General Implications of the Froggatt-Nielsen Mechanism for Leptons

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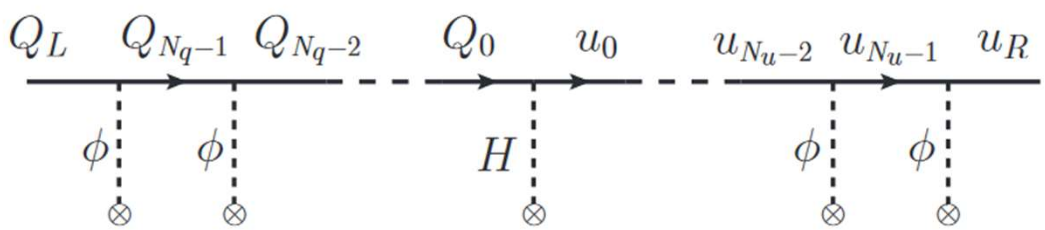
Froggatt-Nielsen



- All fermions are charged under a new $U(1)_{FN}$
- Yukawas generated by effective operators coupled to heavy flavon
- If $\frac{\langle\phi\rangle}{\Lambda} \equiv \epsilon \sim 0.1$, this can result in large hierarchies in low-energy Yukawas

$$\mathcal{L} \supset -c^u_{ij}\bar{Q}_i\tilde{H}u_j\left(\frac{\phi}{\Lambda}\right)^{|X_{Q_i}-X_{u_j}|} - c^d_{ij}\bar{Q}_iHd_j\left(\frac{\phi}{\Lambda}\right)^{|X_{Q_i}-X_{d_j}|}$$

Determining Charges (Textures)

arXiv: 2306.08026

Usual approach: work backwards from masses and mixings

Question: Does this miss phenomenologically viable textures?

Secondary question: Are there experimental signatures of different FN models which could probe these different textures?

Lepton Extension

$$\mathcal{L} \supset -c^{\ell}_{ij} L_i H^{\dagger} \bar{e}_j \left(\frac{\phi}{\Lambda}\right)^{|X_{L_i} + X_{\bar{e}_j}|} \quad + \quad ?$$

- In the lepton sector, need to contend with how neutrinos get mass

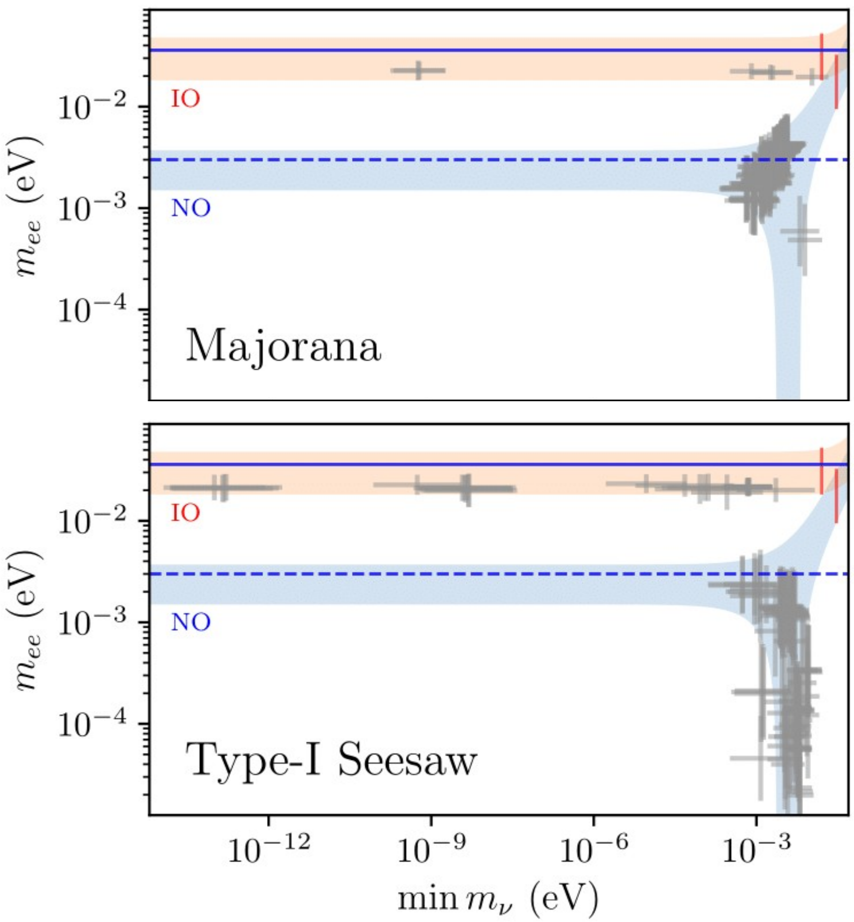
Lepton Extension

$$\mathcal{L} \supset -c^{\ell}_{ij} L_i H^\dagger \bar{e}_j \left(\frac{\phi}{\Lambda}\right)^{|X_{L_i} + X_{\bar{e}_j}|} \quad + \quad ?$$

