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Nucleation rate with out-of-eq effects

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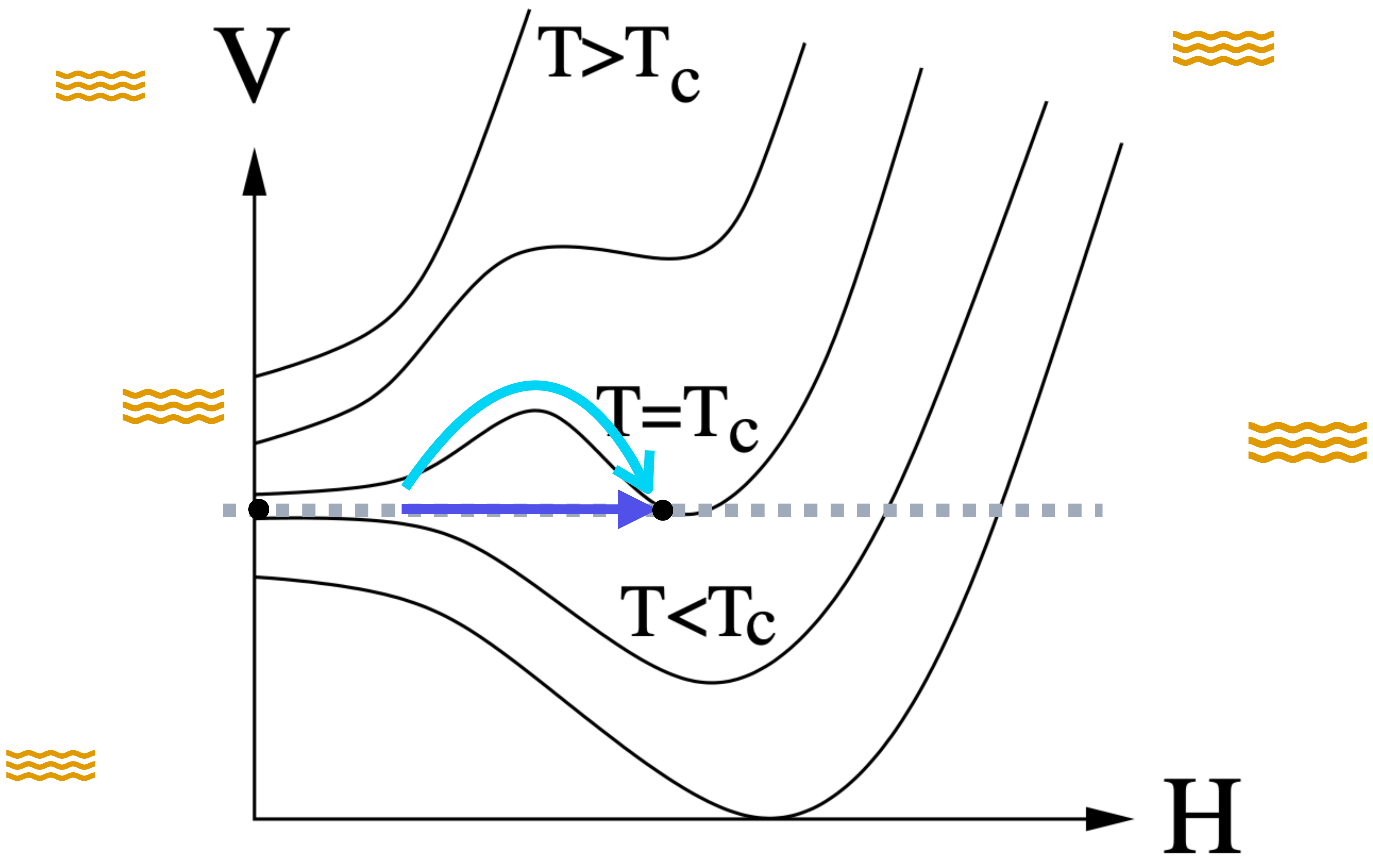
EWPT $\sim 10^{-12}$ s
We are here!
Interesting physics here!
CMB emission
10^{-32} seconds
1 microsecond
3 minutes
380,000 years
200 million years
400 million years
10 billion years
13.8 billion years
Inflation
Initial expansion
First Particles
Neutrons, protons, and electrons form
First Nuclei
Helium and hydrogen form
First Light
The first atoms form
First Stars
Gas and dust condense into stars
Galaxies & Dark Matter
Galaxies form in dark matter cradles
Dark Energy
Expansion accelerates
Today
Humans observe the universe

