# Tackling the Axion Isocurvature Problem with Modulus Field Kination

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# Construct a Cosmological Scenario for the QCD axion* where:

1. QCD axion avoids quality problem
2. QCD axion comprises *all* the dark matter
3. Supports high inflationary energy scale

* QCD axion = axion that solves the strong CP problem

# Construct a Cosmological Scenario for the QCD axion* where:

1. QCD axion avoids quality problem → ***consider axions that come from extra dimensional gauge fields***
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# Extra Dimensional Axions Seed Isocurvature Fluctuations

High Quality Axions → Extra-dimensional axions → around during inflation → seed isocurvature[1,2]

<u>Inflation with Inflaton Only</u>

$$\delta\phi$$

$$\delta_{DM}(t,x) = \delta_b(t,x) = \tfrac{3}{4}\delta_\gamma(t,x)$$

⇒ initial fluctuations are *adiabatic*[3]


*[1] David Marsh. arXiv: arXiv:1510.07633v2 (2016)*

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*[3] Baumann, Daniel. Cosmology. Cambridge University Press, 2022.*

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**Inflation with Inflaton and Spectator Field**

$$\delta\phi \qquad \delta a$$

$$\delta_b(t,x) = \tfrac{3}{4}\delta_\gamma(t,x) \not\sim \delta_{DM}(t,x)$$

⇒ initial fluctuations are *non-adiabatic*

Known as "isocurvature"



*[1] David Marsh. arXiv: arXiv:1510.07633v2 (2016)*
*[2] Ciaran A. J. O'Hare. arXiv:2403.17697v2 (2024)*
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# Isocurvature Constraints from the CMB

$$\left(\frac{H_{inf}}{\pi f \theta_i}\right)^2 \lesssim 8 \cdot 10^{-11}$$ [1]

$$\longrightarrow \boxed{H_{inf} \lesssim \theta_i \left(\frac{f}{10^{12}\, GeV}\right) 10^7\, GeV}$$

For QCD axion to comprise all the dark matter, we need $f \approx 10^{12}$ GeV [2,3]

⇒ Low inflationary Hubble scale

**Problem**: inflation with low $H_{inf}$ tends to end prematurely[4]

“**Axion Isocurvature Problem**”


*[1] Ciaran A. J. O’Hare. arXiv:2403.17697v2 (2024)*

*[2] David Marsh.* arXiv:1510.07633v2 (2016)

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*[4] Clough, Katy, et al. "Robustness of inflation to inhomogeneous initial conditions." Journal of Cosmology and Astroparticle Physics 2017.09 (2017): 025.*

# Allowing a Larger $H_{inf}$ with *Time Varying* Axion Decay Constant

$f$ need *not* be constant *throughout* cosmic history.

We want:

(1) **Constant** $f \approx 10^{12}$ $GeV$ **after reheating** (DM abundance constraints).
(2) $f > 10^{12}$ $GeV$ **during inflation** to allow for larger $H_{inf}$ (isocurvature constraints).

$\Rightarrow$ Need a mechanism that allows $f$ to *decrease* between inflation and reheating

# Decreasing Axion Decay Constant with Bulk Modulus Evolution

Consider: a (4+n)-dimensional spacetime manifold $M = X_{4D} \times Y_{nD}$ with gauge field A

In the 4D EFT we find:

An **axion** $\theta$ coming from the KK zero mode of $A$

Axion decay constant:

$$f^2 \propto \frac{1}{\mathcal{V}_Y}$$

where $\mathcal{V}_Y$ = volume of $Y_{nd}$

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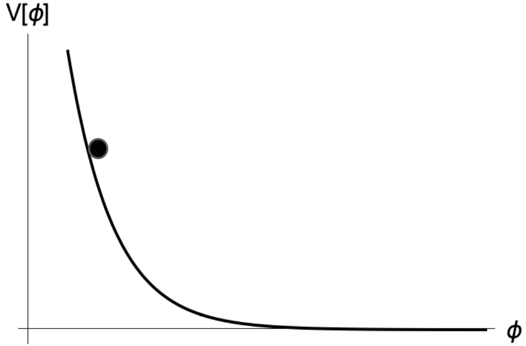
A **bulk modulus field** $\phi$

$\phi$ sets the overall volume of the extra dimensional manifold: $\mathcal{V}_Y \sim \text{Exp}[\#\phi/M_P]$

Has an exponentially decaying potential $V(\phi) \sim \exp(-\phi)$



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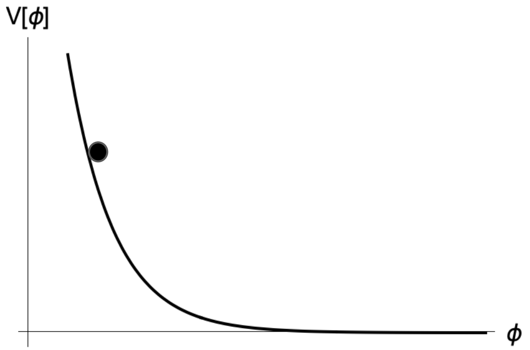



$\Rightarrow f^2 \sim \mathrm{Exp}[-\#\phi/M_P]$

$\phi$ increases $\Rightarrow$ Volume $\mathcal{V}$ increases $\Rightarrow$ *$f$ decreases*