1. Pure Dirac
2. Generic high energy physics: Weinberg operator $\frac{c^W_{ij}}{\Lambda_W}\left(\frac{\phi}{\Lambda}\right)^{|X_{Li}+X_{Lj}|}(L_i H)(L_j H)$
3. Type I Seesaw

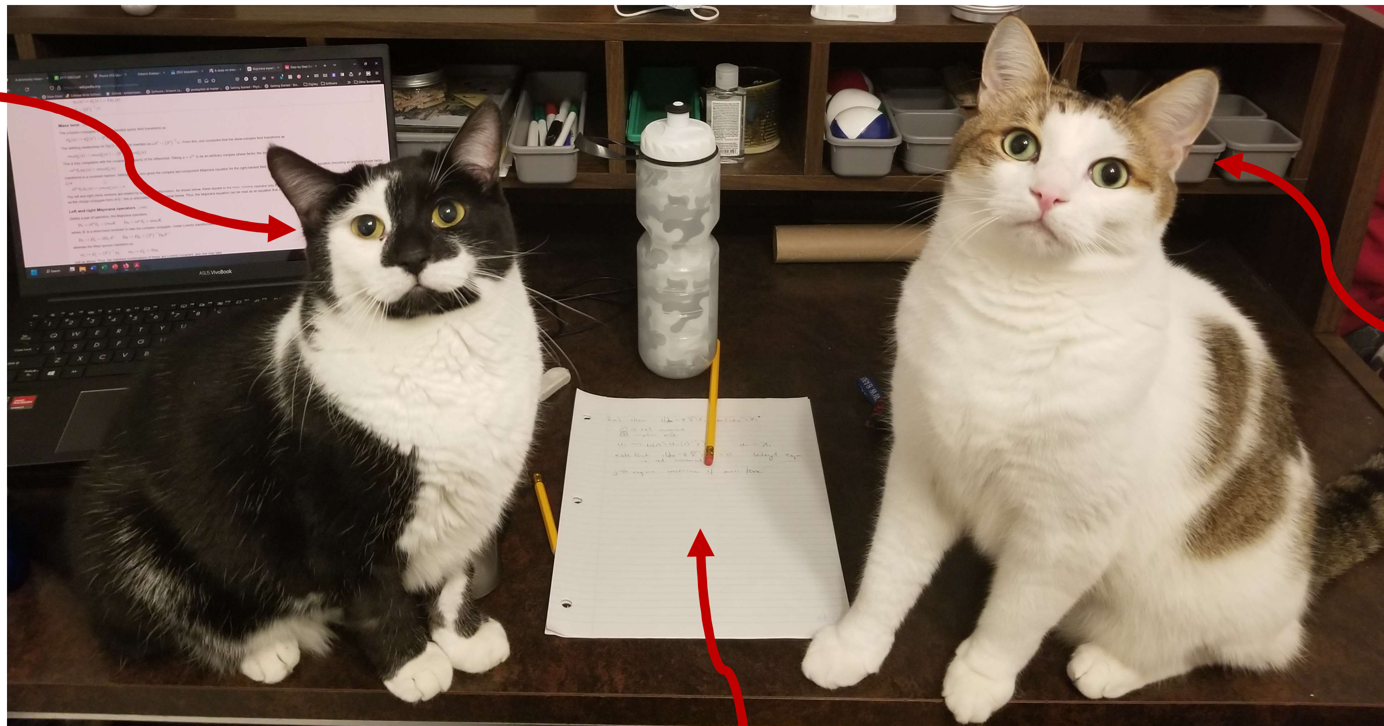
Neutrinoless Double Beta Decay

- Inverted order models all fall below current bounds—not imposed by our method
- Several Majorana type models will be probed in the future
- Many other models lie just below future experimental limits



Thank You! Questions?

