\xi}\bar{\sigma}^\mu)^\alpha \partial_\mu \varphi \\ \vdots \\ \delta D = \frac{i}{2}(\xi\sigma^\mu \partial_\mu \bar{\lambda}) + \frac{i}{2}\bar{\xi}\bar{\sigma}^\mu \partial_\mu \eta = \partial_\mu J^\mu \end{cases}$$

$\Rightarrow \int d^4x\, D$ is invariant ☺

Dimensional considerations $\quad [\theta] = [\xi] = [x]^{1/2}$

$[\varphi] = 0 \quad [\chi] = [\bar{\psi}] = \frac{1}{2} \quad \dots \quad [\bar{\lambda}] = [\eta] = \frac{3}{2} \quad [D] = 2$

$$\Rightarrow \underset{2}{\delta D} \sim \underset{-1/2}{\xi} \times \underset{3/2}{[\bar{\lambda}]} \times \underset{1}{\partial}$$

$$\delta D = \partial_\mu (\dots) \quad \text{unavoidable}$$

- Like a product of fields is itself a field

$\quad$ // $\quad$ " superfields $\quad$ // $\quad$ " superfield

$$\Rightarrow \int [V_1 V_2 \dots V_n]_D \quad \text{non-trivial invariant}$$

- Besides products another recepy to construct another superfield is to use the "right action"

Left $\quad e^{i(Pa+\xi Q+\bar{\xi}\bar{Q})}\, e^{i(Px+\theta Q+\bar{\theta}\bar{Q})} \equiv g_L(x)$

Right $\quad e^{i(Px+\theta Q+\bar{\theta}\bar{Q})}\, e^{i(Pa+\xi Q+\bar{\xi}\bar{Q})} = g_R(x)$

$$g_L \circ g_R = g_R \circ g_L$$

$$\Rightarrow \quad \tilde{\Phi}(x) = \Phi \circ h_R(x)$$

if I SUSY transform Φ, the $\tilde{\Phi}$ transforms as

$$\tilde{\Phi}(x) \to \Phi \circ g_L^{-1} \circ h_R(x) = \Phi \circ h_R \circ g_L^{-1}(x) = \tilde{\Phi} \circ g_L^{-1}(x)$$

i.e. it transforms as a superfield

- infinitesimally: Right action $\Longleftrightarrow$ covariant derivatives

$D_\mu = \partial_\mu \qquad D_\alpha = \partial_\alpha - i(\sigma^\mu \bar\theta)_\alpha \partial_\mu \qquad \bar D_{\dot\alpha} = -\bar\partial_{\dot\alpha} + i(\theta\sigma^\mu)_{\dot\alpha} \partial_\mu$

▲ $D_\alpha, \bar D_{\dot\alpha}$ allow to impose covariant constraints in analogy with ordinary $\partial_\mu A^\mu = 0$, $\not{\partial}\chi = 0$

▣ chiral & anti-chiral superfields

- chiral $\Phi(x,\theta,\bar\theta)$: $\bar D_{\dot\alpha}\Phi = 0 \Longrightarrow$

$$\Phi = \Phi(x - i\theta\sigma\bar\theta, \theta) = e^{-i\theta\not{\partial}\bar\theta}(\varphi + \sqrt{2}\,\theta\chi + \theta^2 F)$$

- analogue of $\partial_{z^*} f = 0 \xrightarrow{\text{holomorphic}} f = f(z)$

- working out algebra

$\delta\Phi \Rightarrow$

$$\delta\varphi = -\sqrt{2}\,\xi\cdot\chi$$

$$\delta\chi_\alpha = -\sqrt{2}\,\xi_\alpha F + \frac{i}{\sqrt{2}}(\bar\sigma^\mu\xi)_\alpha\,\partial_\mu\varphi$$

$$\delta F = -\frac{i}{\sqrt{2}}\,\partial_\mu(\chi\sigma^\mu\bar\xi)$$

- anti-chiral $\bar\Phi$: $D_\alpha\phi \Rightarrow \phi = \phi(x + i\theta\sigma\bar\theta, \bar\theta)$

$$D_\alpha\phi^\dagger = (\underbrace{D_{\dot\alpha}\phi}_{\text{chiral}})^\dagger = 0 \Rightarrow \phi \text{ chiral} \Rightarrow \phi^\dagger \text{ anti-chiral}$$