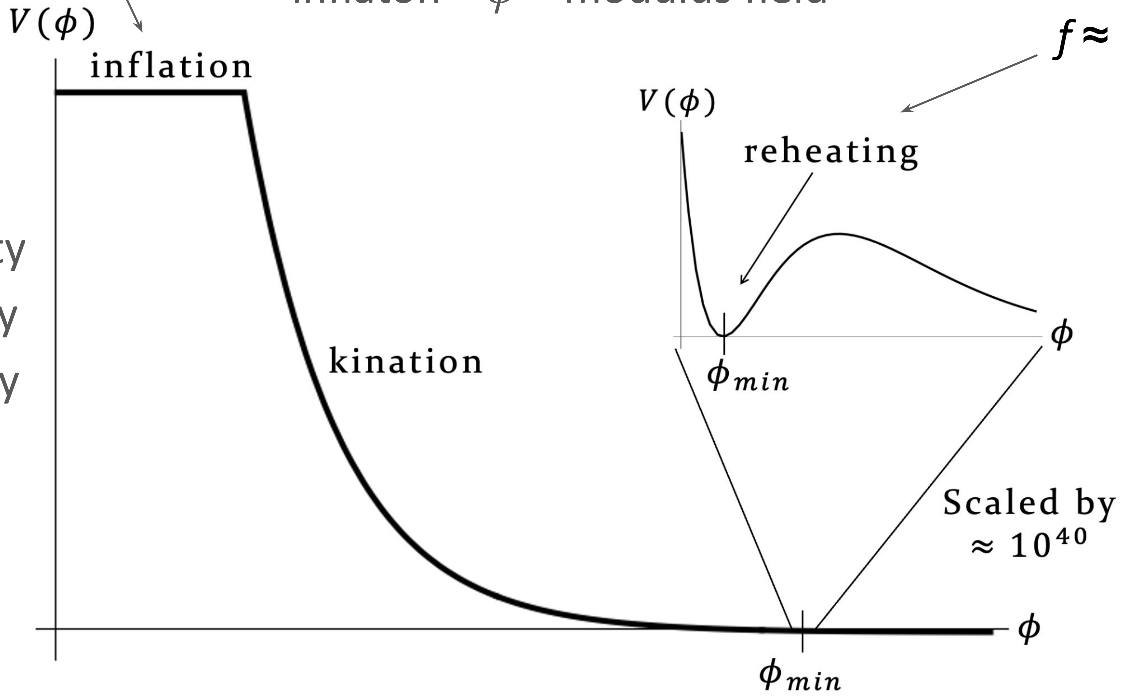
# Cosmic History with Bulk Modulus Field

$f \gg 10^{12}$ GeV

inflaton = $\phi$ = modulus field

$f \approx 10^{12}$ GeV

**Kination** = energy density dominated by kinetic energy of scalar field[1,2]





*[1]* Conlon, Joseph P., and Filippo Revello. "Catch-me-if-you-can: the overshoot problem and the weak/inflation hierarchy." *Journal of High Energy Physics* 2022.

*[2] Fien Apers et al., arXiv:2401.04064 (2024).*

[1] Conlon, Joseph P., and Filippo Revello. "Catch-me-if-you-can: the overshoot problem and the weak/inflation hierarchy." *Journal of High Energy Physics* 2022.
[2] *Joseph Conlon et al.*, arXiv:0806.0809v2 (2008)

# Ending Kination with Matter/Radiation

Need to end kination to avoid decompactification

High kinetic energy
$\Rightarrow$ modulus overshoots the minimum[1]

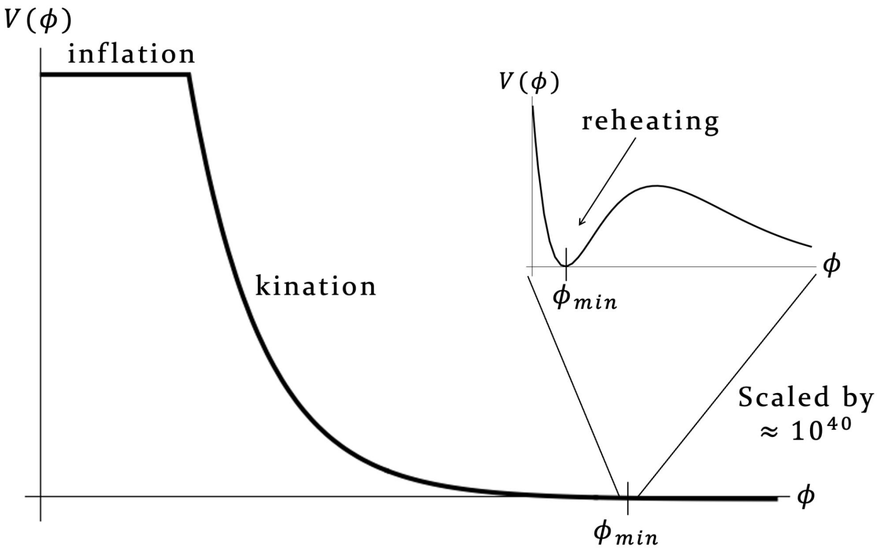
$$\mathcal{L} \supset \frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi - \cancel{V(\phi)}$$

$$\ddot{\phi} + 3H\dot{\phi} = 0$$

$$H^2 = \frac{\dot{\phi}^2}{6M_P^2} = \left(\frac{\dot{a}}{a}\right)^2$$

$$\phi(t) = \phi_0 + \sqrt{\frac{2}{3}}\,M_P \ln(t/t_0)$$

$$\Rightarrow \boxed{\rho_{kination} \sim \dot{\phi}^2 \sim \frac{1}{a^6}}$$




$$\rho_{matter} \sim \frac{1}{a^3} \qquad \rho_{radiation} \sim \frac{1}{a^4}$$

Modulus trapped if kination ends before $\phi_{\text{min}}$[1,2]

# Thank You!

# Backup Slides

# Motivation Overview

QCD Axion of High Quality

Axion from extra dimensions

Axion present during inflation, seeds isocurvature amplitude $(H_{inf}/f)^2$

\+

QCD Axion Comprises *all* the Dark Matter

Axion decay constant $f \approx 10^{12}$ GeV

Low $H_{inf}$, less robust inflation

"**Axion Isocurvature Problem**"

# QCD Axion: A Solution to the Strong CP Problem

*[1] Hook, Anson. arXiv:1812.02669 (2018).*
*[2]* Reece, Matthew. *arXiv:2304.08512* (2023).
*[3] Abel, Christopher, et al. Physical Review Letters 124.8 (2020): 081803.*

The QCD Lagrangian has a CP violating term[1,2]:

$$\mathcal{L}_{QCD} \supset \bar{\theta}\left(\frac{g_s^2}{32\pi^2}\right) G^a_{\mu\nu}\tilde{G}^{a\mu\nu} \qquad \text{where} \quad \tilde{G}^{a\mu\nu} = \varepsilon^{\mu\nu\alpha\beta}G^a_{\alpha\beta}$$