Sylvester



Nismo

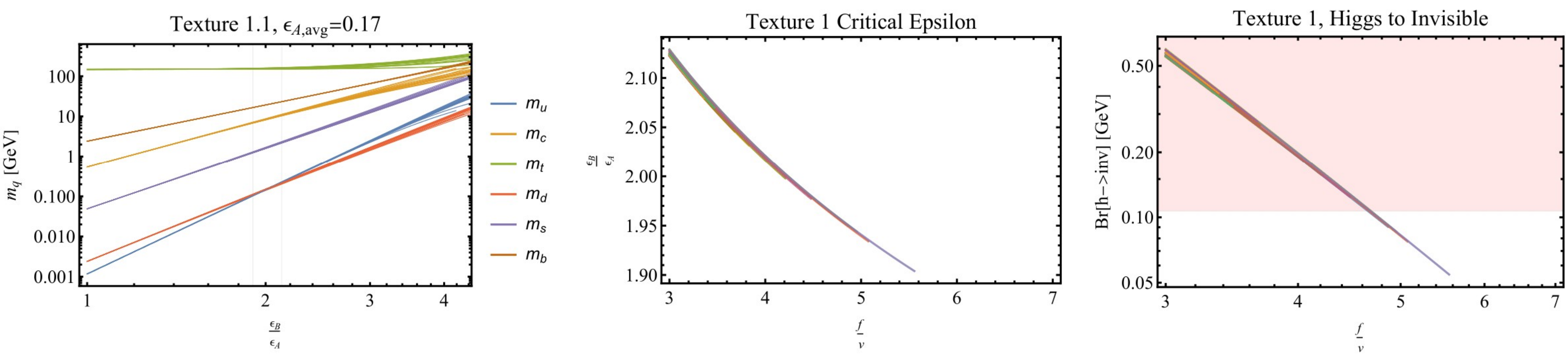
QFT Homework

Backup Slides

Froggatt-Nielsen + the Mirror Twin Higgs

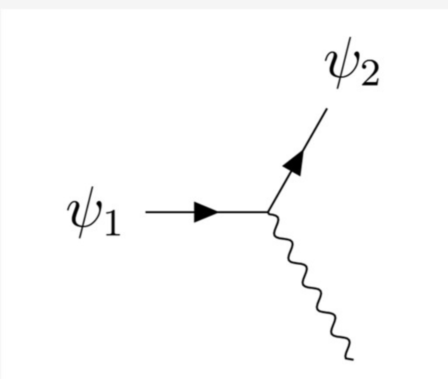
- Small Z2 breaking in the ϵ parameter between the two sectors leads to very large differences in masses
- Can easily achieve a 5 GeV nucleon in the dark sector—coincidence problem
- arXiv: 1706.05548 for an exemplar case

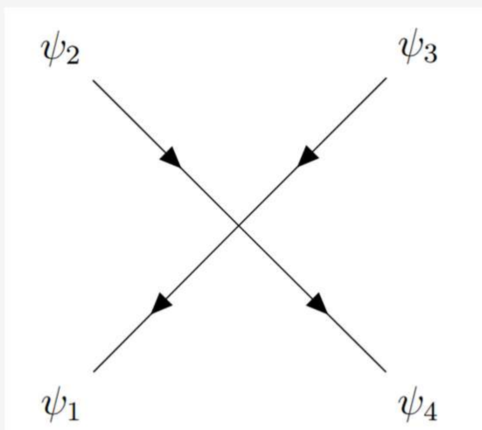
$$X_Q = \{3, 2, 0\},\ X_u = \{-4, -2, 0\},\ X_d = \{-3, -3, -3\}$$



Observables

- Matching using python package wilson: 1804.05033
- Flavour-violating decays using flavio arXiv: 1810.08132
- Charged lepton flavour violating collider observables





$$\frac{c_{ijkl}}{\Lambda^2_{eff}} \left(\bar{\psi}_i \psi_j\right) \left(\bar{\psi}_k \psi_l\right)$$

$$\frac{c_{ij} v}{\sqrt{2}\Lambda^2_{eff}} \bar{\psi}_{Li} \sigma^{\mu\nu} \psi_{Rj} F_{\mu\nu} + h.c.$$

$$\frac{1}{\Lambda^2_{eff}} = \frac{\epsilon^{|X_i - X_j + X_k - X_l|}}{\Lambda^2_F}$$

Dirac Textures

$$\mathcal{L} \supset -c^{\ell}_{ij} L_i H^\dagger \bar{e}_j \left(\frac{\phi}{\Lambda}\right)^{|X_{L_i}+X_{\bar{e}_j}|} - c^{\nu}_{ij} L_i H N_j \left(\frac{\phi}{\Lambda}\right)^{|X_{L_i}+X_{N_j}|}$$

L_1	L_2	L_3	$\bar{e}_1$	$\bar{e}_2$	$\bar{e}_3$	N_1	N_2	N_3	$F_{1.35}$ (%)	F_2 (%)	F_5 (%)	ϵ	NO (%)
6	5	5	-3	-2	0	9	8	8	0.030	3.0	63	0.10	96
3	3	3	2	-1	-6	9	9	8	0.030	1.9	62	0.07	99
3	3	3	2	-5	-6	9	9	8	0.030	1.9	62	0.07	99
7	7	6	-4	-2	0	9	9	9	0.026	1.9	47	0.14	99
7	7	6	-4	-3	-1	9	7	7	0.024	3.1	54	0.11	99
3	3	3	2	0	-5	9	9	8	0.024	2.0	62	0.07	99
3	3	3	2	0	-1	9	9	8	0.024	2.0	62	0.07	99
6	5	5	-3	-2	0	9	7	7	0.023	1.9	48	0.08	97
7	3	3	2	0	-5	9	9	9	0.021	1.7	65	0.08	93
6	6	6	-4	-3	-1	9	6	5	0.020	1.2	53	0.07	99

arXiv: 2501.00629

Majorana

$$\mathcal{L} \supset -c^{\ell}_{ij} L_i H^{\dagger} \bar{e}_j \left(\frac{\phi}{\Lambda}\right)^{|X_{L_i}+X_{\bar{e}_j}|} - \frac{c^W_{ij}}{\Lambda_W} \left(\frac{\phi}{\Lambda}\right)^{|X_{Li}+X_{Lj}|} (L_i H)(L_j H)$$