⚠ Fun building Lagrangians

- **Superpotential**

$$\Phi \text{ chiral} \longrightarrow \Phi^n \text{ is chiral}$$

$$\left(\bar{D}_{\dot\alpha}\Phi^n = n(\bar{D}_{\dot\alpha}\Phi)\Phi^{n-1} = 0\right)$$

$$\Phi^n = \left[\Phi(x - i\theta\sigma\bar\theta, \theta)\right]^n = e^{-i\theta\sigma\bar\theta\partial}\left[\left(\varphi + \sqrt{2}\,\theta\chi + \theta^2 F\right)\right]^n$$

$$= e^{-i\theta\partial\bar\theta}\left(\cdots + \underbrace{\theta^2\left(n\varphi^{n-1}F - \frac{n(n-1)}{2}\varphi^{n-2}\chi^2\right)}_{\text{invariant density}}\right)$$

$$\Rightarrow \boxed{\left[W[\Phi]\right]_F = \underbrace{\frac{\partial W}{\partial \varphi} \cdot F}_{=\text{"potential"}} - \underbrace{\frac{1}{2}\,\partial^2_\varphi W \cdot \chi^2}_{\text{"Yukawa"}}}$$

- Kahler Potential ($\sim$ kinetic term)

$\Phi^\dagger\Phi \equiv$ real vector superfield $\quad [\Phi^\dagger\Phi]_D \equiv$ density

$\Rightarrow$ compute

$$\int[\Phi^\dagger\Phi]_D = \int\left[\Phi^\dagger(x+i\theta\sigma\bar\theta,\bar\theta)\,\Phi(x-i\theta\sigma\bar\theta,\theta)\right]_D d^4x$$

$$= \int\left[\Phi^\dagger(x+2i\theta\sigma\bar\theta,\bar\theta)\,\Phi(x,\theta)\right]_D d^4x$$

$$\Downarrow \qquad\qquad\qquad\qquad \Downarrow$$

$$\varphi\,\frac{1}{2}(2i\theta\not{\partial}\bar{\theta})^2\varphi^\dagger \qquad\qquad 2i(\theta\not{\partial}\bar{\theta}\;\bar{\chi}\bar{\theta})(2\theta\chi)$$

$$= -\theta^2\bar{\theta}^2\,\varphi\Box\varphi^\dagger \qquad\qquad = i\theta^2\bar{\theta}^2\,\chi\not{\partial}\bar{\chi}$$

$$\Rightarrow \boxed{\int [\Phi^\dagger\Phi]_D = \int \partial_\mu\varphi^\dagger\partial^\mu\varphi + i\,\bar{\chi}\,\bar{\not{\partial}}\chi + F^\dagger F}$$

1974

⚠ Simplest example : Wess-Zumino model

$$S = \int [\Phi^\dagger \Phi]_D + \left[\frac{m}{2}\Phi^2 + \frac{\lambda}{3}\Phi^3\right]_F + \left[\frac{m^*\Phi^{\dagger 2}}{2} + \frac{\lambda^*\Phi^{\dagger 3}}{3}\right]_{F^\dagger}$$

$$= \int |\partial\varphi|^2 + i\bar{\chi}\bar{\not{\partial}}\chi + FF^\dagger + \left\{\left[(m\varphi + \lambda\varphi^2)F - \left(\frac{m}{2} + \lambda\varphi\right)\chi^2\right] + h.c.\right\}$$

$F =$ auxilliary $\Rightarrow$ fixed locally by e.o.m.

$$\bar{F} = -(m\varphi + \lambda\varphi^2)$$

$$\Rightarrow \mathcal{L} = |\partial\varphi|^2 + i\bar{\chi}\bar{\not{\partial}}\chi - |m\varphi + \lambda\varphi^2|^2 - \left\{\left(\frac{m}{2} + \lambda\varphi\right)\chi^2 + h.c.\right\}$$

- scalar mass = fermion (Majorana) mass $\equiv m$
- manifest at tree level but true to all orders by SUSY Algebra $[Q_\alpha, H] = 0$

⚠ Now m_χ = fermion mass $\equiv$ complex

$\equiv$ transforms under $\chi \to e^{i\alpha}\chi$

$\Rightarrow \delta m_\chi \propto m_\chi$

$\Rightarrow$ m_χ not affected by power UV divergences

$\Rightarrow$ also m_φ won't be!

Let us check at 1-loop

1-loop correction to m_φ

$+$

$$4\lambda^2\int\frac{d^4p}{(2\pi)^4}\,\frac{1}{p^2-m^2} \qquad -4\lambda^2\int\frac{d^4p}{(2\pi)^4}\,\frac{p^2}{(p^2-m^2)^2}$$

$$\sim\ 4\lambda^2\int\frac{d^4p}{(2\pi)^4}\,\frac{m^2}{(p^2-m^2)^2} \longrightarrow \text{Log div !}$$

Non-renormalization Theorem (Seiberg 1993)

