



Climbing
Frisbee



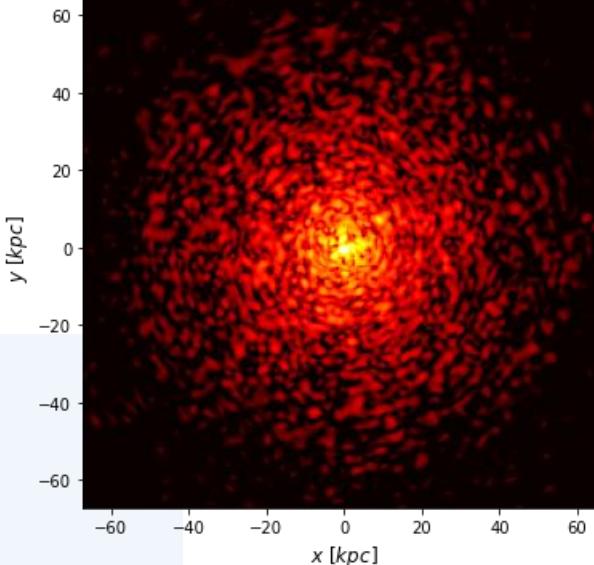
Revisiting Axion Strings

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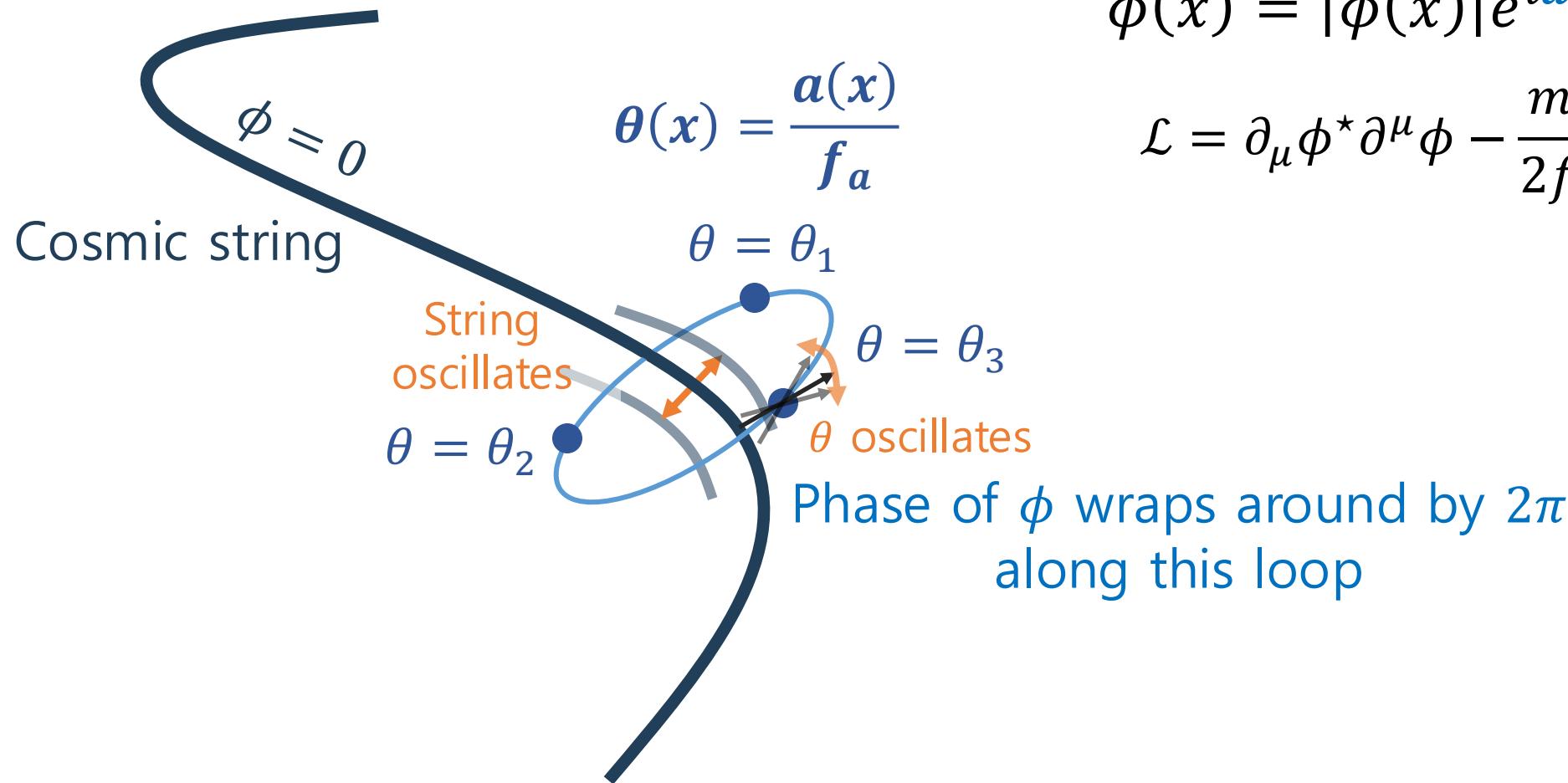


Based on **Kim, H., Park, J. & Son, M. (2024) JHEP07(2024)150**
Kim, H. & Son, M. (2025) JHEP07(2025)052



DM, Bootstrapping, ...

