Functional determinants and lifetime of the Standard Model

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based on *Phys.Rev.Lett.* 134 (2025) 1, 011601, [arxiv: 2406.05180]
and *JHEP* 05 (2025) 100, [arXiv: 2502.15878]
in collaboration with P. Baratella, M. Nemevšek, Y. Shoji, L. Ubaldi

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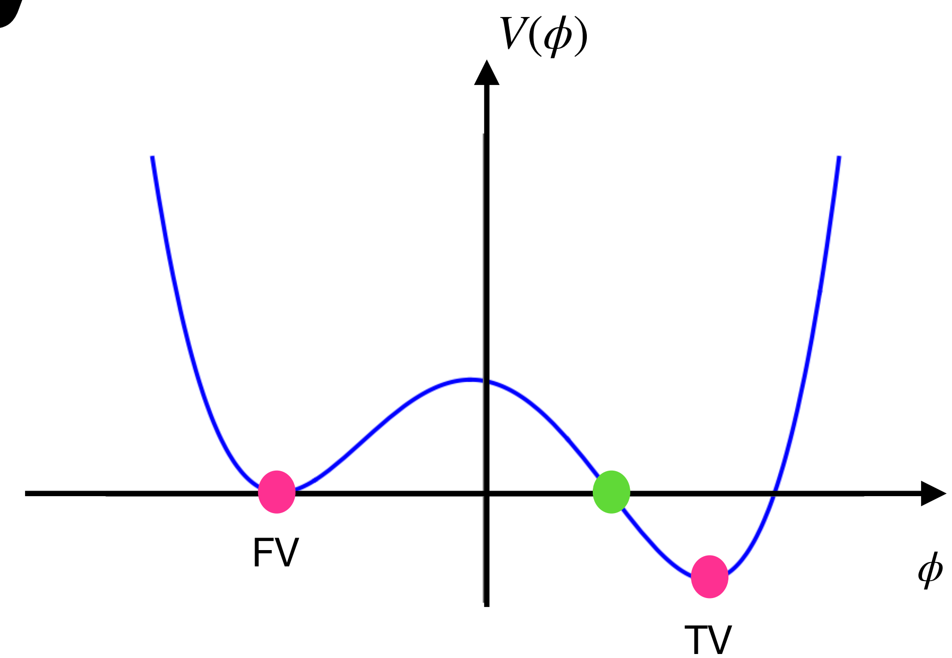
Cargese 2025-BSM odyssey