Preliminary: R-symmetry

$$\theta \to e^{i\gamma}\theta \qquad \bar\theta \to e^{-i\gamma}\bar\theta \qquad [R, Q] = iQ$$

$$[R, \bar Q] = -i\bar Q$$

$$\Phi \to e^{iq\gamma}\Phi(x, e^{-i\gamma}\theta, e^{i\gamma}\bar\theta)$$

$$\Downarrow$$

$$\begin{cases} \varphi \to e^{iq\gamma}\varphi \\ \psi \to e^{i(q-1)\gamma}\psi \\ F \to e^{i(q-2)}F \end{cases} \Rightarrow [\Phi^n]_F \to e^{i(nq-2)}[\Phi^n]_F$$

if $W(\Phi) \longrightarrow e^{i2\gamma} W(\Phi)$ then $U(1)_R$ is a symmetry

- Ex $[\Phi^3]_F$ invariant if $q_\Phi = 2/3$

- $U(1)_{PQ} \times U(1)_R$ on field and couplings

can think of two $U(1)$'s in WZ model

- $U(1)_{PQ}$: $\phi \longrightarrow e^{i\delta} \phi(x, \theta, \bar{\theta})$
- $U(1)_R$: $\phi(x, \theta, \bar{\theta}) \longrightarrow \phi(x, \theta e^{-i\gamma}, \bar{\theta} e^{i\gamma})$

- $U(1)_{PQ} \times U(1)_R$ is broken by superpotential but I can formally extend m & λ to background chiral superfields with suitable $U(1)_{PQ} \times U(1)_R$ charges so as to make W invariant and derive <u>selection rules</u>. (our favorite trick)

- Notice this makes sense because W depends only on m, λ and not m^*, λ^*: <u>holomorphy</u>

$$W = \frac{1}{2} m \phi^2 + \frac{\lambda}{3} \phi^3 \qquad U(1)_{PQ} \times U(1)_R \text{ invariant} \Longrightarrow$$

	ϕ	m	λ
$U(1)_R$	0	2	2
$U(1)_{PQ}$	1	-2	-3

← charges

$$W = m\phi^2\left(\frac{1}{2} + \frac{1}{3}\frac{\lambda\phi}{m}\right)$$

$$\hookrightarrow I = \frac{\lambda\phi}{m}$$

$$\boxed{\begin{array}{c}\text{unique} \\ \text{holomorphic} \\ U(1)_{PQ} \times U(1)_{R} \\ \text{invariant}\end{array}}$$

⚠ What general form the full quantum W^{1PI} can have?

$$W^{1PI} = m\phi^2 f\left(\frac{\lambda\phi}{m}\right) \quad \text{by } U(1)\times U(1)\,!$$

$$\lim_{\lambda\to 0} f\left(\frac{\lambda\phi}{m}\right) = f_{tree} = \frac{1}{2} + \frac{1}{3}\frac{\lambda\phi}{m}$$

but I can take this limit keeping $u = \frac{\lambda\phi}{m}$

fixed $\Rightarrow$ $f(u) = \lim_{\lambda \to 0} f(u) = f_{tree}(u)$

$\Rightarrow$ $f(u)$ coincides with tree level result!

- This derivation seem perhaps too clever so let us consider by absurd $f(u) \neq f_{tree}(u)$

$$f(u) = \underbrace{f_0 + f_1 \frac{\lambda \phi}{m}}_{\lambda^{0,1} \equiv \text{tree}} + \underbrace{f_2 \frac{\lambda^2 \phi^2}{m^2} + f_3 \frac{\lambda^3 \phi^3}{m^3} + \cdots}_{\text{possible loop effects}}$$

$\Rightarrow$ $f_0 = \frac{1}{2}$ $f_1 = \frac{1}{3}$

but consider $f_2 \frac{\lambda^2 \phi^2}{m^2}$: it corresponds to

$$\Delta W = f_2 \frac{\lambda^2}{m} \phi^4 \qquad \equiv \text{2 couplings -- 4 legs}$$

- not a loop diagram!! $\Longrightarrow f_2 = 0$
- this argument extends to all orders

$\boxed{}$ Holomorphy was essential in deriving this result; presence of λ^* would have allowed to write more terms

$U(1)_R \times U(1)_{PQ}$, but $\lambda^* \equiv$ chiral superfield

and supersymmetry does not permit its

presence in superpotential !!

▨ Can check non-ren theorem directly at

1-loop by working off-shell.

$$\int \left\{ |\partial\varphi|^2 + i\bar{\chi}\bar{\not{\partial}}\chi + FF^\dagger + \left[(m\varphi + \lambda\varphi^2)F - \left(\frac{m}{2} + \lambda\varphi\right)\chi^2 \right] + h.c. \right\}$$

$\underline{\underline{Ex}}$ compute 1-loop correction to

φF bilinear

A toy supersymmetric Higgs-top sector

- SM top Yukawa $\quad y_t \, \bar{q}_L \, \tilde{H} \, t_R + h.c.$

$$\tilde{H}_i = \varepsilon_{ij} H_j^* \qquad \bar{q}_L \to (3, 2, \tfrac{1}{3}) \quad \tilde{H} \to (1, 2, -1) \quad t_R \to (3, 1, 4/3)$$