L_1	L_2	L_3	$\bar{e}_1$	$\bar{e}_2$	$\bar{e}_3$	$F_{1.23}$ (%)	F_2 (%)	F_5 (%)	ϵ	$\log \Lambda$	NO (%)
2	0	-1	7	6	4	0.027	4.6	54	0.24	15	91
5	5	-2	7	-2	-3	0.025	5.6	56	0.08	12	3
4	4	3	5	2	0	0.022	5.7	62	0.23	11	96
7	6	5	7	3	0	0.020	6.8	60	0.39	11	97
6	6	5	5	1	-1	0.020	6.2	62	0.30	10	96
7	7	6	2	-1	-3	0.020	5.7	62	0.23	7.6	96
5	5	4	6	2	0	0.019	6.1	62	0.30	11	96
7	7	6	4	0	-2	0.019	6.2	62	0.30	9	96
5	5	-2	7	-2	-7	0.019	5.5	56	0.08	12	3
1	1	-1	-7	-5	-4	0.019	4.9	66	0.18	15	2

Type I Seesaw

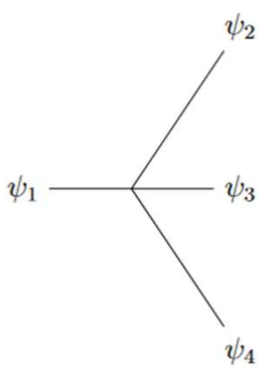
$$\mathcal{L} \supset -c^{\ell}_{ij} L_i H^\dagger \bar{e}_j \left(\frac{\phi}{\Lambda}\right)^{|X_{L_i}+X_{\bar{e}_j}|} - c^{\nu}_{ij} H L_i N_j \left(\frac{\phi}{\Lambda}\right)^{|X_{L_i}+X_{N_j}|} - c^{M}_{ij} \frac{M}{2} N_i N_j \left(\frac{\phi}{\Lambda}\right)^{|X_{N_i}+X_{N_j}|}$$

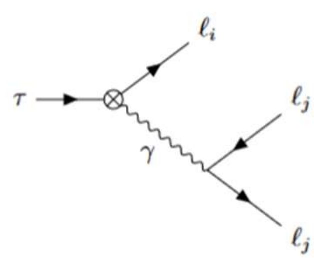
L_1	L_2	L_3	$\bar{e}_1$	$\bar{e}_2$	$\bar{e}_3$	N_1	N_2	N_3	$F_{1.18}$ (%)	F_2 (%)	F_5 (%)	ϵ	$\log\Lambda$	NO (%)
6	1	-1	7	7	6	3	0	-4	0.035	7.2	39	0.36	14	93
6	1	-1	6	6	6	3	0	-4	0.034	7.4	38	0.34	14	93
6	1	-2	7	7	7	5	0	-4	0.031	5.3	34	0.37	14	93
7	2	-1	7	7	7	4	0	-5	0.028	6.7	35	0.40	14	95
6	2	-6	2	1	1	3	2	-4	0.027	6.5	38	0.16	12	90
4	1	-1	6	5	5	6	0	-3	0.026	6.2	34	0.27	14	93
4	1	-1	7	5	5	4	0	-3	0.025	5.8	35	0.29	14	93
7	2	-1	7	7	6	4	0	-5	0.025	5.9	34	0.39	14	95
6	1	-1	7	6	6	3	0	-4	0.024	6.8	38	0.35	14	93
5	1	-1	5	5	5	2	0	-3	0.024	5.8	36	0.27	14	79