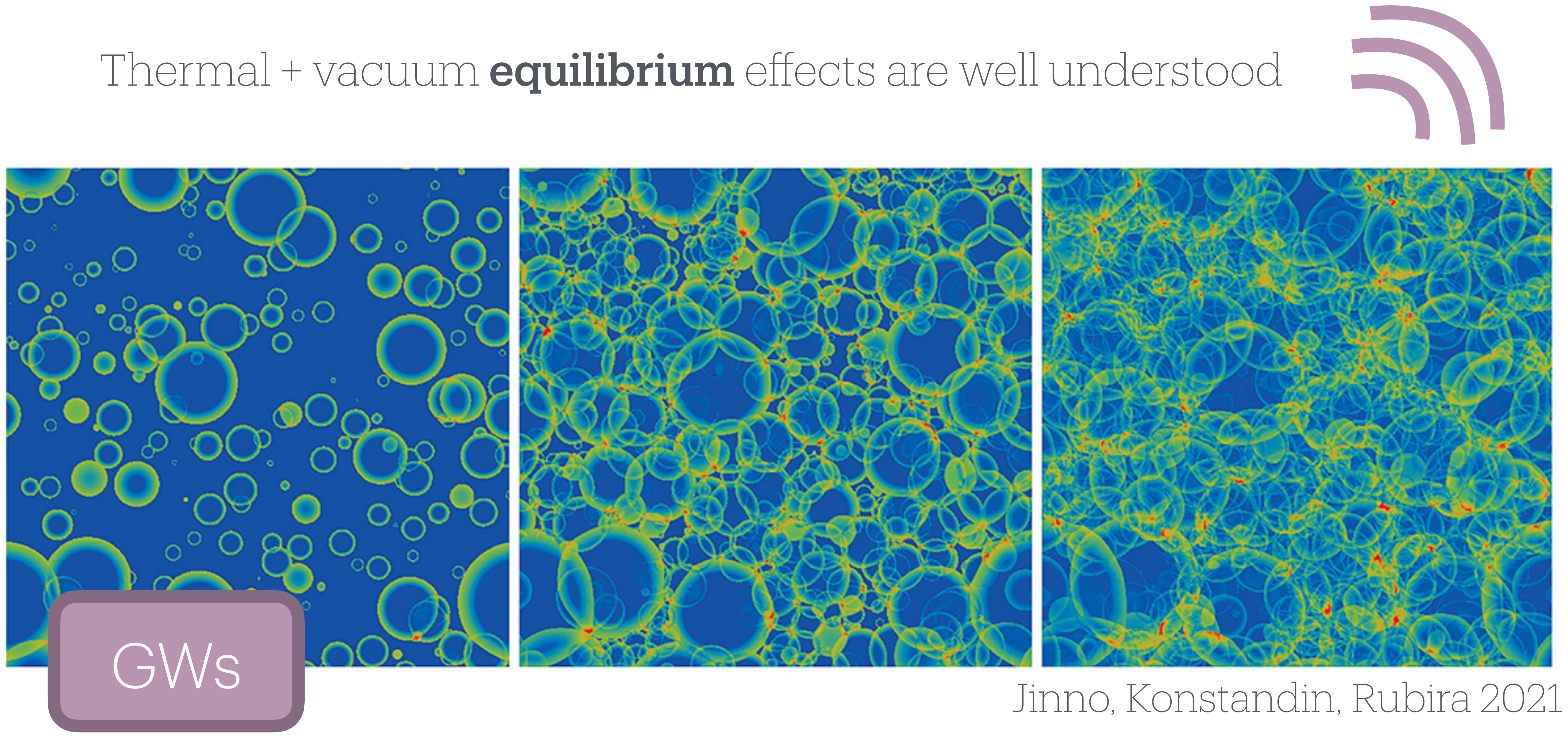
First-order phase transitions

+ dissipative effects
Out-of-eq effects & open quantum systems



Cline 2006

Thermal + vacuum **equilibrium** effects are well understood



Jinno, Konstandin, Rubira 2021

- Nucleation via **Thermal activation**
- Nucleation via **Quantum tunneling**

EFT for nucleation

k

UV

UV modes (shrinking bubbles)

$\Lambda_{\text{nucl}} \sim R_c^{-1}$ (Set by the bubble critical radius)

IR

IR modes (growing bubbles)