- Supersymmetric extension

$$\boxed{\begin{aligned} q_L &\to Q = \begin{pmatrix} T \\ B \end{pmatrix} \\ \bar{t}_R = t_c &\Rightarrow T_c \\ \tilde{H}^\dagger &\longrightarrow H = \begin{pmatrix} H_0 \\ H_1 \end{pmatrix} \end{aligned}}$$

$\left.\begin{matrix} T_{1,2,3} \\ B_{1,2,3} \end{matrix}\right\}$ color triplet chiral superfields

$(T_c)_{1,2,3}$ } color anti-triplet

$$\mathcal{L} = \left[H^\dagger H + Q^\dagger Q + T_c^\dagger T_c \right]_D + \left\{ \left[y_t \underbrace{(H \cdot Q)}_{SU(2)} \cdot T_c \right]_F + h.c. \right\}$$

color

- field components

$$\begin{pmatrix} T \\ B \end{pmatrix} \longrightarrow \begin{matrix} \tilde{T} & t \\ \tilde{B} & b \end{matrix} \qquad T_c \longrightarrow \tilde{T}_c \quad t_c$$

left handed stop & sbottom

right handed stop

$$\begin{pmatrix} H_0 \\ H_1 \end{pmatrix} \longrightarrow \begin{matrix} \tilde{h}_0 & h_0 \\ \tilde{h}_1 & h_1 \end{matrix}$$

higgsinos

higgs doublet

In components

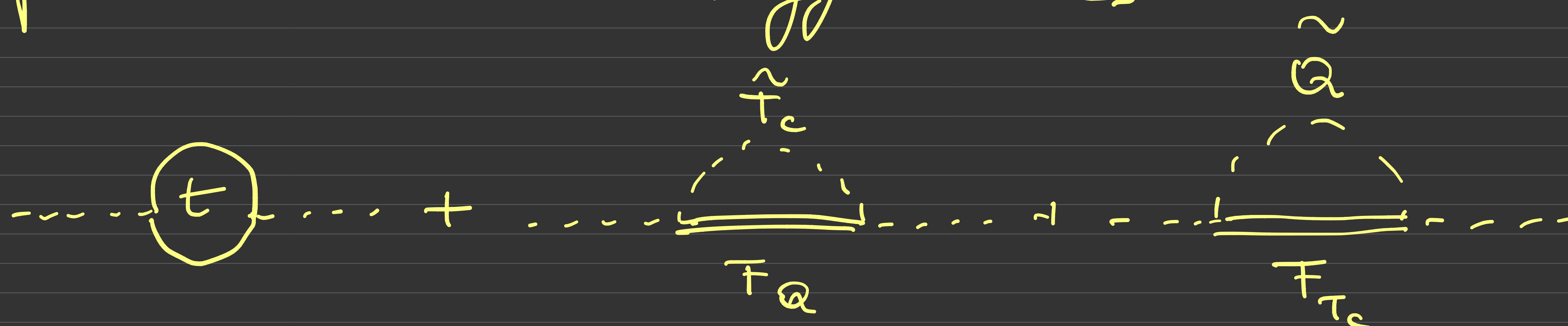
$$|\partial \tilde{Q}|^2 + i\,\bar{q}\,\bar{\partial\!\!\!/}\,q + |F_Q|^2 + |\partial \tilde{T}_c|^2 + i\,\bar{t}_c\,\bar{\partial\!\!\!/}\,t_c + |F_{T_c}|^2$$

$$+ |\partial H|^2 + i\,\bar{\tilde{h}}\,\bar{\partial\!\!\!/}\,\tilde{h} + |F_H|^2$$

$$+ y_t\left[(F_H\cdot\tilde{Q})\tilde{T}_c + (H\cdot F_Q)\tilde{T}_c + (H\cdot\tilde{Q})F_{T_c}\right]$$

$$-\left(y_t(Hq)t_c + y_t\,\tilde{h}\cdot q\,\tilde{T}_c + y_t\,\tilde{h}\,\tilde{Q}\,t_c + h.c.\right)$$

$\boxed{\ }$ 1-loop correction to Higgs mass

$\delta m_H^2 =$ 

sums up to zero as expected from non-renormalization theorem.

