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KSVZ axion model

$$\frac{1}{\sqrt{g}} \mathcal{L}_{\text{KSVZ}} = \underbrace{\frac{1}{2} \partial_\mu \phi^* \partial^\mu \phi}_{\text{SM-neutral complex scalar}} + \underbrace{i \bar{Q} \bar{\sigma}^\mu D_\mu Q}_{\text{SU(3)}_c \text{ triple}} + \underbrace{i \bar{Q} \bar{\sigma}^\mu D_\mu Q}_{\text{SU(3)}_c \text{ anti-triple}} + (y \phi Q \bar{Q} + \text{h.c.}) - V(\phi^* \phi)$$

$Q, \bar{Q}$ might or might not have electroweak gauge interactions; minimal "KSVZ" line on axion constraint plots assumes not.

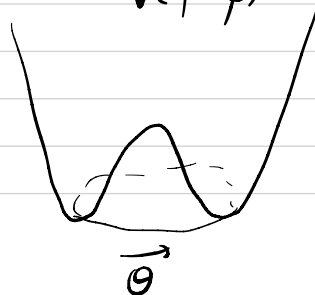
Classical global symmetry:

$$U(1)_{PQ} : \phi \mapsto e^{i\alpha} \phi, \quad Q \mapsto e^{-i\alpha} Q$$

$$\text{also } U(1)_a : Q \mapsto e^{i\beta} Q, \quad \bar{Q} \mapsto e^{-i\beta} \bar{Q}.$$

Any linear comb. of these exact $U(1)_a$ can be called "a" PQ (Peccei-Quinn) symmetry.

Assume $V(\phi^* \phi)$ has a symmetry-breaking form,



$$\phi = \frac{1}{\sqrt{2}} \left(f + s(x) \right) e^{i\theta(x)}$$

$\downarrow$ decay const. $\downarrow$ "saxion" $\downarrow$ "axion"

$\theta(x) \cong \theta(x) + 2\pi$ clearly a periodic scal.

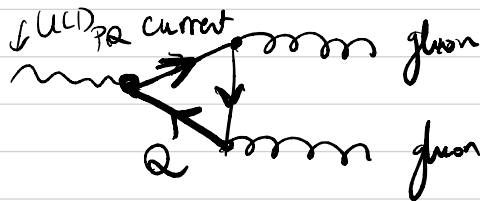
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KSVZ

$y \phi Q \bar{Q}$ means the quarks get a mass at the

PQ-breaking scale, $m_Q = yf/\sqrt{2}$

At low energies, we can integrate out $S, Q, \bar{Q}$ and get an EFT involving the pseudo-Nambu-Goldstone boson θ . Why pseudo? Because the PQ symmetry is chiral.



In other words, we have a mass term

$$m_Q e^{i\theta(x)} Q \bar{Q} + m_Q e^{-i\theta(x)} Q^\dagger Q^\dagger.$$

We want to integrate out Q , but it has this θ -dependence. We can eliminate it by a field redef, like $Q \mapsto e^{-i\theta(x)} Q(x)$. But this means our low-energy EFT picks up a term

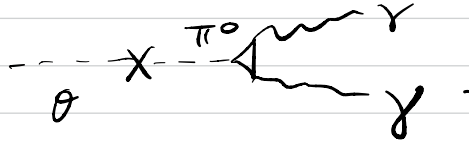
$$\int \frac{\theta(x)^2}{8a^2} \text{tr}(G \wedge G).$$

This is just what we need for θ to solve the Strong CP problem. Canonically normalize: $\theta(x) \rightarrow \frac{a(x)}{f}$, $G \mapsto g_s G$.

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In this model, there are no other couplings —
at energies $\Lambda_{\text{QCD}} \ll E \ll f$,
the axion interacts w/ the SM only via $\Theta \text{tr}(G \wedge G)$.

Below the confinement scale, the axion acquires
an effective interaction w/ photons by mixing with
the π^0 :



In units

$$\frac{\alpha_s}{8\pi} \frac{a}{F_a} G_{\mu\nu}^a \tilde{G}^{a\mu\nu} + \frac{\alpha}{8\pi} \left(\frac{N_Y}{N_G} \right) \frac{a}{F_a} F_{\mu\nu} \tilde{F}^{\mu\nu}$$

Contributes -1.92 to effective N_Y/N_G .