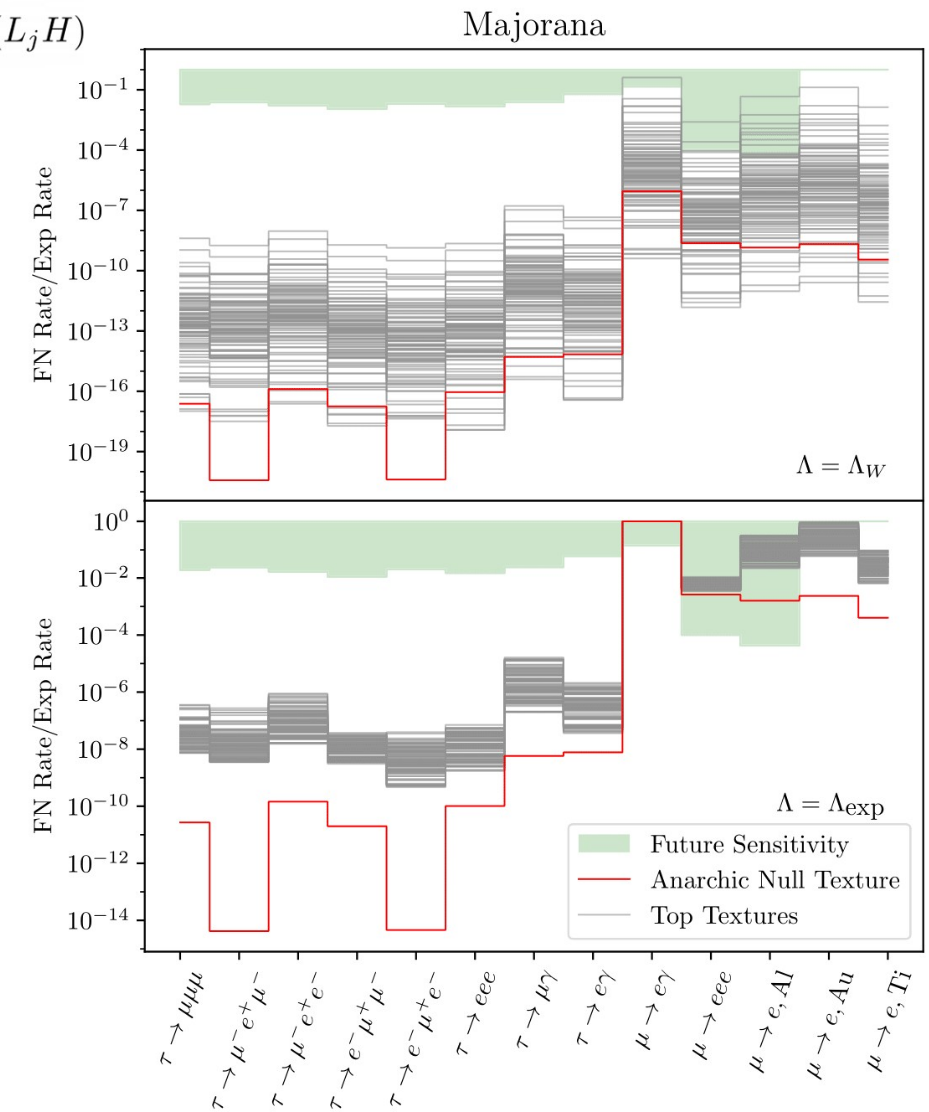
$$\mathcal{L} \supset -c^{\ell}_{ij} L_i H^\dagger \bar{e}_j \left(\frac{\phi}{\Lambda}\right)^{|X_{L_i}+X_{\bar{e}_j}|} - \frac{c^W_{ij}}{\Lambda_W}\left(\frac{\phi}{\Lambda}\right)^{|X_{Li}+X_{Lj}|}(L_i H)(L_j H)$$

Branching Fractions - Majorana

- Either take $\Lambda_W = \Lambda_F$ or assume $\Lambda_W > \Lambda_F = \Lambda_{exp}$
- For first case, prediction is fixed and textures can be entirely excluded
- Second case, similar story to Dirac case



