KSVZ axion model

$$\frac{1}{\sqrt{-g}}\mathcal{L}_{KSVZ} = \underbrace{\frac{1}{2}\partial_\mu\phi^*\partial^\mu\phi}_{\substack{\text{SM-neutral}\\ \text{complex scalar}}} + \underbrace{iQ^\dagger\bar{\sigma}^\mu D_\mu Q}_{SU(3)_c\text{ triplet}} + \underbrace{i\bar{Q}^\dagger\bar{\sigma}^\mu D_\mu \bar{Q}}_{\substack{SU(3)_c\\ \text{anti-triplet}}}$$

$$+\left(y\phi Q\bar{Q} + h.c.\right) - V(\phi^*\phi)$$

$Q, \bar{Q}$ might or might not have electroweak gauge interactions; minimal "KSVZ" line on axion constraint plots assumes not.

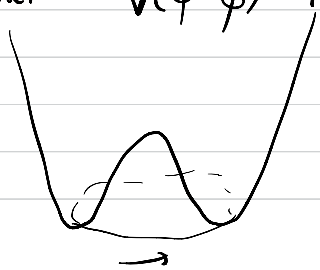
Classical global symmetry:

$$U(1)_{PQ} : \phi \mapsto e^{i\alpha}\phi, \quad Q \mapsto e^{-i\alpha}Q$$

also $\quad U(1)_Q : Q \mapsto e^{i\beta}Q, \quad \bar{Q} \mapsto e^{-i\beta}\bar{Q}.$

Any linear comb. of these except $U(1)_Q$ can be called "a" PQ (Peccei-Quinn) symmetry.

Assume $V(\phi^*\phi)$ has a symmetry-breaking form,



$\overrightarrow{\theta}$

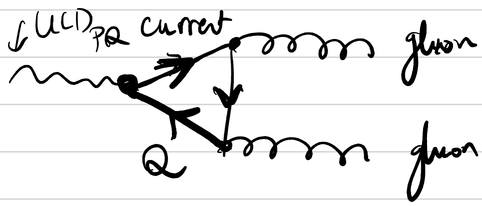
$$\phi = \frac{1}{\sqrt{2}}\Big(\underbrace{f}_{\text{decay const.}} + \underbrace{s(x)}_{\text{"saxion"}}\Big)e^{i\underbrace{\theta(x)}_{\text{"axion"}}}$$

$\theta(x) \cong \theta(x) + 2\pi$ clearly a periodic scalar.

KSVZ

$y\phi Q\bar{Q}$ means the quarks get a mass at the PQ-breaking scale, $m_Q = yf/\sqrt{2}$

At low energies we can integrate out $s, Q, \bar{Q}$ and get an EFT involving the pseudo-Nambu-Goldstone boson θ. Why pseudo? Because the PQ symmetry is **chiral**.



In other words, we have a mass term $\mathcal{L}$

$$m_Q e^{i\theta(x)} Q\bar{Q} + m_Q e^{-i\theta(x)} Q^\dagger \bar{Q}^\dagger .$$

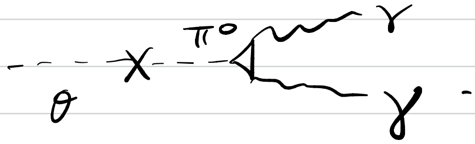
We want to integrate out Q, but it has this θ-dependence. We can eliminate it by a field redef, like $Q \mapsto e^{-i\theta(x)} Q(x)$. But this means our low-energy EFT picks up a term

$$\int \frac{\theta(x)}{8\pi^2} \mathrm{tr}(G \wedge G).$$

This is just what we need for θ to solve the Strong CP problem. Canonically normalize: $\theta(x) \longrightarrow \frac{a(x)}{f}$, $G \to g_s G$.

In this model, there are no other couplings — at energies $\Lambda_{QCD} \ll E \ll f$, the axion interacts w/ the SM only via $\theta\, \mathrm{tr}(G \wedge G)$.

Below the confinement scale the axion acquires an effective interaction w/ photons by mixing with the π^0:



In units

$$\frac{\alpha_s}{8\pi}\frac{a}{F_a} G^a_{\mu\nu}\tilde{G}^{a\mu\nu} + \frac{\alpha}{8\pi}\left(\frac{N_\gamma}{N_G}\right)\frac{a}{F_a} F_{\mu\nu}\tilde{F}^{\mu\nu}$$

Contributes -1.92 to effective N_γ/N_G.