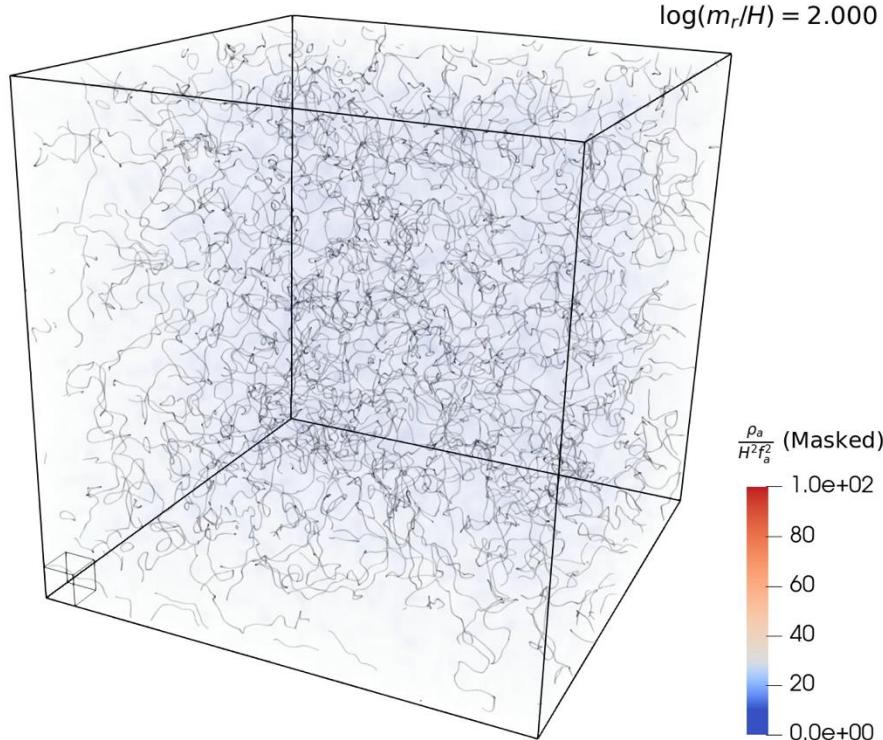
Brief Description of Axion String



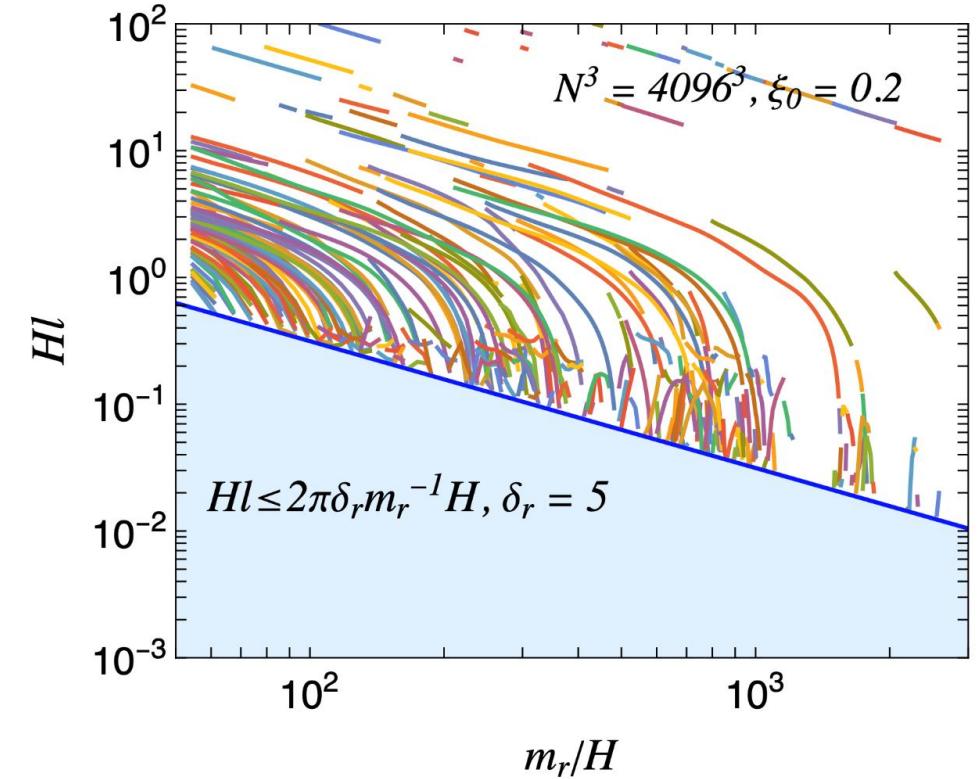
$$\phi(x) = |\phi(x)| e^{i \mathbf{a}(x)/f_a} \quad (\text{Axion})$$

$$\mathcal{L} = \partial_\mu \phi^* \partial^\mu \phi - \frac{m_r^2}{2 f_a^2} \left(|\phi|^2 - \frac{f_a^2}{2} \right)^2$$

Axion Radiation from String Dynamics



- Strings get shorten
- Release energy into axion



- Number of string segments per Hubble patch seems to grow in time.
- Axion radiation seems to evolve toward more IR dominated spectrum.