DFSZ axion model

(Skipping for lack of time)

In this model, the SM is extended to a 2HDM, and the PQ global symmetry acts on the Higgses as well as a heavy complex scalar ϕ .

To act on the Higgses, PQ must also act on SM fermions! So, it's more elaborate than KSVZ.

e.g.: H_u, H_d, ϕ all have PQ charge +1

Quartic interaction $\lambda_{u\phi} H_u \cdot H_d \phi^{\dagger 2}$

Yukawa interactions
(Type II 2HDM, not only option!)

$$y_u H_u Q U^c + y_d H_d Q D^c + y_e H_d L E^c + \text{h.c.}$$

U^c, D^c, E^c have PQ charge +1 as well.

Again, phases of scalars appear in mass terms; chiral rotations eliminate them. Carrying this out produces derivative interactions:

$$\sim \frac{v_u^2}{v_u^2 + v_d^2} \frac{(\partial_\mu f_a)}{f_a} U^{c\dagger} \vec{\sigma}^\mu U^c + \frac{v_d^2}{v_u^2 + v_d^2} \frac{(\partial_\mu f_a)}{f_a} D^{c\dagger} \vec{\sigma}^\mu D^c + \dots$$

as well as the key couplings to gauge fields:

$$\int \left[\frac{3}{8\pi^2} \Theta \text{tr}(G \wedge G) + \frac{8}{8\pi^2} \Theta F \wedge F \right]$$

(check numbers!)

$N_g/N_g: 8/3 - 1.92 \rightarrow \underline{\underline{\text{smaller}}}$ (lower band on plots)

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Axion quality problem

Throughout the above discussion we've been assuming that we can impose a global symmetry. But (as we'll discuss tomorrow), we don't expect symmetries to be fundamental.

Explicit PQ-breaking terms can completely spoil the solution to the Strong CP problem!

e.g., take KSVZ and add a term

$$\frac{1}{\sqrt{g}} \mathcal{L}_{PQ} = \frac{c}{M_{Pl}^{n-4}} \phi^n + \text{c.c.},$$

w/ coefficient c generally having a phase,

$$c = |c| e^{i\phi_{PQ}}.$$

After ϕ gets a VEV, this becomes an effective axion potential,

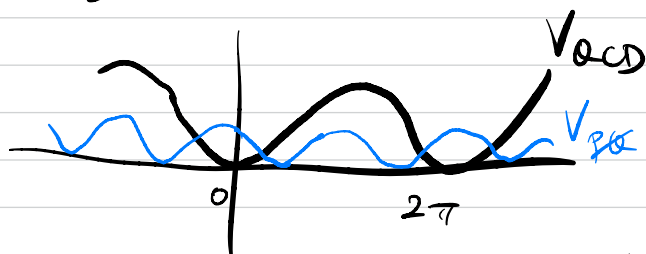
$$\begin{aligned} & \frac{|c|}{M_{Pl}^{n-4}} \left(\frac{f}{\sqrt{2}} \right)^n \left[e^{i(\phi_{PQ} + n\theta)} + e^{-i(\phi_{PQ} + n\theta)} \right] \\ &= 2|c| M_{Pl}^4 \left(\frac{f}{\sqrt{2} M_{Pl}} \right)^n \cos(n\theta + \phi_{PQ}). \end{aligned}$$

There is no reason for ϕ_{PQ} to be a small phase if CP is not a fundamental symmetry.

axion
quality prob

So this can shift the min. of the potential!

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Experimentally, we know $|\theta| < 10^{-10}$!

So V_{PQ} must be very small, or ϕ_{PQ} must.

$V_{QCD}(\theta)$ is naturally exp. small, but V_{PQ} is only power-law small: e.g., if $f \approx 10^{12}$ GeV, then if $n=8$, $V_{PQ} \approx (250 \text{ TeV})^4 \cos(n\theta + \phi_{PQ})$... way too big!

if $n=12$, $V_{PQ} \approx (73 \text{ MeV})^4 \cos(n\theta + \phi_{PQ})$.

Close to QCD size — $\mathcal{O}(1)$ correction. But need at most $\mathcal{O}(10^{-10})$. Need $n=14$.

So, we have to forbid (or strongly suppress) many dangerous operators for a model like KSVZ to work. One approach is to use large discrete gauge symmetries under which ϕ and other fields are charged.