DFSZ axion model

(Skipping for lack of time)

In this model, the SM is extended to a 2HDM, and the PQ global symmetry acts on the Higgses as well as a heavy complex scalar ϕ.
To act on the Higgses, PQ must also act on SM fermions! So, it's more elaborate than KSVZ.

e.g. H_u, H_d, ϕ all have PQ charge $+1$

Quartic interaction $\quad \lambda_{ud\phi} H_u \cdot H_d \phi^{\dagger 2}$

Yukawa interactions $\quad y_u H_u Q U^c + y_d H_d Q D^c$
(Type II 2HDM; not only option!) $\quad + y_e H_d L E^c + h.c.$

U^c, D^c, E^c have PQ charge $+1$ as well

Again, phases of scalars appear in mass terms; chiral rotations eliminate them. Carrying this out produces derivative interactions:

$$\sim \frac{v_u^2}{v_u^2+v_d^2} \frac{(\partial_\mu a)}{f_a} U^{c\dagger} \bar{\sigma}^\mu U^c + \frac{v_d^2}{v_u^2+v_d^2} \frac{(\partial_\mu a)}{f_a} D^{c\dagger} \bar{\sigma}^\mu D^c + \ldots$$

as well as the key couplings to gauge fields:

$$\int \left[\frac{3}{8\pi^2} \theta \, \mathrm{tr}(G \wedge G) + \frac{8}{8\pi^2} \theta \, F \wedge F \right]$$

(check numbers!)

N_γ / N_g : $8/3 - 1.92 \rightarrow$ Smaller (lower band on plots)

Axion quality problem

Throughout the above discussion we've been assuming that we can impose a global symmetry. But (as we'll discuss tomorrow), we don't expect symmetries to be fundamental.

Explicit PQ-breaking terms can completely spoil the solution to the Strong CP problem!

e.g., take KSVZ and add a term

$$\frac{1}{\sqrt{-g}} \mathcal{L}_{\cancel{PQ}} = \frac{c}{M_{Pl}^{n-4}} \phi^n + \text{c.c.},$$

w/ coefficient c generally having a phase,

$c = |c| e^{i\phi_{PQ}}$.

After ϕ gets a VEV, this becomes an effective axion potential,

$$\frac{|c|}{M_{Pl}^{n-4}} \left(\frac{f}{\sqrt{2}}\right)^n \left[e^{i(\phi_{PQ} + n\theta)} + e^{-i(\phi_{PQ} + n\theta)} \right]$$

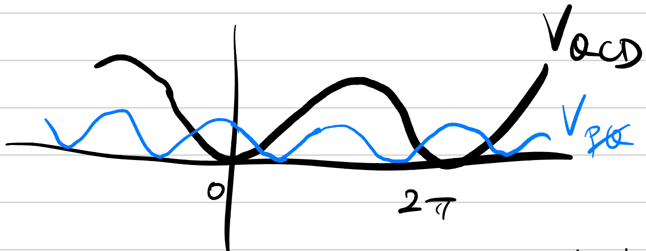
$$= 2|c| M_{Pl}^4 \left(\frac{f}{\sqrt{2} M_{Pl}}\right)^n \cos(n\theta + \phi_{PQ}).$$

There is no reason for ϕ_{PQ} to be a small phase if CP is not a fundamental symmetry.

axion quality prob

2

So this can shift the min. of the potential!



Experimentally, we know $|\theta| < 10^{-10}$!

So V_{PQ} must be very small, or ϕ_{PQ} must.

$V_{QCD}(\theta)$ is naturally exp. small but V_{PQ} is only power-law small: e.g., if $f \simeq 10^{12}$ GeV, then

if $n=8$, $V_{PQ} \approx (250\,\text{TeV})^4 \cos(n\theta + \phi_{PQ}) \ldots$

way too big!

if $n=12$, $V_{PQ} \approx (73\,\text{MeV})^4 \cos(n\theta + \phi_{PQ})$.

Close to QCD size — $\mathcal{O}(1)$ correction. But need at most $\mathcal{O}(10^{-10})$. Need $n = 14$.

So, we have to forbid (or strongly suppress) many dangerous operators for a model like KSVZ to work. One approach is to use large discrete gauge symmetries under which ϕ and other fields are charged.

Another is compositeness: make ϕ fundamentally a high-dim op. My favorite: make θ come from a higher-dim gauge field.