⚠ More realistic Model: gaugeless MSSM

- Need two independent Higgs chiral superfields

$$W = Y_u^{ij}(H_2 Q_i)U_{c_j} + Y_d^{ij}(H_1 Q_i)D_{c_j} + Y_e^{ij}(H_1 L_i)E_{c_j}$$

$\Longrightarrow$ pletora of new scalars with B, L, Flavor quantum numbers .... and possibly new interactions breaking them

Ex $W_{\not{R}_p} = \lambda_{ijk} D_{c_i} D_{c_j} U_{c_k} + \lambda'_{ijk}(L_i Q_j) D_{c_k}$

$$+ \quad \lambda''_{ijk} (L_i L_j) E^c_k$$

$W_{\cancel{R_p}}$ would be disastrous ... but can be forbidden, by a seemingly ad-hoc, matter parity

$$D^c, U^c, Q, L, E^c \longrightarrow -(D^c, U^c, Q, L, E^c)$$

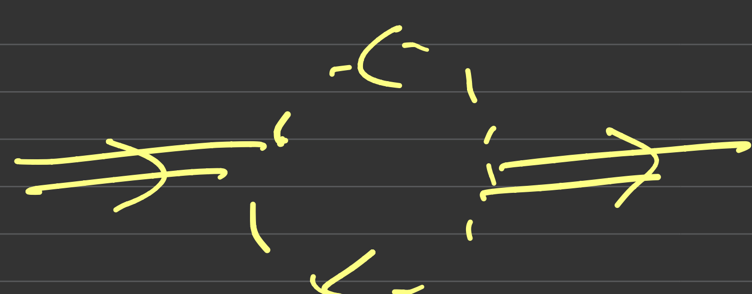
△ 1-loop renormalization

$$\delta W\Big|_{\text{all loops}} = 0 \qquad \delta K \neq 0$$

• $\delta K^{\text{1-loop}} =$  $= \# \left[\Phi^\dagger \Phi\right]_D \ln \Lambda$

enough to compute the $F^\dagger F$ component

• Ex WZ model

 $= (i)^4\, 2\lambda^2 \int \frac{d^4p}{(2\pi)^4} \left(\frac{1}{p^2}\right)^2 = i\frac{\lambda^2}{8\pi^2} \ln\frac{\Lambda^2}{k^2}$

external momentum

$$\Rightarrow \quad \Delta\mathcal{L}_{CT} = -\frac{\lambda^2}{4\pi^2} \ln\frac{\Lambda}{\mu} \left[\phi^\dagger \phi\right]_D$$

$$\mathcal{L}_0 = \left(1 - \frac{\lambda^2}{4\pi^2} \ln\frac{\Lambda}{\mu}\right) \left[\phi^\dagger \phi\right]_D + \left\{ \left[\frac{\lambda}{3} \phi^3\right]_F + h.c. \right\}$$

$$\Rightarrow \quad \lambda_0 = \frac{\lambda(\mu)}{\left(1 - \frac{\lambda^2}{4\pi^2} \ln\frac{\Lambda}{\mu}\right)^{3/2}} \quad \Longrightarrow \quad \beta_\lambda = \frac{3}{8\pi^2} \lambda^3$$

△ A more inspiring way to do things:

- very much like λ can be extended to chiral superfield

we can put a vector superfield Z to source $\phi^\dagger \phi$

$$\Longrightarrow \mathcal{L} = \left[Z \phi^\dagger \phi\right]_D + \left[\frac{a}{3}\phi^3\right] + h.c.$$

$\langle \lambda \rangle \equiv \lambda$ $\qquad \langle Z \rangle \equiv 1$ ... only in the end

◉ Manifest "gauge" redundancy

$$
\begin{cases}
\phi \to e^{\chi}\phi \\
\lambda \to e^{-3\chi}\lambda \\
Z \to Z\, e^{-\chi-\chi^{+}}
\end{cases}
\Rightarrow \quad \text{Obs can only depend on } \bar{\lambda}^2 \equiv \frac{\lambda^{+}\lambda}{Z^3}
$$

We can thus rewrite previous 1-loop result

$$
\mathcal{L}_0 = \Big[\underbrace{Z\Big(1 - \frac{1}{4\pi^2}\frac{\lambda^{+}\lambda}{Z^3}\ln\frac{\Lambda}{\mu}\Big)}_{Z_0 \,\equiv\, \text{bare}}\phi^{+}\phi\Big]_D + \Big[\frac{\lambda}{3}\phi^3\Big] + \ldots
$$

$$
\leftarrow \quad \frac{\lambda_0}{3}
$$

Spontaneously Broken Supersymmetry

- how is Poincaré broken in realistic situations?

- $\underline{\underline{\text{Ex}}}$ superfluid
  - $\exists\ \varphi \to \varphi + c \qquad U(1)_Q$
  - $\varphi = \mu t + \pi$ $\hookrightarrow$ phonon

$\downarrow$

coordinate dependent background

- natural to generalize this to Supersymmetry

$\Longrightarrow$ ~~SUSY~~ $\Longleftrightarrow$ θ dependent superfield background

■ vector superfield $\langle V \rangle = F_0 + F_2 \theta^2 + F_2^* \bar{\theta}^2 + F_4 \theta^2 \bar{\theta}^2$

chiral $\quad$ " $\quad \langle \varphi \rangle = f_0 + f_2 \theta^2$

This breaking could happen in another sector

Many more things could be said here but let us instead go swiftly to the point:

- Assume SUSY breaking manifests itself on the extended SM fields through a background real vector superfield $\langle V \rangle = m^2 \theta^2 \bar{\theta}^2$

$$\mathcal{L}_{Kin} \longrightarrow \left[(1 - c_H V) H^\dagger H + (1 - c_Q V) Q^\dagger Q + (1 - c_T V) T^\dagger T \right]_D$$

+ untouched superpotential

this is the most general possibility at lowest order in $(\partial_\mu, D_\alpha, \bar{D}_{\dot\alpha})$ expansion.

