Examining the anomalous nature of chiral effects in thermodynamics

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Chiral anomaly

Chiral transformation $\delta\psi = i\alpha(x)\gamma_5\psi$

$$Z = \int \mathcal{D}\bar{\psi}\mathcal{D}\psi\, e^{i\int d^4x\, \bar{\psi} i\not{D}\, \psi} = \int J[\alpha]\mathcal{D}\bar{\psi}\mathcal{D}\psi\, e^{i\int d^4x\, \bar{\psi}(i\not{D} - (\not{\partial}\,\alpha)\gamma_5)\psi}$$

$$J = \frac{\det \ldots}{\det \ldots}$$

$$\log J[\alpha] = \mathrm{Tr}(-(\not{\partial}\,\alpha)\gamma_5(i\not{D})^{-1})$$

Chiral Vortical Effect

$$\langle j_5^\mu \rangle \supset \frac{T^2}{6}\Omega^\mu$$

Fluid vorticity $\Omega^\mu = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}u_\nu\partial_\rho u_\sigma$

Related to mixed anomaly?

Partition function

- Imaginary-time formalism

$$Z = \mathrm{Tr} e^{-\beta H} = \int_{AP\ b.c.} \mathcal{D}\bar{\psi}\mathcal{D}\psi\, e^{-\int_0^\beta dt \int d^3x\, \bar{\psi}\not{D}\psi}$$

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- Imaginary-time formalism

$$Z = \mathrm{Tr} e^{-\beta H} = \int_{AP\ b.c.} \mathcal{D}\bar{\psi}\mathcal{D}\psi\, e^{-\int_0^\beta dt \int d^3x\, \bar{\psi} \not{D} \psi}$$

- Curved spacetime

$$Z = \mathrm{Tr}\, e^{-\int d\Sigma\, n_\mu T^{\mu\nu}\beta_\nu}$$

Coordinate choice: simplify foliation, general metric

Computation

$$\log J[\alpha] = \mathrm{Tr}(-(\partial\!\!\!/\,\alpha)\gamma_5(iD\!\!\!\!/\,)^{-1})$$

$$\nabla_\mu \langle j_5^\mu \rangle \supset T \sum_n \int d^3q \; \mathrm{tr}\partial\!\!\!/\,\gamma_5 \Delta i D\!\!\!\!/\,\Delta = \frac{T^2}{6} \nabla_\mu \Theta^\mu$$

$$\Theta^\mu = -\frac{1}{2}\epsilon^{\mu\nu\rho\sigma} u_\nu u^\lambda \omega_{\lambda,\rho\sigma}$$

Conclusion

- More general result
- New contributions to the divergence of the axial current
- Independence of time slice makes it vanish

Other topics of interest: Schwinger effect and derivative corrections

Thank you!