# (Super) Gravity from Positivity

Marcello Romano

*Université Paris-Saclay, CNRS, CEA, Institut de Physique Théorique*

Based on 2507.12535, with B. Bellazzini, A. Pomarol and F. Sciotti

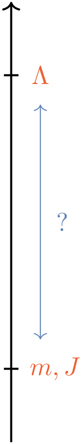
*Cargèse, 31/07/2025*

# Is scale separation always consistent?

# A general question

EFT for a massive particle alone at the bottom of the spectrum

**Q**: *Can the cut-off be parametrically larger than the mass?*




**A.1**: If $J \leq 1$, yes!

**A.2**: If $J \geq \frac{3}{2}$? Shortly, no!

Technically: $0 < m \ll \Lambda$, weakly coupled @ $m \leq E \leq \Lambda$

# (Very roughly) Why?

Expand $\mathcal{M}_{2\to 2}$ at $m \ll E \ll \Lambda$

- For *e.g.* $J = 3/2$ by dimensional analysis

$$\mathcal{M}_{2\to 2} \sim \frac{E^6}{M^6}(1 + \# m^2/E^2 + ...)$$

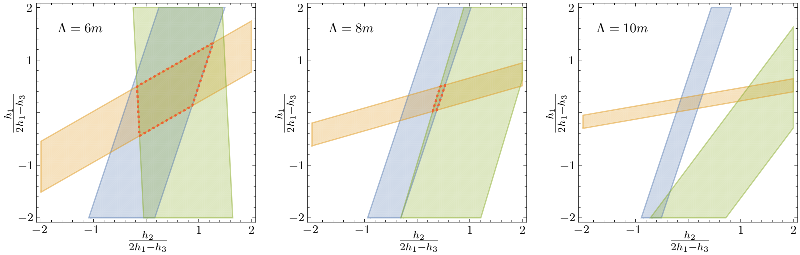
- Causality, unitarity and Lorentz $\implies$ Positivity bounds

$$\mathcal{M}_{2\to 2} \lesssim E^4/M^4 \quad \implies \quad \# m^2/\Lambda^2 \gtrsim O(1)$$

Focus on boundary case $J = 3/2$

# The Majorana spin-3/2 case

Positivity constraints can be made precise




*New dynamics must appear at* $\Lambda \leq 9m$

# Include new light states in the EFT?

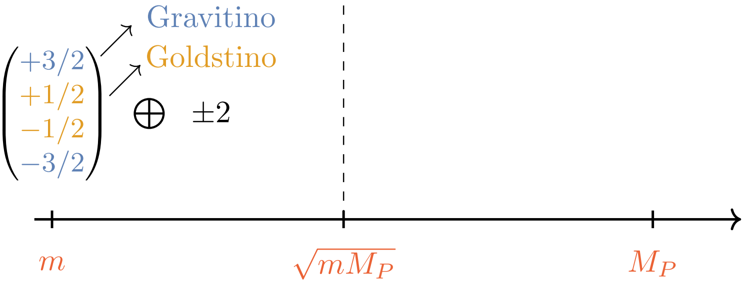
(Consider scalars, vectors and massless graviton)

# Gravity comes to help

In order to have large scale separation

a) *Gravity is necessary for consistency*

b) *Recover $\mathcal{N} = 1$ spontaneously broken supergravity*



# What if we have a global symmetry?

Consider Dirac spin-3/2 particle with $U(1)$ symmetry

# The Dirac spin-$\frac{3}{2}$ case

What does not change

a) *Spin-3/2 alone does not allow for scale separation*

b) *Gravity necessary for consistency*

What is new

c) *A photon gauging the $U(1)$ is necessary*

d) *Recover $\mathcal{N} = 2$ spontaneously broken supergravity*

- $2e^2/m^2 = 1/M_P^2$
- Swampland conjectures

# Thanks!!

# The Majorana spin-$\frac{3}{2}$ case

Isolated particle: only contact terms

- Longitudinal modes $h = \pm\frac{1}{2} \Longrightarrow \psi_\mu \supset \frac{\partial_\mu}{m}\tilde{\psi} \to \left(\frac{E}{m}\right)^{\frac{3}{2}}$ (for simplicity)

$$\frac{mc_1}{M^3}[(\bar{\psi}_\mu\psi^\mu)^2 - (\bar{\psi}_\nu\gamma_5\psi^\nu)]$$

$$\frac{mc_2}{M^3}(\bar{\psi}_\mu\gamma_\rho\psi^\mu)(\bar{\psi}_\nu\gamma^\rho\psi^\nu)$$

$$\frac{mc_3}{M^3}[(\bar{\psi}_\mu\psi^\mu)^2 + (\bar{\psi}_\nu\gamma_5\psi^\nu)]$$