Another is compositeness: make ϕ fundamentally a high-dim op. My favorite: make θ come from a high-dim gauge field.

Extra-dimensional axions

A 4d axion field can arise as a $\textcircled{Z}$ zero mode of a (generalized) higher-dimensional gauge field.

Such modes can exhibit exponentially good control of the axion quality problem.

They are not a pseudo-Nambu-Goldstone boson of a 4d PQ symmetry, except in the trivial sense that $\partial_\mu \theta$ is a symmetry current.

The first example was in string theory: Witten 1984 investigation of the Type I superstring.

Let's instead start with the simplest example, a 5d gauge theory on a circle, S^1 .

Higher-dimensional fields give rise to an infinite tower of 4d fields, called "Kaluza-Klein modes." For circle compactification this is just Fourier analysis, e.g., for a 5d scalar field,

$$\phi(x^\mu, x^5) = \sum_{n=-\infty}^{\infty} \underbrace{e^{in x^5/R}}_{\substack{\text{wavefunction} \\ \text{in } S^1}} \underbrace{\phi_n(x^\mu)}_{\text{4d field}}$$

$\uparrow$ 4d coords $\uparrow$ 5d direction

where the 5th dimension has period $x^5 \cong x^5 + 2\pi R$.

Plugging in to a 5d action: $\int d^5x [\partial_\mu \phi^2 - \frac{1}{2} M^2 \phi^2]$,
the 4d modes have masses

$$m_n^2 = M^2 + \left(\frac{n}{R}\right)^2$$

This is typical even for extra dimensions w/ more complicated ~~geometry~~ geometry: massive KK modes have $m \propto 1/R$ w/ R a characteristic size scale of the extra dimension.

The axion modes of interest to us are instead massless 4d fields, and we will ignore their massive Kaluza-Klein cousins.

Given a 5d gauge field A (for gauge group $U(1)$),
we have a 4d mode

$$\theta(x^4) = \int_{S^1} A = \int_0^{2\pi R} A_5 dx^5.$$

This is a periodic variable. $e^{i \int_{S^1} A}$ is a Wilson line operator, matching to $e^{i\theta}$ in 4d, so $\theta \equiv \theta + 2\pi$.

In more detail, "large" gauge transformations of A , those that wind around the circle:

$$A \mapsto A + d\alpha \quad \text{w/} \quad \alpha(x^5) = \frac{n x^5}{R} \quad \text{not single valued}$$

but with $e^{i\alpha(2\pi R)} = e^{i\alpha(0)}$ and $\int_{S^1} d\alpha = 2\pi n$

descend in 4d to gauge transformations

$$\theta(x) \mapsto \theta(x) + 2\pi n$$

which are the defining feature of a periodic scalar.

Plugging the ansatz
into a 5d action

$$A = \frac{\theta}{2\pi R} dx^5$$

$$\int -\frac{1}{2e_{5d}^2} F \wedge \star_{5d} F$$

(units: A_5 is dimension 1, so
 $\frac{1}{e_{5d}^2}$ has mass dimension 1)

gives the 4d kinetic term

$$\int -\frac{1}{2} f^2 d\theta \wedge \star d\theta$$

where

$$f^2 = \frac{1}{2\pi R e_{5d}^2}.$$

Weak 5d coupling $\Rightarrow$ large f and weakly coupled 4d axion.

Large volume of extra dimensions $\Rightarrow$ smaller f and stronger 4d couplings.

These patterns persist for more complicated geometries.

The other key feature of a 4d axion was its Chern-Simons coupling to gluons, $\int \frac{k_G}{8\pi^2} \text{tr}(G \wedge G)$.

This can originate from a 5d Chern-Simons coupling,
 $\int \frac{k_G}{8\pi^2} A \wedge \text{tr}(G \wedge G)$.

The quantization $k_G \in \mathbb{Z}$ already held in 5d. For this to make sense, we need Standard Model gauge fields to propagate in extra dimensions, like the axion does.

In 5d the gluons have a kinetic term

$$\int_{5d} -\frac{1}{g_5^2} \text{tr}(G \wedge \star_{5d} G)$$

$\int$ reduces w/ w.r.t. z $G = \frac{1}{2} G_{\mu\nu} dx^\mu \wedge dx^\nu$
(no x^5 dependence)

$$\int_{4d} -\frac{1}{g^2} \text{tr}(G \wedge \star G) \quad \text{where} \quad \frac{1}{g^2} = \frac{2\pi R}{g_5^2}$$

Weak 4d gauge couplings can come from large volume of internal dimensions.