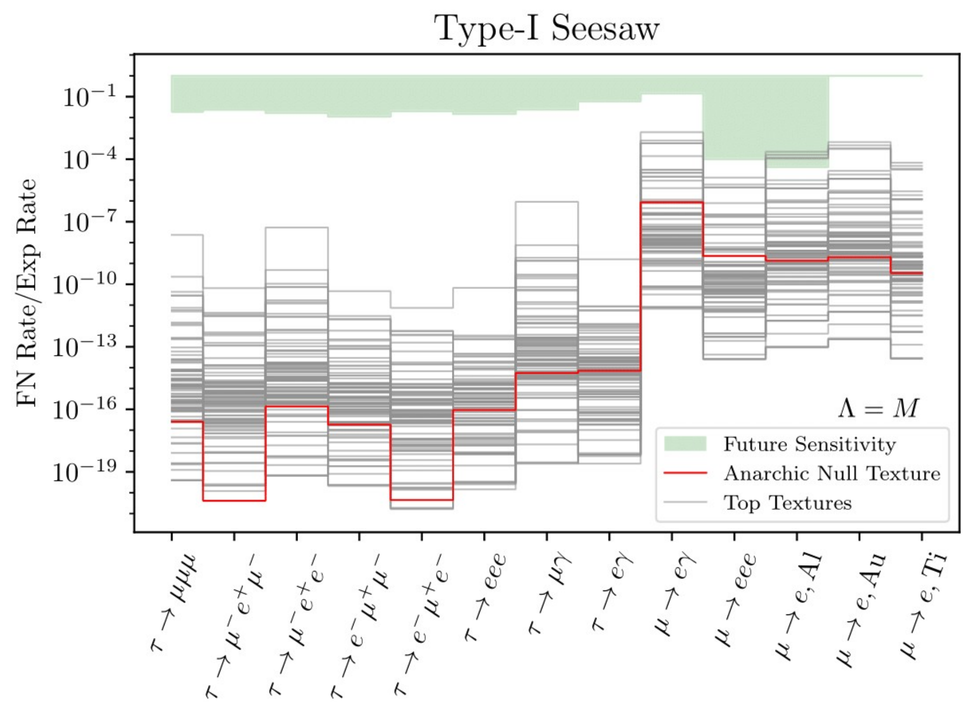


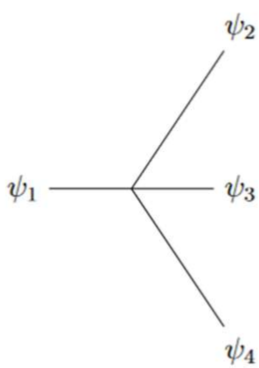


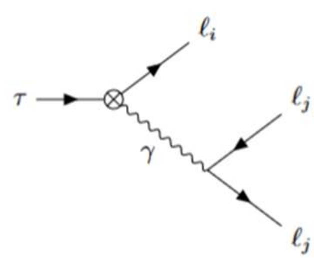
$$\mathcal{L} \supset -c^{\ell}_{ij} L_i H^{\dagger} \bar{e}_j \left(\frac{\phi}{\Lambda}\right)^{|X_{L_i}+X_{\bar{e}_j}|} - c^{\nu}_{ij} H L_i N_j \left(\frac{\phi}{\Lambda}\right)^{|X_{L_i}+X_{N_j}|} - c^{M}_{ij} \frac{M}{2} N_i N_j \left(\frac{\phi}{\Lambda}\right)^{|X_{N_i}+X_{N_j}|}$$

Branching Fractions - Seesaw

- In the seesaw analysis $\mathrm{M} = \Lambda_F = \Lambda_{exp}$
- Prediction is fixed by neutrino scale





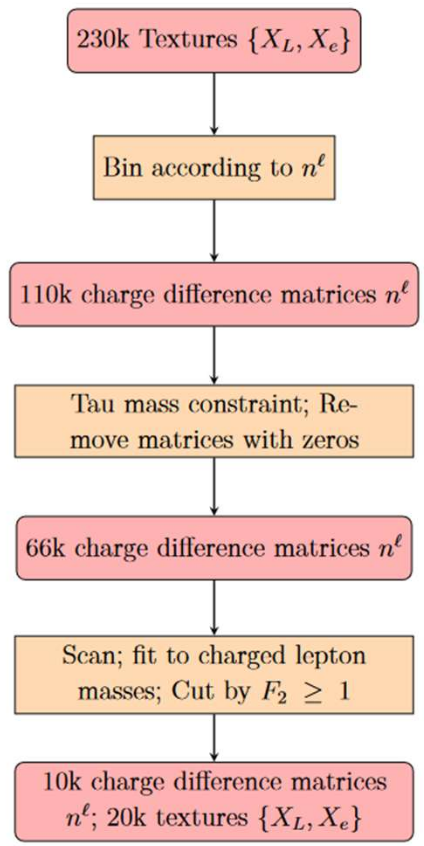


Fractional Deviation

$$\delta_{\mathcal{O}} = \exp\left|\ln\left(\frac{\mathcal{O}^{FN}}{\mathcal{O}^{\exp}}\right)\right|$$

$$\delta_{\mathcal{O}} = \begin{cases} \exp\left|\ln\left(\frac{\mathcal{O}^{\mathrm{FN}}}{\mathcal{O}^{\exp}_{\min}}\right)\right| & \text{if } \mathcal{O}^{\mathrm{FN}} < \mathcal{O}^{\exp}_{\min}, \\ 1 & \text{if } \mathcal{O}^{\exp}_{\min} \leq \mathcal{O}^{\mathrm{FN}} \leq \mathcal{O}^{\exp}_{\max}, \\ \exp\left|\ln\left(\frac{\mathcal{O}^{\mathrm{FN}}}{\mathcal{O}^{\exp}_{\max}}\right)\right| & \text{if } \mathcal{O}^{\mathrm{FN}} > \mathcal{O}^{\exp}_{\max}. \end{cases}$$

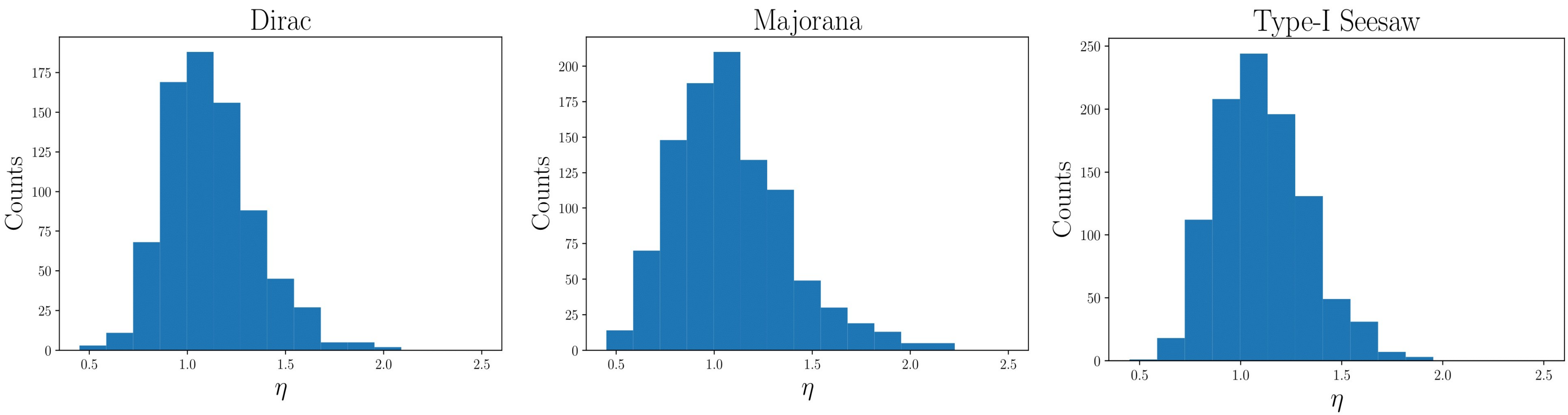
Methods



230k Textures $\{X_L, X_e\}$
Bin according to n^ℓ
110k charge difference matrices n^ℓ
Tau mass constraint; Remove matrices with zeros
66k charge difference matrices n^ℓ
Scan; fit to charged lepton masses; Cut by $F_2 \geq 1$
10k charge difference matrices n^ℓ; 20k textures $\{X_L, X_e\}$