Axion abundance and mass bound

The string network evolution may matter



$$\frac{\Omega_a^{\text{str}}}{\Omega_a^{\text{mis}}} \approx 30$$



$$m_a \geq 400 \text{ }\mu\text{eV}$$

c.f. Misalignment bound: $m_a \geq 30 \text{ }\mu\text{eV}$

String of length l

n^{th} fluctuation mode has wavenumber

$$k_n = \frac{2\pi n}{l}$$

$$(\text{Fluctuation amplitude})^2 \propto \frac{1}{k^p}$$

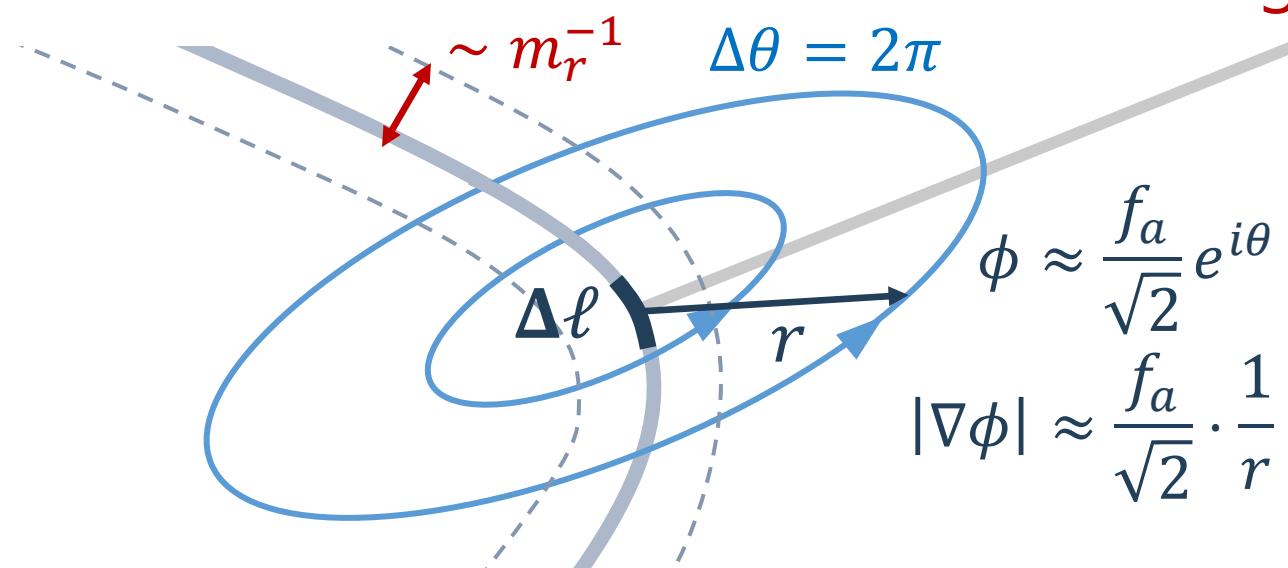
- **Evolution of super-horizon string fluctuation spectrum**
 - Seems to (roughly) match
(analysed with **Kalb-Ramond** theory)
- **Evolution of axion radiation spectrum**

Backup Slides

IR diverging string tension

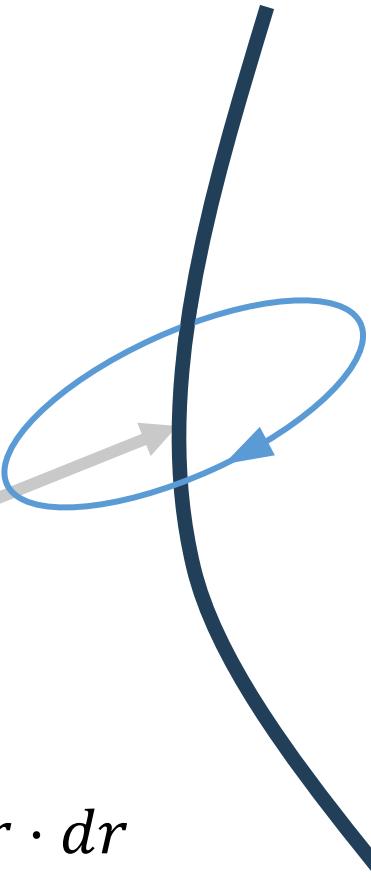
Strings exhibit long-range interaction

$$\mathcal{L}_{PQ} = \partial_\mu \phi^* \partial^\mu \phi - \frac{m_r^2}{2f_a^2} \left(|\phi|^2 - \frac{f_a^2}{2} \right)^2$$