Now, we turn to the axion quality problem in the extra-dimensional context. What effects in the 5d theory can lead to a potential $V(\theta)$ in 4d?

$V(\theta)$ can only come from terms involving θ ~~without a derivative on it~~ $(d\theta)$, which originate in 5d from terms that involve A appearing distinct from $F=dA$.

These come from either:

a) Chern-Simons terms, like the $\int A \wedge (G \wedge G)$ term we already mentioned $\Rightarrow$ needed to get $V_{\text{eff}}(\theta)$ in 4d.

b) Charged particles/fields, coupling to A via the covariant derivative, $D\phi = d\phi + iq A \phi$.

Thus, we expect that if 5d charged particles exist, this should somehow lead to a 4d effective potential for the axion θ .

One way to understand this is the following: the mass spectrum of KK modes depends on θ .

Covariant derivative $D_5 = \frac{\partial}{\partial x^5} + iq \frac{A_5}{2\pi R}$

so the $(n/R)^2$ term in the formula shifts,

$$m_n^2 = M^2 + \frac{1}{R^2} \left(n + \frac{q\theta}{2\pi} \right)^2$$

for modes of a scalar of charge q and mass M in 5d.

This means that integrating out such modes produces an effective potential for θ ,

$$\sum_n \text{---} \bigcirc \text{---}$$

The mass formula is not periodic in θ :

$$m_n^2 = M^2 + \left(n + \frac{q\theta}{2\pi}\right)^2 \rightarrow m_{n+q}^2 = M^2 + \left((n+q) + \frac{q\theta}{2\pi}\right)^2$$

under $\theta \rightarrow \theta + 2\pi$. This is a form of monodromy.
Basically identical to the particle on a circle in QM from the beginning of the semester!

Result: integrating out mode n gives a potential for θ that violates periodicity, but adding them all up gives a manifestly periodic function.

The mathematical trick behind this is Poisson resummation:

$$\sum_n \underbrace{V_n(\theta)}_{\text{not periodic}} = \sum_w \underbrace{\tilde{V}_w(\theta)}_{\text{manifestly periodic, each term} \sim e^{iqw\theta}}$$

Can also think about this from viewpoint of worldline formalism: heavy particle has a worldline action

$$S_{\text{part}} = \int_{\gamma} M d\tau + iq \int_{\gamma} A$$

γ = particle worldline.

The path integral sums over all paths $x(\tau)$.

$$\int D(x) e^{-S_{\text{part}}[x(\tau)]} \xrightarrow{\text{saddle}} \sum_{w \in \mathbb{Z}} (\text{prefactor}) \cdot e^{-2\pi i R \cdot w} e^{iqw\theta},$$

where each $w \in \mathbb{Z}$ corresponds to a saddle for the path integral where the particle worldline wraps the circle w/ winding number $w \in \mathbb{Z}$.

Now, these contributions to the axion potential are exponentially small when $2\pi R M \gg 1$, i.e., when the extra dimensions are large compared to the Compton radius of charged 5d particles.

Exponentially small corrections to $V(\theta)$ are much smaller than the $(f/M_{\text{Pl}})^n$ corrections we see in 4d models.

Effectively, extra-dimensional axion models take the log of the quality prob.

IF TIME ALLOWS:

General case: a theory w/ $d = (4+n)$ spacetime dimensions that has a p -form gauge field $C^{(p)}$, w/ gauge symmetry $C^{(p)} \mapsto C^{(p)} + d\lambda^{(p-1)}$ gives rise to a 4d axion for every independent non-trivial p -cycle in the n extra dimensions. ($p \leq n$)

These cycles are associated to cohomology class $[\omega_i^{(p)}] \in H^p(Y, \mathbb{Z})$, w/ Y = geometry of n internal dimensions.