22.07.2025

False Vacuum Decay



- Metastable state decays via tunnelling to energetically lower true vacuum

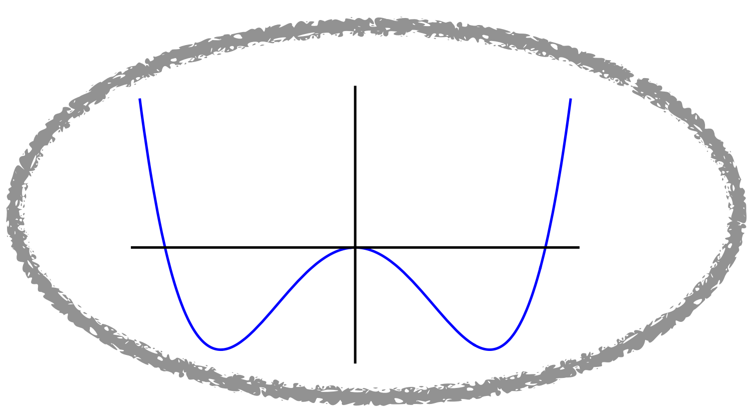
- Described by bounce solution

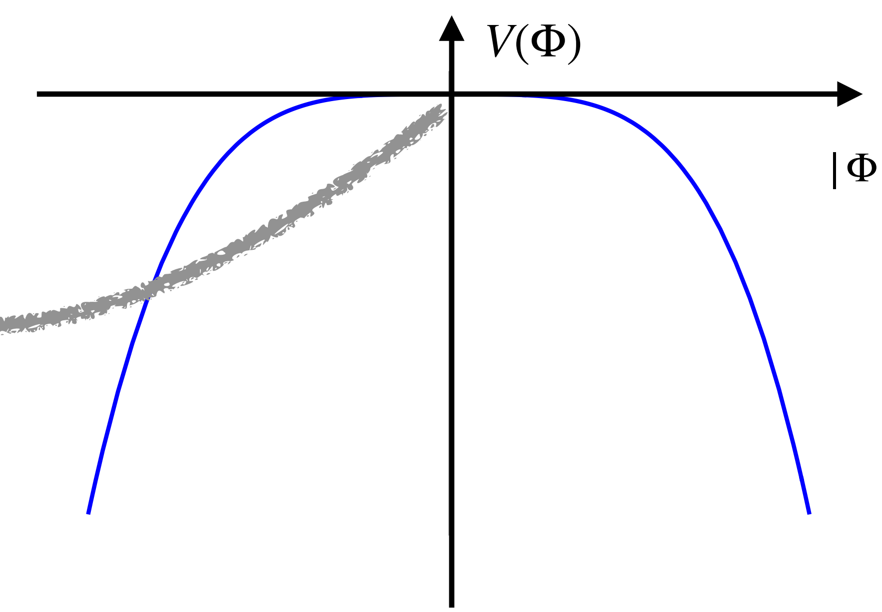
- Example: EW vacuum metastable:

$\lambda < 0$

$$8\pi^2 \frac{d\lambda}{d \log \mu} \simeq -3y_t^4 + 6y_t^2 \lambda$$

$$V(\Phi) = -m^2 |\Phi|^2 + \lambda |\Phi|^4 \simeq \lambda |\Phi|^4$$

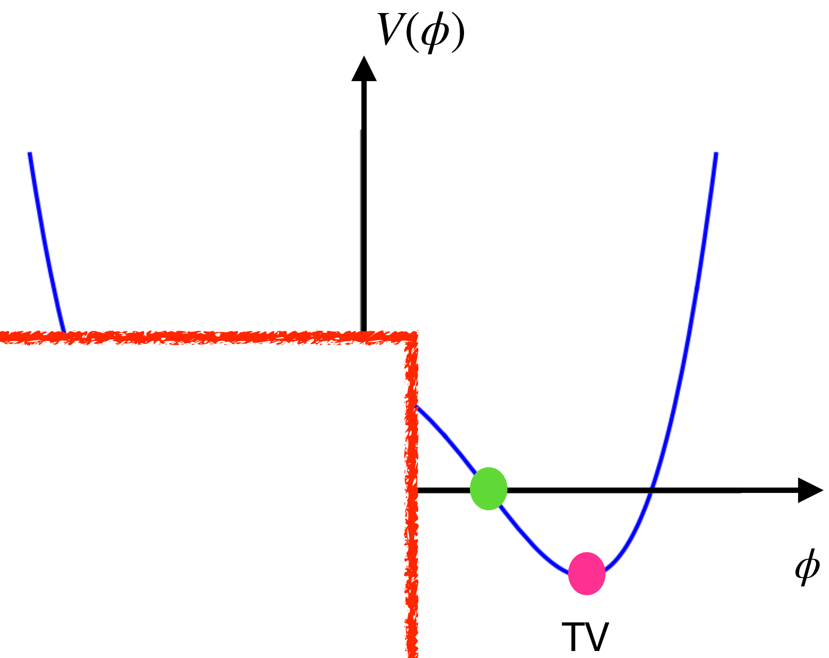




- Decay rate per time per unit volume at 1-loop:

$$\gamma = \mathscr{A} e^{-B} \qquad B = S[\bar{\varphi}] - S[\varphi_{FV}]$$

False Vacuum Decay



✳ Metastable state decays via tunnelling to energetically lower true vacuum

✳ Describe

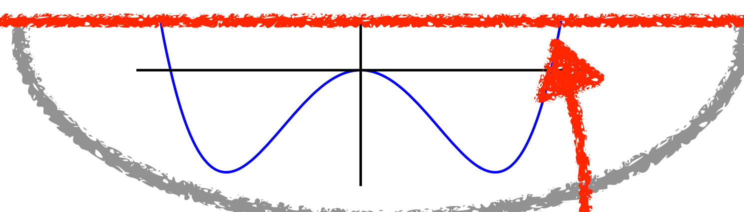
✳ Example

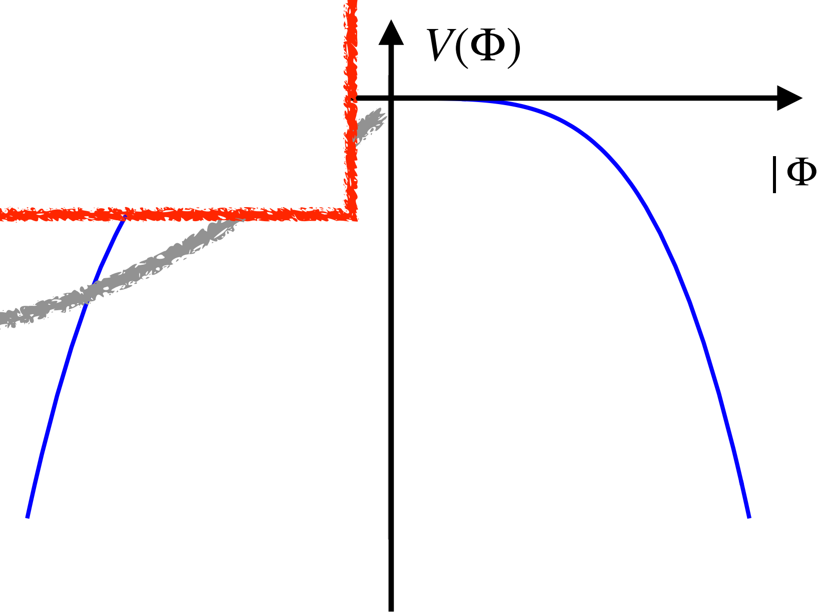
$$V(\Phi) = \ldots - 3y_t^4 + 6y_t^2\lambda$$

$$\mathscr{A} = \mathscr{A}^{\phi}\mathscr{A}^{\psi}\mathscr{A}^{(A,a)}$$

$$S \simeq S[\bar{\phi}] + \underbrace{S'[\bar{\phi}]}_{=0}\ \delta\eta_i + \frac{1}{2}\delta\eta_i\ S''[\bar{\phi}]\ \delta\eta_j \qquad \hat{S}'' \equiv S''[\phi_{\rm FV}]$$

$$\mathscr{A} \sim \frac{\det S''}{\det \hat{S}''}$$





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Regularisation of Determinant

- Regularisation procedure: isolate UV divergent one-loop diagrams
  [Baacke, Lavrelashvili, '03]

$$\ln \frac{\det S''}{\det \hat{S}''} = \sum_\nu \left(d_\nu \ln \frac{\det S''_\nu}{\det \hat{S}''_\nu} - \text{subtraction}_\nu \right) + \left(\text{addback}\right)_{\text{regularised}}$$

- Problem:

$$\int_0^\infty \rho \delta m^2(\rho) K_\nu^2(m\rho)^2 \int_0^\rho \rho' I_\nu^2(m\rho')$$

$$\int \mathrm{d}^D q \, \widetilde{\delta m}^2(q) \widetilde{\delta m}^2(-q) \int \frac{\mathrm{d}^D k}{(k^2 - m^2)((k+q)^2 - m^2)}$$

Final Simple Formulas

[Baratella, Nemevšek, Shoji, K.T., Ubaldi, arxiv: 2502.15878]

✳ scalar fluctuations: [Dunne, Kirsten, '06]

$$\ln \frac{\det' S''}{\det \hat{S}''}\bigg|_{\phi} = \sum_{\nu=1}^{\infty}\left(d_\nu^\phi \ln R_\nu^\phi - \frac{\nu}{2}\int_\rho \rho\, \mathrm{tr}\delta m^2 + \frac{1}{8\nu}\int_\rho \rho^3 \mathrm{tr}\delta m^4\right) - \frac{1}{8}\int_\rho \rho^3 \mathrm{tr}\delta m^4\left(\frac{1}{\epsilon} + 1 + \gamma_E + \ln\frac{\tilde{\mu}\rho}{2}\right)$$