Extent of CP violation parametrized by $\bar{\theta}$

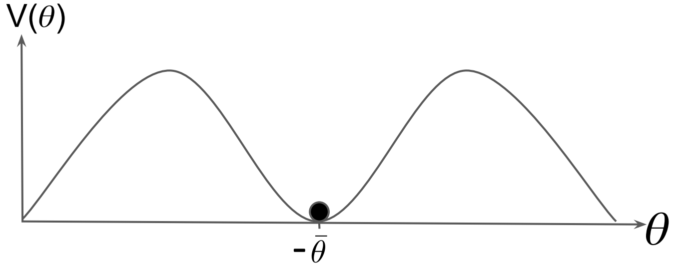
From neutron EDM measurements[3]: $\bar{\theta} \leq 10^{-10}$

Introducing the Axion field, $\theta$, with gluon coupling[1,2]: $\mathcal{L} \supset \mathcal{L}_{QCD} + \frac{f^2}{2}\partial_\mu\theta\,\partial^\mu\theta + \theta\left(\frac{g_s^2}{32\pi^2}\right) G^a_{\mu\nu}\tilde{G}^{a\mu\nu}$

Generates periodic axion potential[1]

$$V(\theta) = -F_\pi^2 m_\pi^2\sqrt{1 - \frac{4m_u m_d}{(m_u+m_d)^2}\sin^2(\bar{\theta}+\theta(x))}$$

*minimized* at $\langle\theta\rangle = -\bar{\theta}$




⇒ Axion dynamically relaxes and **cancels CP violating term** in $\mathcal{L}_{QCD}$

# The Quality Problem

[1] Hook, Anson. arXiv:1812.02669 (2018).
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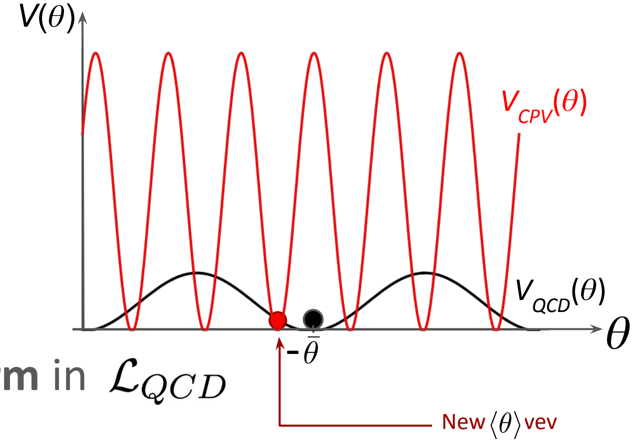
From neutron EDM measurements[3]: $\bar{\theta} \leq 10^{-10}$

Introduce the axion, $\theta$, with periodic potential $V_{QCD}$ *minimized* at $\langle\theta\rangle = -\bar{\theta}$ [1]




⇒ Axion dynamically relaxes and **cancels CP violating term** in $\mathcal{L}_{QCD}$

But, adding higher order axion operators will generically *break* CP symmetry[2]

In order to preserve CP symmetry, we would need to justify neglecting these operators

⇒ known as the **Quality Problem**

# Alleviating the Quality Problem with Extra Dimensional Axions

Set up: $\theta$ comes from higher dimensional gauge field ***A***[1]

Coupling of ***A*** to ***G*** generates $V_{QCD}$ in 4D

To spoil CP, we need non-derivative axion terms

⇒ non-derivative gauge terms in higher dimensions

⇒ **gauge symmetries** permit two kinds of terms

(1) ***A*** couples to other gauge fields

(2) ***A*** couples to charged fields

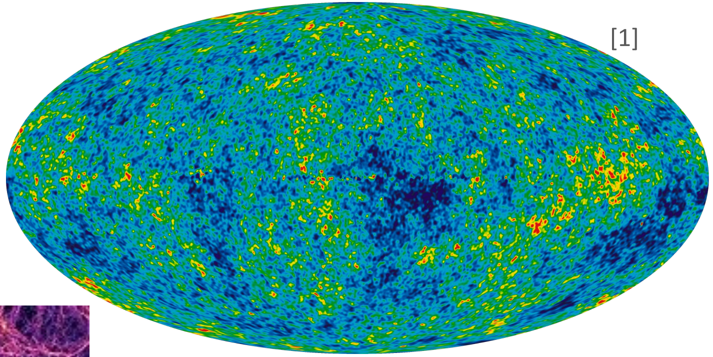
Covariant derivative generates axion potential in 4D

$$V_{extra} \sim V_{QCD} \sim e^{-\#R}$$

Where R is the size of the extra dimensions

⇒ We will be working with **extra dimensional axions**

# QCD Axions as *All* the Dark Matter

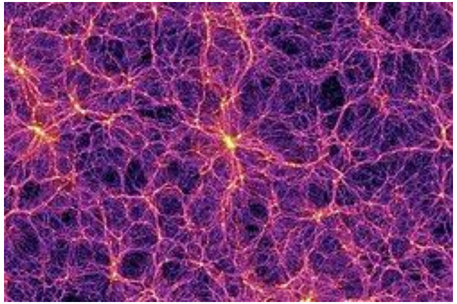

[1]

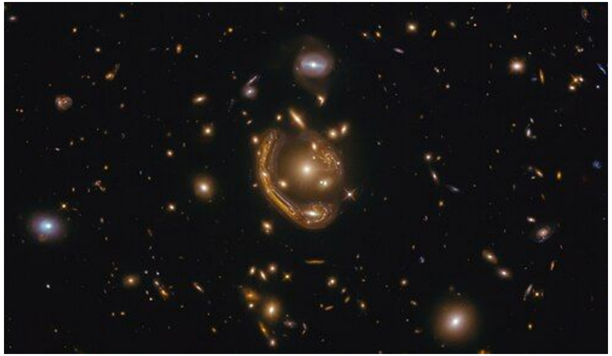
Ample evidence for dark matter (DM) from

CMB

Large Scale Structure

Strong Lensing


[2]


[3]

We know DM comprises ~85% of the mass of the universe.

Interacts weakly with Standard Model.

Could extra dimensional QCD axions comprise *all* the dark matter?

[1] ‘Cosmic Microwave Background’, Wikipedia.
[2] ‘Large-scale structure of the universe’, Wikipedia.    [3] ‘Strong gravitational lensing’, Wikipedia.

# Axions as Dark Matter Through Misalignment Mechanism

Could extra dimensional QCD axions comprise *all* the dark matter?