Extra-dimensional axions

A 4d axion field can arise as a zero mode of a (generalized) higher-dimensional gauge field.

Such modes can exhibit exponentially good control of the axion quality problem.

They are not a pseudo-Nambu-Goldstone boson of a 4d PQ symmetry, except in the trivial sense that $\partial_\mu \theta$ is a symmetry current.

The first example was in string theory: Witten 1984 investigation of the Type I superstring.

Let's instead start with the simplest example, a 5d gauge theory on a circle, S^1.

Higher-dimensional fields give rise to an infinite tower of 4d fields, called "Kaluza-Klein modes." For circle compactification this is just Fourier analysis, e.g., for a 5d scalar field,

$$\underset{\text{4d direction}}{\phi(x^\mu},\ \underset{\text{5d direction}}{x^5)} = \sum_{n=-\infty}^{\infty} \underbrace{e^{inx^5/R}}_{\text{wavefunction in 5d}} \underbrace{\phi_n(x^\mu)}_{\text{4d field}}$$

when the 5th dimension has period $x^5 \cong x^5 + 2\pi R$.

Plugging in to a 5d action: $\int d^5x \left[\frac{1}{2}(\partial\phi)^2 - \frac{1}{2}M^2\phi^2\right]$, the 4d modes have masses

$$m_n^2 = M^2 + \left(\frac{n}{R}\right)^2.$$

This is typical even for extra dimensions w/ more complicated geometry: massive KK modes have $m \propto 1/R$ w/ R a characteristic size scale of the extra dimensions.

The axion modes of interest to us are instead <u>massless</u> 4d fields, and we will ignore their massive Kaluza-Klein cousins.

Given a 5d gauge field A (for gauge group $U(1)$), we have a 4d mode

$$\theta(x^\mu) = \int_{S^1} A = \int_0^{2\pi R} A_5 \, dx^5.$$

This is a periodic variable: $e^{i\int_{S^1} A}$ is a Wilson line operator, matching to $e^{i\theta}$ in 4d, so $\theta \cong \theta + 2\pi$.

In more detail, "large" gauge transformations of A, those that wind around the circle:

$$A \mapsto A + d\alpha \quad \text{w/} \quad \alpha(x^5) = \frac{n x^5}{R} \quad \text{not single valued}$$

but with $e^{i\alpha(2\pi R)} = e^{i\alpha(0)}$ and $\int_{S^1} d\alpha = 2\pi n$

descend in 4d to gauge transformations

$$\theta(x) \mapsto \theta(x) + 2\pi n$$

which are the defining feature of a periodic scalar.

Plugging the ansatz $A = \frac{\theta}{2\pi R} dx^5$ into a 5d action

$$\int -\frac{1}{2e_{5d}^2} F \wedge \star_{5d} F$$

(units: A_μ is dimension 1, so $1/e_5^2$ has mass dimension 1)

gives the 4d kinetic term

$$\int -\frac{1}{2} f^2 \, d\theta \wedge \star \, d\theta$$

where

$$f^2 = \frac{1}{2\pi R e_5^2}.$$

Weak 5d coupling $\Rightarrow$ large f and weakly coupled 4d axion.
Large volume of extra dimensions $\Rightarrow$ smaller f and stronger 4d couplings.

These patterns persist for more complicated geometries.

The other key feature of a 4d axion was its Chern-Simons coupling to gluons, $\int \frac{k_G}{8\pi^2} \theta \, \mathrm{tr}(G \wedge G)$.

This can originate from a $\underline{5d}$ Chern-Simons coupling,

$$\int \frac{k_G}{8\pi^2} A \wedge \mathrm{tr}(G \wedge G).$$

The quantization $k_G \in \mathbb{Z}$ already held in 5d.
For this to make sense, we need Standard Model gauge fields to propagate in extra dimensions, like the axion does.

In 5d the gluons have a kinetic term

$$\int_{5d} -\frac{1}{g_5^2} \mathrm{tr}(G \wedge \star_{5d} G)$$

$\Big\downarrow$ reduce w/ ansatz $\quad G = \frac{1}{2} G_{\mu\nu} dx^\mu \wedge dx^\nu$
(no x^5 dependence)

$$\int_{4d} -\frac{1}{g^2} \mathrm{tr}(G \wedge \star G) \qquad \text{where} \quad \frac{1}{g^2} = \frac{2\pi R}{g_5^2}$$

Weak 4d gauge couplings can come from large volume of internal dimensions.