✳ fermion fluctuations:

$$\ln \frac{\det S''}{\det \hat{S}''}\bigg|_{\psi} = \sum_{\nu=1}^{\infty}\left(d_\nu^\psi \ln R_\nu^\psi - \left(\nu + \tfrac{1}{2}\right)\int_\rho \rho\, \mathrm{tr}\delta m_\psi^2 + \frac{1}{4\nu}\int_\rho \rho^3 \mathrm{tr}\left(\delta m_\psi^4 + \dot{m}_\psi{}^2\right)\right)$$
$$-\frac{1}{4}\int_\rho \rho^3 \mathrm{tr}\left(\delta m_\psi^4 + \dot{m}_\psi{}^2\right)\left(\frac{1}{\epsilon} + 1 + \gamma_E + \ln\frac{\tilde{\mu}\rho}{2}\right)$$

✳ gauge boson fluctuations:

$$\ln \frac{\det' S''}{\det \hat{S}''}\bigg|_{A,a}^{\xi=1} = \sum_{\nu=1}^{\infty}\left(d_\nu^\phi \ln R_\nu^{(Da)} + 2\, d_\nu^T \ln R_\nu^{(T)} - \frac{1}{2}\int_\rho \rho\left(\nu\, \delta m_a^2 + 2(2\nu+1)\,\delta m_A^2\right) + \frac{1}{8\nu}\int_\rho \rho^3\left(\delta m_a^4 + 4\,\delta m_A^4 + 8\,\dot{m}_A^2\right)\right)$$
$$+\frac{1}{2}\int_\rho \rho\, \delta m_A^2 + \frac{1}{8}\int_\rho \rho^3 \delta m_A^4 - \frac{1}{8}\int_\rho \rho^3\left(\frac{1}{\epsilon} + 1 + \gamma_E + \ln\frac{\tilde{\mu}\rho}{2}\right)\left(\delta m_a^4 + 4\,\delta m_A^4 + 8\,\dot{m}_A^2\right)$$

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$$\ln \frac{\det' S''}{\det \hat{S}''}\bigg|_{A,a}^{\xi=1} = \sum_{\nu=1}^{\infty}\left(d_\nu^\phi \ln R_\nu^{(Da)} + 2\, d_\nu^T \ln R_\nu^{(T)} - \frac{1}{2}\int_\rho \rho\left(\nu\, \delta m_a^2 + 2(2\nu+1)\,\delta m_A^2\right) + \frac{1}{8\nu}\int_\rho \rho^3\left(\delta m_a^4 + 4\,\delta m_A^4 + 8\,\dot{m}_A^2\right)\right)$$
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✳ fermion flu…

$$\ln \frac{\det S''}{\det \hat{S}''}\Bigg|_{\psi} = \sum_{\nu=1} \cdots$$

$$\cdots \frac{}{4}\int_\rho \rho^{3}\, \mathrm{tr}\left(\delta m_\psi \cdots m_\psi \cdots\right)\left(\frac{1}{\epsilon} + 1 + \gamma_E + \ln \frac{\cdots}{2}\right)$$

$$\mathscr{A}^{(A,\phi)} = \mathscr{A}^{(D,\phi)}\mathscr{A}^{T}$$

previous: $d_j^T = (j+1)^2 = d_j^D = d_j^{\phi}$

corrected: $d_j^T = j(j+2)$ [Baratella, Nemevšek, Shoji, K.T., Ubaldi, arxiv: 2406.05180]

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Standard Model Lifetime

[Isidori, Ridolfi, Stumia, ’01;
Andreassen, Frost, Schwartz, ’17;
Chigusa, Moroi, Shoji, ’17& ’18]