Scale invariant power spectrum ⇒ inflation in 4D [1,2]

⇒ extra dimensional axions around during inflation

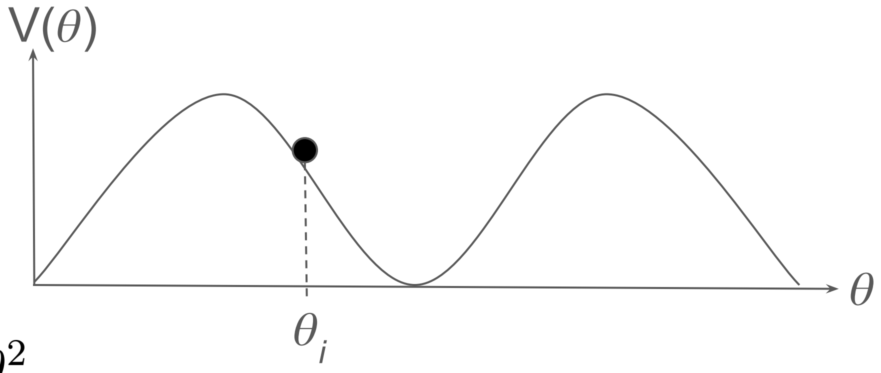

*[1] Anchordoqui, Luis A., and Ignatios Antoniadis. arXiv: 2310.20282v2 (2024)*
*[2] Ignatios Antoniadis et al., arXiv: 2311.17680v2 (2024)*

*[3] David Marsh. arXiv:* arXiv:1510.07633v2 (2016)
[4] Ciaran A. J. O'Hare. arXiv:2403.17697v2 (2024)


Large $H_{inf}$ freezes axion in field space at random $\theta$ value[3,4]

Axion begins to roll when H falls below $m_a$

Oscillations around minimum behave like matter[3,4]




Comoving energy once oscillating: $\rho_a = \frac{1}{2} m_a^2 f^2 \theta_i^2$

Dark matter density bounds ⇒ $f \approx 10^{12}$ *GeV* [3,4]

# Quantifying Axion Isocurvature Fluctuations

In dS, scalar fields with m≪H have fluctuations[1,2]

$$\langle \delta a^2 \rangle = \left( \frac{H_{inf}}{2\pi} \right)^2 \xrightarrow{a = f\theta} \langle \delta \theta^2 \rangle = \left( \frac{H_{inf}}{2\pi f} \right)^2$$

Fluctuations in $\theta$ turn into *isocurvature* matter fluctuations during oscillations:

$$\rho_a = \frac{1}{2} m_a^2 a_i^2 = \frac{1}{2} m_a^2 f^2 \theta_i^2 \qquad \delta\rho_{a,\, iso} = m_a^2 f^2 \theta_i \delta\theta$$

Axion fluctuations obtain adiabatic component due to misalignment mechanism

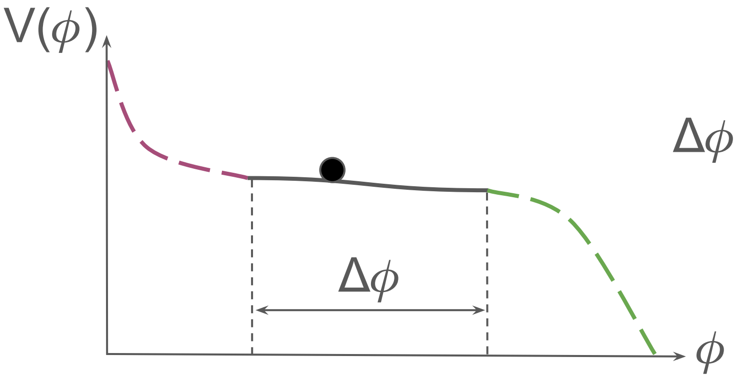
Larger local H ⇒ later oscillations ⇒ larger local DM density

$$\mathcal{P}_a(k) = \mathcal{P}_{a,\, adiabatic}(k) + \mathcal{P}_{a,\, iso}(k)$$

*[1] David Marsh. arXiv: arXiv:1510.07633v2 (2016)*
*[2] Ciaran A. J. O'Hare. arXiv:2403.17697v2 (2024)*

# Low Inflationary Energy Scales ⇒ Less Robust Inflation

Low inflationary Hubble scales ⇒ small inflaton field displacement




$\Delta\phi \approx 10^{-2}\, M_P$

⇒ Small initial inhomogeneities in inflaton field can end inflation too soon

[1] showed that for $H_{inf} \lesssim 10^7\ GeV$, inflation ends prematurely

(with ~$\phi^4$ inflation model)

*[1] Clough, Katy, et al. "Robustness of inflation to inhomogeneous initial conditions." Journal of Cosmology and Astroparticle Physics 2017.09 (2017): 025.*

# Low Inflationary Energy Scale ⇒ Small Field Displacement

$$r \equiv \frac{A_t}{A_s}$$

Tensor power spectrum amplitude

$$A_t = \frac{2H_*^2}{\pi^2 M_P^2}$$

Scalar power spectrum amplitude

$$A_s = \left(\frac{H_*^2}{2\pi\dot{\phi}_*}\right)^2$$

$$\Rightarrow \quad r = \frac{8}{M_P^2}\left(\frac{\mathrm{d}\phi}{\mathrm{d}N}\right)^2$$

⇘ ⇙

$$\frac{H_*}{M_P} = \frac{\pi}{\sqrt{2}}\sqrt{A_s r}$$

⇓

$$\frac{\Delta\phi}{M_P} = N\sqrt{\frac{r}{8}}$$

$$\Rightarrow \quad \frac{\Delta\phi}{H_*} = \frac{N}{2\pi}\sqrt{\frac{1}{A_s}} \quad \Leftarrow$$

⇘

$$N \sim \ln T_R \quad \& \quad T_R \sim \sqrt{M_P H_{inf}}$$

$$\boxed{\Delta\phi \sim H_{inf}\ln\left(\sqrt{H_{inf}}\right)}$$

# Bulk Axions as Seeds of Radiation

In type IIB string theories, the bulk modulus couples to a bulk axion[1]:

$$\mathcal{L} \supset \frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi + \frac{3M_P^2}{4}\exp\left[-2\sqrt{\frac{2}{3}}\frac{\phi}{M_P}\right]\partial_\mu\theta_b\,\partial^\mu\theta_b$$

$\theta_b$ = bulk axion

Assume we start with *zero* bulk axion particles after inflation.