Now, we turn to the $\underline{\text{axion quality problem}}$ in the extra-dimensional context. What effects in the 5d theory can lead to a potential $V(\theta)$ in 4d?

$V(\theta)$ can only come from terms involving θ w/out a derivative on it $(d\theta)$, which originate in 5d from terms that involve A appearing distinct from $F = dA$.

These come from either:

a) Chern-Simons terms, like the $\int A \wedge \mathrm{tr}(G \wedge G)$ term we already mentioned $\Rightarrow$ needed to get $V_{QCD}(\theta)$ in 4d.

b) Charged particles/fields, coupling to A via the covariant derivative, $D\phi = d\phi + iq A\phi$.

Thus, we expect that if 5d charged particles exist, this should somehow lead to a 4d effective potential for the axion θ.

One way to understand this is the following: the mass spectrum of KK modes depends on θ.

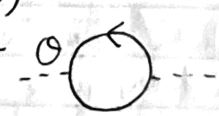
Covariant derivative $D_5 = \underbrace{\partial_5}_{in/R} + \underbrace{iqA_5}_{iq\frac{\theta}{2\pi R}}$

So the $(n/R)^2$ term in the formula shifts,

$$M_n^2 = M^2 + \frac{1}{R^2}\left(n + \frac{q\theta}{2\pi}\right)^2$$

for modes of a scalar w/ charge q and mass M in 5d.

This means that integrating out such modes produces an effective potential for θ,

$\sum_n$ 

The mass formula is not periodic in θ:

$$m_n^2 = M^2 + \left(n + \frac{q\theta}{2\pi}\right)^2 \longmapsto m_{n+q}^2 = M^2 + \left((n+q) + \frac{q\theta}{2\pi}\right)^2$$

under $\theta \to \theta + 2\pi$. This is a form of monodromy.
Basically identical to the particle on a circle in QM from the beginning of the semester!

Result: integrating out mode n generates a potential for θ that violates periodicity, but adding them all up gives a manifestly periodic function.

The mathematical trick behind this is Poisson resummation:

$$\sum_n \underbrace{V_n(\theta)}_{\text{not periodic}} = \sum_w \underbrace{\tilde{V}_w(\theta)}_{\text{manifestly periodic, each term} \sim e^{iqw\theta}}$$

Can also think about this from viewpoint of worldline formalism: heavy particle has a worldline action

$$S_{\text{particle}} = \int_\gamma M\, d\tau + iq\int_\gamma A$$

γ = particle worldline.

The path integral sums over all paths $x(\tau)$.

$$\int \mathcal{D}x(\tau)\, e^{-S_{\text{particle}}[x(\tau)]} \xrightarrow[\text{semiclassical}]{} \sum_{w\in\mathbb{Z}} (\text{prefac})\cdot e^{-2\pi MR\cdot w} e^{iqw\theta},$$

where each $w \in \mathbb{Z}$ corresponds to a saddle for the path integral where the particle worldline wraps the circle w/ winding number $w \in \mathbb{Z}$.

Now, these contributions to the axion potential are exponentially small when $2\pi RM \gg 1$, i.e., when the extra dimensions are large compared to the Compton radius of charged 5d particles.

Exponentially small corrections to $V(\theta)$ are much milder than the $(f/M_{Pl})^n$ corrections we see in 4d models.

Effectively, extra-dimensional axion models take the log of the quality problem.

$\boxed{\text{IF TIME ALLOWS:}}$

General case: a theory w/ $d = (4+n)$ spacetime dimensions that has a p-form gauge field $C^{(p)}$, w/ gauge symmetry $C^{(p)} \mapsto C^{(p)} + d\lambda^{(p-1)}$, gives rise to a 4d axion for <u>every</u> <u>independent</u> <u>non-torsion</u> <u>p-cycle</u> in the n extra dimensions. $(p \leq n)$

These cycles are associated to cohomology class $[\omega_i^{(p)}] \in H^p(Y, \mathbb{Z})$, w/ Y = geometry of n internal dimensions

Ansatz: $C^{(p)} = \sum_i \theta^i(x)\, \hat{\omega}_i^{(p)}(y)$

$\Longrightarrow$ 4d eff. action $\quad -\int \frac{1}{2} K_{ij}\, d\theta^i \wedge \star d\theta^j$,

$\theta^i \cong \theta^i + 2\pi$,

$K_{ij} = \frac{1}{e_p^2} \cdot \int_Y \hat{\omega}_i^{(p)} \wedge \star_Y \hat{\omega}_j^{(p)}$.

e_p: gauge coupling of $C^{(p)}$

Fine point: $\hat{\omega}_i^{(p)}$ is a unique representative of the equivalence class $[\omega_i^{(p)}]$ that is <u>harmonic</u>, equivalently $d\star\hat{\omega}_i^{(p)} = 0$.