String Tension, μ

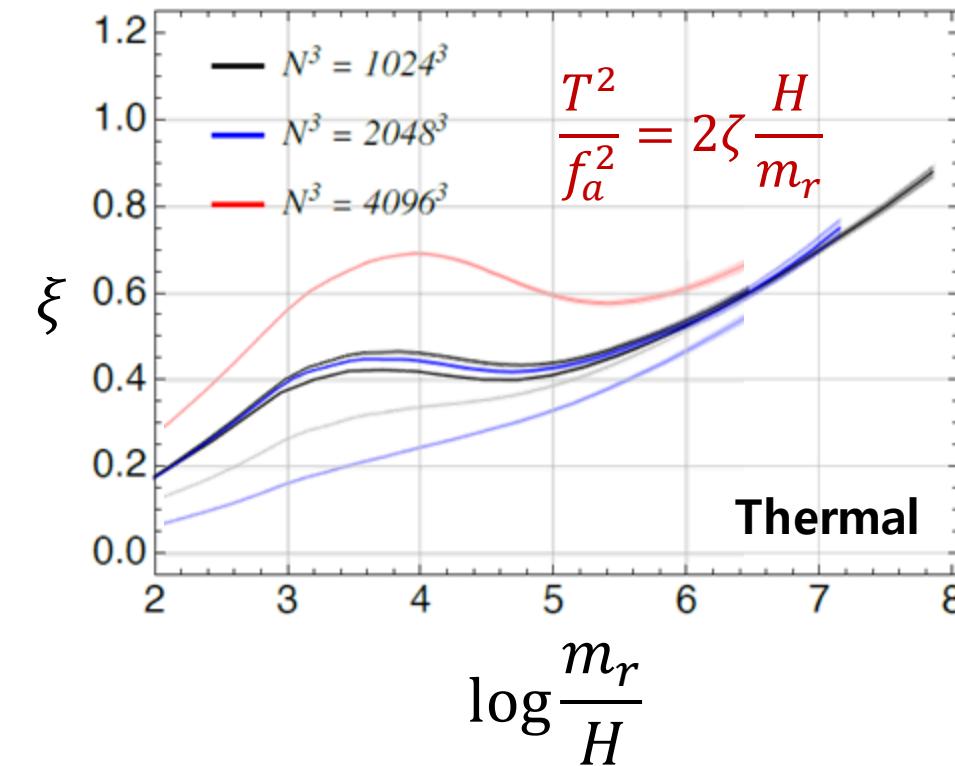
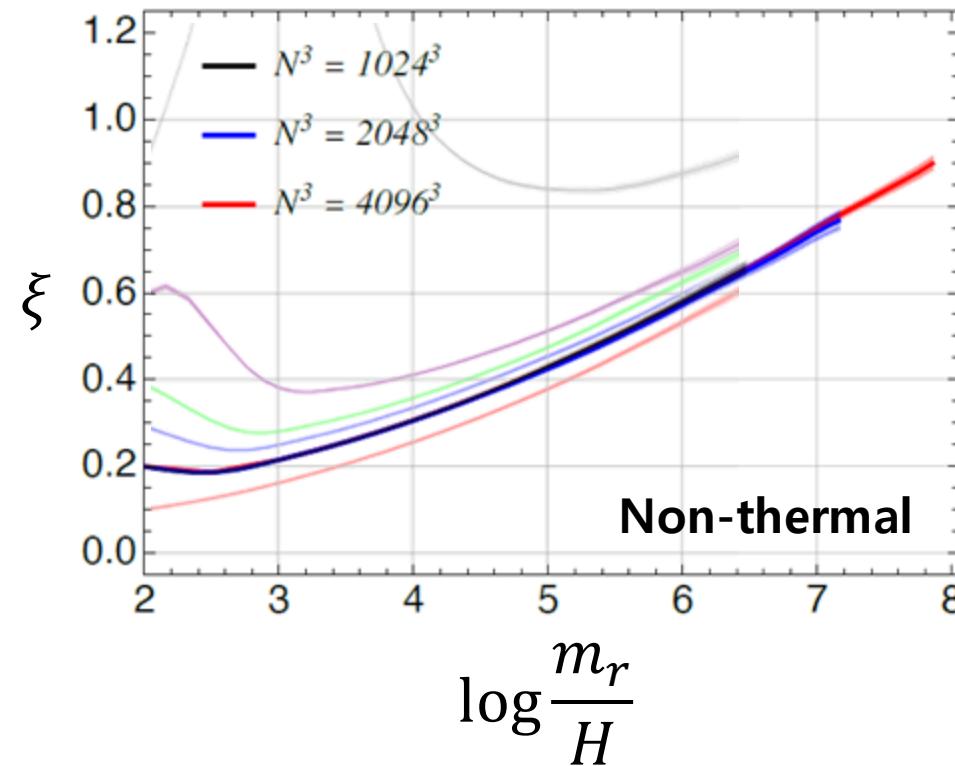
$$\begin{aligned} \frac{\Delta E}{\Delta \ell} &\approx \int_{m_r^{-1}}^d |\nabla\phi|^2 \cdot 2\pi r \cdot dr \\ &= \pi f_a^2 \log(m_r d) \end{aligned}$$