Two ways to produce bulk axions:

1) Cosmological particle production
2) Bulk modulus “decays” into bulk axions

Enough to trap bulk modulus?



*[1] Michele Cicoli et al., arXiv:1208.3562v2 (2012)*

# Bulk Axion Mode Equation of Motion

Mode EOM:

$$\ddot{\chi}_k + \left(3H(t) - \frac{2}{M_P}\sqrt{\frac{2}{3}}\,\dot{\phi}\right)\dot{\chi}_k + \frac{k^2}{a^2}\chi_k = 0$$

$$\downarrow \quad H^2 = \frac{\dot{\phi}^2}{6M_P^2}$$

$$\ddot{\chi}_k - H\,\dot{\chi}_k + \frac{k^2}{a^2}\,\chi_k = 0 \quad \xrightarrow{\chi_k(t) = a^{1/2}(t)\,X_k(t)} \quad \boxed{\ddot{X}_k + \omega_k^2\,X_k = 0}$$

where $$\boxed{\omega_k^2(t) \equiv \frac{k^2}{a^2(t)} - \frac{3}{4}H^2(t) + \frac{1}{2}\frac{\ddot{a}(t)}{a(t)}}$$

# Calculating Bulk Axion Particle Number and Energy Density

$$\ddot{X}_k + \omega_k^2 X_k = 0 \xrightarrow{\text{ansatz}} X_k(t) = \frac{\alpha_k(t)}{\sqrt{2\omega}} e^{i\int \omega dt} + \frac{\beta_k(t)}{\sqrt{2\omega}} e^{-i\int \omega dt}$$

$$|\alpha|^2 - |\beta|^2 = 1$$

$\alpha$ and $\beta$ are known as Bogoliubov coefficients, and we can determine particle number density $n_k(t)$ from $\beta$:

$$n_k(t) = |\beta_k(t)|^2 = \frac{\omega_k}{2}\left(\frac{|\dot{X}_k|^2}{\omega_k^2} + |X_k|^2\right) - \frac{1}{2}$$

$$\rho_\gamma \sim \int k^2 dk \, (n_k \omega_k)$$

⇒ solve for mode functions $X_k(t)$

⇒ determine energy density stored in bulk axions ( $\varrho_\gamma$ )

# p-cycles and $C^{(p)}$ compactifications

“p-cycles” are p-dimensional submanifolds which:

- Have no boundary
- Are not a boundary themselves

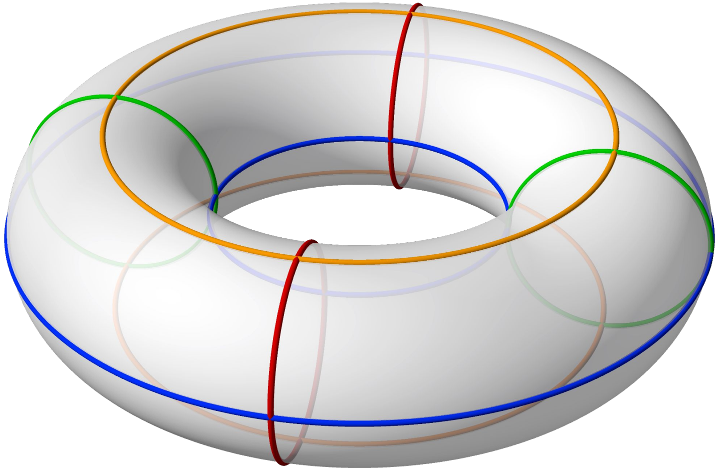


Image credit: Wikipedia

# Preliminary Results

# Scenario with Largest Possible $H_{inf}$

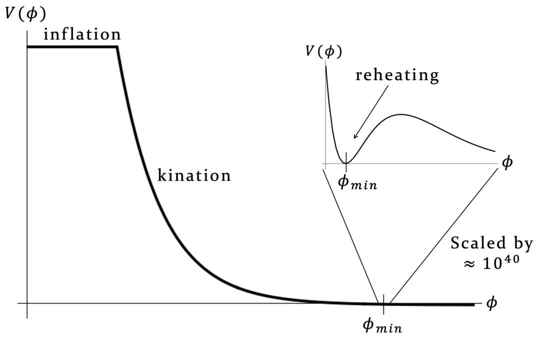
$f \lesssim M_P$ —— isocurvature constraint ——→ $H_{inf} \lesssim 5 \cdot 10^{13}\,\mathrm{GeV}$

—— taking $f_{RH} \approx 10^{12}\, GeV$ ——→ $\Delta\phi \lesssim 24$

In the *Large Volume Scenario* (LVS) of Type IIB Compactifications, modulus potential is given by[1]:

$$V(\phi) = V_0\left((1-\epsilon\,(\phi/M_P)^{3/2})e^{-\sqrt{\frac{27}{2}}\phi/M_P} + Ce^{-10\phi/\sqrt{6}M_P} + De^{-11\phi/\sqrt{6}M_P} + \delta e^{-\sqrt{6}\phi/M_P}\right)$$

- $(1-\epsilon\,(\phi/M_P)^{3/2})e^{-\sqrt{27/2}\,\phi/M_P}$: Generates local AdS minimum at $\phi \sim \epsilon^{-2/3}$
- $Ce^{-10\phi/\sqrt{6}M_P} + De^{-11\phi/\sqrt{6}M_P}$: Generates flat portion ⇒ inflation epoch
- $\delta e^{-\sqrt{6}\phi/M_P}$: Uplifts local minimum to dS, V>0




In this scenario, volume given by $\mathcal{V} = \exp\left(\sqrt{\frac{3}{2}}\phi/M_P\right)$

Chose LVS parameters that yield $H_{inf} \approx 2 \cdot 10^{13}\,\mathrm{GeV}$ and $\Delta\phi \approx 23$



*[1] Conlon, Joseph P., et al. "Volume modulus inflation and the gravitino mass problem." Journal of Cosmology and Astroparticle Physics 2008.09 (2008): 011.*