$$d_j^T = j(j+2)$$

[Baratella, Nemevšek,
Shoji, K.T., Ubaldi,
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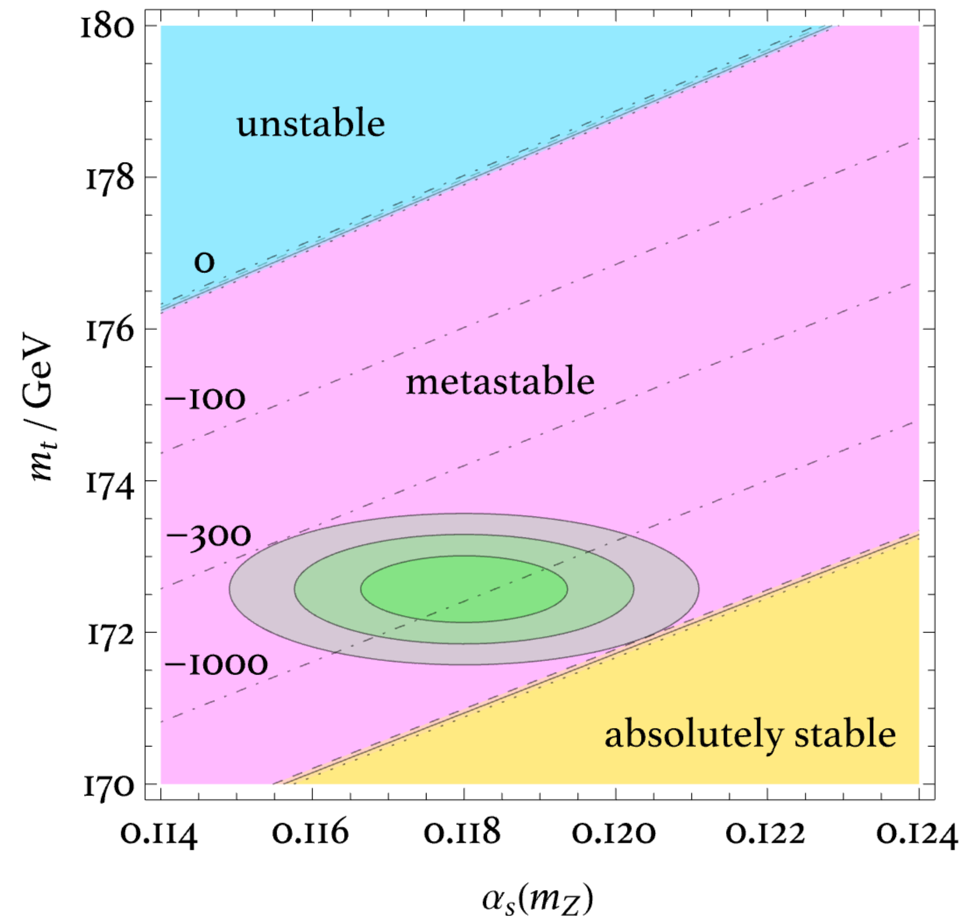
$$\frac{\det S''}{\det \hat{S}''} = \prod_{\nu=1}^{\infty} \left(\frac{\det S''_\nu}{\det \hat{S}''_\nu} \right)^{d_\nu}$$

$$\gamma$$

Updated EW decay rate:

$$\log_{10} \frac{\gamma}{\mathrm{Gyr}^{-1}\mathrm{Gpc}^{-3}} = -877 \;\rightarrow\; -871^{+35\;+175\;+209}_{-37\;-253\;-330}$$



Thank you

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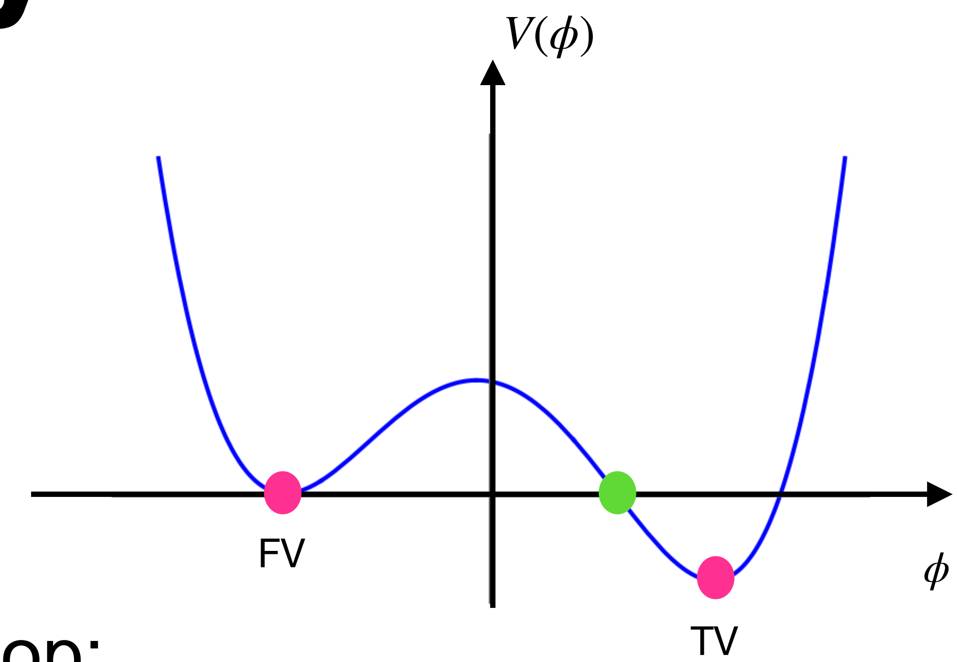
- Decay rate per time per unit volume at 1-loop:



$$\gamma = e^{-S} \left(\frac{S}{2\pi}\right)^2 \mathrm{Im} \sqrt{\left.\frac{\det \hat{S}''}{\det' S''}\right|_{\phi}} \left.\frac{\det S''}{\det \hat{S}''}\right|_{\psi} V_G \, J_G \sqrt{\left.\frac{\det \hat{S}''}{\det' S''}\right|_{A,a}} \left.\frac{\det S''}{\det \hat{S}''}\right|_{c\bar{c}}$$

tree level

fluctuation operators at bounce/FV

group volume

zero mode removal

Jacobians for zero modes

Scalar Fluctuations

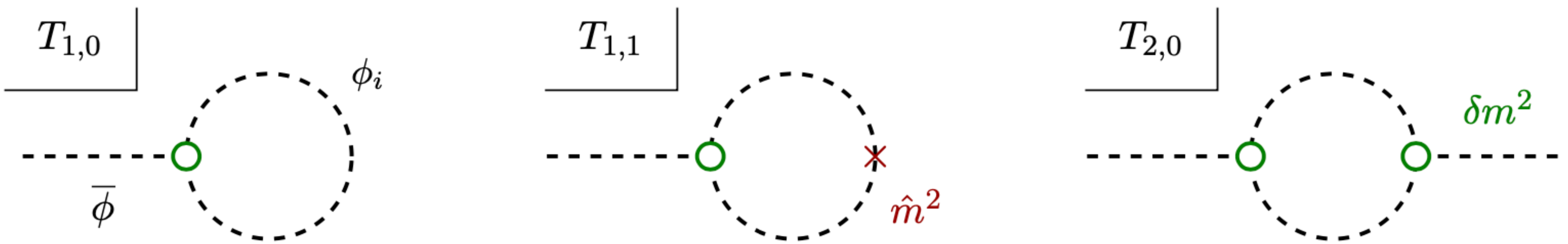
"field dependent mass" $m^2(\rho) = \frac{d^2V}{d\phi^2}|_{\bar{\phi}}$

$$S''[\bar{\phi}] = -\partial^2 + m^2(\rho)$$

$\hat{S}'' \equiv S''[\phi_{FV}]$

- Euclidean action:

$$S[\phi] = \int d^4x \left(\frac{1}{2}(\partial_\mu \phi)^2 + V(\phi)\right) \simeq S[\bar{\phi}] + \underbrace{S'[\bar{\phi}]}_{=0}\,\delta\phi + \frac{1}{2}\delta\phi S''[\bar{\phi}]\delta\phi$$



$\delta m^2 \equiv m^2 - \hat{m}^2$

$$\ln \frac{\det S''}{\det \hat{S}''} = \mathrm{tr}\ln\left(1 + \frac{1}{-\partial^2 + \hat{m}^2}\delta m^2\right) = -\sum_{n=1}^{\infty}\frac{(-1)^n}{n}\mathrm{tr}\left(\frac{1}{-\partial^2 + \hat{m}^2}\delta m^2\right)^n$$

$$= \underbrace{\mathrm{tr}\frac{1}{-\partial^2}\delta m^2}_{T_{1,0}} - \underbrace{\mathrm{tr}\frac{1}{-\partial^2}\hat{m}^2\frac{1}{-\partial^2}\delta m^2}_{T_{1,1}} - \frac{1}{2}\underbrace{\mathrm{tr}\frac{1}{-\partial^2}\delta m^2\frac{1}{-\partial^2}\delta m^2}_{T_{2,0}} + O\left((\delta m^2)^3, \hat{m}^4\right)$$