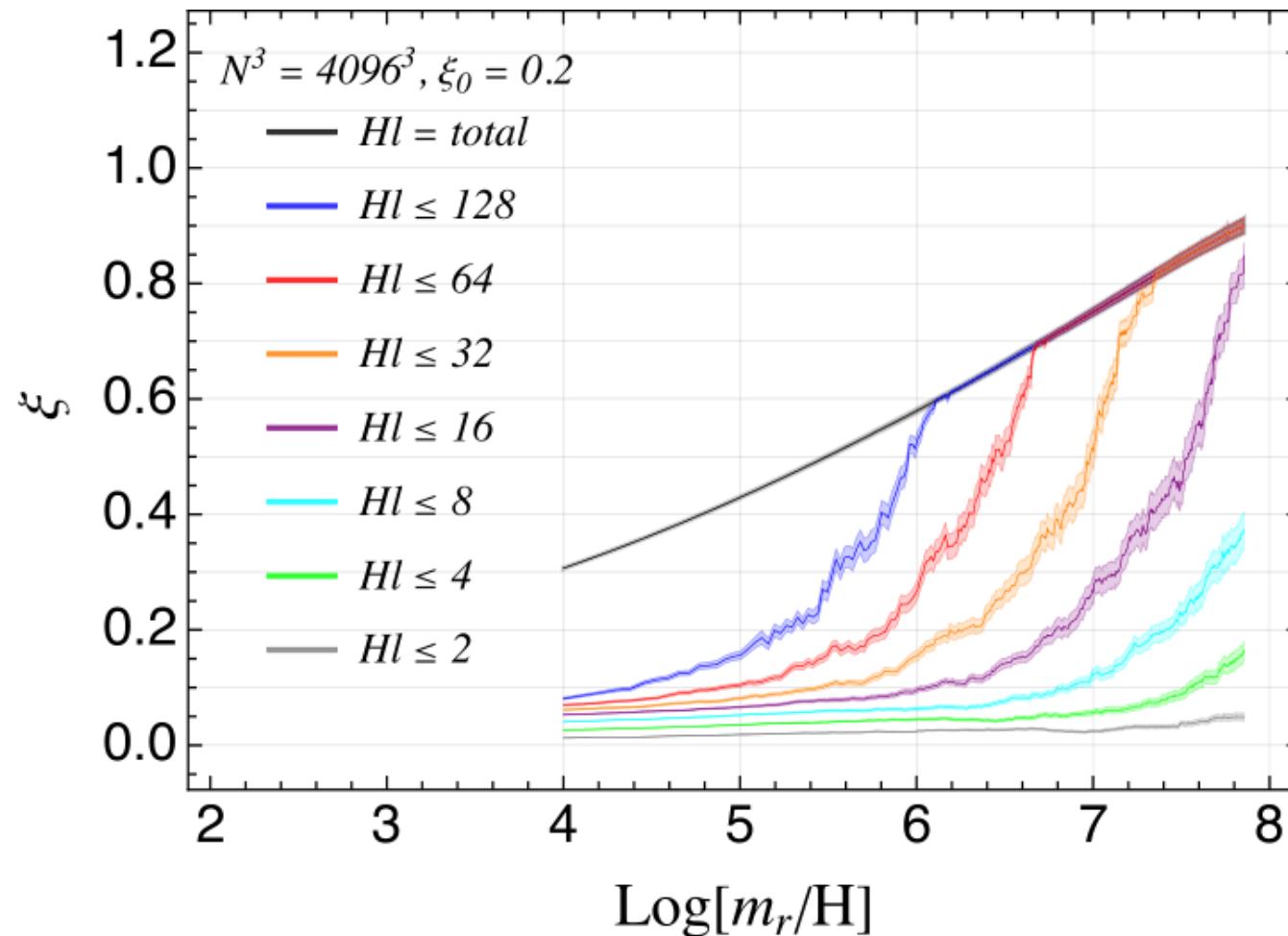
Strings May Be More Abundant Than Expected