- What does this modification imply?

$$[V\Phi^\dagger\Phi]_D = m^2|\varphi|^2 \longrightarrow \begin{aligned} m_H^2 &= c_H m^2 \\ m_Q^2 &= c_Q m^2 \\ m_T^2 &= c_T m^2 \end{aligned}$$

$\Rightarrow$ ~~SUSY~~ offers a "theory for the origin of mass"

- unlike in the SM, $m_H^2 \neq 0$ is here associated to the breaking of a symmetry.

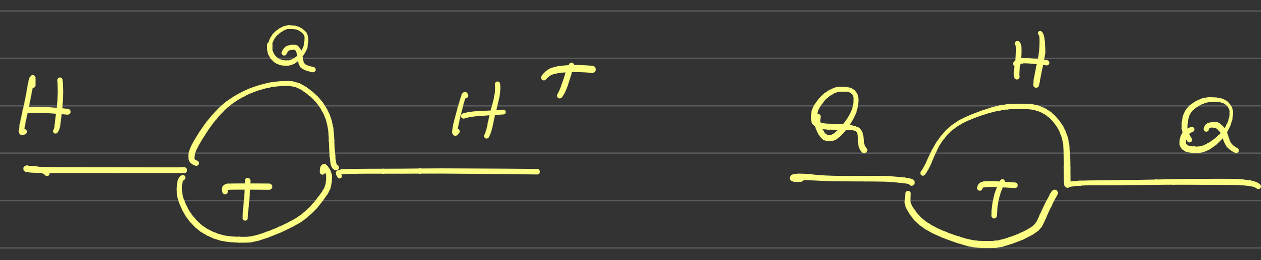
• The Lagrangian $\equiv \mathcal{L}(\mu) \longrightarrow$ RG scale

$\longrightarrow c_{H,Q,T} \longrightarrow c_H(\mu),\ c_Q(\mu),\ c_T(\mu)$

and similarly $m^2_{H,Q,T}$

• Like for WZ model we can express that by

$$\mathcal{L}(\mu) = \underbrace{\left[Z_H(\mu) H^\dagger H + Z_Q(\mu) Q^\dagger Q + Z_T(\mu) T^\dagger T \right]_D}_{\text{All renormalization happens here}} + \left[y_t H Q T \right]_F + \text{h.c.}$$



etc.

$$\mathcal{Z}_H(\mu) \equiv \underbrace{Z_H(\mu)}_{\text{c-number}} \left(1 - c_H(\mu) V\right) \equiv \text{superfield}$$

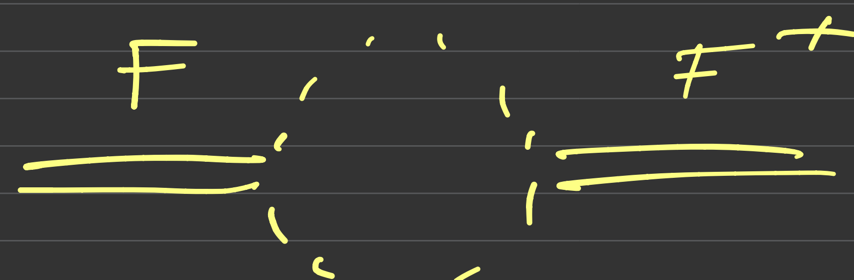
• background reparametrizations

$$\boxed{\begin{aligned} H &\to e^{x_H} H \\ Q &\to e^{x_Q} Q \\ T &\to e^{x_T} T \end{aligned}} \Longrightarrow \begin{aligned} Z_H &\to Z_H\, e^{-x_H - x_H^\dagger} \\ Z_Q &\to Z_Q\, e^{-x_Q - x_Q^\dagger} \\ Z_T &\to Z_T\, e^{-x_T - x_T^\dagger} \end{aligned}$$

$$\Downarrow$$

$$y_t \to e^{-(x_H + x_Q + x_T)}\, y_t$$

$\Longrightarrow$ invariant $\bar{y}_t^2 \equiv y_t^\dagger y_t / Z_H Z_Q Z_T$ controls corrections



fixes coeffs.