This also generalizes to warped compactifications.

Axion mass: 1) Chern-Simons terms, like $\frac{k_G}{8\pi^2}\int_\Sigma C^{(p)} \wedge \mathrm{tr}(G \wedge G)$

Σ = 4d spacetime $\times$ p-cycle, interaction + gluons live on <u>brane</u> on p-cycle

[illegible]

<u>Mention</u>: Ahmed's argument: $f \gtrsim \frac{1}{R}$ ~ M_{KK} as condition for weakly coupled description @ compactification scale

Expt. measure $f \Rightarrow$ sharp contrast in interp between 4d models and extra dim models! My recent work: quantitative claim re cutoff, $\Lambda_{UV} \lesssim 2\pi\sqrt{S_{inst}}\, f$.

Known-ish for QCD, ~80.

2) <u>branes</u> charged under $C^{(p)}$, Euclidean action

$$S_{brane} = \int_{\Gamma^{(p)}} \left(T_{(p)}\, \mathrm{vol}(\Gamma) + i q\, C^{(p)} \right)$$

brane tension: generalization of mass M (mass / unit volume)

Similar semi-classical analysis:

$$V(\theta) \propto e^{-T_{(p)} \mathrm{Vol}(\Gamma^{(p)}) + i q \theta}$$

corrections <u>exponentially suppressed</u> when volume is large in units of brane tension.

Symmetry structure: in 4d, massless axion $\Rightarrow f^2 d \star d\theta = 0$.

$f^2 \star d\theta$ is the conserved current of a continuous shift symmetry $\theta \mapsto \theta + \text{const}$.

This is inherited from the "electric p-form symmetry" of the higher dimensions: if action lacks C-S term or branes,

$$d\left[\frac{1}{e_p^2} \star dC^{(p)}\right] = 0$$

or more generally $\frac{\delta L}{\delta(dC^{(p)})}$, the electric flux density of $C^{(p)}$.

$\int_{\Sigma} \frac{1}{e_p^2} \star dC^{(p)}$ measures the <u>electric flux</u> through the $(d-p-1)$-surface Σ.

No charged object $\Rightarrow$ electric flux conserved

Charged objects $\Rightarrow$ screening, electric flux not conserved.

This is the higher dimensional origin of the symmetry solving the quality problem.

Axions as gauge fields

$$\int \left[\frac{1}{2} f^2 d\theta \wedge \star d\theta + \frac{n\theta}{8\pi^2} \mathrm{tr}(G \wedge G) \right]$$

$\Rightarrow$ eqn. of motion:

$$d\left(f^2 \star d\theta \right) = \frac{n}{8\pi^2} \mathrm{tr}(G \wedge G).$$

Integrate both sides:

$$0 = n \cdot N_{inst}.$$

$\boxed{\text{Axion} \Rightarrow \text{no instanton number!}}$

Compare Maxwell's eqns:

$$d\left(\frac{1}{e^2} \star F \right) = J_{EM}$$

$$\xRightarrow{\text{integrate}} \quad 0 = Q_{EM}$$

(Gauss's law: no net charge on compact space)

Just as A gauges sym. J_{EM},

θ gauges "symmetry" $\mathrm{tr}(G \wedge G)$.

Quantum gravity hates any global symmetry:
axion has an important job!

In 4d, $\mathrm{tr}(G \wedge G)$ is a little funny because we integrate it over spacetime.

In 5d, $\mathrm{tr}(G \wedge G)$ is integrated over 4d space, not time.

And, (Chern-Weil) $d\left[\mathrm{tr}(G \wedge G)\right]$

$= 2\,\mathrm{tr}(dG \wedge G) = 2\,\mathrm{tr}(DG \wedge G)$ covariant

$= 0$ by non-abelian Bianchi $DG = 0$.

Instanton number is a global symmetry in 5d!

Chern-Simons term $A \wedge \mathrm{tr}(G \wedge G)$ is required, by QG, to banish it.

$\Rightarrow$ axion in 4d.

Higher-dim analogues exist.

Takeaway: higher-dim axions
(a) good for quality problem
(b) good for eliminating symmetries.