String number density

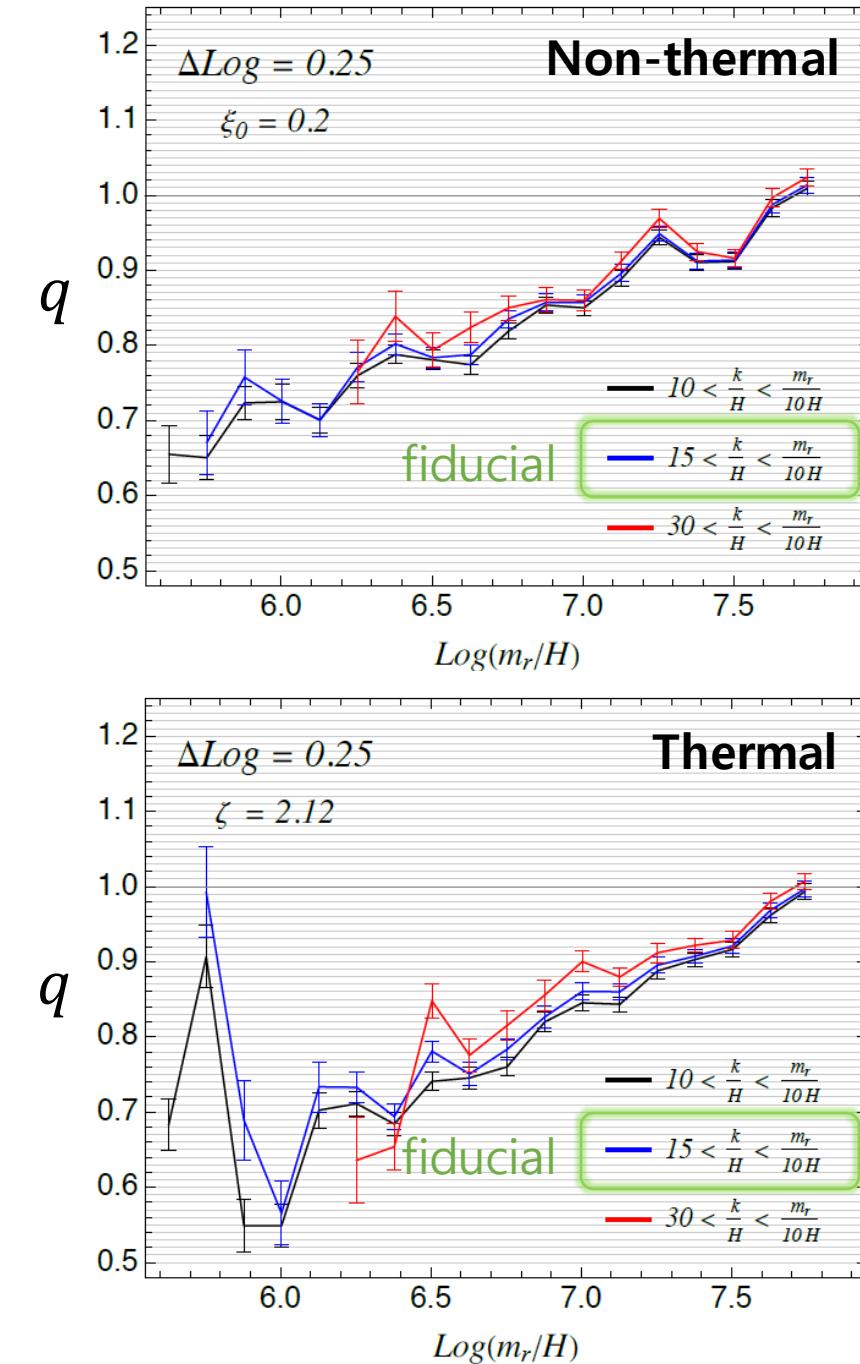
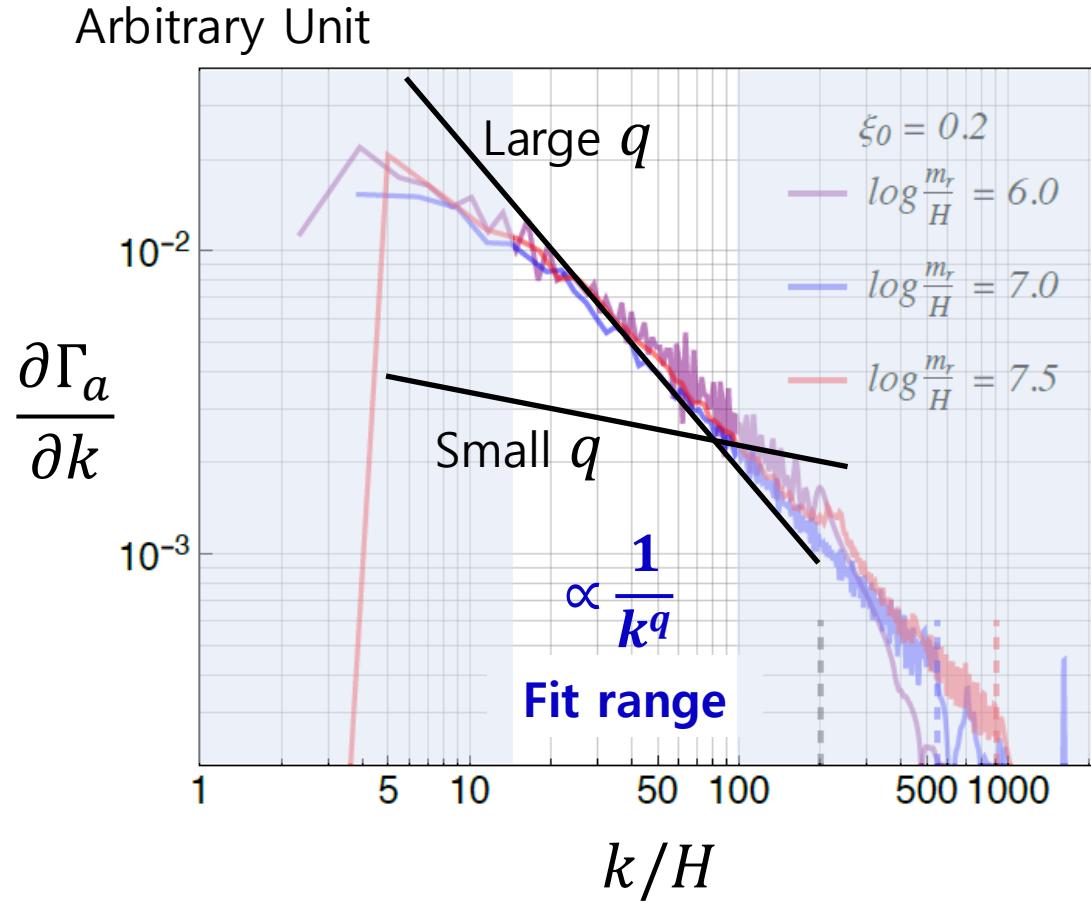
$$\xi = \frac{l_{\text{string}}/t}{V_{\text{univ}}/t^3}$$