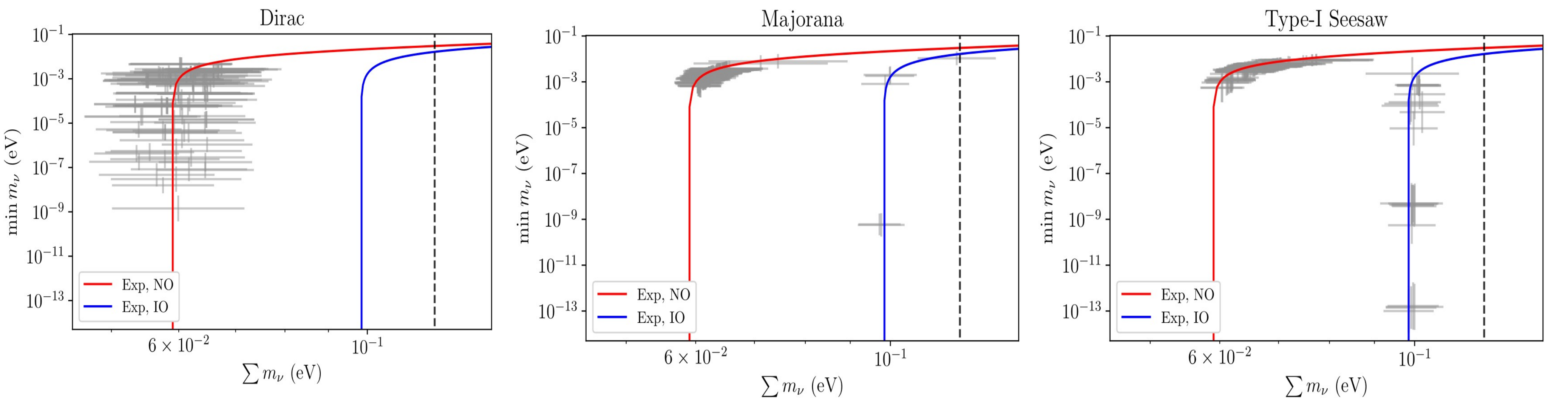
Coefficient Distributions

$$\eta = \log_{10} \frac{\max |c|}{\min |c|}$$



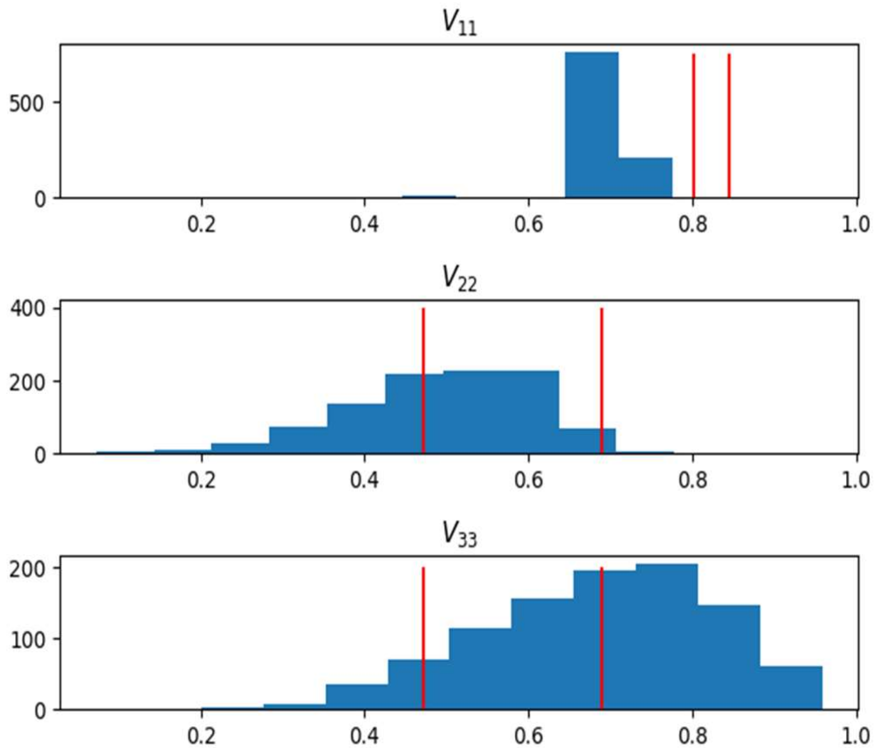
- For random coefficient draws within a factor of 2 of all SM observables, refine coefficients to get exact SM fit
- Can easily get exact fits with maximum coefficient variation of 1 order of magnitude

