How Cool is Yang-Mills Theory? 😎

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Thoughts based on work with:
Prateek Agrawal, Vazha Loladze, John March-Russell and Mario Reig

2508.xxxxx (coming soon (hopefully))
arXiv: 2504.00199 (Tangentially related)

Special thanks to Paolo for lending me a pair of glasses!

Transitions in strongly coupled theories.

FOPT–bubbles–collisions–**GW waves**

Strength depends on multiple factors– one of then is **super-cooling**:

$$\epsilon = 1 - \frac{T}{T_{\rm cr}}$$

Large supercooling and far-from-quasi-equilibrium dynamics

Pure $SU(N)$ Yang-Mills (YM)–FOPT $\mathbb{Z}_N^{(1)}$ broken $\leftrightarrow$ $\mathbb{Z}_N^{(1)}$ restoring.

$$\Gamma \sim e^{-S_{\rm b}} \propto H^4$$

Lattice [hep-lat/0502003, 2502.01396]

$$\sigma_{\rm BW} \approx 0.019 N^2 T_{\rm cr}^3$$
$$Q_{\rm h} \approx 0.35 N^2 T_{\rm cr}^4$$
$$\frac{S_3}{T} \approx \frac{16\pi\sigma_{\rm BW}^3}{3\epsilon^2 T_{\rm cr} Q_{\rm h}^2} \approx \frac{8.8\times 10^{-4} N^2}{\epsilon^2}$$

Observation: small tension $\rightarrow$ small action– why ?

Hints from Holography– Maximal Supercooling limit– eg. Hawking-Page transition $\leftrightarrow$ $\mathcal{N}=4$ SYM on $S^3\times S^1$ [hep-th/9803131, 0903.3605, 2210.1821, 2309.10090]

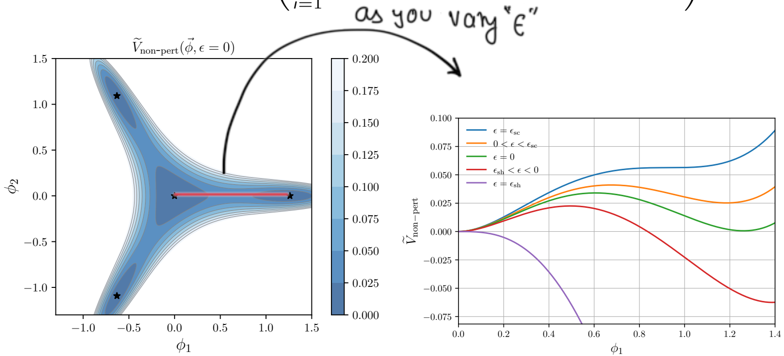
👉 Order Parameter- Polyakov Loop– winds around the S^1 (thermal circle for thermal YM)

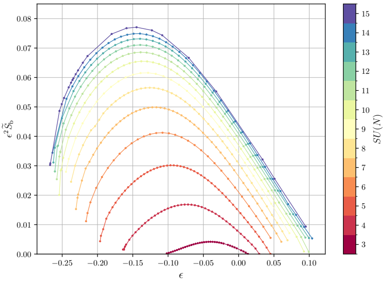
👉 Softly broken $\mathcal{N} = 1$ SYM on $\mathbb{R}^3 \times S^1$– Has $\mathbb{Z}_N^{(1)}$-breaking $\leftrightarrow$ $\mathbb{Z}_N^{(1)}$-preserving FOPT

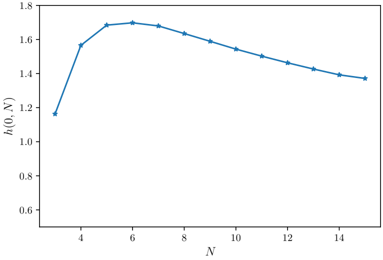
Continuity conjecture: *this transition is qualitatively same as Thermal YM transition* [1205.0290, 1212.1238, 1302.2641, 1710.06509]

👉 Non-Perturbative potential– instantons and bions

$$V(\vec{\phi}, \epsilon) = V^0_{\mathrm{bion}} \left(\sum_{i=1}^{N} e^{-\vec{\alpha}_i \cdot \vec{\phi}} \left(e^{-\vec{\alpha}_i \cdot \vec{\phi}} - e^{-\vec{\alpha}_{i+1} \cdot \vec{\phi}} - c(\epsilon) \right) \right)$$







For Softly-Broken $\mathcal{N} = 1$ SYM

Use `FindBounce` [arXiv:2002.00881]

$$\epsilon^2 S_\mathrm{b} = -\frac{N^2 \Lambda^3}{L_\mathrm{cr} \sigma^2_\mathrm{str}} (1-\epsilon)^2 \times \underbrace{h(\epsilon, N)(\epsilon - \epsilon_\mathrm{sc}(N))(\epsilon - \epsilon_\mathrm{sh}(N))}_{\epsilon^2 \widetilde{S}_\mathrm{b}}$$

For YM

Use the ansatz:

$$\epsilon^2 S_\mathrm{b}|_{\epsilon \to 0} \sim -\mathcal{O}(1) \frac{T^4_\mathrm{cr} N^2}{\sigma^2_\mathrm{str}} \epsilon_\mathrm{sc} \epsilon_\mathrm{sh}$$

Small maximal supercooling– $\epsilon_{\rm sc} \lesssim 0.1$

👉 Reheating of plasma around – Wall velocity (?)

👉 What happens if you go to these instability points in strongly coupled theories.

👉 Some interesting directions– effect on maximal supercooling having a non-zero θ, effect of matter, understanding dynamics of bubbles

(Picture credit: Megan and the kind stranger who took the picture)

Thanks to everyone for making these couple of weeks full of fun and amazing learning experience!!

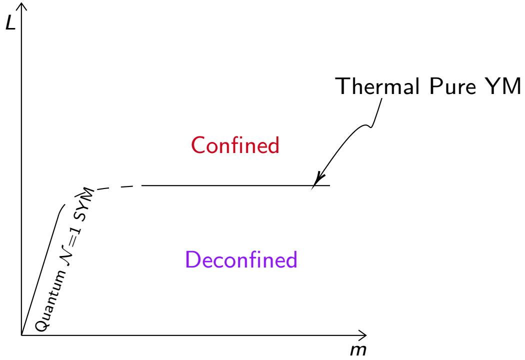
Extra slides 🤓

Continuity Conjecture

👉 The conjecture [1205.0290, 1212.1238] can be written as:

$$\mathrm{Tr}\left[e^{-LH}(-1)^F\right] = Z_{\mathrm{G}}[L,m] = \begin{cases} I_{\mathrm{G}}, & m=0 \\ \mathcal{Z}_{\mathrm{G}}[\beta = L], & m \gg \Lambda \end{cases}$$

where I_{G} is the Witten index, $\mathcal{Z}$ is the thermal partition function and G is the gauge group.



Maximal Supercooling and Superheating in the softly broken $\mathcal{N} = 1$ SYM on $\mathbb{R}^3 \times S^1$