# Solving for Modulus Evolution

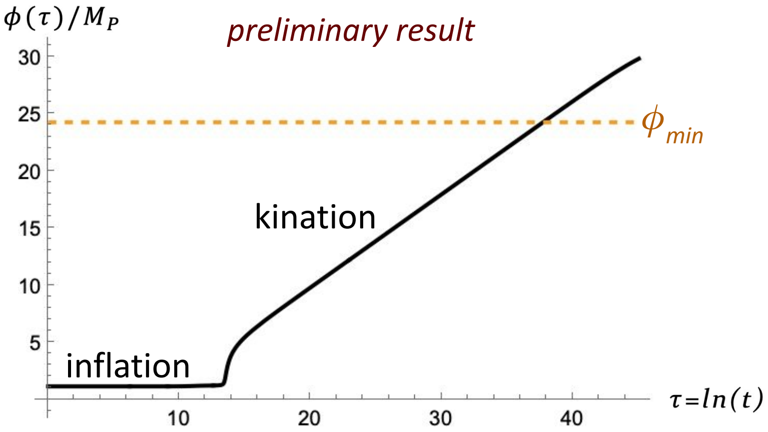
With $H_{inf} \approx 2 \cdot 10^{13}\,\mathrm{GeV}$ we source $\rho_{\gamma,K} \approx 10^{-12}\rho_{inf}$ in bulk axions

Modulus evolution given by:

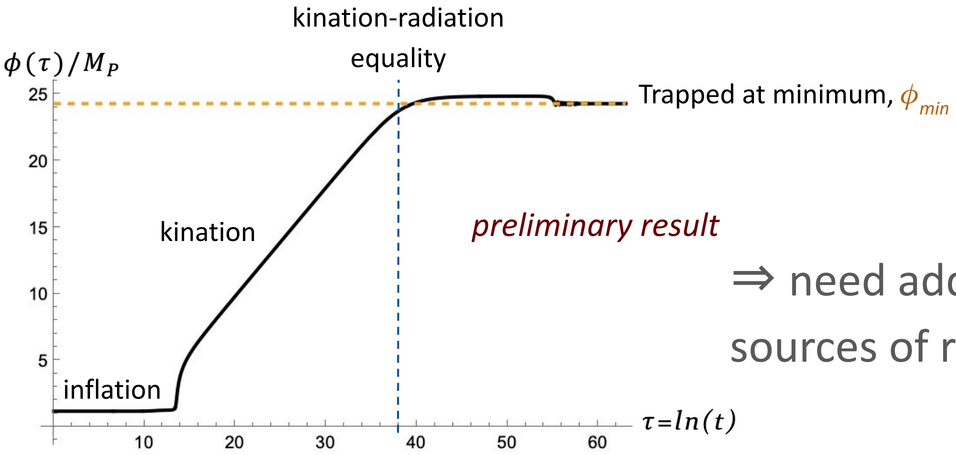
$$\ddot{\phi} + 3H(t)\dot{\phi} + \frac{\mathrm{d}V}{\mathrm{d}\phi} = 0 \qquad \& \qquad H(t) = \frac{1}{\sqrt{3}M_P}\sqrt{\boxed{\frac{\rho_{\gamma,K}}{a^4}} + \frac{1}{2}\dot{\phi}^2 + V(\phi)}$$

# Preliminary Result: Not Enough Bulk Axion Production to Trap Modulus

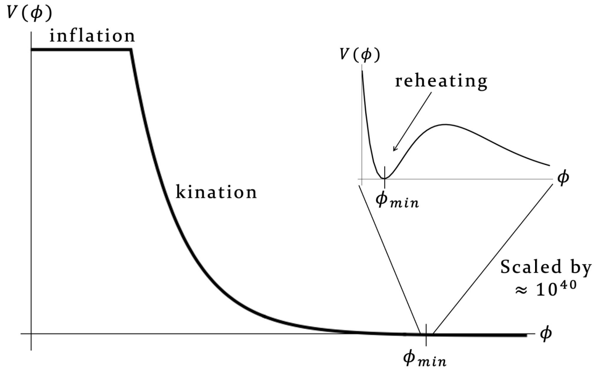
We find that the modulus does *not* get trapped




However, if we increase $\varrho_{\gamma,K}$ by a factor of ~250, we *can* trap the modulus field




⇒ need additional sources of radiation



# Next Steps

Consider perturbations in modulus field

These redshift like radiation[1]

⇒ could help trap modulus field[2]

Consider additional axions (the supersymmetric partners of stabilized moduli)

Consider LVS potentials with *two* minima[3]

⇒ could slow down the modulus field

[1] *Eröncel, Cem, et al.* "A universal bound on the duration of a kination era." *arXiv: 2501.17226* (2025).

[2] *Martin Mosny, Joseph Conlon, Edmund Copeland.* "Self-Tracking Solutions for Asymptotic Scalar Fields." *arXiv: 2507.04161v1* (2025).

[3] *AbdusSalam, Shehu, et al.* "Coexisting Flux String Vacua from Numerical Kahler Moduli Stabilisation." *arXiv:2507.00615* (2025).

# Extra Dimensional Compactifications and 4D Dynamics

# Axions from Extra Dimensional Gauge Fields

Consider: 5 dimensional manifold with 5th dimension a compact circle of radius R,
M = $X_{4d}$ x $S^1$

with 1-form gauge field *A* living in 5D $\vec{A} = A_\mu x^\mu + A_5 x^5$

How does the 5D field *A* manifest in 4D?

5D kinetic term: $S = \int_{X_{4d}\times S^1} \sqrt{\det[g_{MN}]}\, \mathrm{d}^5x \left(-\frac{1}{4e_5^2} F_{MN}F^{MN}\right)$

where $F_{MN} = \partial_M A_N - \partial_N A_M \sim (\partial_\mu A_\nu - \partial_\nu A_\mu) + (\partial_\mu A_5 - \partial_5 A_\mu)$

$$A_\mu(x_\mu, x_5) = \sum_{n=-\infty}^{\infty} e^{inx_5/R}\, \tilde{A}_\mu^{(n)}(x^\mu) \qquad A_5(x_\mu, x_5) = \sum_{n=-\infty}^{\infty} e^{inx_5/R}\, \phi^{(n)}(x^\mu)$$

$$\tilde{A}^{(-n)} = \left(\tilde{A}^{(n)}\right)^*$$

$$\phi^{(-n)} = \left(\phi^{(n)}\right)^*$$

*Define* **axion** as zero mode of *$A_5$*[1]: $\theta(x_\mu) \equiv \int_0^{2\pi R} A_5(x_\mu, x_5)\, \mathrm{d}x_5 = 2\pi R\, \phi^{(0)}(x^\mu)$



*[1]* Matthew Reece. *arXiv:2304.08512* (2023).