The result for the bare $\mathcal{L}_0$ at 1-loop is

$$\mathcal{L}_0 = \Big[Z_H(\mu)\Big(1 - \frac{y_t^+ y_t}{Z_H Z_Q Z_T}\,\frac{3}{8\pi^2}\,\ln\frac{\Lambda}{\mu}\Big) H^+ H \quad +$$

$$Z_Q(\mu)\Big(1 - \bar{y}_t^{\,2}\,\frac{1}{8\pi^2}\,\ln\frac{\Lambda}{\mu}\Big) Q^+ Q$$

$$Z_T(\mu)\Big(1 - \bar{y}_t^{\,2}\,\frac{2}{8\pi^2}\,\ln\frac{\Lambda}{\mu}\Big) T^+ T \Big]_D \quad + \ldots$$

$$Z_H^0 \equiv Z_H(\mu)\big(1 - \bar{y}_t^{\,2}\big) \equiv \text{bare wave function}$$

$\Longrightarrow$ RG evolution of the $Z(\mu)$'s encapsulates the running of both coupling and masses!

◎ Now then

$$0 = \mu \frac{d}{d\mu} \mathcal{Z}_H^0 \longrightarrow \frac{d}{d\ln\mu} Z_H(\mu) = -\frac{3}{8\pi^2} Z_H \frac{y_t^\dagger y_t}{Z_H Z_Q Z_T}$$

$$\Longrightarrow \begin{cases} \dfrac{d}{d\ln\mu} \ln Z_H(\mu) = -\dfrac{3}{8\pi^2} \bar{y}_t^2 \equiv -\dfrac{3}{8\pi^2} \dfrac{y_t^\dagger y_t}{Z_H Z_Q Z_T} \\ \dfrac{d}{d\ln\mu} \ln Z_Q = -\dfrac{1}{8\pi^2} \bar{y}_t^2 \\ \dfrac{d}{d\ln\mu} \ln Z_T = -\dfrac{2}{8\pi^2} \bar{y}_t^2 \end{cases}$$

and similarly (*)

- define $Z_X \equiv Z_H Z_Q Z_T$

$\Rightarrow (*) \Rightarrow \frac{d}{d\ln\mu} Z_x = -\frac{3}{4\pi^2} y_t^2$

$y_t \equiv$ number

$$\Rightarrow \boxed{Z_x(\mu) = Z_x(\mu_0) - \frac{3}{4\pi^2} y_t^2 \ln\frac{\mu}{\mu_0}}$$

superfield

number

remember

$Z_i = \underbrace{Z(\mu)}_{\text{number}}(1 - m_i^2\theta^2\bar\theta^2)$

- This eq. contains the running of every thing

△ coupling $y_t^2(\mu) = \frac{y_t^2}{Z_x(\mu)} = \frac{y_t^2}{Z_x(\mu_0)} \frac{1}{1 - \frac{3}{4\pi^2}\frac{y_t^2}{Z_x(\mu_0)}\ln\frac{\mu}{\mu_0}}$

$$= \frac{y_t^2(\mu_0)}{1 - \frac{3y_t^2(\mu_0)}{4\pi^2}\ln \mu/\mu_0}$$

⚠ masses

$$\left[\ln Z_x(\mu)\right]_D = \left[\ln\left(Z_x(\mu_0) - \frac{3}{4\pi^2} y_t^2 \ln\frac{\mu}{\mu_0}\right)\right]_D$$

$$\Rightarrow X(\mu) = X(\mu_0)\left(\frac{y_t(\mu)}{y_t(\mu_0)}\right)^2 \qquad X(\mu) \equiv m_H^2 + m_Q^2 + m_T^2$$

$$\frac{Z_H(\mu)}{Z_Q^3(\mu)} = \frac{Z_H(\mu_0)}{Z_Q^3(\mu_0)} \qquad \frac{Z_T}{Z_Q^2}(\mu) = \frac{Z_T}{Z_Q^2}(\mu_0)$$

$$\Downarrow \qquad\qquad \Downarrow$$

$$\left(m_H^2 - 3m_Q^2\right)(\mu) = \left(m_H^2 - 3m_Q^2\right)(\mu_0) \qquad \left(m_T^2 - 2m_Q^2\right)(\mu) = \left(\quad\right)(\mu_0)$$

Ex Universal initial condition $m_H^2 = m_Q^2 = m_T^2 = m_0^2$

at $\mu = \mu_0$

$$r(\mu) = \left(\frac{y_t(\mu)}{y_t(\mu_0)} \right)^2$$

$$\left. \begin{aligned} m_H^2(\mu) &= m_0^2 \left(\frac{3r(\mu) - 1}{2} \right) \end{aligned} \right\}$$ can easily cross to negative $\Rightarrow SU(2)$ broken

$$\left. \begin{aligned} m_Q^2 &= m_0^2 \left(\frac{r(\mu)+1}{2} \right) \\ m_T^2 &= m_0^2 \, r(\mu) \end{aligned} \right\}$$ always ≥ 0 ✓