Ansatz: $C^{(p)} = \sum_i \theta^i \omega_i^{(p)}(y)$

$\Rightarrow$ 4d eff. act $-\int \frac{1}{2} K_{ij} d\theta^i \wedge d\theta^j$,

$\theta^i \cong \theta^i + 2\pi$,
 $K_{ij} = \frac{1}{e_p^2} \int_Y \hat{\omega}_i^{(p)} \wedge \hat{\omega}_j^{(p)}$
 $\uparrow$
 gauge cycles of $C^{(p)}$

Five point: $\hat{\omega}_i^{(p)}$ is a unique representative of the equivalence class $[\omega_i^{(p)}]$ that is harmonic, equivalently $d \star \hat{\omega}_i^{(p)} = 0$.

This also generalizes to warped compactifications.

Axion mass: 1) Chern-Simons term, like $\frac{k_C}{8\pi^2} \int C^{(p)} \wedge (F \wedge F)$

$\downarrow$
 = 4d spectrum $\times$ p -cycle, interactions + gluons live on brane on p -cycle

Newton: Ahn's argument: $f \geq \frac{1}{R}$. M_{Pl} as counter for weakly coupled

disruption @ compactified scale

Expt. means $f \Rightarrow$ sharp contrast in step between 4d models and extra d models. Myrner work: quantum class is cut off, $\Lambda_{\text{oc}} \leq \frac{2\pi\sqrt{S_{\text{max}}}}{f}$.
Known rule for $\alpha_{\text{GUT}} \sim 80$.

2) branes charged under $C^{(p)}$, Euclidean action

$$S_{\text{brane}} = \int_{I^{(p)}} (T_{(p)} \text{vol}(I) + i q C^{(p)})$$

↓
brane tension: generalization of mass M
(mass / unit volume)

Similar semiclassical analysis:

$$V(\theta) \propto e^{-T_{(p)} \text{Vol}(I^{(p)}) + i q \theta}$$

Corrections exponentially suppressed when volume is large in units of brane tension.

Symmetry structure: in 4d, massless axion $\Rightarrow f^2 d \star d\theta = 0$.

$f^2 d \star d\theta$ is the conserved current of a continuous shift symmetry $\theta \mapsto \theta + \text{const.}$

This is inherited from the "electric p-form symmetry" of the higher dimension: if action lacks C-S term or branes

$$d \left[\frac{1}{e_p} \star dC^{(p)} \right] = 0$$

↓
or more generally $\frac{\delta L}{\delta(dC^{(p)})}$, the electric flux density of $C^{(p)}$.

$\int_{\Sigma} \frac{1}{e_p} \star dC^{(p)}$ measures the electric flux through the $(d-p-1)$ -surface Σ .

No charged object $\Rightarrow$ electric flux conserved

Charged objects $\Rightarrow$ screening, electric flux not conserved.

This is the higher dimensional origin of the symmetry solving the quality problem.

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①

Axions as gauge fields

$$\int \left[\frac{1}{2} f^2 d\theta \wedge \star d\theta + \frac{n\theta}{8\pi^2} \text{tr}(G \wedge G) \right]$$

$\Rightarrow$ eqn. of motion:

$$d(f^2 \star d\theta) = \frac{n}{8\pi^2} \text{tr}(G \wedge G)$$

Integrate both sides:

$$0 = n \cdot N_{\text{inst.}}$$

Axion $\Rightarrow$ no instanton number!

Compare Maxwell's eqns:

$$d\left(\frac{1}{e^2} \star F\right) = J_{\text{EM}}$$

integrate

$$\Rightarrow 0 = Q_{\text{EM}}$$

(Gauss's law: no net charge on compact space)

Just as a gauge sym. J_{EM} ,

θ gauge "symmetry" $\text{tr}(G \wedge G)$.

Quantum gravity hates any global symmetry:
axion has an important job!

②

In 4d, $\text{tr}(G \wedge G)$ is a little funny because we integrate it over spacetime.

In 5d, $\text{tr}(G \wedge G)$ is integrated over 4d space, not time.

And, (Chern-Wil) $d[\text{tr}(G \wedge G)]$ ↙ covariant

$$= 2 \text{tr}(dG \wedge G) = 2 \text{tr}(D G \wedge G)$$

$$= 0 \quad \text{by non-abelian Bianchi: } D G = 0.$$

Instanton number is a global symmetry in 5d!

Chern-Simons term $A \wedge \text{tr}(G \wedge G)$ is required, by QG, to banish it.

$\Rightarrow$ axion in 4d.

Higher-dim analogues exist.

Takeaway: higher-dim axions

② good for quality problem

⑥ good for eliminating symmetries.