# Axions from Extra Dimensional Gauge Fields

Consider: 5 dimensional manifold with 5th dimension a compact circle of radius R,

M = $X_{4d}$ x $S^1$

with 1-form gauge field *A* living in 5D $\vec{A} = A_\mu x^\mu + A_5 x^5$

5D kinetic term: $S = \int_{X_{4d}\times S^1} \sqrt{\det[g_{MN}]}\, \mathrm{d}^5x \left(-\frac{1}{4e_5^2} F_{MN}F^{MN}\right)$

ansatz: $A_\mu(x_\mu, x_5) = \sum_{n=-\infty}^{\infty} e^{inx_5/R}\, \tilde{A}_\mu^{(n)}(x^\mu)$ and $A_5(x_\mu, x_5) = \sum_{n=-\infty}^{\infty} e^{inx_5/R}\, \phi^{(n)}(x^\mu)$

*Define* **axion** as zero mode of $A_5$[1]: $\theta(x_\mu) \equiv \int_0^{2\pi R} A_5(x_\mu, x_5)\, \mathrm{d}x_5 = 2\pi R\, \phi^{(0)}(x^\mu)$



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# Extra Dimensional Axions have Decay Constants that Depend on Extra Dimensional Geometry

Recall: $$A_5(x_\mu, x_5) \; = \sum_{n=-\infty}^{\infty} e^{inx_5/R}\, \phi^{(n)}(x^\mu)$$

$$\mathcal{L}_{4d} \supset -\frac{1}{4e_5^2} \int_{S^1} (\partial_\mu A_5)^2 \; = \; \frac{2\pi R}{4e_5^2} \sum_n \left(\partial_\mu \phi^{(n)}\right) \left(\partial^\mu \phi^{(n)}\right)$$

$\Downarrow$ Zero mode, using $\theta(x_\mu) = 2\pi R\, \phi^{(0)}(x^\mu)$

$$\boxed{\mathcal{L}_{4d} \supset \frac{1}{2}\, f^2 \partial_\mu \theta\, \partial^\mu \theta}$$ where $$\boxed{f^2 \; = \; \frac{1}{2\pi R e_5^2}}$$

More *generally*, for a higher dimensional manifold[1]

$M = X_{4D} \times Y_{nD}$,

$$f^2 \propto \frac{1}{\mathcal{V}_Y}$$

where $\mathcal{V}_Y$ = volume of $Y_{nd}$



*[1] Matthew Reece. arXiv: 2406.08543 (2024).*

# Extra Dimensional Axions have Decay Constants that Depend on Extra Dimensional Geometry

$A_5 \to A_5 + \partial_5 \alpha$ where $\alpha = x_5/R$

$\theta(x_\mu) \equiv \int_0^{2\pi R} A_5(x_\mu, x_5)\, \mathrm{d}x_5 \to \theta(x_\mu) + 2\pi$ $\Rightarrow \theta$ is $2\pi$ periodic ✓

Recall: $A_5(x_\mu, x_5) = \sum_{n=-\infty}^{\infty} e^{inx_5/R}\, \phi^{(n)}(x^\mu)$

$$\mathcal{L}_{4d} \supset -\frac{1}{4e_5^2} \int_{S^1} (\partial_\mu A_5)^2 = \frac{2\pi R}{4e_5^2} \sum_n \left(\partial_\mu \phi^{(n)}\right) \left(\partial^\mu \phi^{(n)}\right)$$

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# Compactifying Extra Dimensions Generically Leads to a Scalar Field with Exponential Potential in 4D

Again start with higher dimensional manifold $M = X_{4D} \times Y_{nD}$

$$ds^2 = g_{MN}(X)\, dX^M dX^N = g_{\mu\nu}(x)\, dx^\mu dx^\nu + \boxed{\lambda^2(x)}\, g_{mn}(y)\, dy^m dy^n$$

Overall volume of *Y* depends on position in 4D

$$S_{EH} = \int_M \sqrt{\det[g_{MN}]}\; \boxed{\mathcal{R}_{(4+n)D}}\; M_{(4+n)}^{2+n}\, d^4x\, d^ny$$

$$\boxed{\mathcal{R}_{(4+n)D}} = \mathcal{R}_{4D} + \underbrace{\frac{\mathcal{R}_{nD}(y)}{\lambda^2(x)}}_{\text{Potential term}} + \underbrace{\#\frac{1}{\lambda^2(x)}\, g^{\mu\nu}(x)\, \partial_\mu\lambda(x)\, \partial_\nu\lambda(x)}_{\text{Kinetic term}}$$

Kinetic term → $\sim \partial_\mu \Phi\, \partial^\mu \Phi$

Potential term → $\boxed{V(\Phi) \sim (e^{-\Phi})^{n+2}}$

Canonical field: $\lambda(x) = e^{\Phi(x)}$

bulk modulus field: $\phi(x) = M_P \Phi(x)$

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Higher dimensional Planck constant

$$\det[g_{MN}] = \lambda^{2n}(x) \det[g_{\mu\nu}(x)] \det[g_{mn}(y)]$$

$$\mathcal{R}_{(4+n)D} = \mathcal{R}_{4D} + \underbrace{\frac{\mathcal{R}_{nD}(y)}{\lambda^2(x)}}_{\text{Potential term}} + \underbrace{\# \frac{1}{\lambda^2(x)}\, g^{\mu\nu}(x)\, \partial_\mu \lambda(x)\, \partial_\nu \lambda(x)}_{\text{Kinetic term}}$$