- Decoupling limit: $m \ll E \ll M$, $m \cdot M = f^2$ fixed

$$\mathcal{M}_{long} \sim \frac{E^6}{f^6} + O\left(\frac{m^2 E^4}{f^6}\right)$$

# The Majorana spin-$\frac{3}{2}$ case

- Match to massless $h = \pm\frac{1}{2}$ amplitudes

$$\mathcal{M}^{IR}_{--++}(s,t) = \langle 12\rangle[34](c_{1,0}s + c_{2,0}s^2 + c_{2,1}tu + \ldots)$$
$$\mathcal{M}^{IR}_{----}(s,t) = \langle 12\rangle\langle 34\rangle(\tilde{c}_{2,1}tu + \ldots)$$

- Find

Causality!

$$\frac{|c_{2,0}|}{c_{1,0}}, \frac{|c_{2,1}|}{c_{1,0}}, \frac{|\tilde{c}_{2,1}|}{c_{1,0}} \sim \frac{O(1)}{m^2} \quad \Longleftrightarrow \quad \frac{|c_{2,i}|}{c_{1,0}} \lesssim \frac{O(1)}{\Lambda^2}$$

*Isolated Majorana spin-$\frac{3}{2}$ is inconsistent* (as soon as $m \ll \Lambda$)

# The Majorana spin-$\frac{3}{2}$ case

Decoupling limit: $m \to 0,\ M_{Pl} \to +\infty,\ m^2 \cdot M_{Pl} = f_G^3$ fixed

$$\mathcal{M}^{IR}_{--++}(s,t) = \left(c_{2,0} + \frac{2}{9f_G^6}\right) s^3 + \left(c_{2,1} + \frac{12}{9f_G^6}\right) stu$$

$$\mathcal{M}^{IR}_{----}(s,t) = \tilde{c}_{2,1} stu$$

- Consistency with positivity bounds

$$c_{2,0} = -\frac{2}{9m^4 M_{Pl}^2}\,,\ c_{2,1} = -\frac{12}{9m^4 M_{Pl}^2}\,,\ \tilde{c}_{2,1} = 0$$

# The Majorana spin-$\frac{3}{2}$ case

Purely gravitational EFT: spin-$\frac{3}{2}$ $\Longrightarrow$ Gravitino

$$\mathcal{M}_{long} = s^2 \left( \frac{1}{3m^2 M_{Pl}^2} - \frac{1}{t M_{Pl}^2} \right) + \dots$$

- Decoupling limit: $m \to 0$, $M_{Pl} \to +\infty$

$$\mathcal{M}_{long} \to \frac{s^2}{F^2}$$

- EFT of goldstinos, $F^2 = 3m^2 M_{Pl}^2 \Longrightarrow$ SUSY breaking

# The Majorana spin-$\frac{3}{2}$ case

Purely gravitational EFT: spin-$\frac{3}{2}$ $\Longrightarrow$ Gravitino

$$\mathcal{M}_{long} = s^2 \left( \frac{1}{3m^2 M_{Pl}^2} - \frac{1}{t M_{Pl}^2} \right) + \dots$$

- Decoupling limit: $m \to 0$, $M_{Pl} \to +\infty$

$$\mathcal{M}_{long} \to \frac{s^2}{F^2}$$

- EFT of goldstinos, $F^2 = 3m^2 M_{Pl}^2 \Longrightarrow$ SUSY breaking

**Q.3**: *Explore the bounds on the goldstinos EFT?*

# The Dirac spin-$\frac{3}{2}$ case

**What's the same?**

- Isolated particle must be free
- Gravity required by consistency

**What's new?**

- $U(1)$ global symmetry
- Photon required by consistency

Causality requires minimal coupling in the sense of [Arkani-Hamed, T.-C. Huang, and Y.-t. Huang 2017] with

$$\frac{e}{m} = \frac{1}{\sqrt{2}M_{Pl}}$$

- Saturates the weak gravity conjecture bound
- Example of no global symmetry with gravity
- Match truncated $\mathcal{N} = 2$ supergravity [Freedman and Das 1977]

# Conclusions

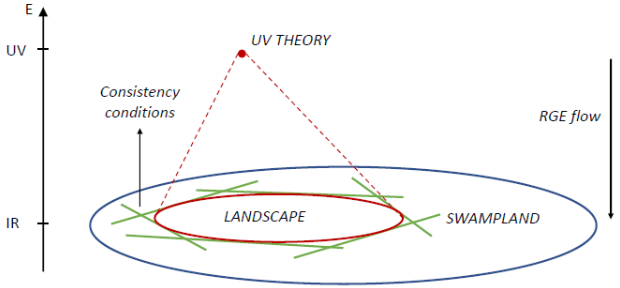
What did we learn?