1. System-Environment splitting

$$\phi(x) = \phi_{\text{IR}}(x) + \phi_{\text{UV}}(x)$$

$$S[\phi] = S[\phi_{\text{IR}}] + S[\phi_{\text{UV}}] + S_{\text{mix}}[\phi_{\text{IR}}, \phi_{\text{UV}}]$$

$$\hat{\rho}\,(t_0) = \hat{\rho}_{\text{IR}}\,(t_0)\,\hat{\rho}_{\text{UV}}\,(t_0)$$

$$\hat{\rho}_{\text{IR}}(t) = \text{Tr}_{\text{UV}}\{\text{U}(\text{t},\text{t}_0)\,\hat{\rho}\,(\text{t}_0)\text{U}^\dagger(\text{t},\text{t}_0)\}$$

Sebastian Ellis
IR-UV couplings affect the IR physics

2. Initial conditions

$$\rho_{\text{UV}}[\phi^+_{\text{UV}}, \phi^-_{\text{UV}}; t] \equiv \langle \phi^+_{\text{UV}}(\vec{x})\,|\,\hat{\rho}_{\text{UV}}(t)\,|\,\phi^-_{\text{UV}}(\vec{x})\rangle$$

In-In formalism

Example:

$$\hat{\rho}_{\text{UV}}(t_0) = |\,\Omega\rangle\langle\Omega\,| \Rightarrow \Psi[\phi^+_{\text{UV}}]\Psi[\phi^-_{\text{UV}}]^*$$

Daniel Baumann
Vacuum wavefunctional

IR partition function:

$$Z_{\mathrm{IR}} = \int_{\mathrm{B.C.}} D\phi^+_{\mathrm{IR}} D\phi^-_{\mathrm{IR}} \ \rho_{\mathrm{IR}}[\phi^+_{\mathrm{IR}}, \phi^-_{\mathrm{IR}}; t_0] \ e^{\frac{i}{\hbar} S_{\mathrm{eff}}[\phi^+_{\mathrm{IR}}, \phi^-_{\mathrm{IR}}]}$$

$$G^{\mathrm{F}}(x, x') = \langle \mathcal{T}\{\phi(x)\phi(x')\}\rangle$$
$$G^{>}(x, x') = \langle \phi(x)\phi(x')\rangle$$
$$G^{<}(x, x') = \langle \phi(x')\phi(x)\rangle$$
$$G^{\mathrm{D}}(x, x') = \langle \mathcal{A}\{\phi(x)\phi(x')\}\rangle$$

Effective action:

$$S_{\mathrm{eff}}[\phi^+_{\mathrm{IR}}, \phi^-_{\mathrm{IR}}] = \sum_a a \left(S[\phi^a_{\mathrm{IR}}] + S_{\mathrm{IF}}[\phi^+_{\mathrm{IR}}, \phi^-_{\mathrm{IF}}] \right)$$