$SU(3)_c$ unbroken

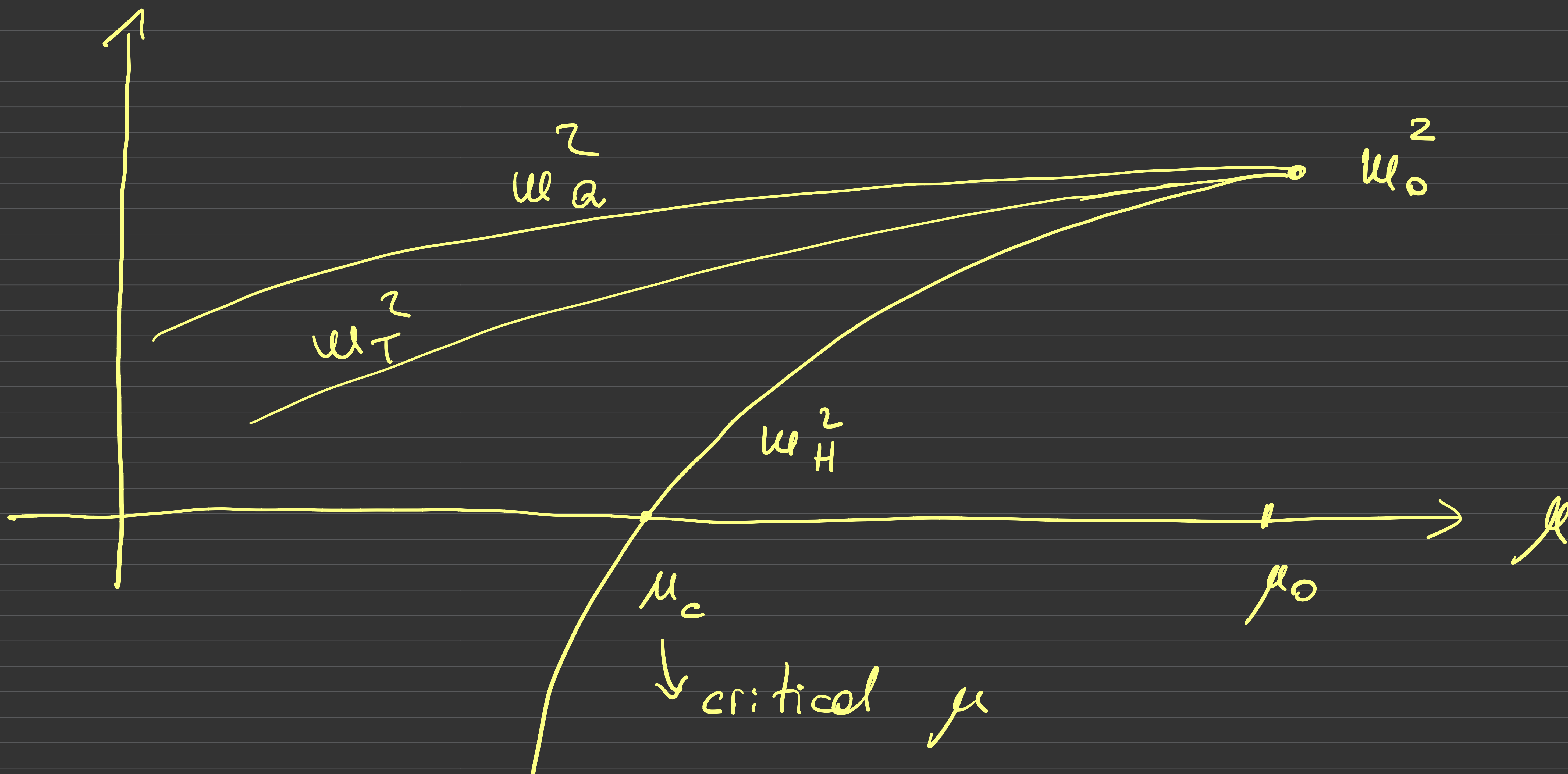
Notice $r(\mu) = \dfrac{1}{1 - \frac{3}{4\pi^2} y_t^2(\mu_0) \ln\frac{\mu}{\mu_0}}$

If expanded in y_t^2 shows corrections to m_H^2 in agreement with the selection rules previously discussed

Ex

$$m_H^2(\mu) = m_H^2(\mu_0) - \frac{3}{8\pi^2} y_t^2 \left(m_H^2 + m_Q^2 + m_T^2 \right) \ln\frac{\mu_0}{\mu} + \cdots$$

We can concretely see what tuning would look like



if m_0 chosen right at μ_c RG evolution yelds $m_H\big|_{phys} = 0$

- but for generic choices

$$m_H^2 \sim -m_0^2 \sim -m_{\tilde{q}}^2$$

$\Longrightarrow$ squarks at weak scale