Dominance of Super-Horizon strings



Axion Radiation is getting Softer



Radiation from String

String releases energy while getting shortened

If strings do not constantly disappear:

$$\xi = \frac{l_{tot} t^2}{V_{univ}} \propto \frac{R(t) t^2}{R(t)^3} \propto t$$

Radiation dominated universe ($R \propto \sqrt{t}$)

String releases its energy

(for $\xi \sim c_0 + c_1 \log \frac{m_r}{H}$)

$$\Gamma \quad \text{radiation power}$$

$$\sim \frac{\text{string number density} \times \text{string tension}}{t^3}$$



$$\xi \sim c_0 + c_1 \log(m_r/H)$$

Saikawa et al. [2401.17253]

Kim et al. [2402.00741]

Safdi et al. [2412.08699], [2108.05368]

Gorghetto et al. [2007.04990]

Axion Radiation Spectrum from Super-Horizon String Fluctuations

Kalb-Ramond theory

$$H^{\alpha\beta\gamma} = \frac{f_a}{\sqrt{2}} \epsilon^{\mu\alpha\beta\gamma} \partial_\mu \left(\frac{a}{f_a} \right)$$

$$S = \int -\mu \sqrt{-g} d^2\zeta + \frac{1}{6} \int H^2 d^4x + \sqrt{2}\pi f_a \int B_{\mu\nu} d\sigma^{\mu\nu}$$

$$H_{\mu\nu\rho} = \partial_\mu B_{\nu\rho} + \partial_\nu B_{\rho\mu} + \partial_\rho B_{\mu\nu}$$

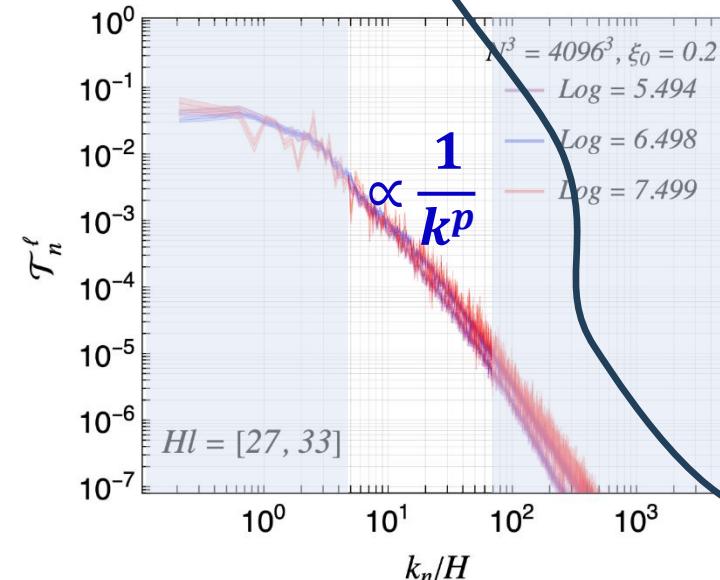
String of length l

n^{th} mode has wavenumber

$$k_n = \frac{2\pi n}{l}$$

$$\frac{\partial \Gamma_a}{\partial k} \propto \frac{1}{k^{p-1} - O(0.1)} \text{ correction}$$