Unitary evolution of IR modes

EFT corrections

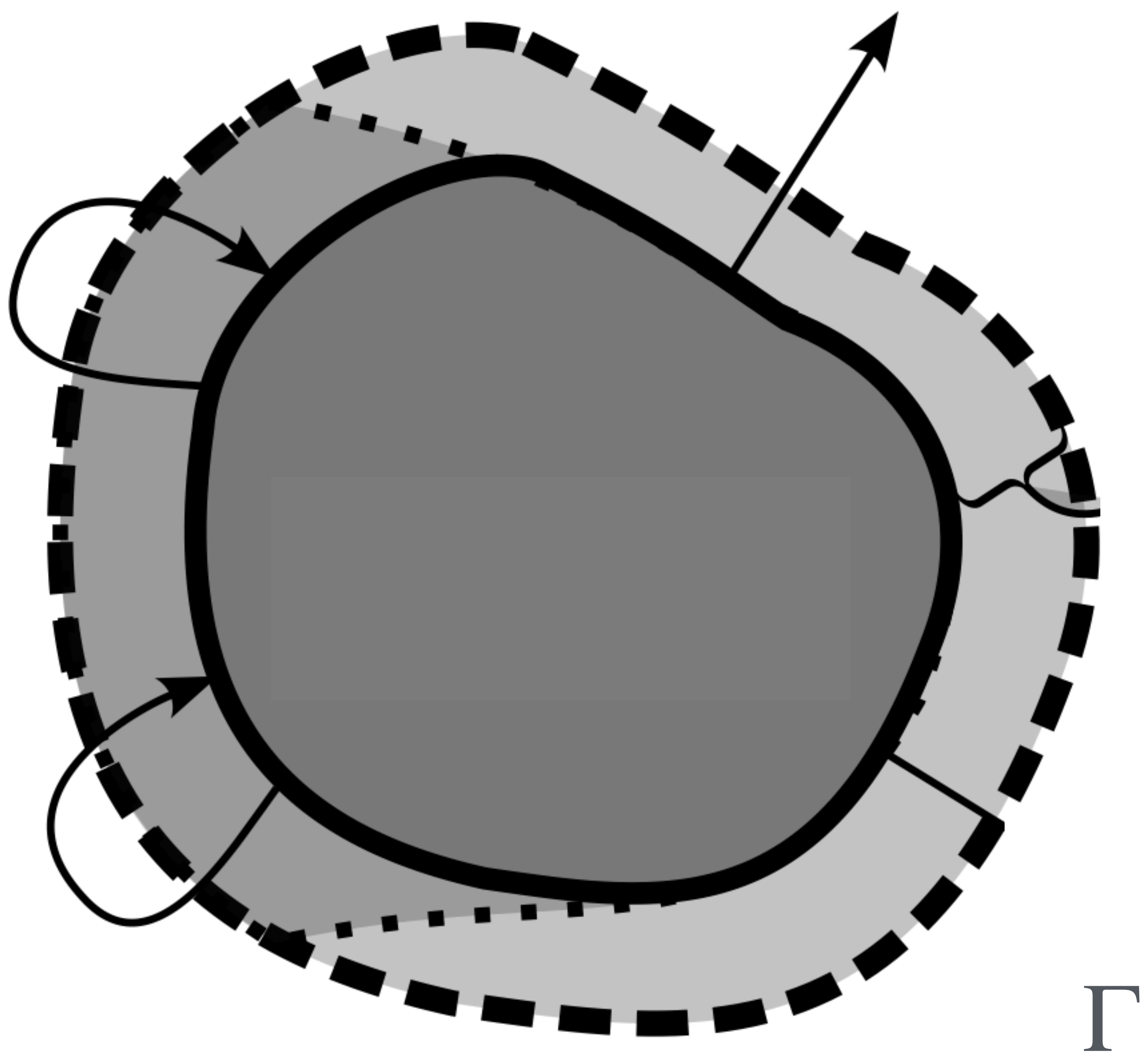
Influence functional:

$$e^{\frac{i}{\hbar} S_{\mathrm{IF}}[\phi^+_{\mathrm{IR}}, \phi^-_{\mathrm{IR}}]} = \int_{\mathrm{B.C.}} D\phi^+_{\mathrm{UV}} D\phi^-_{\mathrm{UV}} \ \rho_{\mathrm{UV}}[\phi^+_{\mathrm{UV}}, \phi^-_{\mathrm{UV}}; t_0] \ e^{\sum_a \frac{i}{\hbar} a \left(S[\phi^a_{\mathrm{UV}}] + S_{\mathrm{mix}}[\phi^a_{\mathrm{IR}}, \phi^a_{\mathrm{UV}}]\right)}$$

Sebastian Ellis

In fact, we can map this information into a “reduced Wigner function”

Out-of-eq effects, Memory/non-local effects, Collision terms, etc



$$W[\phi, \pi; t]$$

$$\frac{dW}{dt} = -\vec{\nabla} \cdot \vec{J} = -\vec{\nabla}_\phi \cdot \vec{J}_\phi - \vec{\nabla}_\pi \cdot \vec{J}_\pi$$ (Schematically)

$$\Gamma \propto \int_\Omega \vec{J} \cdot \hat{n}$$

$$\Gamma = \frac{\omega_0}{2\pi\omega}\left[\left(\frac{\bar{\gamma}^2}{4} + \bar{\omega}^2\right)^{1/2} - \frac{\bar{\gamma}}{2}\right] \exp\left(-\frac{V_b - V_{\text{f.v.}}}{kT}\right) .$$

Nucleation rate in Quantum mechanics

Hanggi-Mojtabai (1982)

Function of the dissipation kernel

Negative eigenmode (Now corrected by dissipative kernels)

Take-home message

- Γ depends on the details of the dissipation.
- The dissipation kernel can be computed directly from the *Influence action*.
- Strong memory effects lead to sizable effects in the nucleation rate (deviations from the traditional result).
- Theoretical uncertainties in the GW spectrum due to dissipation.

Thank you