Lightest Neutrino Mass

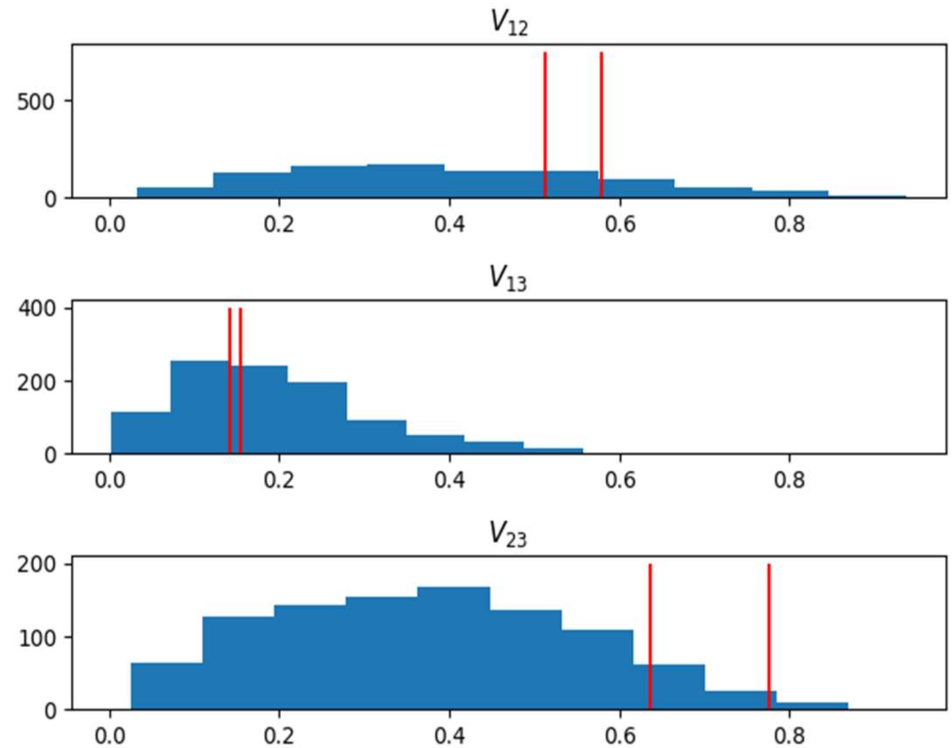


Parameter Distributions

Bad Texture



Better Texture



Operators

- Warsaw basis: 1008.4884
- Matching using python package wilson: 1804.05033
- Phenomenology using flavio: 1810.08132

Q_{LL}	$\left(\bar{L}_p\gamma_\mu L_r\right)\left(\bar{L}_s\gamma^\mu L_t\right)$	$Q_{LQ}^{(1)}$	$\left(\bar{L}_p\gamma_\mu L_r\right)\left(\bar{Q}_s\gamma^\mu Q_t\right)$
Q_{ee}	$\left(\bar{e}_p\gamma_\mu e_r\right)\left(\bar{e}_s\gamma^\mu e_t\right)$	$Q_{LQ}^{(3)}$	$\left(\bar{L}_p\gamma_\mu\tau^I L_r\right)\left(\bar{Q}_s\gamma^\mu\tau^I Q_t\right)$
Q_{Le}	$\left(\bar{L}_p\gamma_\mu L_r\right)\left(\bar{e}_s\gamma^\mu e_t\right)$	Q_{He}	$\left(H^\dagger i\overleftrightarrow{D}_\mu H\right)\left(\bar{e}_p\gamma_\mu e_r\right)$
Q_{ed}	$\left(\bar{e}_p\gamma_\mu e_r\right)\left(\bar{d}_s\gamma^\mu d_t\right)$	$Q_{HL}^{(1)}$	$\left(H^\dagger i\overleftrightarrow{D}_\mu H\right)\left(\bar{L}_p\gamma_\mu L_r\right)$
Q_{eu}	$\left(\bar{e}_p\gamma_\mu e_r\right)\left(\bar{u}_s\gamma^\mu u_t\right)$	$Q_{HL}^{(3)}$	$\left(H^\dagger i\overleftrightarrow{D}^I_\mu H\right)\left(\bar{L}_p\tau^I\gamma_\mu L_r\right)$
Q_{Lu}	$\left(\bar{L}_p\gamma_\mu L_r\right)\left(\bar{u}_s\gamma^\mu u_t\right)$	Q_{eW}	$\left(\bar{L}_p\sigma^{\mu\nu}e_r\right)\tau^I H W^I_{\mu\nu}$
Q_{Ld}	$\left(\bar{L}_p\gamma_\mu L_r\right)\left(\bar{d}_s\gamma^\mu d_t\right)$	Q_{eB}	$\left(\bar{L}_p\sigma^{\mu\nu}e_r\right)H B_{\mu\nu}$
Q_{Qe}	$\left(\bar{Q}_p\gamma_\mu Q_r\right)\left(\bar{e}_s\gamma^\mu e_t\right)$		