With some caveats

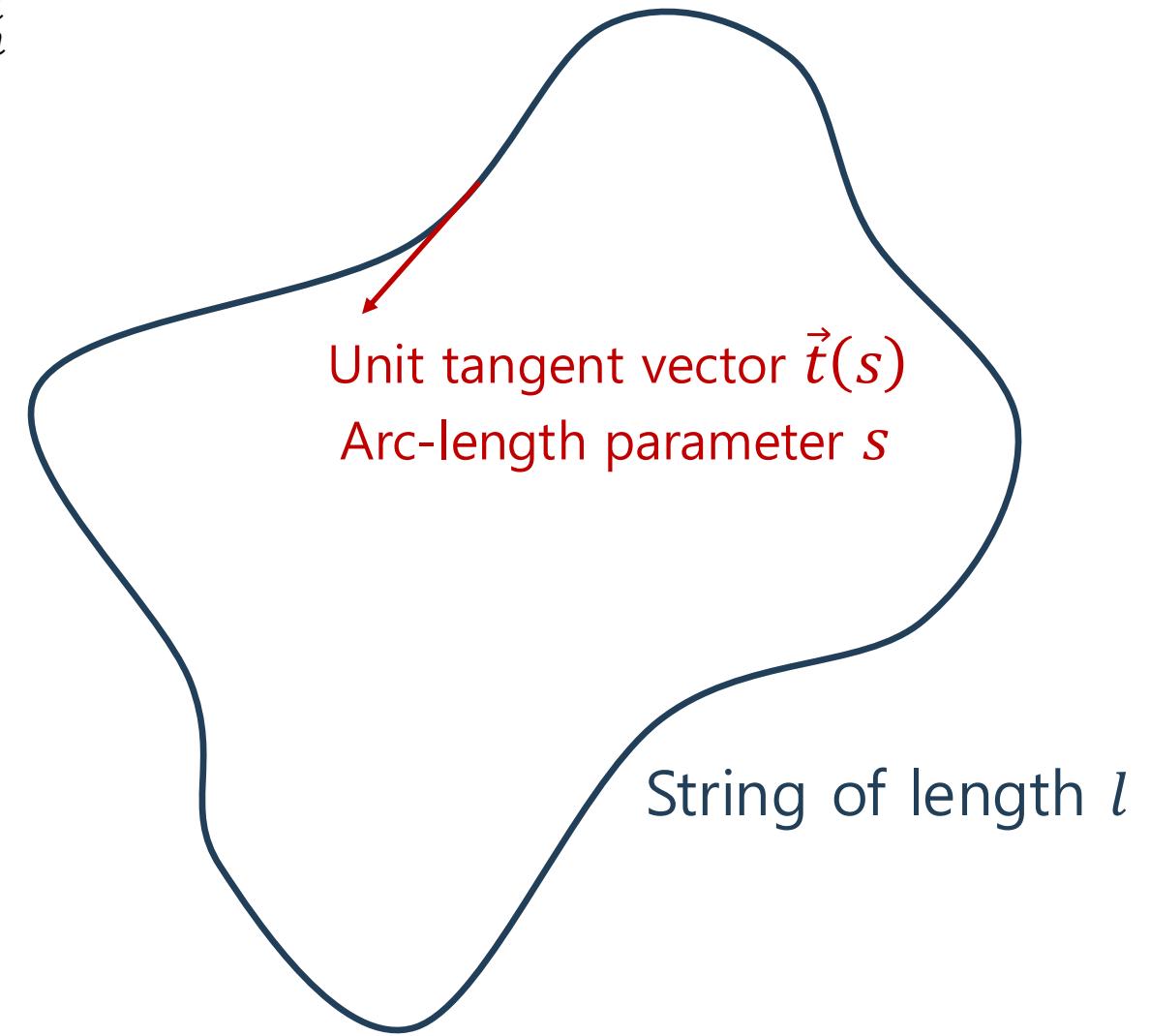


Closer look at String fluctuation

String Fluctuation profile (spectrum) \vec{T}_n^l

$$\vec{T}_n^l = \int_0^l \frac{ds}{l} \vec{t}(s) e^{-2\pi ins/l}$$

$$\langle \vec{T}_{n_1}^l \cdot \vec{T}_{n_2}^l \rangle = \mathcal{T}_{|n_1|}^l \delta_{n_1+n_2}$$



Connecting Spectrum: “string” & “axion”

Caveats in our analytic calculation

$$\frac{1}{m_r^2 f_a^2} \frac{\partial \Gamma_a}{\partial k} = 8\pi^3 \xi \left(\frac{m_r}{H}\right)^{-2} \int_0^\infty \frac{d(Hl)}{H l_{avg}} \frac{\rho(l)}{H} \sum_{n=1}^{\infty} n H \delta\left(k - \frac{2\pi n}{l}\right) \sum_{\substack{|m| < n \\ m+n \text{ even}}} \frac{\mathcal{T}_{n+m}^l}{2} \frac{\mathcal{T}_{n-m}^l}{2}$$

- Low momentum fluctuation modes are involved even for high momentum axion radiation
- Neglected Hubble expansion plays significant role in low momentum fluctuation mode dynamics
- Small fluctuation assumption may break down for low momentum modes