Kinetic term → $\sim \partial_\mu \Phi\, \partial^\mu \Phi$

Potential term → $V(\Phi) \sim (e^{-\Phi})^{n+2}$

bulk modulus field: $\phi = M_P \Phi$

Weyl rescaling: $\tilde{g}_{\mu\nu} = \lambda^n(x) g_{\mu\nu}$

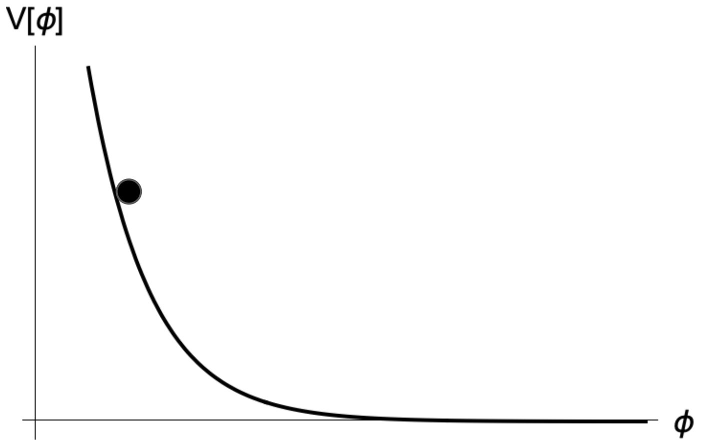
Canonical field: $\lambda(x) = e^{\Phi(x)}$

# Bulk Modulus Induces Dynamical Axion Decay Constant

Extra dimensional compactifications also generically lead to 'modulus fields'

*Bulk* modulus field, $\phi$:

- Characterizes overall volume of the extra dimensional manifold: $\mathcal{V}_Y$ ~ Exp[#$\phi/M_P$]
- Has an exponential potential V($\phi$) ~ exp(-$\phi$)




$f^2 \sim 1/\mathcal{V}_Y \sim$ Exp[ - #$\phi/M_P$]

Dynamical $\phi$ ⇒ dynamical $f$ !

Steep potential ⇒ fast roll ⇒ kination

⇒ large $\Delta\phi$ ⇒ large $\Delta\mathcal{V}_Y$ ⇒ large $\Delta f$

# Bulk Axions as Seeds of Radiation

In type IIB string theories, the bulk modulus couples to a bulk axion[1]:

$$\mathcal{L} \supset \frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi + \frac{3M_P^2}{4}\exp\left[-2\sqrt{\frac{2}{3}}\frac{\phi}{M_P}\right]\partial_\mu\theta_b\,\partial^\mu\theta_b$$

$\theta_b$ = bulk axion

Assume we start with *zero* bulk axion particles after inflation.

Two ways to produce bulk axions:

1) Cosmological particle production
2) Bulk modulus “decays” into bulk axions

Enough to trap bulk modulus?



[1] *Michele Cicoli et al., arXiv:1208.3562v2 (2012)*

# Bulk Axion Mode Equation of Motion

$$\ddot{\theta}_b + \left(3H(t) - \frac{2}{M_P}\sqrt{\frac{2}{3}}\,\dot{\phi}\right)\dot{\theta}_b - \frac{1}{a^2}\nabla^2\theta_b = 0 \quad \xrightarrow{H^2 = \frac{\dot{\phi}^2}{6M_P^2}} \quad \ddot{\theta}_b - H(t)\,\dot{\theta}_b - \frac{1}{a^2}\nabla^2\theta_b = 0$$

$$\hat{\theta}_b(x,t) = \int d^3k\,\left[\hat{a}_k\chi_k(t)e^{-ikx} + \hat{a}_k^\dagger\chi_k^*(t)e^{ikx}\right]$$ where $$\left[\hat{a}_k, \hat{a}_k^\dagger\right] = 1$$

Mode EOM:

$$\ddot{\chi}_k - H\,\dot{\chi}_k + \frac{k^2}{a^2}\,\chi_k = 0 \quad \xrightarrow{\chi_k(t) = a^{1/2}(t)\,X_k(t)} \quad \boxed{\ddot{X}_k + \omega_k^2\,X_k = 0}$$

where $$\boxed{\omega_k^2(t) \equiv \frac{k^2}{a^2(t)} - \frac{3}{4}H^2(t) + \frac{1}{2}\frac{\ddot{a}(t)}{a(t)}}$$

# Particle Number in Curved Spacetimes

Bulk axion quantum field in terms of rescaled mode functions:

$$\hat{\theta}_b(x,t) = \int d^3k\, a^{1/2} \left[ \hat{a}_k X_k(t) e^{-ikx} + \hat{a}_k^\dagger X_k^*(t) e^{ikx} \right]$$

$$\ddot{X}_k + \omega_k^2 X_k = 0 \xrightarrow{\text{ansatz}} X_k(t) = \frac{\alpha_k(t)}{\sqrt{2\omega}} e^{i\int \omega dt} + \frac{\beta_k(t)}{\sqrt{2\omega}} e^{-i\int \omega dt}$$

$$|\alpha|^2 - |\beta|^2 = 1$$

Rewriting mode expansion in terms of $\alpha_k$ and $\beta_k$:

$$\hat{\theta}_b(x,t) = \int \frac{d^3k\, a^{1/2}}{\sqrt{2\omega}} \left[ \left( \alpha_k(t)\, \hat{a}_k + \beta^*_{-k}(t) \hat{a}^\dagger_{-k} \right) e^{-i\int \omega dt} e^{-ikx} + \left( \beta_{-k}(t) \hat{a}_{-k} + \alpha_k^*(t) a_k^\dagger \right) e^{i\int \omega dt} e^{ikx} \right]$$

$$\hat{b}_k(t) \equiv \alpha_k(t) \hat{a}_k + \beta^*_{-k}(t) \hat{a}^\dagger_{-k}$$

$$\hat{b}_k^\dagger(t) \equiv \alpha_k^*(t) \hat{a}_k^\dagger + \beta_{-k}(t) \hat{a}_{-k}$$

$$\left[ \hat{b}_k, \hat{b}_k^\dagger \right] = 1$$

"Bogoliubov Transformations"

# Particle Number in Curved Spacetimes Cont'd

$$\hat{a}_k|\Omega\rangle = 0$$

$$n_k(t) \equiv \langle\Omega\,|\,\hat{N}_k(t)\,|\,\Omega\rangle = \langle\Omega\,|\,\hat{b}_k^\dagger(t)\hat{b}_k(t)\,|\,\Omega\rangle = |\beta_k(t)|^2$$

$$= \frac{\omega_k}{2}\left(\frac{|\dot{X}_k|^2}{\omega_k^2} + |X_k|^2\right) - \frac{1}{2}$$

⇒ solve for mode functions $X_k(t)$

⇒ determine comoving $n_k(t)$

⇒ physical number density given by $n_k(t)/a^3(t)$