1. Isolated spin-$\frac{3}{2}$ inconsistent with causality as $m \ll \Lambda$
2. Way out: include gravity!
3. If $U(1)$ symmetric: gauge! $\implies$ Saturate WGC

# Positivity bounds

Consistency conditions on EFTs from UV physics

- ***Not*** *all that is consistent (in the IR) is allowed*

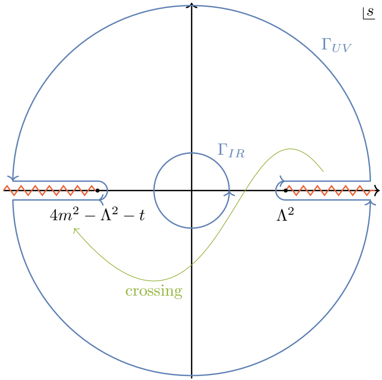



New understanding in the last 20 years (mostly after [Adams et al. 2006])

# Positivity bounds

Connect UV and IR through $2 \to 2$ scattering amplitudes

- Impose causality ($\to$ analyticity), unitarity ($\to$ positivity) and Lorentz invariance in the UV




Arcs

$$a_n(t) = \oint_{\Gamma_{IR}} \frac{ds}{2\pi i} \frac{\mathcal{M}(s,t)}{(s - 2m^2 + \frac{t}{2})^{3+n}}$$

- IR: Wilson coefficients
- UV: $\int_{UV} d\mu(s,t) \text{Disc}\, \mathcal{M}(s,t)$

# Positivity bounds

Ex. Massless $h = \pm\frac{1}{2}$ fermion

$$\mathcal{M}^{IR}_{--++}(s,t) = \langle 12\rangle[34](c_{1,0}s + c_{2,0}s^2 + c_{2,1}tu + ...)$$
$$\mathcal{M}^{IR}_{----}(s,t) = \langle 12\rangle\langle 34\rangle(\tilde{c}_{2,1}tu + ...)$$

- IR: Wilsons* $a_n^{--++}(0) = c_{n+1,0}$
- UV: Moments of a positive measure

$$a_n^{--++}(0) = \int_{\Lambda^2}^{+\infty} \frac{ds}{\pi} \frac{\text{Im}\mathcal{M}_{--++}(s,0) + (-1)^n \text{Im}\mathcal{M}_{-++-}(s,0)}{s^{3+n}}$$

- Imply *e.g.*

$$0 < |c_{2,0}| \leq \frac{c_{1,0}}{\Lambda^2}$$

# Positivity bounds

Ex. Massless $h = \pm\frac{1}{2}$ fermion

- Further bounds from $\left|\hat{T}(\alpha\,|A\rangle + \beta\,|B\rangle)\right|^2 \geq 0$
  [Bellazzini et al. 2024]

$$2|\text{Disc}\,\mathcal{M}_{A\to B}| \leq \text{Disc}\,\mathcal{M}_{A\to A} + \text{Disc}\,\mathcal{M}_{B\to B}$$

- Imply *e.g.*

$$\left|a_0^{--++}(t)\right| \leq \frac{a_0^{--++}(0)}{(1+\frac{2t}{\Lambda^2})^3}$$

# Positivity bounds

Ex. Massless $h = \pm\frac{1}{2}$ fermion

$$\mathcal{M}^{IR}_{--++}(s,t) = \langle 12\rangle[34](c_{1,0}s + c_{2,0}s^2 + c_{2,1}tu + \ldots)$$
$$\mathcal{M}^{IR}_{----}(s,t) = \langle 12\rangle\langle 34\rangle(\tilde{c}_{2,1}tu + \ldots)$$

- With a (slightly) more refined analysis

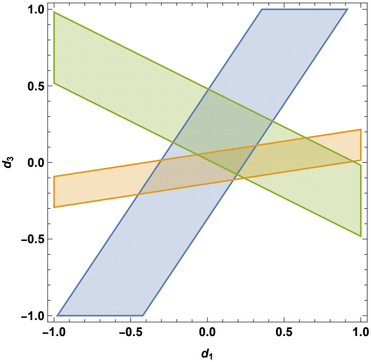
$$0 < \frac{|c_{2,0}|}{c_{1,0}}, \frac{|c_{2,1}|}{c_{1,0}}, \frac{|\tilde{c}_{2,1}|}{c_{1,0}} \leq \frac{O(1)}{\Lambda^2}$$

- In this sense $\mathcal{M} \lesssim E^4$

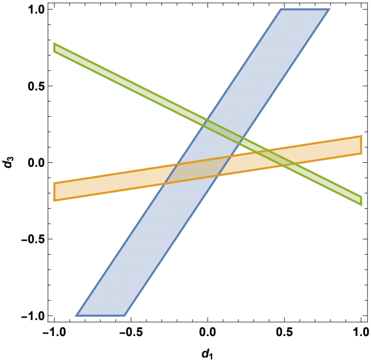
# The Majorana spin-$\frac{3}{2}$

Finite mass effects: work at $m^2 \ll -t \ll \Lambda^2$

$$|a_0(t)| \leq \frac{a_0(0)}{(1+\frac{2t}{\Lambda^2})^3 +} + \underbrace{O\left(\frac{m^2}{\Lambda^2}, \frac{-t}{\Lambda^2}\right)}_{\textit{known}}$$




$t = -10m^2$ , $\Lambda = 5m$




$t = -10m^2$ , $\Lambda = 10m$

# The golstino EFT-hedron

How: Arcs $\Longrightarrow$ 2D moment problem $(s, J)$ [Bellazzini et al. 2021; Caron-Huot and Van Duong 2021; Arkani-Hamed, T.-C. Huang, and Y.-t. Huang 2021]

- Bounds implemented numerically and optimized